# A Real-Coded Genetic Algorithm with System Reduction and Restoration for Rapid and Reliable Power Flow Solution of Power Systems 

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#### Abstract

The paper presents a highly accurate power flow solution, reducing the possibility of ending at local minima, by using Real-Coded Genetic Algorithm (RCGA) with system reduction and restoration. The proposed method (RCGA) is modified to reduce the total computing time by reducing the system in size to that of the generator buses, which, for any realistic system, will be smaller in number, and the load buses are eliminated. Then solving the power flow problem for the generator buses only by real-coded GA to calculate the voltage phase angles, whereas the voltage magnitudes are specified resulted in reduced computation time for the solution. Then the system is restored by calculating the voltages of the load buses in terms of the calculated voltages of the generator buses, after a derivation of equations for calculating the voltages of the load busbars. The proposed method was demonstrated on 14-bus IEEE test systems and the practical system 362-busbar IRAQI NATIONAL GRID (ING). The proposed method has reliable convergence, a highly accurate solution and less computing time for on-line applications. The method can conveniently be applied for on-line analysis and planning studies of large power systems.


Keywords: Load flow analysis, Load modeling, Power system modeling, Real Coded Genetic algorithms, Simulation, Voltage measurement

الحل السريع والموثوق لسريان الحمل الكهربائي بأستخدام الخوارزمية الجينية ذات التثففير الحقيقي مع أختزال الثبكة وأعادتها
أ.م. د. حسن عبالله كية

## الخلاصة

يقدم البحث طريقة عالية الدقة لحساب سريان الحمل الكهربائي و تقليل أحتمالية الانتهاء في الحدود الدنيا المحلية بأستخدام الخوارزمية الجينية ذات التشفير الحقيقي مع أختز ال الثبكة وأعادتها وتم تطوير الطريقة المقترحة ( الخوارزمية الجينية ذات التشفير الحقيقي) لتقليل زمن الحساب الكلي بتقليل حجم النظام الى عدد محطات النوليد فقط، بعد أختز ال عدد محطات الأحمـال في النظام الحقيقي أو الو اقحي لتقليل الزمن الللام للحساب ، ومن ثم يتم حساب زاوية طور الفولتية بعد تحدبد مقدار الفولتية لكل محطة نوليد بأَستخدام الخوارزمية الجينية ذات التشفير الحقيقي ، بعد ذللك يتم أعادة تمثيل النظام ككل وحساب مقدار وزاوية طور الفولتيات لكل محطات الأحمال بأستخدام النتائج المستحصلة لمقدار وزاوية طور الفولتيات لكل محطات التوليد بعد أنشتقاق المعادلات المطلوبة لحساب مقدار وزاوية طور فولتيات لكل محطات ألأحمال بصيغة مقدار وزاوية فولتيات محطات اللوليد ، الطريقة المقترحة تم تطبيقها للعمل على اللبكة الوطنية العر اقية.الطريقة المقترحة عالية الدقة، موثوقة والزمن اللازم للوصول الى الحل قليل وكذلك مككن تطبيقها في دراسات التحليل و التخطيط للأنظمة الكهربائية كبيرة الحجم وأثناء أشتغال المنظومة
الكلمات الرئيسية: تحليل تدفق الحمل ، تمثيل الحمل ، تمثيل نظام القدرة الكهربائية ، الخو ارزمية الجينية ذات التمثيل الحقيقي ، المحاكاة ، وحساب الفولتية.

$$
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& \text { قسم الهندسة الكهربائية } \\
& \text { كلية الهندسة/جامعة بغداد }
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& \text { قسم الهندسة الكهربائية } \\
& \text { كلية الهنسسة/جامعة بغداد }
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## 1. INTRODUCTION

The power flow problem, which is to determine the power system static states (voltage magnitudes and voltage phase angles) at each busbar to find the steady state operating condition of a system, is very important and the most frequently carried out study by electrical power utilities for power system on-line operation, planning and control. The mathematical formulation of the electrical power flow problem results in a set of non-linear algebraic equations. The optimization numerical methods such as Newton-Raphson method or the artificial intelligence methods such as Genetic Algorithm (GA) are applied to solve the power flow problem. The power flow problem has multiple solutions, Kubba 1991.The numerical methods and some of the artificial intelligence methods suffer from the local minima problem. Also there are many criteria which should be taken into consideration such as the speed of solution, storage requirement and the degree of solution accuracy. With increasing computer speeds, researchers are increasingly applying artificial and computational intelligence techniques, especially in power system problems. These methods offer several advantages over traditional numerical methods. Among these techniques is that of genetic algorithm. Genetic algorithms (GAs) are efficient stochastic search techniques that emulate natural phenomena. They have been used successfully to solve a wide range of optimization problems. Because of existence of local minima, these algorithms offer promise in solving large-scale problems. A genetic algorithm mimics Darwin's evolution process by implementing "survival of the fittest" strategy. Genetic algorithm solves linear and nonlinear problems by exploring all regions of the search space and exponentially exploiting promising areas through selection, crossover, and mutation operations. They have been proven to be an effective and flexible optimization tool that can find optimal or near-optimal solutions, Wong, et al., 1999. In this study, an improved genetic algorithm solution of the load flow problem is presented in order to minimize the total active and reactive power mismatches of the given systems, a real-coded genetic algorithm has been implemented. The proposed method has been demonstrated on a typical test system, and was used to solve the Iraqi National Grid load flow problem.

## 2. THE REAL-CODED (CONTINUOUS) GENETIC ALGORITHM (RCGA)

The binary genetic algorithm is conceived to solve many optimization problems that stump traditional techniques. But, the attempting to solve a problem where the values of the variables are continuous and want to define them to the full machine precision. In such a problem, each variable requires many bits to represent it. If the number of variables is large, the size of the chromosome is also large. In principle, any conceivable representation could be used for encoding the variables. When the variables are naturally quantized, the binary genetic algorithm fits nicely. However, when the variables are continuous, it is more logical to represent them by floating-point numbers, i.e., real number. In addition, since the binary genetic algorithm has its precision limited by the binary representation of variables, using floating-point numbers instead easily allows representation to the machine precision. This continuous genetic algorithm also has the advantage of requiring less storage than the binary genetic algorithm because a single floating-point number represents the variable instead of $\mathrm{N}_{\text {bits }}$ integers. The continuous genetic algorithm is inherently faster than the binary genetic algorithm, because the chromosomes do not have to be decoded prior to the evaluation of the cost function (objective function), Ippolito, et al., 2006. Since the continuous GA is implemented using floating point numbers, i.e., real numbers we have called this as Real-Coded GA (RCGA).

## 3. MATHEMATICAL DESCRIPTION \& COMPONENTS OF A CONTINUOUS GENETIC ALGORITHM (RCGA)

The real-coded genetic algorithm is very similar to the binary genetic algorithm, but the primary difference is the fact that variables are no longer represented by bits of zeros and ones, but instead by floating-point real numbers over whatever range is deemed appropriate. However, this simple fact adds some nuances to the application technique that must be carefully considered. In particular, we will present the RCGA operators, which are used in this research.

### 3.1 The Variables and Cost Function

A cost function generates an output from a set of input variables (a chromosome). The cost function may be a mathematical function, or from experiment. The objective is to modify the output in some desirable fashion by finding the appropriate values for the input variables. The goal is to solve some optimization problem where we search for an optimum (minimum) solution in terms of the variables of the problem. The term fitness is extensively used to designate the output of the objective function in the genetic algorithm literature. Fitness implies a maximization problem. Fitness has a closer association with biology than the term cost, and thus we have adopted the term cost, since most of the optimization literature deals with minimization, hence cost. They are equivalent. If the chromosome has $\mathrm{N}_{\mathrm{var}}$ variables (a 2 N -dimensional optimization problem) given by ( $\mathrm{b}_{1}, \mathrm{~b}_{2}, \ldots \ldots, \mathrm{~b}_{\text {Nvar }}$ ) where N is the number of buses, then the chromosome is written as an array with $\left(1 \times N_{\mathrm{var}}\right)$ elements so that:

$$
\begin{equation*}
\text { chromosome }=\left[b_{1}, b_{2}, b_{3}, \ldots \ldots \ldots, b_{\text {Nvar }}\right] \tag{1}
\end{equation*}
$$

In power flow problem, the chromosome is written in terms of the voltages magnitudes and voltages phase angles variables of all the buses as follows:

$$
\begin{equation*}
\text { chromosome }=\left[V_{1}, V_{2}, \ldots, V_{N}, \theta_{1}, \theta_{2}, \ldots ., \theta_{N}\right] \tag{1.1}
\end{equation*}
$$

In this case, the variable values are represented as floating-point numbers. Each chromosome has a cost found by evaluating the cost function $(f)$ at the variables $\left(\mathrm{V}_{1}, \mathrm{~V}_{2}, \ldots, \mathrm{~V}_{\mathrm{N}}, \theta_{1}, \theta_{2}, \ldots \ldots, \theta_{\mathrm{N}}\right)$.

$$
\begin{equation*}
\text { cost }=f(\text { chromosome })=f\left(b_{1}, b_{2}, \ldots, b_{\text {Nvar }}\right) \tag{2}
\end{equation*}
$$

Equations (1) and (2) along with applicable constraints constitute the problem to be solved. Our primary problem in this research is the continuous functions introduced below. The two cost functions are:

$$
\begin{gather*}
N \\
\Delta P_{i}=P_{i}^{s p}-V_{i} \sum_{k=1} V_{k}\left(G_{i k} \cos \theta_{i k}+B_{i k} \sin \theta_{i k}\right) \tag{3}
\end{gather*}
$$

Where $\mathrm{P}_{\mathrm{i}}{ }^{\mathrm{sp}}$ is the specified active power at bus i , eqn. 3 is for " PV " (generator buses), and "PQ" (load buses),

$$
\Delta Q_{i}=Q_{i}^{s p}-V_{i} \sum_{k=1}^{N} V_{k}\left(G_{i k} \sin \theta_{i k}-B_{i k} \cos \theta_{i k}\right)
$$

Where $Q_{i}{ }^{s p}$ is the specified reactive power at bus i, eqn. 4 is for PQ buses only, Where $\theta_{i k}=\theta_{i}-\theta_{k}$ and, $\left(\Delta \mathrm{P}_{\mathrm{i}}\right)$ is the mismatch active power at bus (i) and $\left(\Delta \mathrm{Q}_{\mathrm{i}}\right)$ is the mismatch reactive power at bus (i). $\left(\mathrm{V}_{\mathrm{i}}, \mathrm{V}_{\mathrm{k}}, \theta_{\mathrm{i}}, \theta_{\mathrm{k}}\right)$ are the voltage magnitude and angle at buses (i) and (k) respectively, which are the variables of the two cost functions and ( N ) is the number of buses, Kubba, 2008.

### 3.2 Variable Encoding, Precision, and Bounds

Here, the difference between binary and continuous genetic algorithms is shown. It is no longer needed to consider how many bits are necessary to represent accurately a value. Instead, (V) and ( $\theta$ ) have continuous values that are limited between appropriate bounds which are in our problem, $0.9 \leq \mathrm{V} \leq 1.1$ and $-20 \leq \theta \leq 20$. Since the genetic algorithm is a search technique, it must be limited to exploring a reasonable region of variable space. Sometimes, this is done by imposing a constraint on the problem. If one does not know the initial search region, there must be enough diversity in the
initial population to explore a reasonably sized variable space before focusing on the most promising regions.

### 3.3 Initial Population

The genetic algorithm starts with a group of chromosomes known as the population. A matrix represents the population with each row in the matrix being a $\left(1 \times \mathrm{N}_{\text {var }}\right)$ array (chromosome) of continuous values. Given an initial population of $\mathrm{N}_{\text {ind }}$ chromosomes, the full matrix of ( $\mathrm{N}_{\mathrm{ind}} \times \mathrm{N}_{\mathrm{var}}$ ) random values is generated. All variables are normalized to have values between 0 and 1 , the range of a uniform random number generator. The values of a variable are "unnormalized" in the cost function. If the range of values is between $b_{l o}$ and $b_{h i}$, then the unnormalized values are given by:

$$
\begin{equation*}
b=\left(b_{h i}-b_{l o}\right) b_{n o r m}+b_{l o} \tag{5}
\end{equation*}
$$

where, $b_{h i}$ is highest number in the variable range, $b_{l o}$ is lowest number in the variable range, and $\mathrm{b}_{\text {norm }}$ is normalized value of variable. This society of chromosomes is not a democracy; the individual chromosomes are not all created equal. Each one's worth is assessed by the cost function. So at this point, the chromosomes are passed to the cost function for evaluation. In this research, we had used a population size (initial population) of 20 individuals (chromosomes) for 14-bus IEEE system power flow solution and 500 individuals for 362 -bus Iraqi National Grid (ING) power flow solution which depends on the number of variables for each system. These population sizes are kept constant throughout the whole solution process.

### 3.4 Natural Selection

Survival of the fittest translates into discarding the chromosomes with the higher costs. First, the $\mathrm{N}_{\text {ind }}$ costs and associated chromosomes are ranked from lowest cost to highest cost. Then, only the best are selected to continue, while the rest are deleted. The selection rate, $\mathrm{X}_{\text {rate }}$, is the fraction of $\mathrm{N}_{\text {ind }}$ that survives for the next step of mating. The number of chromosomes that are kept each generation is:

$$
\begin{equation*}
N_{\text {keep }}=X_{\text {rate }} \cdot N_{\text {ind }} \tag{6}
\end{equation*}
$$

Natural selection occurs each generation or iteration of the algorithm. Of the $\mathrm{N}_{\text {ind }}$ chromosomes, only the top $\mathrm{N}_{\text {keep }}$ survive for mating, and the bottom ( $\mathrm{N}_{\text {ind }}-\mathrm{N}_{\text {keep }}$ ) are discarded to make room for the new offspring. Deciding how many chromosomes to keep is somewhat arbitrary. Letting only a few chromosomes survive to the next generation limits the available genes in the offspring. Keeping too many chromosomes allows bad performers a chance to contribute their traits to the next generation. We use $50 \%\left(\mathrm{X}_{\text {rate }}=0.5\right)$ in the natural selection process. Another approach to natural selection is called thresholding (Truncation Selection) is used in this research. In this approach, all chromosomes that have a cost function lower than some truncation threshold survive. The threshold must allow some chromosomes to continue in order to have parents to produce offspring. Otherwise, a whole new population must be generated to find some chromosomes that pass the test. At first, only a few chromosomes may survive. In later generations, however, most of the chromosomes will survive unless the threshold is changed. An attractive feature of this technique is that the population does not have to be sorted.

### 3.5 Selection

In this process, two chromosomes are selected from the mating pool of $\mathrm{N}_{\text {keep }}$ chromosomes to produce two new offspring. Pairing takes place in the mating population until $\left(\mathrm{N}_{\text {ind }}-\mathrm{N}_{\text {keep }}\right)$ offspring are born to replace the discarded chromosomes. Pairing chromosomes in a genetic algorithm can be as interesting and varied as pairing in an animal species. Two types of selection are used in this research, which are:
3.5.1. Rank-weighted roulette wheel: This approach uses a uniform random number generator to select chromosomes. The row numbers of the parents are found using:
$m a=\operatorname{ceil}\left(N_{\text {keep }} * \operatorname{rand}\left(1, N_{\text {keep }} / 2\right)\right)$
$p a=\operatorname{ceil}\left(N_{\text {keep }} * \operatorname{rand}\left(1, N_{\text {keep }} / 2\right)\right)$,
Where ceil rounds the value to the next highest integer and rand generates arrays of random numbers whose elements are uniformly distributed in the interval $(0,1)$. This approach is problem independent and finds the probability from the rank of the chromosome. Rank weighting is slightly more difficult to program than the other selection types. Small populations have a high probability of selecting the same chromosome. The probabilities only have to be calculated once. We tend to use rank weighting because the probabilities do not change each generation. This approach of selection had been used in 14-bus IEEE-system.
3.5.2. Tournament selection: Another approach that closely mimics mating competition in nature is to randomly pick a small subset of chromosomes (two or three) from the mating pool, and the chromosome with the lowest cost in this subset becomes a parent. The typical value accepted by many applications is $k=2$ (so-called tournament size). The tournament repeats for every parent needed. Thresholding and tournament selection make a nice pair, because the population never needs to be sorted. Tournament selection works best for large population sizes because sorting becomes timeconsuming for large populations. Each of the parent selection schemes results in a different set of parents. As such, the composition of the next generation is different for each selection scheme. Rankweighted Roulette-wheel and tournament selection are standard for most genetic algorithms. It is very difficult to give advice on which selection scheme works best. In our problem, we follow the roulettewheel and tournament parent selection procedures for 14-bus IEEE-system and 362-bus ING respectively, Younes, and M. Rahli, 2006.

### 3.6 Crossover (Recombination)

As for the binary algorithm, two parents are chosen, and the offspring are some combination of these parents. Many different approaches have been tried for crossing over in continuous genetic algorithm. The simplest methods choose one or more points in the chromosome to mark as the crossover points. Then the variables between these points are merely swapped between the two parents. For example, consider the two parents to be:

$$
\begin{aligned}
& \text { parent } 1=\left[b_{m 1}, b_{m 2}, b_{m 3}, b_{m 4}, b_{m 5}, b_{m 6}, \ldots \ldots, b_{m N v a r}\right] \\
& \text { parent } 2=\left[b_{d 1}, b_{d 2}, b_{d 3}, b_{d 4}, b_{d 5}, b_{d 6}, \ldots \ldots, b_{d N v a r}\right]
\end{aligned}
$$

Crossover points are randomly selected (at points (3, 4)), and then the variables in between are exchanged:

$$
\begin{aligned}
& \text { offspring } 1=\left[b_{m 1}, b_{m 2}, b_{d 3}, b_{d 4}, b_{m 5}, b_{m 6}, \ldots \ldots, b_{m N v a r}\right] \\
& \text { offspring } 2=\left[b_{d 1}, b_{d 2}, b_{m 3}, b_{m 4}, b_{d 5}, b_{d 6}, \ldots \ldots, b_{d N v a r}\right]
\end{aligned}
$$

The extreme case is selecting $\mathrm{N}_{\text {var }}$ points and randomly choosing which of the two parents will contribute its variable at each position. Thus, one goes down the line of the chromosomes and, at each variable, randomly chooses whether or not to swap information between the two parents. This method is called uniform crossover:

$$
\begin{aligned}
& \text { offspring } 1=\left[b_{m 1}, b_{d 2}, b_{m 3}, b_{m 4}, b_{d 5}, b_{m 6}, \ldots \ldots, b_{d N v a r}\right] \\
& \text { offspring } 2=\left[b_{d 1}, b_{m 2}, b_{d 3}, b_{d 4}, b_{m 5}, b_{d 6}, \ldots \ldots, b_{m N v a r}\right]
\end{aligned}
$$

The problem with these point crossover methods is that no new information is introduced; each continuous value that was randomly initiated in the initial population is propagated to the next generation, only in different combinations. Although this strategy works fine for binary representations, there is now a continuum of values, and in this continuum we are merely interchanging two data points. These approaches totally rely on mutation to introduce new genetic material. The blending methods remedy this problem by finding ways to combine variable values from the two parents into new variable values in the offspring. A single offspring variable value $b_{\text {new }}$ comes from a combination of the two corresponding parents variable values:

$$
\begin{equation*}
b_{\text {new }}=\beta b_{m n}+(1-\beta) b_{d n} \tag{7}
\end{equation*}
$$

Where, $\beta$ is a random number on the interval $[\mathbf{0 , 1}], \mathrm{b}_{\mathrm{mn}}=\mathrm{n}^{\text {th }}$ variable in the mother chromosome, $\mathrm{b}_{\mathrm{dn}}=\mathrm{n}^{\text {th }}$ variable in the father chromosome.
The same variable of the second offspring is merely the complement of the first (i.e. replacing $\beta$ by 1 $-\beta$ ). If $\beta=1$, then $b_{\mathrm{mn}}$ propagates in it's entirely and $b_{\mathrm{dn}}$ dies. In contrast, if $\beta=0$, then $\mathrm{b}_{\mathrm{dn}}$ propagates in it's entirely and $b_{m n}$ dies. When $\beta=0.5$, the result is an average of the variables of the two parents. This method has been demonstrated to work well on several interesting problems. Choosing which variables to blend is the next issue. Sometimes, this linear combination process is done for all variables to the right or to the left of some crossover point, Woon, 2004. Any number of points can be chosen to blend, up to $\mathrm{N}_{\text {var }}$ values where all variables are linear combinations of those of the two parents. The variables can be blended by using the same $\beta$ for each variable or by choosing different $\beta$ 's for each variable. These blending methods effectively combine the information from the two parents and choose values of the variables between the values bracketed by the parents; however, they do not allow introduction of values beyond the extremes already represented in the population. Of course, the factor ( 0.5 ) is not the only one that can be used in such a method. Heuristic crossover is a variation where some random number $\beta$ is chosen on the interval $[0,1]$ and the variables of the offspring are defined by:

$$
\begin{equation*}
b_{\text {new }}=\beta\left(b_{m n}-b_{d n}\right)+b_{d n} \tag{8}
\end{equation*}
$$

Variations on this theme include choosing any number of variables to modify and generating different $\beta$ for each variable. This method also allows generation of offspring outside of the values of the two parent variables. Sometimes, values are generated outside of the allowed range. If this happens, the offspring is discarded and the algorithm tries another $\beta$. In our problem, we want to find a way to closely mimic the advantages of the binary genetic algorithm scheme. It begins by randomly selecting a variable c in the first pair of parents to be the crossover point, Yin, 1993:

$$
\begin{equation*}
c=\text { round up }\left\{\text { random } * N_{\text {var }}\right\} \tag{9}
\end{equation*}
$$

Where, (round up) is rounding mode that rounds to the nearest allowable quantized value. We'll let: parent $1=\left[b_{m 1}, b_{m}, \ldots \ldots ., b_{m c}, \ldots . ., b_{m N v a r}\right]$ parent $2=\left[b_{d 1}, b_{d 2}, \ldots \ldots ., b_{d c}, \ldots . ., b_{d N v a r}\right]$, Where (m) and (d) subscripts discriminate between the mom and dad parent. Then, the selected variables are combined to form new variables that will appear in the children:

$$
\begin{aligned}
& b_{\text {new } 1}=b_{m c}-\beta\left(b_{m c}-b_{d c}\right) \\
& b_{\text {new } 2}=b_{d c}+\beta\left(b_{m c}-b_{d c}\right)
\end{aligned}
$$

Where, $\beta$ is also a random value between 0 and 1 . The final step is to complete the crossover with the rest of the chromosome as in binary genetic algorithm:

$$
\begin{aligned}
& \text { offspring } 1=\left[b_{m 1}, b_{m 2}, \ldots \ldots, b_{\text {new } 1}, \ldots \ldots, b_{d N v a r}\right] \\
& \text { offspring } 2=\left[b_{d 1}, b_{d 2}, \ldots \ldots, b_{\text {new } 2}, \ldots \ldots, b_{m N v a r}\right]
\end{aligned}
$$

If the first variable of the chromosomes is selected, then only the variables to the right of the selected variable are swapped. If the last variable of the chromosomes is selected, then only the variables to the left of the selected variable are swapped. This method does not allow offspring variables outside the bounds set by the parent unless $\beta>1$, Younes, and Rahli, 2006-Jain, and Martin1, 1998.

### 3.7 Mutation

Random mutations alter a certain percentage of the genes in the list of chromosomes. If care is not taken, the genetic algorithm can converge too quickly into one region of the cost surface. If this area is in the region of the global minimum, that is good. However, some functions, such as the one we are modeling, have many local minima. If nothing is done to solve this tendency to converge quickly, it may end up in a local rather than a global minimum. To avoid this problem of overly fast convergence (premature convergence), the routine is forced to explore other areas of the cost surface by randomly introducing changes, or mutations, in some of the variables. Mutation points are randomly selected from the $\left(\mathrm{N}_{\mathrm{ind}} \times \mathrm{N}_{\text {var }}\right)$, total number of genes in the population matrix.
Increasing the number of mutations increases the algorithm's freedom to search outside the current region of variable space. It also tends to distract the algorithm from converging on a popular solution. With the process of the crossover and mutation taking place, there is a high chance that the optimum solution could be lost as there is no guarantee that these operators will preserve the fittest string. To counteract this, elitist models are often used. In an elitist model, the best individual in the population is saved before any of these operations take place. After the new population is formed and evaluated, it is examined to see if this best structure has been preserved. If not, the saved copy is reinserted back into the population. The genetic algorithm then continues on as normal, Ibrahim, 2005- Vasconcelos, et al., 2002.

## 4. PROPOSED TECHNIQUE

In the proposed method the load busbars are eliminated, retaining only generator busbars for the iterative process. The system equations in terms of generator busbars and load busbars can be written as:

$$
\binom{I_{G}}{I_{L}}=\left(\begin{array}{ll}
Y_{11} & Y_{1}  \tag{10}\\
Y_{21} & Y_{23}
\end{array}\right] \quad\left[\begin{array}{l}
V_{G} \\
V_{L}
\end{array}\right)
$$

If the voltage of the $\mathrm{K}^{\text {th }}$ load busbar is initially assumed to be $\mathrm{V}_{\mathrm{Lk}}=1.00^{\circ}$. Then the current in the busbar to the load is:

$$
\begin{equation*}
I_{L k}=\frac{P_{L k-j Q_{L k}}}{V *_{L k}} \tag{11}
\end{equation*}
$$

From the second row of eqn. 10, we have

$$
\begin{equation*}
V_{L}=-Y_{2}^{-1} Y_{21} V_{G}+Y_{22}^{-1} I_{L} \tag{12}
\end{equation*}
$$

Substituting eqn. 12 in the first row of eqn. 10 , we get

$$
\begin{equation*}
I_{G}=Y_{11} V_{G}+Y_{12}\left(-Y_{22}^{-1} Y_{21} V_{G}+Y_{22}^{-1} I_{L}\right) \tag{13}
\end{equation*}
$$

The above equation is written as

$$
\begin{equation*}
I_{G}=Y_{G G} V_{G}+Y_{G L} I_{L} \tag{14}
\end{equation*}
$$

Where: $Y_{G G}=Y_{11}-Y_{12} Y_{22}^{-1} Y_{21} \quad$ and $\quad Y_{G L}=Y_{12} Y_{22}^{-1}$
From eqn. 14, the $i^{\text {th }}$ generator busbar is:

$$
\begin{equation*}
I_{i}=\sum_{k=1}^{m} Y_{i k} V_{k}+a_{i}, \text { for } \mathrm{i}=1,2, \ldots \ldots, \mathrm{~m} . \tag{16}
\end{equation*}
$$

Where $\mathrm{a}_{\mathrm{i}}$ is the $\mathrm{i}^{\text {th }}$ element of the column vector A given by

$$
\begin{equation*}
A=Y_{G L} I_{L} \tag{17}
\end{equation*}
$$

The complex power at the busbar is

$$
\begin{equation*}
S_{i}=V_{i}^{*} I_{i}=V_{i}^{*} \sum_{k=1}^{m} Y_{i k} V_{k}+V_{i}^{*} a_{i} \tag{18}
\end{equation*}
$$

for $i=1,2, \ldots \ldots, \mathrm{~m}$.
The real power injection at the busbar is
$P_{i}=\operatorname{Re} S_{i}=\sum_{k=1}^{m} e_{i}\left(e_{k} G_{i k}-f_{k} B_{i k}\right)+\sum_{k=1}^{m} f_{i}\left(e_{k} B_{i k}+\quad f_{k} G_{i k}\right)+L_{i}$
for $i=1,2, \ldots, \mathrm{~m}$.
Where $L_{i}=e_{i} c_{i}+f_{i} d_{i}$
( $L_{i}$ ) can be considered as an equivalent local load at generator busbar $i$ due to elimination of the load busbars, Mithulananthan, et al., 2004 .

## 5. COMPUTER ALGORITHM OF THE PROPOSED METHOD

The computer algorithm for the proposed method is as follows:

1. Read the lines data and form the nodal admittance matrix.
2. Read the busbars data, such as the specified active power, voltage magnitude of the generator buses, specified active and reactive power of the load buses, slack bus voltage, and initial estimate of the voltage of the load buses, assuming ( 1.0 p.u., $0.1 \mathrm{MW} / \mathrm{MVAr}$ )
3. Eliminate the load busbars and reduce the network to the size of that of the generators busbars.
4. Compute ( $I_{L}$ ) using Eqn. (11) for all load buses, form the column vector (A) given by Eqn. (17), then form $\left(L_{i}\right)$ assuming $\left(e_{i}\right)$ equal to the specified values, and $\left(f_{i}\right)$ initially is zero.
5. Execute the Real-Coded Genetic Algorithm on the generator buses only to find the most recent value of the voltages, implementing all the GA operators such as Selection with Rank-Weighting Roulette Wheel, Tournament selection with truncation threshold, Single-point Crossover with blending method, and Mutation (rate of Mutation=0.2), we use initial population of 20 chromosomes for 14-bus IEEE system and 500 chromosomes for Iraqi National Grid (ING) system. At each generation (iteration) of the GA, we calculate the most recent values of ( $V_{L}$ ) from Eqn. (12), ( $I_{L}$ ) from Eqn. (11) and $\left(L_{i}\right)$, then calculate $\left(P_{i}\right)$ from Eqn. (19).
6. Convergence Test: The mismatch active powers for the generator buses (cost function) are calculated at each GA generation (iteration) according to the following equation:

$$
\begin{equation*}
\Delta P_{i}=P_{i}^{s p}-P_{i}^{c a l}, \text { for } i=1,2, \ldots, \mathrm{~m} . \tag{21}
\end{equation*}
$$

When the mismatch active powers (cost function) for all generator buses except the slack bus are less than a small tolerance value (usually 0.001 ), $0.1 \mathrm{MW} / \mathrm{MVAR}$ then the solution has converged.
7. Restore the system and calculate the load busbars voltages using Eqn. (12).
8. Print results and end.

## 6. IMPLEMENTATIONS AND RESULTS

Two test systems were used to demonstrate the performance of the proposed method, namely: 1. 14 busbars IEEE International test system, the lines and buses data are present in, Kubba, 1991. The "14- bus" test system consists of: 1 slack bus, 4 generator buses (PV) and 9 load buses(PQ). 2. The Iraqi National Grid (ING) which consists 362 busbars, 1 slack bus, 29 generator buses (PV) and 332 load buses (PQ), Al-Bakri, 1994.

The load flow solution using real-coded genetic algorithm programs with and without the method of Reduction and Restoration have been developed by the use of MATLAB version 7, and tested with a Pentium 4, 3GHz (Cache 2M) PC with 2GB RAM. Table 1 illustrates the power flow solution for a 14-bus IEEE test system using conventional RCGA with two objective functions, which are the mismatch active and reactive powers at each bus according to its constraints except the slack bus. The sum of weighted cost multi-objective functions is used. The most straightforward approach to multiobjective optimization is to weight each function and add them together, Abido, 2003.

$$
\cos t=\sum_{i=1}^{h} w_{i} f_{i}
$$

Where $f_{i}$ is the cost function (i), $w_{i}$ is the weighting factor, $h$ is the number of objective functions, and

$$
\sum_{i=1}^{h} w_{i}=1
$$

Implementing this multiple objective optimization approach in a real-coded genetic algorithm only requires modifying the cost function to fit the form of Eqn. (22) and does not require any modification to the genetic algorithm. Thus, Eqn. (22) becomes:

$$
\begin{equation*}
\cos t=w f_{1}+(1-w) f_{2} \tag{24}
\end{equation*}
$$

Where $f_{1}$ and $f_{2}$ are the mismatch active and reactive powers respectively, and have the same rank of importance. This approach is adopted in this research for its simplicity, easy of programming and gives us the required accuracy. Here, $(w)$ is chosen to be ( 0.5 ), Riccieri, and Falcao, 1999.
Because of the stochastic nature of the genetic algorithm process, each independent run will probably produce a different number of generations and consequently the computation time and the best amongst these should be chosen. The best of the 10 implementations runs are shown in the tables. The total computation time was 7.156 sec . Table 2 illustrates the power flow solution of the same IEEE test system using RCGA with the method of system Reduction and Restoration (Proposed Method). Since, we only retain the generator buses for the GA process, so a single objective function (mismatch active power) is needed. The total computation time for the whole load flow solution was 0.18 second. The power flow solution results for the Iraqi National Grid (362-bus) by using RCGA with the method of system reduction and restoration were tabulated in Table 3 and Table 4. Since the proposed method (RCGA with system Reduction and Restoration) implements the complete cycles of the genetic algorithm on the generator busbars only which are the first thirty buses of the system, then Table 3 shows the results and number of generations for each generator busbar and the power flow solution for the total Iraqi National Grid are presented. Table 4 shows the voltages of load buses which are calculated after restoring the system, also the mismatch active and reactive powers of load buses are presented. The total computation time with conventional RCGA method was more than 72 hours, while the total computation time for the proposed method was 519 seconds for the whole load flow solution of 362 -bus ING with the same accuracy. A ranked-weighted roulette wheel and Tournament selection process were used for 14-bus IEEE and ING respectively. Figure1 shows the evaluation process of the genetic algorithm for bus 2 of 14-bus IEEE system, the dotted curve represents the
minimum cost of the solution (chromosome) which is converged with 15 generations and the solid curve represents the average value of the costs amongst generations versus the number of generations.

## 7. CONCLUSIONS

The proposed method which had presented in this paper is very much faster than the simple realcoded genetic algorithm, since the system is reduced to the size of that of the generator busbars which for any realistic system is small as we see for the 362 -bus Iraqi National Grid, only 30 buses are generator busbars. We must take into consideration that the main drawback of the genetic algorithm is the large computation time. So, this contribution is especially for GA as an optimization technique. The objective function (cost function) for the generator buses is only the mismatch active power, so that multi-objective function techniques are not needed. Thus, it can be concluded that the proposed method is suitable for on-line implementation for small and medium-scale power systems and it can be used for planning study for large-scale systems. The proposed method has reliable convergence and high accuracy of solution. Whereas the traditional numerical techniques (Gauss-Seidel, NewtonRaphson, Fast decoupled,...etc.) use the characteristics of the problem to determine the next sampling point (e.g. gradient, linearity and continuity), genetic algorithm makes no such assumptions. Instead, the next sampled point is determined based on stochastic sampling or decision rules rather than on a set of deterministic decision rules. Genetic algorithms with the method of system reduction and restoration have been used to solve difficult problems with objective functions that possess properties such as continuity, differentiability and so forth. Also, whereas the traditional numerical techniques mentioned above use single point at a time to search the problem space, genetic algorithm uses a population of candidate solutions for solving the problem, thus reducing the possibility of ending at a local minima.

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## NOMENCLATURE

$\mathrm{N}=$ number of busbars in the system.
$\mathrm{m}=$ number of generator busbar in the system.
$V_{G}=$ M-dimensional vector of voltages of generator busbars.
$I_{G}=\mathrm{M}$-dimensional vector of currents of generator busbars.
$V_{L}=(\mathrm{N}-\mathrm{M})$ dimensional vector of voltages of load busbars.
$I_{L}=(\mathrm{N}-\mathrm{M})$ dimensional vector of currents of load busbars.
$Y=$ admittance matrix of order NxN .
$Y_{11}, Y_{12}, Y_{21}, Y_{22}=$ sub-matrices of $Y$ of appropriate order.
$V_{k}^{*}=$ conjugate of $\mathrm{k}^{\text {th }}$ busbar voltage $V_{\mathrm{k}}$.
$e_{\mathrm{k}}, f_{\mathrm{k}}=$ inphase and quadrature components of $V_{\mathrm{k}}$.
$c_{\mathrm{i}}, d_{\mathrm{i}}=$ real and imaginary parts of $a_{\mathrm{i}}$.
$s p=$ specified value .
cal $=$ calculated.
$\mathrm{G}_{\mathrm{ik}}, \mathrm{B}_{\mathrm{ik}}=$ real and imaginary parts of the admittance $Y_{i k}$


Figure1 Evaluation process for busbar (2), 14- bus IEEE test system

Table 1 Power Flow Solution (14-Bus IEEE) Test System with accuracy (0.001p.u.), using RCGA Without Reduction \& Restoration

| Bus | Active <br> power | Reactive <br> power | Voltage | Voltage | No. of |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No | mismatch | mismatch | Magnitude <br> (p.u) | Angle <br> (deg) | generations |
| 1 | Slack | Slack | 1.06 | 0.00 | - |
| 2 | 0.000329 | PV | 1.045 | 3.2117 | 17 |
| 3 | 0.000131 | PV | 1.010 | -4.3582 | 7 |
| 4 | 0.000484 | PV | 1.070 | -6.1436 | 21 |
| 5 | 0.000890 | PV | 1.090 | -12.423 | 47 |
| 6 | 0.000798 | 0.000481 | 1.057131 | 6.30252 | 95 |
| 7 | 0.000365 | 0.000060 | 1.0773818 | -4.6541 | 107 |
| 8 | 0.000222 | 0.000773 | 1.0565362 | -1.7120 | 193 |
| 9 | 0.000185 | 0.000682 | 1.0456395 | 1.44081 | 172 |
| 10 | 0.000273 | 0.000322 | 1.045163 | -9.0031 | 18 |
| 11 | 0.000950 | 0.000223 | 1.057696 | -5.4828 | 90 |
| 12 | 0.000411 | 0.000535 | 1.061725 | 7.67754 | 43 |
| 13 | 0.000770 | 0.000521 | 1.0482889 | -11.028 | 29 |
| 14 | 0.000209 | 0.000762 | 1.0588537 | -3.3446 | 47 |
|  | Total | Computational | Time: |  | 7.156 sec. |

Table 2 Power Flow Solution (14-Bus IEEE) Test System with accuracy (0.001p.u.), using RCGA With Reduction \& Restoration

| Bus | Active power | Reactive power | Voltage | Voltage | No. of |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. | mismatch | mismatch | magnitude(p.u) | angle(deg.) | Generations |
| 1 | Slack | Slack | 1.06 | 0.00 | - |
| 2 | 0.000373 | PV | 1.045 | 3.2117 | 15 |
| 3 | 0.000130 | PV | 1.010 | -3.35826 | 5 |
| 4 | 0.000374 | PV | 1.070 | -6.10062 | 21 |
| 5 | 0.000890 | PV | 1.090 | -11.0235 | 40 |
| 6 | 0.000678 | 0.000444 | 1.0476131 | 6.30252 | - |
| 7 | 0.000360 | 0.0006 | 1.0573888 | -4.6999 | - |
| 8 | 0.000223 | 0.000788 | 1.065092 | -2.71203 | - |
| 9 | 0.000109 | 0.0005 | 1.0558895 | 1.63081 | - |
| 10 | 0.000223 | 0.000321 | 1.0551690 | -9.03316 | - |
| 11 | 0.000850 | 0.000221 | 1.0476990 | -4.08283 | - |
| 12 | 0.000407 | 0.000546 | 1.0762725 | 6.60054 | - |
| 13 | 0.000660 | 0.000512 | 1.0482889 | -11.0208 | - |
| 14 | 0.000205 | 0.000769 | 1.0688837 | -3.34469 | - |
| Total | Computational | Time: |  |  | 0.18 sec. |

*Table 3 Power Flow Solution For "IRAQI NATIONAL GRID" with accuracy (0.001p.u.), using RCGA with the method of Reduction and Restoration (Only the Generator Busbars)

| Bus No. | Active power mismatch (p.u) | Reactive power mismatch (p.u) | $\begin{aligned} & \text { Voltage } \\ & \text { magnitude } \\ & \text { (p.u) } \end{aligned}$ | Voltage <br> Angle(deg) | No. of Generations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Slack | Slack | 1.04 | 0 | - |
| 2 | 0.0005 | PV | 1 | 18.3805 | 262 |
| 3 | 0.00021334 | PV | 1 | 2.8233 | 57 |
| 4 | 0.0008081 | PV | 1 | -9.5400 | 319 |
| 5 | 0.00011245 | PV | 1 | 13.6445 | 521 |
| 6 | 0.00043106 | PV | 1 | -11.8520 | 34 |
| 7 | 0.0018487 | PV | 1 | 4.1875 | 500 |
| 8 | 0.00066843 | PV | 1 | 7.5529 | 244 |
| 9 | 0.00023882 | PV | 1 | 12.3150 | 30 |
| 10 | 0.00016648 | PV | 1 | 4.0006 | 134 |
| 11 | 0.0003391 | PV | 1 | -19.7704 | 88 |
| 12 | 0.00045458 | PV | 1 | -6.3530 | 266 |
| 13 | 0.00013682 | PV | 1 | 4.5221 | 424 |
| 14 | 0.00058912 | PV | 1 | -6.9794 | 76 |
| 15 | 0.00054176 | PV | 1 | -8.1968 | 353 |
| 16 | 0.00021063 | PV | 1 | 13.5898 | 42 |
| 17 | 0.00078201 | PV | 1 | 4.5766 | 39 |
| 18 | $2.4477 * 10^{-6}$ | PV | 1 | 11.1094 | 41 |
| 19 | 0.00090163 | PV | 1 | 7.0672 | 47 |
| 20 | 0.00089409 | PV | 1 | -7.0275 | 9 |
| 21 | 0.00037127 | PV | 1 | -3.2876 | 159 |
| 22 | 0.00014522 | PV | 1 | -10.7986 | 24 |
| 23 | 0.00093387 | PV | 1 | 2.0421 | 216 |
| 24 | 0.00084462 | PV | 1 | 9.0268 | 47 |
| 25 | 0.00038532 | PV | 1 | 2.9669 | 17 |
| 26 | 0.00023586 | PV | 1 | 3.8338 | 52 |
| 27 | $7.2047 * 10^{-6}$ | PV | 1 | -6.8666 | 88 |
| 28 | 0.00011686 | PV | 1 | 0.0252 | 50 |
| 29 | 0.00026843 | PV | 1 | 7.3612 | 333 |
| 30 | 0.0005791 | PV | 1 | 9.1833 | 134 |

*Total computing time (Genetic Algorithm without the method of Reduction and Restoration): more than 72 hours.
*Total computing time (Genetic Algorithm with the method of Reduction and Restoration): 519 sec ., this time is for the total load flow solution of 362-bus ING system.

Table 4 Power Flow Solution For "IRAQI NATIONAL GRID" (Load Buses) After System Restoration

| Bus | Active power | Reactive <br> power <br> mismatch | Voltage | (pagnitude (p.u) |
| :---: | :---: | :---: | :---: | :---: |


| 64 | 0.00052304 | 0.0006 | 0.956323 | 0.126932 |
| :---: | :---: | :---: | :---: | :---: |
| 65 | 0.00039148 | 0.00027503 | 0.959269 | -3.93418 |
| 66 | 0.0003338 | 0.00048038 | 0.959048 | 5.45821 |
| 67 | 0.00045369 | 0.00074015 | 1.03691 | 8.05995 |
| 68 | 0.00077681 | 0.00053799 | 0.962027 | -0.556099 |
| 69 | 0.00027057 | 0.00061177 | 1.0712 | 5.11859 |
| 70 | 0.00040663 | 0.00054506 | 0.959477 | -13.4194 |
| 71 | 0.00015602 | 0.00084029 | 1.0151 | 4.6136 |
| 72 | 0.00085314 | 0.0000044 | 0.9505 | -9.4481 |
| 73 | 0.00082757 | 0.00056498 | 0.9587 | 14.8084 |
| 74 | 0.0000066 | 0.00028761 | 1.0205 | -0.5232 |
| 75 | 0.00045213 | 0.00089712 | 0.9606 | 16.254 |
| 76 | 0.00026312 | 0.00099152 | 1.0235 | -0.4562 |
| 77 | 0.0003 | 0.0002 | 0.9989 | -15.9269 |
| 78 | 0.00060173 | 0.00081968 | 0.9569 | 9.3835 |
| 79 | 0.00097288 | 0.00093161 | 0.9615 | -9.6373 |
| 80 | 0.00030775 | 0.00026214 | 1.0172 | -11.3539 |
| 81 | 0.00028197 | 0.0003666 | 0.9567 | 10.958 |
| 82 | 0.0001 | 0.0007 | 0.9709 | -9.6128 |
| 83 | 0.00017501 | 0.00031678 | 0.9538 | 0.5662 |
| 84 | 0.00060463 | 0.00055344 | 0.9689 | 12.4227 |
| 85 | 0.00049479 | 0.00025939 | 0.9917 | -17.9393 |
| 86 | 0.00074739 | 0.000061 | 0.9976 | 11.2733 |
| 87 | 0.00073692 | 0.00062359 | 1.0459 | 3.6541 |
| 88 | 0.0002 | 0 | 0.9665 | 7.1323 |
| 89 | 0.000081 | 0.00089968 | 1.0154 | -0.9266 |
| 90 | 0.00051316 | 0.00093613 | 1.0007 | -15.3443 |
| 91 | 0.000031 | 0.00072166 | 1.0017 | 18.7953 |
| 92 | 0.00062835 | 0.00047428 | 0.9977 | 12.0889 |
| 93 | 0.0000047 | 0.00047299 | 1.0034 | 11.0409 |
| 94 | 0.00031952 | 0.00085516 | 0.9855 | 1.9537 |
| 95 | 0.00028333 | 0.00050642 | 1.0368 | 18.9362 |
| 96 | 0.0008504 | 0.00096063 | 1.0821 | -15.1242 |
| 97 | 0.00038635 | 0.00026579 | 1.0445 | -5.8019 |
| 98 | 0.00069584 | 0.00012365 | 0.9702 | -8.2354 |
| 99 | 0.0007 | 0.0009 | 0.9562 | 16.788 |
| 100 | 0.00010717 | 0.0000085 | 0.9727 | -10.4169 |
| 101 | 0.0006727 | 0.00077553 | 0.9557 | 8.5754 |
| 102 | 0.00053227 | 0.00015283 | 1.0827 | -10.9894 |
| 103 | 0.00044692 | 0.00081571 | 0.9729 | 0.6583 |
| 104 | 0.00016048 | 0.000073 | 1.0319 | 0.9266 |
| 105 | 0.000025 | 0.00047994 | 0.9625 | -2.6931 |
| 106 | 0.000048 | 0.00079433 | 0.9284 | -10.2436 |
| 107 | 0.00084716 | 0.00015576 | 0.9641 | 4.4674 |
| 108 | 0.00068356 | 0.000055 | 1.0552 | 1.2291 |
| 109 | 0.00026455 | 0.00064387 | 1.0413 | 11.6445 |
| 110 | 0.00058321 | 0.00015476 | 1.02897 | 9.3654 |
| 111 | 0.00071167 | 0.00048416 | 1.089 | 16.4406 |
| 112 | 0.00094005 | 0.00097359 | 0.9736 | 12.9073 |
| 113 | 0.00084364 | 0.00061624 | 1.0071 | 19.928 |
| 114 | 0.00021032 | 0.00017141 | 0.9618 | 3.9724 |


| 115 | 0.00084827 | 0.00097626 | 0.9523 | 0.8467 |
| :---: | :---: | :---: | :---: | :---: |
| 116 | 0.00083 | 0.000547 | 0.9523 | 4.5623 |
| 117 | 0.00047 | 0.0007656 | 1.0258 | -15.256 |
| 118 | 0.000455 | 0.00072 | 0.96136 | 16.2351 |
| 119 | 0.00092 | 0.0009962 | 0.992564 | -9.2541 |
| 120 | 0.00012 | 0.0004547 | 1.03654 | 0.06541 |
| 121 | 0.001 | 0.0008 | 0.9618 | 2.2392 |
| 122 | 0.00013234 | 0.00093924 | 0.9884 | 11.3193 |
| 123 | 0.00096448 | 0.0005687 | 0.9523 | -7.3612 |
| 124 | 0.00081065 | 0.0000031 | 1.0828 | -6.4949 |
| 125 | 0.00030709 | 0.000083 | 0.9998 | -11.0119 |
| 126 | 0.000048 | 0.00071905 | 0.9618 | 3.1288 |
| 127 | 0.00096313 | 0.00098343 | 0.9757 | -4.9987 |
| 128 | 0.00093397 | 0.00077209 | 1.0843 | -0.3262 |
| 129 | 0.00062329 | 0.00064654 | 1.0182 | 12.9015 |
| 130 | 0.00094189 | 0.00037681 | 0.9539 | -8.8013 |
| 131 | 0.00014215 | 0.00057406 | 0.9571 | 1.6474 |
| 132 | 0.00041524 | 0.0005684 | 0.9583 | 10.7085 |
| 133 | 0.00020296 | 0.00046611 | 0.9638 | 6.3202 |
| 134 | 0.00037946 | 0.00062747 | 0.9987 | 9.103 |
| 135 | 0.00095745 | 0.00082409 | 0.9785 | -1.7189 |
| 136 | 0.00027374 | 0.00046965 | 0.9706 | 1.4742 |
| 137 | 0.00020975 | 0.00085845 | 0.971 | 7.2577 |
| 138 | 0.00039402 | 0.00048299 | 1.0127 | 7.7591 |
| 139 | 0.00035319 | 0.00036285 | 0.9632 | 5.5677 |
| 140 | 0.0006075 | 0.0009984 | 0.95154 | 6.3214 |
| 141 | 0.0008155 | 0.0002237 | 0.95214 | -14.2365 |
| 142 | 0.00011 | 0.0009845 | 1.0564 | 0.98745 |
| 143 | 0.0002734 | 0.0002717 | 0.9654 | 3.2145 |
| 144 | 0.0001812 | 0.000567 | 0.9628 | -17.149 |
| 145 | 0.0004911 | 0.0006331 | 0.9752 | -4.2187 |
| 146 | 0.0005208 | 0.000486 | 0.9962 | 0.05871 |
| 147 | 0.0005119 | 0.000886 | 1.0547 | 2.0154 |
| 148 | 0.0002753 | 0.0004816 | 0.9614 | 13.2974 |
| 149 | 0.0004286 | 0.0004054 | 1.0893 | 1.5647 |
| 150 | 0.0002753 | 0.0001582 | 0.9512 | 6.2354 |
| 151 | 0.0007 | 0 | 0.9544 | 3.5375 |
| 152 | 0.00013698 | 0.00027886 | 1.007 | -0.8344 |
| 153 | 0.00020208 | 0.00023032 | 0.9965 | -0.5292 |
| 154 | 0.00040253 | 0.00020923 | 0.9701 | 5.2631 |
| 155 | 0.00046085 | 0.00096044 | 0.9731 | 5.815 |
| 156 | 0.00029987 | 0.000035 | 0.9753 | 4.4393 |
| 157 | 0.00080446 | 0.00053457 | 0.9485 | -8.8882 |
| 158 | 0.00035469 | 0.00015957 | 0.9687 | 13.5985 |
| 159 | 0.0003943 | 0.000029 | 0.9585 | -1.0752 |
| 160 | 0.00073661 | 0.00017162 | 0.9279 | 13.6502 |
| 161 | 0.00065884 | 0.00028923 | 0.9548 | -6.1351 |
| 162 | 0.00035578 | 0.00029898 | 0.9587 | -2.3739 |
| 163 | 0.00075604 | 0.00092751 | 0.9521 | 2.898 |
| 164 | 0.000035 | 0.00038695 | 1.0067 | 8.9077 |
| 165 | 0.00099806 | 0.00071947 | 1.0691 | -10.2774 |
| 166 | 0.00012425 | 0.000052 | 0.955 | -8.262 |


| 167 | 0.00092219 | 0.00019175 | 0.9984 | 13.4656 |
| :---: | :---: | :---: | :---: | :---: |
| 168 | 0.00076628 | 0.00081641 | 0.9775 | 17.2191 |
| 169 | 0.00092571 | 0.000057 | 0.973 | 0.6329 |
| 170 | 0.00054766 | 0.00069843 | 0.9653 | 0.4959 |
| 171 | 0.00099334 | 0.00079967 | 0.9611 | 10.6166 |
| 172 | 0.00049608 | 0.00022697 | 0.9946 | -1.0459 |
| 173 | 0.00055849 | 0.00042155 | 0.9421 | -4.9939 |
| 174 | 0.00048618 | 0.000055 | 1.0338 | 3.6292 |
| 175 | 0.00014611 | 0.00072444 | 0.9827 | 10.898 |
| 176 | 0.00051741 | 0.00064903 | 0.9628 | 6.5469 |
| 177 | 0.00071172 | 0.00036796 | 0.9539 | -1.5042 |
| 178 | 0.00019378 | 0.000039 | 0.9512 | 6.5171 |
| 179 | 0.00060168 | 0.0008397 | 1.0102 | -7.9275 |
| 180 | 0.00059232 | 0.00022898 | 0.9634 | 6.5271 |
| 181 | 0.0001 | 0.0002 | 1.0054 | 4.2552 |
| 182 | 0.00044569 | 0.000618 | 1.0125 | -6.0294 |
| 183 | 0.00055934 | 0.00041766 | 0.9865 | 3.0486 |
| 184 | 0.00020306 | 0.00096151 | 0.9548 | 7.0504 |
| 185 | 0.00088243 | 0.00045767 | 0.9413 | -0.5076 |
| 186 | 0.00069729 | 0.00070518 | 0.9528 | 9.3272 |
| 187 | 0.000044 | 0.000333 | 0.9537 | 4.4252 |
| 188 | 0.000013 | 0.00044238 | 0.9582 | 2.137 |
| 189 | 0.00050756 | 0.00076265 | 0.9543 | 4.8151 |
| 190 | 0.00066192 | 0.00075837 | 0.9592 | 19.2249 |
| 191 | 0.0002513 | 0.00044497 | 0.9583 | 0.8323 |
| 192 | 0.00020302 | 0.00080563 | 0.9555 | 2.266 |
| 193 | 0.00030057 | 0.00094464 | 1.0024 | -17.265 |
| 194 | 0.00046703 | 0.00042967 | 0.9994 | 11.0973 |
| 195 | 0 | 0 | 0.9507 | 18.5179 |
| 196 | 0.00093542 | 0.000085 | 0.9738 | 10.7321 |
| 197 | 0.00051919 | 0.00062756 | 0.9848 | 1.4284 |
| 198 | 0.00013828 | 0.00084404 | 1.067 | 6.9025 |
| 199 | 0.00052945 | 0.0002323 | 1.027 | -17.4657 |
| 200 | 0.00054335 | 0.00038057 | 0.9445 | -15.5822 |
| 201 | 0.00061187 | 0.00075132 | 0.9957 | -18.0073 |
| 202 | 0.00096005 | 0.00091358 | 0.9778 | -2.2563 |
| 203 | 0.00079194 | 0.00022214 | 0.9552 | -3.3525 |
| 204 | 0.00026439 | 0.00095589 | 0.9802 | 0.5317 |
| 205 | 0.00056456 | 0.00087312 | 0.9946 | 18.2447 |
| 206 | 0.00054948 | 0.00055895 | 0.9555 | 8.0271 |
| 207 | 0.00077144 | 0.001 | 0.9546 | 3.8581 |
| 208 | 0.00075889 | 0.00027959 | 1.0049 | 8.4842 |
| 209 | 0.00056002 | 0.00040952 | 1.0468 | -13.798 |
| 210 | 0.0006285 | 0.00057178 | 0.9503 | -5.8999 |
| 211 | 0.00019025 | 0.00095178 | 0.9528 | -5.8964 |
| 212 | 0.00059117 | 0.00056099 | 0.9572 | -0.4872 |
| 213 | 0.00037677 | 0.00016316 | 0.9714 | 17.869 |
| 214 | 0.00088221 | 0.00063269 | 0.9555 | -11.1881 |
| 215 | 0.00065945 | 0.00093676 | 0.9865 | -2.8314 |
| 216 | 0.00087711 | 0.00089287 | 0.988 | -0.0197 |
| 217 | 0.00093717 | 0.00011334 | 0.9586 | -9.8487 |


| 218 | 0.00048357 | 0.0005098 | 1.0667 | 16.3454 |
| :---: | :---: | :---: | :---: | :---: |
| 219 | 0.000074 | 0.00050823 | 0.9713 | 8.419 |
| 220 | 0.00023902 | 0.00097386 | 0.9738 | 8.5508 |
| 221 | 0.00020321 | 0.00085756 | 1.0454 | 15.2306 |
| 222 | 0.00011518 | 0.00095031 | 1.0223 | 6.236 |
| 223 | 0.0009373 | 0.00031901 | 0.9452 | 1.1682 |
| 224 | 0.00063283 | 0.00047536 | 0.99 | -4.223 |
| 225 | 0.00019285 | 0.000067 | 0.9426 | -12.6611 |
| 226 | 0.000078 | 0.00023635 | 0.9599 | -3.5077 |
| 227 | 0.00054555 | 0.000043 | 0.963 | 5.5577 |
| 228 | 0.0007854 | 0.000023 | 0.9515 | 2.3654 |
| 229 | 0.00021656 | 0.00064519 | 1.0196 | -10.1812 |
| 230 | 0.00045404 | 0.00078516 | 1.0707 | -17.4119 |
| 231 | 0.00026644 | 0.00016696 | 0.9708 | -3.5614 |
| 232 | 0.00032806 | 0.0001135 | 1.0165 | -17.2337 |
| 233 | 0.0008132 | 0.00026348 | 1.0561 | 0.7447 |
| 234 | 0.00049081 | 0.00047598 | 0.9979 | 6.2127 |
| 235 | 0.00042958 | 0.00037536 | 0.9598 | 3.4219 |
| 236 | 0.00099515 | 0.00014891 | 1.0225 | 6.0696 |
| 237 | 0.00084 | 0.00054738 | 0.9921 | -3.3138 |
| 238 | 0.00019869 | 0.0005315 | 1.0221 | 5.5529 |
| 239 | 0.00069827 | 0.00070673 | 0.9505 | -0.6187 |
| 240 | 0.00073886 | 0.0006194 | 0.9452 | 14.327 |
| 241 | 0.000012 | 0.00047631 | 1.0173 | -11.6858 |
| 242 | 0.00052499 | 0.000039 | 0.945 | -2.4259 |
| 243 | 0.00044988 | 0.00058049 | 0.9533 | 16.2874 |
| 244 | 0.00033758 | 0.00014153 | 0.9937 | 6.6786 |
| 245 | 0.00068628 | 0.00084693 | 0.9597 | -4.9778 |
| 246 | 0.0004596 | 0.00037358 | 0.9885 | 9.4525 |
| 247 | 0.00051515 | 0.00045483 | 0.9546 | 4.6063 |
| 248 | 0.00032144 | 0.00068358 | 1.0714 | 7.7469 |
| 249 | 0.00043791 | 0.00041416 | 0.9966 | -0.7901 |
| 250 | 0.00028647 | 0.00022116 | 0.9556 | -2.497 |
| 251 | 0.0007 | 0.0003 | 0.95315 | 1.1601 |
| 252 | 0.000095 | 0.00054734 | 0.95733 | -8.8202 |
| 253 | 0.00079549 | 0.00077536 | 0.98058 | -5.4324 |
| 254 | 0.00032922 | 0.00065235 | 1.0836 | -17.168 |
| 255 | 0.000085 | 0.00037146 | 0.959 | 8.3101 |
| 256 | 0.00094778 | 0.000036 | 0.98041 | -19.965 |
| 257 | 0.00078819 | 0.00080088 | 0.95607 | -13.439 |
| 258 | 0.0001091 | 0.00023452 | 1.0162 | -4.7631 |
| 259 | 0.0008952 | 0.00047521 | 0.9842 | 1.5236 |
| 260 | 0.0009 | 0.0003 | 0.9574 | -2.2963 |
| 261 | 0.0004 | 0.0008 | 1.0025 | -3.0548 |
| 262 | 0.00066294 | 0.000013 | 0.9891 | -1.1725 |
| 263 | 0.00048644 | 0.0001791 | 0.94889 | 8.6261 |
| 264 | 0.00015727 | 0.00054346 | 0.95678 | -18.469 |
| 265 | 0.00033058 | 0.00075487 | 0.9502 | 8.8319 |
| 266 | 0.00060066 | 0.00054603 | 1.0306 | 6.2556 |
| 267 | 0.00052942 | 0.00042094 | 1.0529 | 14.874 |


| 268 | 0.00088617 | 0.00069782 | 1.0911 | -0.47424 |
| :---: | :---: | :---: | :---: | :---: |
| 269 | 0.0004741 | 0.00045141 | 1.0366 | 5.3008 |
| 270 | 0.00034368 | 0.00003 | 1.0076 | -13.849 |
| 271 | 0.001 | 0.0005 | 1.052 | 5.453 |
| 272 | 0.00012802 | 0.00052728 | 1.0162 | -0.30575 |
| 273 | 0.00092958 | 0.00022451 | 0.9701 | -1.8693 |
| 274 | 0.000074 | 0.00096697 | 0.99916 | -0.018933 |
| 275 | 0.00072001 | 0.00030789 | 0.99985 | 14.765 |
| 276 | 0.00063061 | 0.00031637 | 1.0273 | 3.0481 |
| 277 | 0.00095612 | 0.000019 | 1.0046 | -7.9187 |
| 278 | 0.00068346 | 0.00018341 | 0.95131 | 3.5014 |
| 279 | 0.0004 | 0.0007 | 0.9827 | -15.4074 |
| 280 | 0.000053 | 0.00026706 | 1.0474 | -14.6568 |
| 281 | 0.00035039 | 0.00018333 | 1.0241 | -4.0515 |
| 282 | 0.0004435 | 0.00023507 | 0.964 | 7.934 |
| 283 | 0.000093 | 0.00028498 | 0.9423 | -3.4764 |
| 284 | 0.00049599 | 0.000075 | 1.0241 | 9.6195 |
| 285 | 0.001 | 0.0002 | 0.9153 | 14.1537 |
| 286 | 0.00087371 | 0.00037862 | 0.912 | -7.0177 |
| 287 | 0.00036547 | 0.00042879 | 0.9512 | -5.2367 |
| 288 | 0.0008 | 0.0009 | 1.0031 | -2.86 |
| 289 | 0.00076513 | 0.00057713 | 0.9934 | 1.0543 |
| 290 | 0.0003 | 0 | 0.9345 | 1.6056 |
| 291 | 0.00072 | 0.00029 | 0.9136 | -6.1 |
| 292 | 0.0001864 | 0.00039723 | 0.9789 | -10.8992 |
| 293 | 0.00012961 | 0.00028 | 0.976 | -9.4627 |
| 294 | 0.00082929 | 0.00050417 | 1.0373 | 13.1924 |
| 295 | 0.00021605 | 0.00030426 | 0.935 | 7.205 |
| 296 | 0.00087376 | 0.00048 | 0.9006 | -16.6558 |
| 297 | 0.00044415 | 0.00067544 | 0.9385 | 11.6025 |
| 298 | 0.001 | 0.0006 | 0.962 | 17.4074 |
| 299 | 0.0002 | 0.0007 | 0.9108 | 4.4968 |
| 300 | 0.0006 | 0.00023838 | 0.9914 | -7.3521 |
| 301 | 0.00021 | 0.00077513 | 0.9295 | -3.7163 |
| 302 | 0.00060162 | 0.00080401 | 0.9947 | 11.3228 |
| 303 | 0.00083567 | 0.00077209 | 1.0395 | 5.0055 |
| 304 | 0.00031004 | 0.00057108 | 1.0016 | 9.8521 |
| 305 | 0.00034026 | 0.00077 | 0.9113 | -19.9432 |
| 306 | 0.0005 | 0.0001 | 0.9574 | 9.9693 |
| 307 | 0.00082069 | 0.00072388 | 0.9175 | -13.1628 |
| 308 | 0.00078933 | 0.00012857 | 0.9248 | -2.9038 |
| 309 | 0.00047276 | 0.00056897 | 0.915 | 3.2155 |
| 310 | 0.00051363 | 0.00048865 | 0.9966 | 1.6473 |
| 311 | 0.0005 | 0.0005 | 0.9932 | 5.0575 |
| 312 | 0.00045 | 0.000501 | 0.9141 | 10.7649 |
| 313 | 0.0009075 | 0.00069747 | 0.9307 | -9.4417 |
| 314 | 0.00059802 | 0.00083978 | 0.9472 | -2.4195 |
| 315 | 0.00090878 | 0.00080899 | 0.9357 | -11.4126 |
| 316 | 0.0004 | 0.0002 | 1.0021 | 12.7353 |
| 317 | 0.00079075 | 0.00051686 | 0.9498 | 14.4289 |
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| 319 | 0.00033893 | 0.00020428 | 0.9459 | 18.9635 |
| :---: | :---: | :---: | :---: | :---: |
| 320 | 0.00073845 | 0.00032157 | 0.9683 | -11.3016 |
| 321 | 0.0008 | 0.0009 | 0.911 | 4.4709 |
| 322 | 0.00089603 | 0.00069157 | 0.9326 | 1.409 |
| 323 | 0.00062809 | 0.00088 | 1.0397 | 15.4222 |
| 324 | 0.00075763 | 0.00013265 | 1.0234 | 7.1817 |
| 325 | 0.00034566 | 0.00027051 | 0.9664 | -8.6569 |
| 326 | 0.00068725 | 0.00094826 | 1.0478 | -0.6129 |
| 327 | 0.00098619 | 0.00011221 | 0.9242 | 10.1916 |
| 328 | 0.00078934 | 0.00017901 | 0.9116 | -14.6921 |
| 329 | 0.00053819 | 0.00015669 | 1.0548 | 16.6016 |
| 330 | 0.00078625 | 0.00091999 | 1.0757 | 12.2295 |
| 331 | 0.0004 | 0.0002 | 0.9059 | -8.4352 |
| 332 | 0.00068151 | 0.00040504 | 0.9118 | -6.1132 |
| 333 | 0.00030002 | 0.00088672 | 0.9854 | -18.1334 |
| 334 | 0.00031717 | 0.00071215 | 1.0076 | -10.6794 |
| 335 | 0.00078608 | 0.00072425 | 0.9117 | 5.1871 |
| 336 | 0.00057 | 0.00029 | 0.9127 | -3.4872 |
| 337 | 0.00035894 | 0.00064906 | 0.9453 | 0.4137 |
| 338 | 0.00054271 | 0.00040283 | 0.9783 | 19.1633 |
| 339 | 0.0004782 | 0.00019485 | 0.9615 | 7.4683 |
| 340 | 0.00019572 | 0.00018078 | 1.0245 | 8.2345 |
| 341 | 0.0002 | 0.0001 | 0.9202 | 15.3665 |
| 342 | 0.00071403 | 0.00057652 | 0.9462 | 5.148 |
| 343 | 0.00041883 | 0.00016 | 0.9531 | 8.1265 |
| 344 | 0.00050634 | 0.00018 | 1.0171 | -5.4776 |
| 345 | 0.00083654 | 0.00070874 | 0.9417 | 3.9911 |
| 346 | 0.00045 | 0.00046 | 1.0612 | 4.6101 |
| 347 | 0.00068 | 0.0005833 | 0.954 | 4.7473 |
| 348 | 0.00070461 | 0.00033 | 1.0019 | 14.4764 |
| 349 | 0.00015384 | 0.00074427 | 0.9648 | 6.3369 |
| 350 | 0.00057962 | 0.00013103 | 0.9975 | 4.7811 |
| 351 | 0.0009 | 0.0005 | 0.9572 | 17.3267 |
| 352 | 0.00071164 | 0.00015677 | 0.9489 | -7.1507 |
| 353 | 0.0007123 | 0.00071494 | 0.9817 | -10.7106 |
| 354 | 0.00074257 | 0.00035 | 0.9236 | -3.1823 |
| 355 | 0.00083908 | 0.00066149 | 0.9129 | -2.6764 |
| 356 | 0.00084867 | 0.00028569 | 1.019 | -3.3266 |
| 357 | 0.00030526 | 0.00062 | 0.9299 | -12.2346 |
| 358 | 0.00073194 | 0.00088 | 0.9218 | -12.8536 |
| 359 | 0.00013875 | 0.00012092 | 0.9438 | -9.7586 |
| 360 | 0.00048704 | 0.00048171 | 0.9875 | 1.5297 |
| 361 | 0.00062871 | 0.00068218 | 1.023 | -5.9336 |
| 362 | 0.00059909 | 0.00069565 | 0.9912 | -1.5367 |

