

## Survey Study of Fractional Order Controllers

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### ABSTRACT

This is a survey study that presents recent researches concerning fractional controllers. It presents several types of fractional order controllers, which are extensions to their integer order counterparts. The fractional order PID controller has a dominant importance, so thirty-one papers are presented for this controller. The remaining types of controllers are presented according to the number of papers that handle them; they are fractional order sliding mode controller (nine papers), fuzzy fractional order sliding mode controller (five papers), fractional order lag-lead compensator (three papers), fractional order state feedback controller (three papers), fractional order fuzzy logic controller (three papers). Finally, several conclusions were drawn from the results that were given in these papers.

**Keywords:** fractional order PID controller, fractional order sliding mode controller, fuzzy fractional order sliding mode controller, fractional order lag-lead compensator, fractional order state feedback controller, fractional order fuzzy logic controller.

## دراسة استقصائية للمسيطرات ذات الرتبة الكسرية

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### الخلاصة

يقدم هذا البحث دراسة استقصائية للأبحاث الحديثة المتعلقة بالمسيطرات ذات الرتبة الكسرية، حيث تقوم عدة أنواع من هذه المسيطرات والتي هي امتدادات لنظرائها من المسيطرات ذات الرتبة الصحيحة. يعد المسيطر التناسبي والتكاملي والتفاضلي ذو الرتبة الكسرية أهم هذه المسيطرات، لذلك فقد تم ذكر واحدًا وثلاثين بحثًا لهذا النوع. أما بقية الأنواع فقد تم ترتيب ذكرها وفقًا لعدد البحوث التي تناولتها، وهي المسيطر الانزلاقي ذو الرتبة الكسرية (تسعة بحوث)، والمسيطر الضبابي الانزلاقي ذو الرتبة الكسرية (خمسة بحوث)، ومسيطر تعويض التأخر و/أو التقدم ذو الرتبة الكسرية (ثلاثة بحوث)، ومسيطر التغذية الاسترجاعية لمتغيرات الحالة ذو الرتبة الكسرية (ثلاثة بحوث)، ومسيطر المنطق الضبابي ذو الرتبة الكسرية (ثلاثة بحوث). في النهاية تم الوصول إلى عدة استنتاجات من خلال النتائج التي توصلت إليها هذه البحوث.

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## 1. INTRODUCTION

Fractional control has become a research topic in control theory and application recently. Fractional control relies on fractional calculus, which extends the meaning of the derivative  $\frac{d^\alpha f(t)}{dt^\alpha}$  to every  $\alpha$ , real or complex. In fractional framework, the change from derivation to integration is continuous; thus, a unique operator for differentiation and integration can be defined. This operator is called differintegrator, and is given by

$$D_c^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \text{if } \alpha \in \mathbb{N} \\ 1, & \text{if } \alpha = 0 \\ \int_c^t (d\tau)^{-\alpha} = I_c^{-\alpha}, & \text{if } \alpha \in \mathbb{Z}^- \end{cases}$$

where  $\alpha \in \mathbb{R}$  is the order of differentiation or integration and  $c$  and  $t$  are the lower and upper limits, respectively.

The three most commonly used definitions for the general fractional differintegral are Riemann-Liouville (RL), Caputo, and Grunwald-Letnikov (GL) definitions (Petras, 2011), (Padula, 2015), and (Zhou, 2017). A fractional controller has fractional order differential equation dynamics. Fractional control adds a degree of freedom by extending the domain of the differentiation and/or integration to real or complex numbers such that it best fits the required specifications. In the past decade, research efforts related to applying fractional calculus to control theory increased. There are many types of fractional order controllers used to control different systems according to the requirements of these system. Some of these types of fractional order controllers with a survey of related papers are given in the following sections.

## 2. FRACTIONAL ORDER PID CONTROLLER

The most common fractional order controller is the fractional order PID controller (also called  $PI^\lambda D^\mu$  controller), proposed by (Podlubny, 1994) and (Podlubny, 1999). It is a generalization of the conventional PID controller, where the integer order derivative and integral actions are replaced by fractional order derivative and integral actions. Some of the recent works that utilize this controller are:

- **Sadati et al. (2007)** presented a novel approach to design an optimal  $PI^\lambda D^\mu$  controller using Particle Swarm Optimization (PSO) algorithm. In this paper, a new performance criterion in the time domain was proposed. This performance criterion includes overshoot, rising time, settling time, steady state error, and Integral Absolute Error (IAE). The PSO algorithm was employed to search for the optimal  $PI^\lambda D^\mu$  controller for Linear Time Invariant (LTI) SISO and MIMO systems. The proposed approach was applied to an electromagnetic suspension system (unstable LTI system). Simulation results showed that this system is more robust and it outperforms the PID control system.
- **Zamani et al. (2009)** designed a  $PI^\lambda D^\mu$  controller for an automatic voltage regulator (LTI system). In this paper, a new performance criterion in time domain and frequency domain was proposed. This performance criterion includes the overshoot, rising time, settling time, steady state error, IAE, integral of squared input, gain margin, and phase margin. The PSO algorithm is used to design the controller. Comparisons with a PID controller showed that the proposed  $PI^\lambda D^\mu$  controller can improve the system robustness against model uncertainties.



- **Metu (2013)** designed a  $PI^\lambda D^\mu$  controller for a liquid level control of a spherical tank, which is a fractional order LTI system. The  $PI^\lambda D^\mu$  controller was designed by specifying gain margin, phase margin, ISO damping property which gives robustness against variations in the gain of the plant by making the phase curve of the Bode plot of the open loop transfer function flat at the gain crossover frequency, noise rejection, and disturbance rejection. Simulation results showed that the  $PI^\lambda D^\mu$  controller gives better results than the PID controller.
- **Han et al. (2013)** designed a  $[PI]^\lambda$  controller and a PID controller for vertical takeoff and landing of unmanned aerial vehicles to control the pitch angle, which is a First Order Plus Time Delay (FOPTD) system. The  $[PI]^\lambda$  controller differs from the  $PI^\lambda$  controller in the sense that in the  $[PI]^\lambda$  controller, the *sum* of the proportional term and integral term is raised to the fractional order  $\lambda$ , rather than raising only the integral term in the  $PI^\lambda$  controller. The controller was designed by specifying gain crossover frequency, phase margin, and ISO damping property. Simulation results showed that the  $[PI]^\lambda$  control system has less overshoot, less rising time and is more robust compared with the PID control system.
- **Daou (2013)** carried out a comparison between three types of controllers: PID controller,  $PI^\lambda D^\mu$  controller, and a Command Robust Order Noninteger (CRONE) controller. These controllers are applied to a hydro-electromechanical test bench (LTI system). The results showed that the CRONE controller is more robust than the PID and  $PI^\lambda D^\mu$  controllers against variations in the plant parameters.
- **Tepljakov et al. (2013)** investigated the design and implementation of  $PI^\lambda D^\mu$  controllers for a nonlinear MIMO system. The plant was treated as a SISO system by manipulating one input, setting to zero the other inputs, and observing the associated output. The results were comparable to other nonlinear control approaches.
- **Tepljakov et al. (2014)** designed  $PI^\lambda D^\mu$  controllers for a laboratory model of a magnetic levitation system (unstable nonlinear system). Stabilizing  $PI^\lambda D^\mu$  controllers were designed by linearizing the nonlinear system around an operating point. Then these controllers were evaluated using the IAE performance index, and those with best performance were selected to enhance the performance of the closed loop system.
- **Bucanovic (2014)** designed a  $PI^\lambda D^\mu$  controller for a cryogenic air separation process (nonlinear MIMO system). After linearizing and decoupling the system, the controller parameters were obtained by minimizing a performance index that involves the IAE, the overshoot, and the rise time. Simulation results showed that the  $PI^\lambda D^\mu$  controller enhances the transient response and that it is more robust than the PID controller against external disturbances.
- **Tajbakhsh et al (2014)** presented a robust  $PI^\lambda D^\mu$  controller to control the speed of a DC motor (LTI system) with parameter uncertainty. The controller was designed by specifying phase margin, gain crossover frequency, ISO damping property, noise rejection, and disturbance rejection. Simulation results revealed that the proposed controller enhances the performance of the control system and is more robust for model uncertainty.
- **Bhisikar et al. (2014)** presented an approach to design a  $PD^\mu$  controller for unstable and integrated systems. The controller was designed by using Bode's ideal transfer function



to stabilize the system and to improve its performance. The proposed controller outperformed the PD and PID controllers with respect to rise time, percentage overshoot, and settling time and the improvements in these quantities are significant.

- **Ozkan (2014)** designed a  $PI^\lambda D^\mu$  controller for an electromechanical actuation system (LTI system). The controller was designed by placing the poles of the closed loop control system on the complex plane by specifying bandwidth and damping ratio values. Simulations showed that the proposed controller enhances the system stability and robustness against the disturbance.
- **Divya et al. (2014)** designed a PI controller and a  $PI^\lambda$  controller for two interacting tank level process (nonlinear system). The performance of the control system investigated with the Integral Squared Error (ISE) performance index. Simulation results showed that the  $PI^\lambda$  controller outperforms the PI controller in terms of reference input tracking, robustness against variations in the plant parameters, and disturbance rejection.
- **Junyi (2015)** proposed a  $PI^\lambda D^\mu$  controller for hydroturbine governing system (nonlinear system with time varying and non-minimum phase characteristics). Investigation of the control system demonstrated that the  $PI^\lambda D^\mu$  controller outperforms the PID controller in terms of reducing the oscillations and reducing the settling time.
- **Sharma (2015)** designed a  $PI^\lambda D^\mu$  controller for a two-link planar rigid robotic manipulator (coupled and highly nonlinear MIMO system) for trajectory tracking task. The performance of the proposed controller was compared with that of a PID controller. The robustness of the control system was tested for model uncertainties, payload variations with time, external disturbance and random noise; simulation results revealed that the proposed controller is more robust than the PID controller.
- **Ijaz et al. (2015)** designed an optimal  $PI^\lambda D^\mu$  controller for Electro-Hydraulic servo system (nonlinear system). The controller parameters are designed using PSO algorithm and the performance index is a linear combination of the overshoot, steady state error, settling time, rise time, IAE, and gain margin. Simulation results demonstrated that the  $PI^\lambda D^\mu$  controller outperforms the PID controller in terms of transient response and frequency domain characteristics for various operating conditions.
- **Badri et al. (2015)** studied the design methods for  $PI^\lambda$  controllers recently proposed. The performance region was also found in the gain crossover frequency-phase margin plane. Practically, some experiments on velocity control of a DC motor (LTI system) were carried out.
- **Kesarkar et al. (2015)** presented novel design methods for three parameter  $[PI]^\alpha$  and  $[PD]^\beta$  controllers, where these methods can be applied to any LTI system, integer or fractional. In a similar manner to the  $[PI]^\alpha$  controller, in the  $[PD]^\beta$  controller, the sum of the proportional term and derivative term is raised to the fractional order  $\beta$ .
- **Zhong et al. (2015)** designed  $PI^\lambda D^\mu$  and PID controllers and applied them to stabilize a solid core magnetic bearing system (LTI FO system). Simulations and experiments were implemented to compare the performance of the PID and the  $PI^\lambda D^\mu$  controllers. The results showed that the  $PI^\lambda D^\mu$  control system outperforms the PID control system by giving smaller overshoot, less oscillation, and less settling time. Also, the  $PI^\lambda D^\mu$  controller achieves larger stability margin, higher closed loop bandwidth, and better robustness for gain variation.



- **Tepljakov et al (2015a)** proposed a  $PI^\lambda D^\mu$  controller for a FFOPTD system. The  $PI^\lambda D^\mu$  controller was designed by first designing a conventional PID controller, sweeping the fractional order parameters  $\lambda$  and  $\mu$  within a specified interval, and then choosing the best controller that minimizes a performance index that is a linear combination of phase margin, gain margin, and ISO damping.
- **Tepljakov et al. (2015b)** designed a  $PI^\lambda D^\mu$  controller for a FFOPDT plant. The  $PI^\lambda D^\mu$  controller was designed by first determining the values of the fractional order parameters  $\lambda$  and  $\mu$  in terms of the plant parameters. Then the gain parameters  $K_P$ ,  $K_I$ , and  $K_D$  were designed to satisfy frequency domain specifications that consist of gain crossover frequency, phase margin, and ISO damping property.
- **Xinsheg (2015)** et al. proposed a  $[PD]^\nu$  controller for satellite attitude system (nonlinear system). This controller was designed by specifying gain crossover frequency, phase margin, and ISO damping property. A traditional integer order lead controller is also designed for comparison purposes. The  $[PD]^\nu$  control system gave a larger bandwidth, a larger phase margin, and a faster response than the integer order lead control system.
- **Fola et al. (2016)** designed an  $I^\lambda D^{1-\lambda}$  controller an unstable nonlinear system. The controller was designed by a linearization method. The advantage of the proposed design procedure is that the controller parameters are computed directly without optimization. Experimental results showed that the closed loop system is stable at different operating points and is robust to plant uncertainties.
- **Nankar (2016)** designed a  $PI^\lambda D^\mu$  controller to control the speed of a DC motor (LTI system). Results showed that the  $PI^\lambda D^\mu$  controller outperforms the PID controller in giving better control effect. The values of  $\lambda$  and  $\mu$  were determined by selecting certain values and ranges for them, evaluating the unit step performance of the control system, and then choosing the combination that minimizes the percentage overshoot, peak time, and settling time.
- **Isabela (2016)** et al. proposed a design procedure of a  $PI^\lambda D^\mu$  controller for vibration suppression in an airplane wing (LTI system). The parameters of the  $PI^\lambda D^\mu$  controller were determined by specifying the magnitude of the closed loop transfer function at certain frequencies. The designed controller has good disturbance rejection and improves the settling time up to 92% compared with the uncompensated system.
- **Nagarajan et al. (2016)** designed a PID controller using Ziegler-Nichols method and a  $PI^\lambda D^\mu$  controller using PSO algorithm for heat exchanger system (LTI MIMO system). Comparisons showed that the  $PI^\lambda D^\mu$  controller highly outperforms the PID controller in terms of steady state error, settling time, ISE, and IAE. However, in order to carry out a fair comparison, both controllers must be designed by the same approach; thus, in this paper both controllers should have been designed using PSO algorithm.
- **Singh et al. (2017)** designed a PID controller and a  $PI^\lambda D^\mu$  controller to control the swing angle and the position for a gantry crane system, which is a nonlinear Single-Input Multi-Output (SIMO) system. The performances of both systems were compared and the  $PI^\lambda D^\mu$  controller outperformed the PID controller in terms of the settling time.
- **Takloo et al. (2017)** designed a PID controller and a  $PI^\lambda D^\mu$  controller for a helicopter gearbox (LTI system) to maintain the torque at a constant set point value. Comparisons revealed that the  $PI^\lambda D^\mu$  controller outperforms the PID controller in terms of the



overshoot, the rise time, and the settling time.

- **Coelho et al. (2008)** presented the modeling and control of a laboratory Twin Rotor Aerodynamic System (TRAS). PID and  $PID^\mu$  controllers were designed to stabilize this system. The controller parameters were tuned using PSO algorithm and ISE performance index. The  $PID^\mu$  controller outperforms the PID controller by faster response, smaller overshoot and smaller errors. Also, the  $PID^\mu$  controller reduces the strong effect of the input and output cross coupling.
- **Mishra et al. (2014)** proposed a method to design optimal PID and  $PID^\mu$  controllers for the TRAS. The controller parameters were tuned using PSO algorithm and ISE performance index. Both controllers stabilized the TRAS successfully by tracking the desired reference angle, but the  $PI^\lambda D^\mu$  control system has much less error and less control effort than the PID control system.
- **Ijaz et al. (2016)** designed a PID and  $PI^\lambda D^\mu$  controllers for the TRAS. A fractional order model of the TRAS was proposed and system identification was carried out using input output data. The parameters of the controllers were tuned using PSO algorithm with a performance index that is a linear combination of rise time, settling time, percentage overshoot, steady state error, IAE, gain margin, and phase margin. Simulation results revealed that the  $PI^\lambda D^\mu$  controller outperforms the PID controller in terms of rise time, settling time, percentage overshoot, and steady state error.
- **Abdulwahhab and Abbas (2017a)** designed a fractional order PID controller for a TRAS. The controller was designed by linearizing the nonlinear state equation of the TRAS, decoupling the coupled system, and designing the controller using specifications in frequency domain. Then, a proof was given to show that this controller stabilizes the original nonlinear system. Simulation results demonstrated that the fractional order PID controller enhances the Integral Time Absolute Error (ITAE), Integral Time Squared Error (ITSE), ISE, and IAE performance indices and is more robust against variation in the plant parameters.
- **Abdulwahhab and Abbas (2018)** designed a fractional order PID controller for the Differential Drive Mobile Robot (DDMR). Three trajectories are taken as case studies for the DDMR to track: Circle, Lemniscate of Bernoulli, and Bowditch. The controller was designed by minimizing the ITSE performance index. A stability analysis method (that utilizes indirect Lyapunov theorem) was developed to show that this controller stabilizes the DDMR. Simulation results demonstrated that the fractional order PID controller enhances the ITAE, ITSE, ISE, and IAE performance indices and it is more robust than the integer order PID controller against external disturbance.
- **Abdulwahhab (2017)** proposed a new method to tune a fractional order PID controller. This method utilized both the analytic and numeric approach to determine the controller parameters. The control design specifications were gain crossover frequency, phase margin, and peak magnitude at the resonant frequency. As a case study, a third order linear time invariant system was taken to be controlled, and the resultant control system exactly fulfilled the control design specification.



### 3. FRACTIONAL ORDER SLIDING MODE CONTROLLER

Fractional calculus can be utilized to extend the Integer Order Sliding Mode Controller (IOSMC) to be a fractional controller, named Fractional Order Sliding Mode Controller (FOSMC). This is achieved when the sliding surface contains fractional order derivative and/or integral of the state variables. According to how the fractional order derivative/integral appears in the sliding surface equation and similar to the fractional order PID controller terminology, FOSMC can be classified to  $PI^\alpha D^\beta$  SMC,  $PI^\alpha$  SMC, and  $PD^\beta$  SMC. Some of the recent works that utilize this controller are:

- **Zhang et al. (2012)** proposed a  $PD^\beta$  FOSMC for a servo control system (LTI system). The parameters of the sliding surface were designed by specifying crossover frequency and phase margin. Furthermore, the switching gain was determined using fuzzy logic system. Simulations and experiments revealed that the proposed FOSMC outperforms the IOSMC and is robust against external disturbances.
- **Gao (2013)** et al. proposed a  $PI^\alpha D^\beta$  FOSMC for a gun control system (LTI system). The performances of the FOSMC system that consist of chattering suppression, positioning accuracy and robustness are investigated and compared with that of the IOSMC system. Simulation results showed that the FOSMC can reduce the chattering effects of the IOSMC system and can give more accurate positioning and better robustness.
- **Mujumdar et al. (2013)** proposed a  $PI^\alpha$  FOSMC for a single link flexible manipulator (LTI system). The performance of the FOSMC was compared with the IOSMC and simulation results showed that the proposed FOSMC outperforms the IOSMC in achieving better control performance, reducing the chattering, and is more robust to external load disturbance and variations in plant parameters.
- **Jagdale et al. (2013)** designed a  $PI^\alpha D^\beta$  FOSMC for DC-DC buck converter (LTI system). Simulation results demonstrated that the FOSMC gives fast response and eliminates the chattering compared to IOSMC.
- **Tang (2013) et al.** designed a  $PD^\beta$  FOSMC for an antilock braking system, which is nonlinear and which includes variation and uncertainties in its parameters due to change in vehicle loading and/or road condition. Experimental results showed that the proposed FOSMC outperforms the IOSMC in giving less slip tracking time, less braking time, and less braking distance, and being more robust against changes in the road conditions.
- **Tianyi (2015) et al.** designed a  $PD^\beta$  FOSMC for a spacecraft attitude system (nonlinear system). Simulation results demonstrated that this controller makes the spacecraft attitude system have good performance. However, this paper did not compare its proposed control strategy with the other ones.
- **Jakovljevic et al. (2015)** dealt with applications of  $\sum I^{\alpha_i}$  SMC techniques to address tracking and stabilization control tasks for a commensurate fractional order linear multivariable square system. However, this paper did not compare its proposed control strategy with the other ones.
- **Zhang et al. (2017)** proposed a new strategy to design a  $PD^\beta$  FOSMC based on a Linear Quadratic Regulator (LQR) for uncertain nonlinear systems. Input/output feedback linearization was used to linearize the nonlinear system and decouple tracking error dynamics, an LQR was designed to stabilize the system so that the tracking error converges to zero as soon as possible. A  $PD^\beta$  FOSMC was designed to achieve system robustness. Then the two outputs produced by the two controllers were added. Simulation results showed that the proposed controller achieves high performance and robustness with system uncertainties and that this controller reduces the



high frequency chattering of the control input compared to the IOSMC.

- **Abdulwahhab (2017b)** designed a fractional order sliding mode controller for a TRAS. The controller was designed by defining a fractional order surface, proving that the state trajectory reaches this surface in a finite time, and that in the sliding phase the equilibrium point is asymptotically stable. Simulation results demonstrated that the fractional order sliding mode controller enhances the rise time, delay time, percentage overshoot, settling time, IAE, ITAE, ISE, and ITSE performance indices.

#### 4. FUZZY FRACTIONAL ORDER SLIDING MODE CONTROLLER

Hybridization between fuzzy logic and sliding mode control can be achieved. The resultant controller is called Fuzzy Integer Order Sliding Mode Controller (FIOSMC). If the SMC is fractional, this controller is called Fuzzy Fractional Order Sliding Mode Controller (FFOSMC). As in FOSMC, FFOSMC can be classified to fuzzy  $PI^\alpha D^\beta$  SMC, fuzzy  $PI^\alpha$  SMC, and fuzzy  $PD^\beta$  SMC. Some of the recent works that utilize this controller are:

- **Zhang et al. (2011)** designed a fuzzy  $PD^\beta$  SMC for a LTI system. A fuzzy system was designed with the sliding surface variable and its derivative as inputs and the control signal as output. Simulation and experimental results revealed that the proposed controller gives better reference input tracking and is robust against variations in the plant parameters compared with the FIOSMC.
- **Bouarroudj et al. (2013)** designed a fuzzy  $PI^\alpha D^\beta$  SMC for an inverted pendulum (nonlinear system). To attenuate the chattering in the control signal, a Takagi-Sugeno FLC was designed instead of signum function, where the input to the fuzzy system is the sliding surface variable and the output is the control signal. Simulation results showed that the FFOSMC outperforms the FIOSMC in achieving fast response, trajectory tracking, and less overshoot. However, the magnitude of the control signal is smaller in the case of FIOSMC.
- **Bouarroudj et al. (2014)** designed a fuzzy  $PD^\beta$  SMC for a nonlinear system. The same design approach that was adopted in (Bouarroudj et al., 2013) was also used in this paper and similar results were obtained for these two papers.
- **Bouarroudj et al. (2015)** designed a  $PI^\alpha D^\alpha$  for a coupled double pendulum (nonlinear system). The same design approach that was adopted in (Bouarroudj et al., 2013) and (Bouarroudj et al., 2014) was also used in this paper and similar results were obtained for these three papers.
- **Long et al. (2015)** proposed a fuzzy  $PD^\beta$  SMC for a vehicle clutch driving system (nonlinear system). In this paper, the input to the fuzzy system is the sliding surface variable and the output is the switch gain. Theoretical analysis and numerical simulations demonstrated that FFOSMC outperforms the FIOSMC in position control and is more robust against load disturbance and other uncertainties.

#### 5. FRACTIONAL ORDER LAG/LEAD COMPENSATOR

A Fractional Order Lead-Lag Compensator (FOLLG) is a generalization of the classical lead-lag compensator. In this thesis, the fractional order lead lag compensator is classified into two types according to the way the fractional order derivative/integral is introduced to the classical compensator. If the whole transfer function of the classical compensator is raised to a fractional power, this is called type 1, while if only the complex frequency variable  $s$  is raised to the fractional power (the same power in both the numerator and denominator), this is called type 2. Some of the





recent works that utilizes this compensator are:

- **Tavazoei et al. (2014)** introduced simple formulas to design type 2 FOLLC to fulfill the specified magnitude and phase at a specified frequency for LTI system. The phase-magnitude plane regions that can be accessed by these compensators were determined. Numerical results showed that these compensators can be applied in control system design.
- **N. Sayyaf et al. (2015)** presented a type 3 fractional-order compensators to achieve the required magnitudes and phases at two given frequencies (for example, to achieve desired phase and gain margins with adjustable cross frequencies). In this generalization, at first some basic analysis of the phase behavior of this introduced type of fractional order compensators was presented. Also, exact formulas were found for designing this family of compensators in order to provide the aforementioned control objective. Finally, a numerical example was presented to confirm the effectiveness of the proposed design method in control systems. Moreover, by a numerical example it was shown that the introduced compensator can be used in control system design for obtaining desired phase and gain margins with adjustable cross-over frequencies.
- **Jadhav et al. (2017)** proposed a generalized analytical method to design robust type 2 Fractional-Order Lead Compensator (FOLC) for LTI system. The proposed compensator adjusts the system's Bode phase curve to achieve the required phase margin at a given frequency. This compensator satisfies the specifications on static error constant, gain crossover frequency and phase margin. Simulation results showed that the proposed type 2 FOLC gives robust and stable performance compared to type 1 FOLC and Integer Order Lead Compensator (IOLC).

## 6. FRACTIONAL ORDER STATE FEEDBACK CONTROLLER

Fractional calculus can be utilized in a state feedback controller so that the control law is a feedback of the fractional derivative of the states. This is called Fractional Order State Feedback Controller (FOSFC). Some of the recent works that utilize this controller are:

- **Hosseinnia et al. (2010)** proposed a FOSFC to control an unstable nonlinear system. The fractional controller converted the system from integer order to fractional order. Simulation results showed that the proposed controller outperforms the Integer Order State Feedback Controller (IOSFC).
- **Huang et al. (2014)** proposed a FOSFC that feedbacks only one state variable to stabilize an unstable fractional order nonlinear system. By this fractional order controller, the unstable equilibrium points in the fractional order system could be asymptotically stable. However, this paper did not compare its proposed control strategy with the other ones.
- **Abdulwahhab and Abbas (2018)** designed a fractional order state feedback controller for the DDMR. Three trajectories are taken as case studies for the DDMR to track: Circle, Lemniscate of Bernoulli, and Bowditch. The controller was designed by minimizing the ITSE performance index. A stability analysis method (that utilizes indirect Lyapunov theorem) was developed to show that this controller stabilizes the DDMR. Simulation results demonstrated that the fractional order state feedback controller enhances the ITAE, ITSE, ISE, and IAE performance indices.

## 7. Fractional Order Fuzzy Logic Controller

Fractional calculus can be utilized to extend the traditional Integer Order Fuzzy logic controller (IOFLC) to be Fractional Order Fuzzy Logic Controller (FOFLC). This is achieved when the



inputs to the fuzzy system are the error and its fractional order derivative and the output is integrated with a fractional order. Some of the recent works that utilize this controller are:

- **Jesus et al. (2014)** designed an optimal FOFLC using Genetic Algorithm (GA). Several LTI plants were taken as case studies. Results showed that The FOFLC gives better results than the IOFLC.
- **Rebai et al. (2015)** designed an FOFLC for piezoelectric actuators (LTI system) using PSO algorithm. Simulation results showed that the proposed controller presents better performances compared to PID controller and IOFLC controller.
- **Abdulwahhab (2018)** designed fractional order FLC for a TRAS. The controller was designed by minimizing the ITAE performance index. Simulation results demonstrated that the fractional order fuzzy logic controller enhances the rise time, delay time, settling time, IAE, ITAE, ISE, and ITSE performance indices at the expense of a larger percentage overshoot.

## 8. CONCLUSIONS

Several conclusions can be drawn from the results that were obtained in each paper given in this survey. They can be summarized in the following points:

- The  $PI^\lambda D^\mu$  controller is more robust than the PID controller against uncertainty in plant parameters. The reason is that the  $PI^\lambda D^\mu$  controller has two extra parameters than the PID controller which means two more degree of freedom to best fit the control design specifications.
- Unlike the PID controller which is a minimum phase system, the  $PI^\lambda D^\mu$  controller and for certain values of its parameters may be a nonminimum phase system.
- For a sliding mode control system, whether it is integer order or fractional order, two issues must be proved to demonstrate its stability; first, during the reaching phase it must be proved that the trajectory reaches the sliding surface in a finite time and second, during the sliding phase, the reduced order system that results when the trajectory slides along the sliding surface must be proved to be stable.
- Since the sliding mode control system is a robust control system, thus designing a controller for the TRAS, which is a MIMO system, does not require a decoupling of the TRAS since the sliding mode controller treats the coupling effect as a disturbance and rejects its effect.
- The FOFLC is more robust than the IOFLC against changes in the operating point. Also, the stability can be enhanced with the FOFLC.

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