Non-Linear Behavior of Strengthened Steel-Concrete Composite Beams with Partial Interaction of Shear Connectors

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ABSTRACT

In this research a theoretical study has been carried out on the behavior and strength of simply supported composite beams strengthened by steel cover plate taking into consideration partial interaction of shear connectors and nonlinear behavior of the materials and shear connectors. Following the procedure that already has been adopted by **Johnson (1975)**, the basic differential equations of equilibrium and compatibility were reduced to single differential equation in terms of interface slip between concrete slab and steel beam. Furthermore, in order to consider the nonlinear behavior of steel, concrete and shear connectors, the basic equation was rearranged so that all terms related to materials are isolated in the equation from the main variable (interface slip). The exact solution was obtained by considering appropriate boundary conditions according to load types and location. A computer program has been written using MATLAB R2013a to simplify the process of computation of section properties where the load applied iteratively from zero to ultimate capacity of the beam, and the results are compared with available experimental results which show good agreement.

As the composite section reaches its ultimate capacity in bending and lower flange start yielding due to excessive loading, cover plate are furnished in order to increase load carrying capacity of beam. In the process of strengthening, using of cover plate as a percent of the area of lower flange of steel section equal to 41%, 82% and 164% will increase the beam carrying capacity by 15%, 30% and 43% respectively; also using the same above mentioned area of cover plate will reduce the central deflection by 59%, 72% and 80% respectively.

Keywords: composite beams, partial interaction theory, nonlinear materials.

التصرف اللاخطي للعتبات المقواة والمركبة من الحديد والخرسانة المتصلة جزئياً باستخدام رباطات القص

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الخلاصة

يقدم البحث نموذجاً نظرياً لسلوك العتبات المركبة البسيطة الإسناد والمقواة باستخدام الصفائح الفولاذية, أخذاً بنظر الاعتبار التصرف اللاخطي لعناصر المقطع المركب (الخرسانة والفولاذ ورباطات القص) بالإضافة إلى اعتماد نظرية التداخل الجزئي لرباطات القص. متبعا نظرية جونسون تم اختصار المعادلات التفاضلية الرئيسية للتوازن والتوافق بين عناصر المقطع المركب إلى معادلة تفاضلية واحدة بدلالة الانزلاق بين الحديد والخرسانة. بالاضافة الى ذلك ، تم فصل المتغيرات المتعلقة بخصائص المقطع المركب من اجل اخذ التصرف اللاخطي للمواد بنظر الاعتبار.

عند وصول المقطع المركب إلى تحمله الأقصى نتيجة الأحمال المسلطة عليه تصل مادة الشفة السفلى للعتبة الفولاذية الى الخضوع لذا يتم إضافة الشرائح الفولاذية أسفل العتبة لزيادة قابلية تحملها. تم كتابة برنامج بلغة الماتلاب بنسخة R2013a من اجل تسهيل حساب خصائص المواد حيث تمت مقارنة النتائج المستحصلة من الدراسة الحالية مع النتائج العملية المنشورة في البحوث سابقة وقد وجد تقارب جيد معها. إن تقوية الشفة السفلى في مقطع الحديد بزيادة السمك يؤدي الى زيادة في مقدار تحمل العتبة المركبة وانخفاض في قيمة كلٍ من الانزلاق الأقصى في النهاية العتبة والهطول في منتصف العتبة المركبة في نفس المستوى من التحميل (بدون التقوية). حيث وجد ان زيادة مساحة مقطع حديد التقوية (الصفائح الفولاذية) بمقدار 41% ,

82% و164% (كنسبة من مساحة مقطع الشفة السفلى) يؤدي إلى زيادة تحمل المقطع بنسب 15% ,30% و 43% على التوالي, وانخفاض في الهطول عند منتصف العتبة المركبة بمقدار 59%, 72% و80% على التوالي.

1. INTRODUCTION

Composite construction, especially for multi-store buildings and bridges, has achieved a high market share in several European countries, USA, Canada and Australia. This is mainly due to a reduction in construction depth, to savings in steel weight and to rapid construction programs. The simplest form of composite steel – concrete beam consists of steel I–beams and an upper concrete slab connected together by shear connectors as shown in **Fig. 1**.

A fundamental point for the structural behavior and design of composite beams is the level of connection and interaction between the steel section and the concrete slab. The term "full shear connection" relates to the case in which the connection between the components is able to fully resist the forces applied to it. Partial composite action occur when the number of shear connectors is less than required to achieves full interaction between slab and steel beam**.**

Mechanical shear connectors are required at the steel-concrete interface and these connectors are necessary to transmit longitudinal shear along the interface, and to prevent separation of steel beam and concrete slab at the interface. The first interaction theory that takes account of slip effects was initially formulated by **Newmark in 1951**, based on elastic analysis of composite beams assuming linear material and shear connector behavior. Using the same element presented by Newmark, **Johnson in 1975** proposed a partial interaction theory for simply supported beams, in which the analysis was based on elastic theory. The composite beam was assumed to be in linear elastic materials. The discrete connection was assumed to be smeared along the beam, so that the connector strength and stiffness can be quoted per unit length of beam. **In 1985, Roberts** tackle the problem of incomplete interaction for composite beams in more details. The solution arrived at the four simultaneous differential equations in terms of four independent displacements which are the horizontal and vertical displacements in each component of the composite section. **In 1990, Al-Amery and Roberts** made a new development in this field where allowance has been made for the simultaneous slip and separation at the interface and the nonlinear behavior for the concrete, steel and connectors.

The researches in this field was continued to cover sandwich composite beams. **In 2007 Al-Amery et al** presented two researches on behavior of multi-layer composite beams with partial interaction of shear connectors, while **Sayhood and Mahmood (2011)** tackle the problem of non-linear behavior of composite slim floor beams with partial interaction. The main differential equations were written in term of interface slip while **Dogan and Roberts (2010)** drive the basic equations in term of axial forces that transmitted between composite section layers.

The aim of this research is developing analytical solution for nonlinear behavior of simply supported composite beam with partial connection.

2. ASSUMPTIONS

The basic assumptions of conventional beam theory were used in which plane sections are assumed to remain plane after bending. The connection is assumed to have negligible thickness and possess finite normal and tangential stiffness, and it will resist the uplift forces and prevent the separation between the steel and concrete. Also, the concrete is assumed to have no tensile strength and all the tensile forces beyond neutral axis are transmitted through connectors to steel beam.

3. THE MODELS

In this study non-linear analysis of composite beam subjected to different loads has been adopted. The reason for choosing typical composite beam is that it is one of the most common flexure elements, and at the same time is simple enough to allow for closed-form analysis.

The second order differential equations obtained by **Johnson in 1975**, were derived as follows. The material properties are isolated in separate terms in order to simplify the process of computing the inelastic properties of these components.

3.1 Equilibrium Equations

An element of a composite steel and concrete beam, length (dx), shown in **Fig. 2**, is subjected to moments (M), shear forces (V) and axial forces (F). Subscripts (c) and (s) denote the concrete and steel respectively, and the local (x-z) axes pass through the centroids of the two materials.

Longitudinal equilibrium of either the steel or concrete gives,

$$\frac{dF}{dx} = -q \qquad (1)$$

Taking moments about the center of the concrete element alone gives,

$$\frac{dM_c}{dx} + V_c = q.\frac{h_c}{2} \qquad (2)$$

The term ($dv_c/2$) has been neglected because it very small value.

Similarly for the steel element

$$\frac{dM_s}{dx} + V_s = q.\frac{h_s}{2} \qquad (3)$$

Also, the term ($dv_s/2$) has been neglected for the same reason above mentioned. Hence,

$$V_c + V_s = N \qquad (4)$$

Where N is the total vertical shear at a section, distance (x) from the support

From Eq. (2), Eq. (3) and Eq. (4),

$$\frac{dM_c}{dx} + \frac{dM_s}{dx} + N = q.\, d_1 \qquad (5)$$

in which (d_1) is the distance between the centroids of the two components. The applied external moment (M) is equal to the sum of the individual moments that each element can carry together with the composite couple, so,

$$M = M_c + M_s + F.d_1 \qquad (6)$$

3.2 Compatibility Equations

Assuming equal curvatures for the two materials gives,

$$\frac{d^2w_c}{dx^2} = \frac{d^2w_s}{dx^2} = \frac{d^2w}{dx^2} \qquad (7)$$

From elastic beam theory,

$$\frac{d^2w}{dx^2} = \frac{d^2w_c}{dx^2} = \frac{M_c}{E_cI_c} \qquad (8)$$

$$\frac{d^2w}{dx^2} = \frac{d^2w_s}{dx^2} = \frac{M_s}{E_sI_s} \qquad (9)$$

Differentiating Eq. (8) and Eq. (9) once with respect to (x) and substituting for ($\frac{dM_c}{dx}$) and ($\frac{dM_s}{dx}$) in Eq. (5), and rearranging gives,

$$\frac{d^3w}{dx^3}.(E_c.I_c + E_s.I_s) + N = q \,.\, d_1 \qquad (10)$$

The shear flow q is related to the slip (u_{cs}) by the equation,

$$q = \frac{K \,.\, u_{cs}}{S} \qquad (11)$$

Substituting for q into Eq. (10) and rearranging, give,

$$\frac{d^3w}{dx^3} = \frac{1}{(E_c.I_c + E_s.I_s)}\left(\frac{K \,.\, u_{cs} .\, d_1}{S} - N\right) \qquad (12)$$

Strains of the concrete and steel, (ε_c) and (ε_s), at the interface can be expressed as,

$$\varepsilon_c = \frac{1}{2}\, h_c \,.\frac{d^2w}{dx^2} - \frac{F}{E_c.A_c} \qquad (13)$$

$$\varepsilon_s = \frac{1}{2}\, h_s \,.\frac{d^2w}{dx^2} + \frac{F}{E_s.A_s} \qquad (14)$$

The interface slip strain ($\frac{du_{cs}}{dx}$) is given by,

$$\frac{du_{cs}}{dx} = \varepsilon_c - \varepsilon_s \qquad (15)$$

Eq. (13) and Eq. (14), give,

$$\frac{du_{cs}}{dx} = d_1.\frac{d^2w}{dx^2} - F\left(\frac{1}{E_c.A_c} + \frac{1}{E_s.A_s}\right) \qquad (16)$$

Differentiating Eq. (16) once with respect to (x), yield into;

$$\frac{d^2u_{cs}}{dx^2} = d_1.\frac{d^3w}{dx^3} - \frac{dF}{dx}\left(\frac{1}{E_c.A_c} + \frac{1}{E_s.A_s}\right) \qquad (17)$$

Substituting for ($\frac{d^3w}{dx^3}$) and ($\frac{dF}{dx}$) from Eq. (12), Eq. (1) and Eq. (11) and rearranging Eq. (17) becomes.

$$\frac{d^2u_{cs}}{dx^2} - \frac{K \,.\, u_{cs}}{S}\left[\frac{1}{E_c.A_c} + \frac{1}{E_s.A_s} + \frac{d_1^2}{(E_c.I_c + E_s.I_s)}\right] = -N.\left(\frac{d_1}{(E_c.I_c + E_s.I_s)}\right) \qquad (18)$$

Let,

$$\alpha_1^2 = \frac{K}{S}\left[\frac{1}{E_c.A_c} + \frac{1}{E_s.A_s} + \frac{d_1^2}{(E_c.I_c + E_s.I_s)}\right] \qquad (19)$$

$$\beta_1 = \frac{d_1}{(E_c.I_c + E_s.I_s)}.\frac{1}{\alpha_1^2} \qquad (20)$$

Hence, substituting in Eq. (18), will give the basic differential equation in terms of interface slip:-

$$\frac{d^2u_{cs}}{dx^2} - \alpha_1^2.u_{cs} = -\alpha_1^2.\beta_1.N \qquad (21)$$

The iterative solution of this equation considering material non linearity will give the values of the interface slip along the beam span.

In which (α_1) and (β_1) are functions of the section and material properties, (N) is the vertical shear force in the beam at a distance (x) from the support.

4. THE SOLUTION OF THE DIFFERENTIAL EQUATION

The general solution for the differential Eq. (21) can be obtained mathematically as usual; then, the constants can be defined after specifying the suitable boundary conditions.

4.1 Case1: Point Loads:

For a simply supported beam subjected to a point load (P) applied at distance (L_1) and (L_2) from the ends of the beam, as shown in **Fig. 3**.

The differential equation obtained by Johnson can be solved after expressing the shear forces to the left and right of the load, as given below:

$$N^L = -P\frac{L_2}{L} \quad \text{(to the left of point load)} \qquad (22)$$

$$N^R = +P\frac{L_1}{L} \quad \text{(to the right of point load)} \qquad (23)$$

where superscripts L and R denote left and right of the load. Solving Eq. (21) for the two parts of the beam (to the left and right of the point load) gives:

$$u_{cs}^L = K_1.\sinh(\alpha_1.x)+K_2.\cosh(\alpha_1.x) - \beta_1.P(\frac{L_2}{L}) \qquad (24)$$

$$u_{cs}^R = K_3.\sinh(\alpha_1(x-L_1))+K_4.\cosh(\alpha_1(x-L_1)) + \beta_1.P(\frac{L_1}{L}) \qquad (25)$$

in which (K_1) to (K_4) are arbitrary constants of integration. The four boundary conditions needed to determine these constants are:

$$u_{cs}^L = u_{cs}^R \quad \text{when } x = L_1 \qquad (26)$$

$$\frac{du_{cs}^L}{dx} = \frac{du_{cs}^R}{dx} \quad \text{when } x = L_1 \qquad (27)$$

$$\frac{du_{cs}^L}{dx} = 0 \quad \text{when } x = 0 \qquad (28)$$

$$\frac{du_{cs}^R}{dx} = 0 \quad \text{when } x = L \qquad (29)$$

Applying these boundary conditions, (K_1) to (K_4) can be determined as follows:

$$K_1 = 0 \qquad (30)$$

$$K_2 = \beta_1.P\frac{\sinh(\alpha_1.L_2)}{\sinh(\alpha_1.L)} \qquad (31)$$

$$K_3 = \beta_1.P\frac{\sinh(\alpha_1.L_1).\sinh(\alpha_1.L_2)}{\sinh(\alpha_1.L)} \qquad (32)$$

$$K_4 = -\beta_1.P\frac{\sinh(\alpha_1.L_1).\cosh(\alpha_1.L_2)}{\sinh(\alpha_1.L)} \qquad (33)$$

Substituting for (K_1) to (K_4) into Eq. (24) and Eq. (25) gives the final solution for a simply supported beam subjected to concentrated load, as shown below:

$$u_{cs}^{L} = \beta_1.P\frac{\sinh(\alpha_1.L_2)}{\sinh(\alpha_1.L)}.\cosh(\alpha_1.x) - \beta_1.P(\frac{L_2}{L}) \qquad (34)$$

$$u_{cs}^{R} = -\beta_1.P\frac{\sinh(\alpha_1.L_1)}{\sinh(\alpha_1.L)}.\cosh(\alpha_1(L-x)) + \beta_1.P(\frac{L_2}{L}) \qquad (35)$$

4.2 Case2: Uniformly Distributed Loads:

Using Johnson's approach for a simply supported beam of span (L) subjected to a uniformly distributed load (ρ) , the vertical shear force at distance (x) from the left hand support as shown in **Fig. 4**, is given by,

$$N = \rho.x - \rho\frac{L}{2} \qquad (36)$$

Substituting for (N), the general solution of the differential Eq. (21) can be obtained as:

$$u_{cs} = K_5.\sinh(\alpha_1.x)+K_6.\cosh(\alpha_1.x) + \beta_1.\rho(x-\frac{L}{2}) \qquad (37)$$

in which (K_5) and (K_6) are arbitrary constants. The boundary conditions can be expressed in terms of the slip (u_{cs}) as the axial strains and curvature equal to zero at supports:

$$\frac{du_{cs}}{dx} = 0 \qquad \text{when } x = 0 \text{ and } x = L \qquad (38)$$

Alternatively due to symmetry, one of the conditions expressed by Eq. (38) can be replaced by:

$$u_{cs} = 0 \qquad \text{when } x = L/2 \qquad (39)$$

Substituting these boundary conditions into Eq. (37), the constants K_5 and K_6 can be determined and the particular solution for this case is:

$$u_{cs} = -\frac{\beta_1.\rho}{\alpha_1}\sinh(\alpha_1.x) + \frac{\beta_1.\rho}{\alpha_1}\tanh(\frac{\alpha_1.L}{2}).\cosh(\alpha_1.x) + \beta_1.\rho(x-\frac{L}{2}) \qquad (40)$$

5. BASIC DIFFERENTIAL EQUATIONS

When the material properties (E) are considered to be constant (linear analysis) Eq. (40) can be solved directly. The equation can be used to get the slip and axial force along the beam. Alternatively, if the material properties are non-linear functions of strain, the above equations require specifying these properties at each loading stage. This can be achieved by dividing the cross sectional area of the concrete and steel shape into a number of layers having areas (A_c^i) and (A_s^j) at a distance (z_c^i) and (z_s^j) from the origin of coordinates, respectively, and using the summation over the appropriate area. The appropriate values of (E_c^i) and (E_s^j), for the layers (A_c^i) and (A_s^j) are the secant values determined from the assumed stress-strain curves, corresponding to the strains (ε_c^i) and (ε_s^j) in center of layers (A_c^i) and (A_s^j), respectively. Therefore, for concrete, (E_c^i) corresponds to a strain (ε_c^i) at the center of the corresponding layer and is given by,

$$\varepsilon_c^i = z_c^i \ . \ \frac{d^2w}{dx^2} - \frac{F}{\sum_{i=1}^{n_1} E_c^i.A_c^i} \qquad (41)$$

Similarly for steel shape,

$$\varepsilon_s^j = -z_s^j \cdot \frac{d^2w}{dx^2} + \frac{F}{\sum_{j=1}^{n_2} E_s^j.A_s^j} \tag{42}$$

in which n_1 and n_2 are the total number of layers in the concrete and steel shape respectively.

The corresponding stress (σ) for each layer can be obtained using appropriate secant (E) values at the center of each layer. Hence,

$$\sigma_c^i = E_c^i.\varepsilon_c^i \tag{43}$$

$$\sigma_s^j = E_s^j.\varepsilon_s^j \tag{44}$$

The non-linear behavior of the stud connectors has been introduced in this formulation through Eq. (53) in which shear stiffness of the connectors (Ks) can be defined from the relative value of the slip. After substituting Eq. (41) and Eq. (42) into the basic equilibrium and compatibility equations Eq. (18) and rearranging them, the new form of the differential Eq. (21), in term of interface slip including the influence of the non-linear material and shear connector behavior is,

$$\frac{d^2u_{cs}}{dx^2} - \frac{K_s.u_{cs}}{S}.\left[\frac{1}{\sum_{i=1}^{n_1} E_c^i.A_c^i} + \frac{1}{\sum_{j=1}^{n2} E_s^j.A_s^j} + \frac{d_1^2}{\left(\sum_{i=1}^{n_1} E_c^i.I_c^i + \sum_{j=1}^{n2} E_s^j.I_s^j\right)}\right] = N.\left[\frac{d_1}{\left(\sum_{i=1}^{n_1} E_c^i.I_c^i + \sum_{j=1}^{n_2} E_s^j.I_s^j\right)}\right] \tag{45}$$

Now letting

$$\alpha_2^2 = \frac{K_s}{S}.\left[\frac{1}{\sum_{i=1}^{n_1} E_c^i.A_c^i} + \frac{1}{\sum_{j=1}^{n_2} E_s^j.A_s^j} + \frac{d_1^2}{\left(\sum_{i=1}^{n_1} E_c^i.I_c^i + \sum_{j=1}^{n_2} E_s^j.I_s^j\right)}\right] \tag{46}$$

$$\beta_2 = \left[\frac{d_1}{\left(\sum_{i=1}^{n_1} E_c^i.I_c^i + \sum_{j=1}^{n_2} E_s^j.I_s^j\right)}\right].\frac{1}{\alpha_2^2} \tag{47}$$

Then, the final form of the differential Eq. (21) becomes,

$$\frac{d^2u_{cs}}{dx^2} - \alpha_2^2.u_{cs} = -\alpha_2^2.\beta_2.N \tag{48}$$

and Eq. (40) becomes,

$$u_{cs} = -\frac{\beta_2.\rho}{\alpha_2}\sinh(\alpha_2.x) + \frac{\beta_2.\rho}{\alpha_2}\tanh(\frac{\alpha_2.L}{2}).\cosh(\alpha_2.x) + \beta_2.\rho(x - \frac{L}{2}) \tag{49}$$

6. MATERIAL PROPERTIES

The performance of any structure under load depends to a large degree on the stress-strain relationship of the material from which it is made. In the following a description of the constitutive relationships for the materials comprising composite beams is given.

Concrete

In American code for structural concrete, the compressive strength of the concrete is usually based on standard (150 mm * 300 mm) cylinders cured under standard laboratory conditions and tested at a specified rate of loading at 28 days of age. Another widely used model for the stress strain relationship of the concrete in compression is that proposed in the **BS8110**, as shown in **Fig. 5** where the variation of the curved portion of the stress-strain relationship is given by,

$$\sigma = 5500\sqrt{f_{cu}}\,.\,\varepsilon - 11.3*10^6.\varepsilon^2 \quad (f_{cu} \text{ is in N/mm}^2) \tag{50}$$

The above equation is used in this study

Steel section

Hot rolled steel sections were used extensively as structural material because of their significant properties such as high strength as compared to any other building material and also ductility which is the ability to deform substantially in either tension or compression before failure. **Fig. 6** shows a suitable mathematical model for representing the behavior of structural steel as a practical strain limit. This model which is proposed by **BS8110**, consists of elastic, perfectly plastic portions. In the present study the strain hardening is not taken into account.

Shear connectors

In composite beam design, headed stud shear connectors are commonly used to transfer longitudinal shear forces across steel-concrete interface. Present knowledge on the load–slip behavior and the shear capacity of the shear stud in composite beam is limited to data obtained from the experimental push-off tests, **Lam, and El-Lobody, 2005**.

Al-Amery, and Roberts, 1990, developed a model for shear connectors and carried out parametric study to end up with the following exponential form in which one point on the load-slip curve is required (values of slip and corresponding load) to model that curve as follows,

$$Q = Q_u\ .\ \left(1- e^{-\phi.u_{cs}}\right) \tag{51}$$

$$\phi = \left(1/\overline{u_{cs}}\right).\ \ln\left[Q_u / (Q_u - \overline{Q})\right] \tag{52}$$

in which (ϕ) is a constant and ($\overline{Q}$) is the midrange load from experimental curve and ($\overline{u_{cs}}$) is the corresponding slip. The tangent stiffness, K_s, is given by differentiating Eq. (51) once with respect to slip value,

$$K_s = dQ/du_{cs} = Q_u.\phi\left(1- e^{-\phi.u_{cs}}\right) \tag{53}$$

This model has been used in the present study, as shown in **Fig. 7**.

7. APPLICATIONS

In order to validate the theoretical representation of the partial interaction behavior of simply-supported composite beam, the formulation presented in the previous sections has been applied to a practical section and the results are compared with the experimental values reported by previous researchers.

The computer program presented in this study for non-linear analysis of composite beams with partial connection is used to investigate various loading conditions. Also, a convergence study is carried out since a numerical method has been employed in the solution. A computer program has been written using MATLAB R2013a to simplify the process of computation the section properties. The full loading history was covered in the program to describe the complete behavior of the composite beam from zero to full yielding loading of strengthened beam.

7.1 Example 1

The example which is presented by **Al-Amery, and Roberts, 1990** was used herein for validating the accuracy of the proposed method of analysis and to examine the effect of some properties on the behavior of simply supported composite beam.

A study was made on the behavior of 9 m span simply supported composite beam having the cross-section dimensions shown in **Fig. 8** and **Fig. 4**. For simplicity, the beam was assumed to be supported during construction, so that the construction strains ε_c, was zero, and subjected to a uniformly distributed load. The free strains ε_f, were also assumed zero.

Concrete slab: Width of concrete slab = 1800 mm, depth of concrete slab = 150 mm, cross sectional area = 270000 mm^2, d_1 =281 mm, initial modulus of elasticity = 30000 MPa, ultimate compressive strength=30 MPa.

Steel beam: Flange width = 153mm, flange thickness = 16 mm, web height = 380 mm, web thickness = 9.4 mm, cross sectional area = 8500 mm^2, initial modulus of elasticity = 200000 MPa, yield stress = 280 Mpa.

Shear connector: The connection between the concrete slab and steel beam was assumed to be produced by pairs of headed studs of 19-mm diameter, l00-mm long, with a spacing of 240 mm. The ultimate shear strength of a single stud, Q_u, was taken as 100 kN while the slip $\bar{u}_{cs}$ corresponding to a shear force $\bar{Q}$ = 62 kN was taken as 0.5 mm **(see Fig. 7)**. This gave an initial tangent value of Ks = 1.935 kN/mm.

It should be noted that the concrete slab was divided into ten equal strips. Each flange of the steel beam was divided into eight equal strips and the web of the steel beam was divided into twenty equal strips.

To illustrate the application of the formulation that was developed at the previous sections, a comparison is made between current study model and Al-Amery and Roberts model. Load-deflection relationships and slip distributions between the concrete slab and steel beam obtained from analysis are shown in **Fig. 9** and **Fig. 10**, respectively, for a simply supported composite beam with partial interaction.

These curves show good agreement between two types of analysis of composite beam where the method developed by Al-Amery and Roberts depends on four independent variables (tow horizontal displacement u_c and u_s) and (two vertical displacement w_c and w_s) while Johnson approach depends on only one independent variable that is slip u_{cs}.

From the examination of load-deflection curve in **Fig. 9** conclusion can be made that beam behaves in almost linear shape up to 50 kN/m, then partial yielding starts in the steel section from lower stand which causes respective increasing in deflection with small increasing in beam load carrying capacity, maximum load recorded was (ρ_u=78 kN/m), at 0.93%ρ_u deflection equal to 175 mm at mid span.

Examination of **Fig.10** represents the relation of applied external load with maximum slip which occurs at the end of span. The vertical applied load is converted to horizontal shear flow that is transmitted between steel section and concrete slab according to traditional beam theory. In the beginning the connector shows linear response to applied load until ρ = 60 kN/m then the nonlinear effect starts between ρ = 60 kN/m and 78 kN/m.

Strain and stress profiles throughout the depth of the simply supported composite beam corresponding to different levels of the applied uniformly load are shown in **Fig. 11** and

Fig. 12.The load increments are taken as percentage of ultimate load (78 kN/m) as; 90% and 93%.

The discontinuous strain profiles indicate the existence of slip at the interface between the concrete and steel, while the stress profiles indicate the spread of plasticity.

7.2 Example 2

Chapman and Balakrishnan, 1964 tested a series of simply supported composite beams. EII is one of the tested beam, with 5.5 m, which was used herein to carry out a second validation test of the proposed analysis method and **Fig. 13** illustrates the cross-section of this beam.

Concrete slab: Width of concrete slab = 1220 mm, depth of concrete slab = 153 mm, cross sectional area = 186660 mm2, d_1 = 162.9 mm, Initial modulus of elasticity = 26700 MPa, ultimate compressive strength = 50 MPa.

Steel beam: flange width = 153 mm, flange thickness = 18.2 mm, web height = 268.6 mm, web thickness = 10.16 mm, cross sectional area =8300 mm^2, initial modulus of elasticity = 205000 MPa, Yield stress = 265 Mpa.

Shear connector: the connection between the concrete slab and steel beam was assumed to be produced by pairs of headed studs of 12.5mm diameter, 50mm long, with a spacing of 110 mm.

The concrete slab was divided into ten equal layers. Each flange of steel beam was divided into eight equal layers and web of steel beam was divided into twenty equal layers.

Fig.14 shows the load-deflection curves for the beam. The results show very good agreement between the experimental test and the theoretical nonlinear analysis as the difference is less than 5% in ultimate load carrying capacity or corresponding deflection.

On the other hand examining the slip behavior along the beam as shown in **Fig.15** clarifies obvious difference between theoretical and experimental test results. This can be explained due to the behavior of point load, where both maximum values of shear force (equal to P/2) and bending moment (equal to pL/4) occur at the region of mid-span simultaneously resulting in high principle stresses. At the early stage of loading, the connectors show almost equal horizontal displacement as the equal shear force is transmitted in the interface of concrete and steel. As the load increases the maximum principle stresses occur in the inclined axis depending on the value of two types of stresses (shear and bending), therefore the inclined principle forces which slope up ward will bend the connector in the experimental tested beam and may cause crashing of concrete at the region of mid-span. Therefore this difference between experimental and theoretical analysis is recorded as shown in **Fig. 15**.

8. DESCRIPTION OF THE BEHAVIOR OF COMPOSITE BEAM OF EXAMPLE 1

After the proposed method has been checked by comparison of results with another model the full behavior of simply supported composite beam can be described as follows.

8.1 Deflection

The deflection along the composite beam can be computed taking into account the nonlinear behavior of steel, concrete, connector, and partial interaction theory. The differences are computed at different load increments. The load increments are taken as percentage of ultimate load capacity of beam (ρ_u =78 kN/m) as follows; 87% and 93%, in order to clarify the result in different scales; the results are shown in the **Fig. 16**.

The maximum deflection is located at mid span. The central deflection is increased with increasing applied uniform load as 122.7 and 175.1 mm for 87% and 93% ρ_u percentage of ultimate load, respectively.

8.2 Interface Slip

The distribution of slip is computed for different load increments. The load increments are as follows; 87% and 93% ρ_u as shown graphically in **Fig. 17**.

The value of maximum slip occurs at the supports and it increases with increasing of applied uniform load as, 4.12 and 11.1 mm for load 87% and 93% ρ_u.

9. BEHAVIOR COMPOSITE BEAM OF EXAMPLE 2 STRENGTHENED WITH COVER PLATE

In some circumstances as, after designing and using a structure for many years it is required to increase load carrying capacity due to different reasons. In the other hand yielding of beam may occur due an expected additional loading. In both cases strengthening of composite beam is required by any available producers to withstand the new conditions taking in consideration the load history of the composite the existing pre-slip at interface and yielding the bottom flange of steel beam which is indicate the failure of composite structure.

In the design of composite structure according to related design code, the design is often made of the section in the bases of tension failure of steel component of the cross section. The tension failure allows for high ductility of structure members and prevents sudden failure of the beams. Therefore if it is required to strengthen any member, it is essential to increase the area of steel in the process of increasing load carrying capacity of the beam. Different techniques of strengthening of structure members have been used such as using CFRP sheets that are glued to the lower flange of the steel section by special type of epoxy or using external prestressing technique of composite member applying force on straight or inclined strand and provide required anchorage at the ends of the beams.

In this research the strengthening of composite member was carried out by welding steel cover plate to lower flange of steel beams. The steel beam and plate are fully connected while partial interaction of the shear connector is considered in the analysis of overall section. Also it's assumed that the plate is attached to the beam after the full yielding of lower flange and the welded plate while the web is the elastic zone.

A computer program is designed to stop temporarily after reaching yield stress and strain, and then the required thickness of plate is added along the beam taking into consideration the transverse dimension to the centroid of cross section. After that the program will continue running taking in consideration all previous loading steps, strains and stresses in the member. The results are shown in **Fig. 18**.

Load-deflection curve for simply supported composite beam had already been tested by **Chapman, and Balakrishnan, 1964.** The original load carrying capacity of beam was 530 kN with central deflection equal to 76.41 mm. Strengthening the beam with cover plate of (153 x 7.5), (153 x 15) and (153 x 30) mm will increase beam capacity by 15 %, 30%, 43% respectively, as shown in **Fig. 19**

Also the distribution of slip with the applied point load (530 kN) is found for different thicknesses of cover plate as shown graphically in **Fig. 19** The maximum slip at the support decreases with increasing of thickness of cover plate as follow: 0.577, 0.511, 0.473, 0.446 and 0.424 mm for 0, 7.5, 15, 22.5 and 30 mm, respectively.

10. CONCLUSIONS

Based on results of the present study, the following conclusions can be drawn,

1. Nonlinear analysis of steel, concrete and shear connecter has been derived for strengthened simply supported composite member, and verified with experimental work.
2. In the process of strengthening, using area of steel equal to 41%, 82% and 164% of the existing lower flange area will increase the beam carrying capacity by 15%, 30% and 43%.
3. Also using the same above mentioned area of steel cover plate will reduce the central total deflection by 59%, 72% and 80%.
4. It is found by increasing the area of cover plate at the same applied load value of 530 kN. The value of maximum deflection will reduce by, 76.4 , 67.76 ,61.57 and 52.94mm for plate thickness 0 ,7.5 ,15 and 30 mm respectively
5. Also it is found by increasing the area of cover plate at the same applied load value of 530 kN the value of maximum slip will be reduced as fallows; 0.5772, 0.5113, 0.4732 and 0.4239 mm respectively.

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NOMENCLATURE

A_c	cross section area of concrete slab.
A_s	cross section area of steel beam.
d_1	distance between centroids of concrete and steel section in a composite beam.
E_c , E_s	young modulus of the concrete and steel beam respectively.
f_{cu}	cubic compressive strength of concrete.
f_y	yield strength of steel.
h_c	total depth of concrete beam.
h_s	total depth of steel beam.
I	second moment of inertia.
I_c	second moment of inertia of concrete.
I_s	second moment of inertia of steel beam.
K	shear stiffness of the connection in the composite beam.
K_1-K_6	constant of integration.
K_s	normal and shear stiffness per unit length
L	span of composite beam.
M	applied bending moment.
M_c	bending moment in the concrete.
M_s	bending moment in the steel beam.
N	applied shear force at a section of the composite beam.
n_1	number of layers of concrete.
n_2	number of layers of steel beam.
P	applied point load.
q	longitudinal shear force per unit length at the interface.
S	spacing of shear connectors
T	normal force per unit length at the interface.
u_{cs}	slip at the interface.
V	applied shear force.
V_c	shear force at the concrete slab.
V_s	shear force at the steel beam.
w	displacement in z direct ion
α & β	function of section and material properties of a composite beam.
ε	strain.

ε_{cu} ultimate concrete crushing strain.
ε_o strain at maximum stress.
ε_y steel yield strain.
ϕ constant.
ρ applied load per unit length.
σ stress.

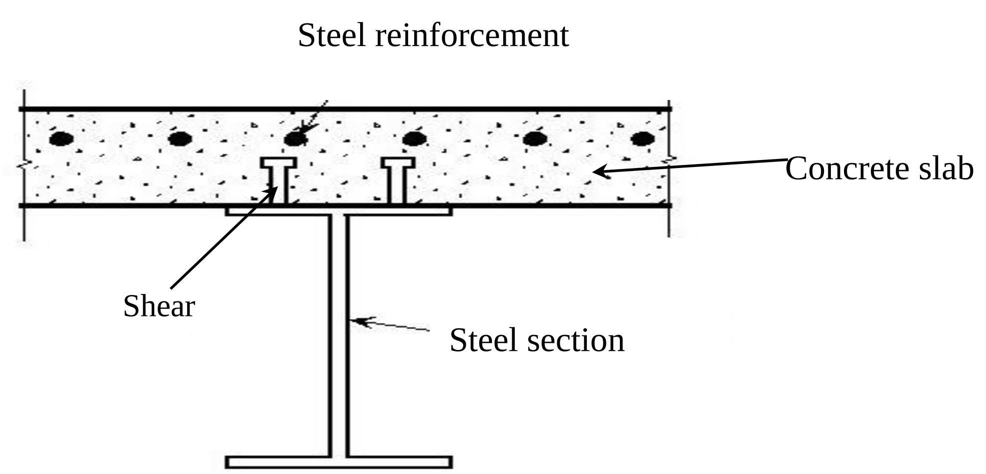

Figure 1 .Typical composite beam consist of steel section and concrete slab.

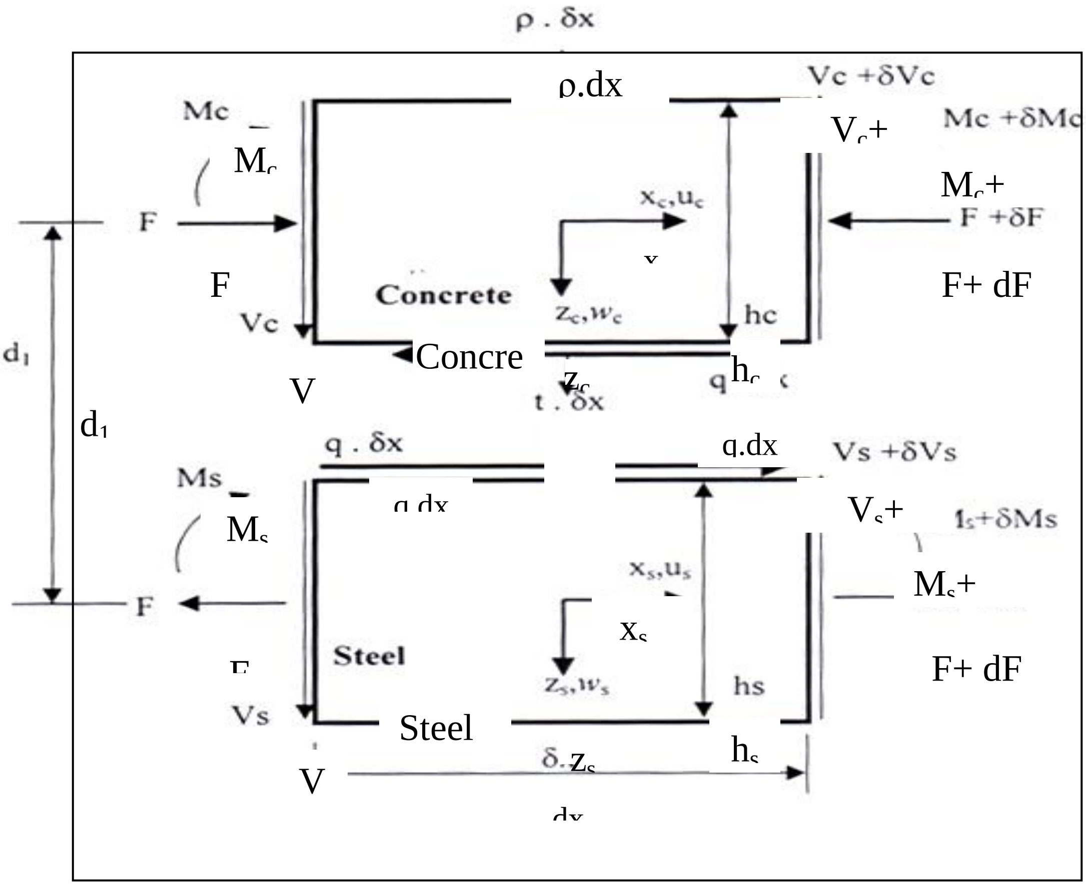

Figure 2 Elevation of element of composite beam

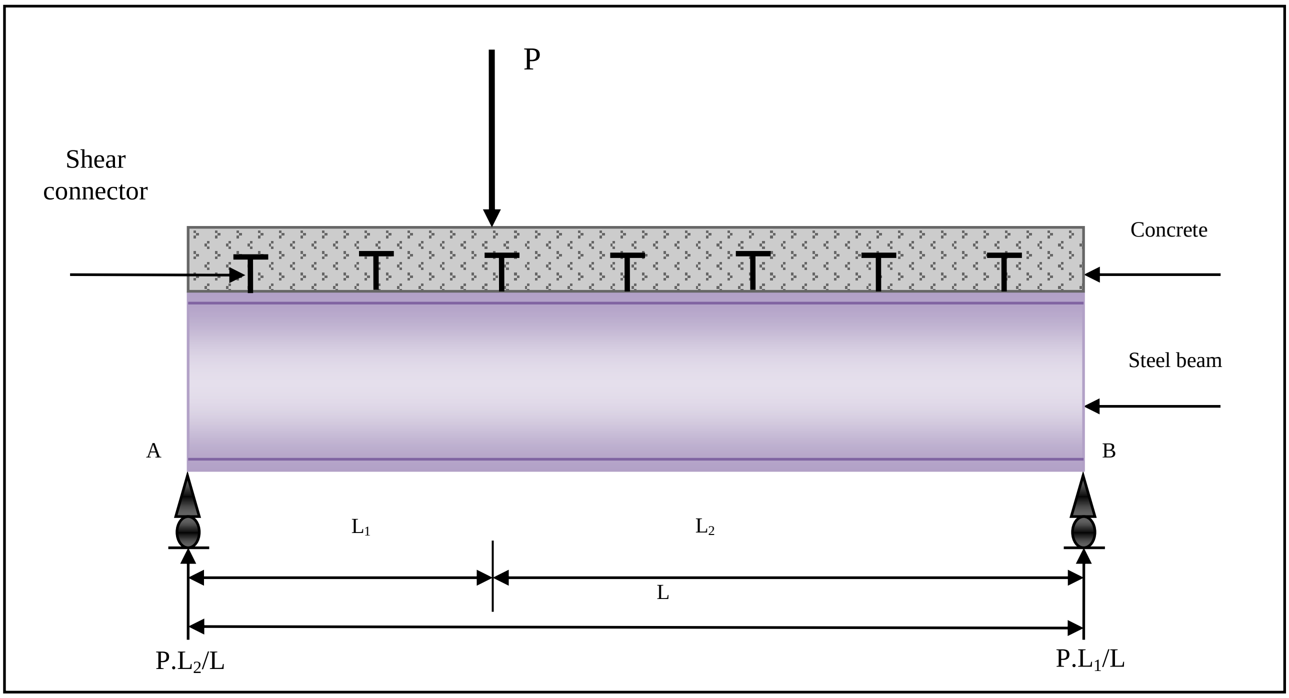

Figure 3. Typical composite beam subjected to point load.

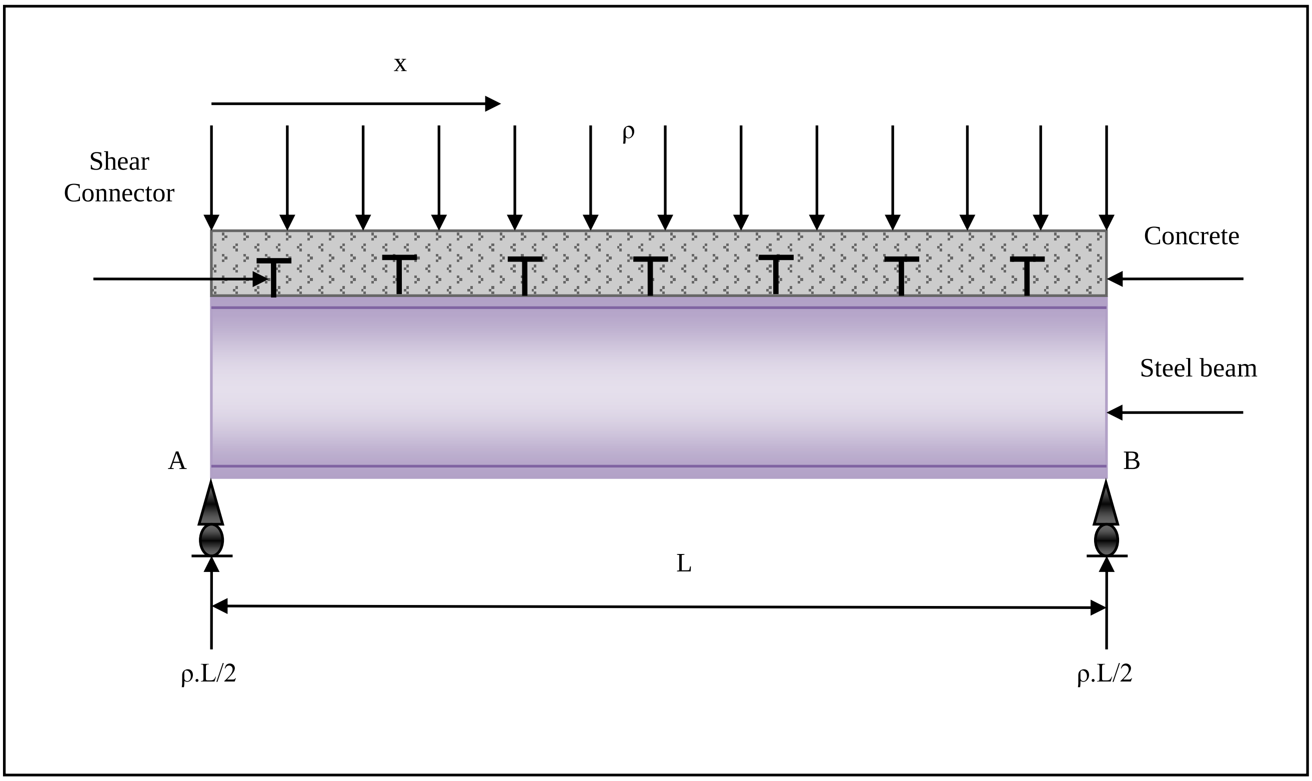

Figure 4 Typical composite beam subjected to uniformly distributed load.

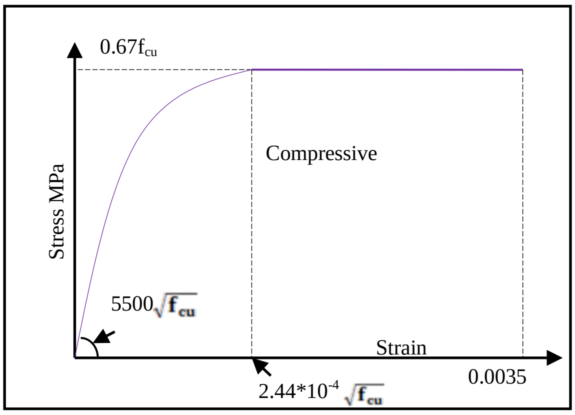

Figure 5 BS 8110 compressive stress-strain curve for concrete.

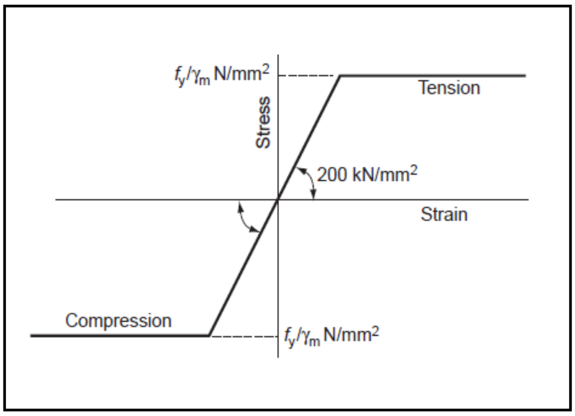

Figure 6 Idealized stress-strain curve for steel.

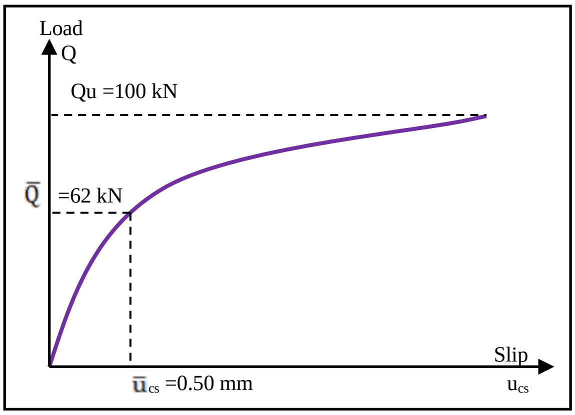

Figure.7 Shear force versus slip curve for shear connectors. **Al-Amery, and Roberts, 1990**

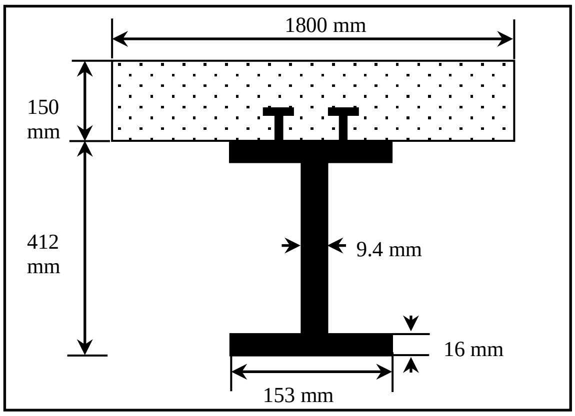

Figure 8 Composite beam-cross-section dimensions of example one

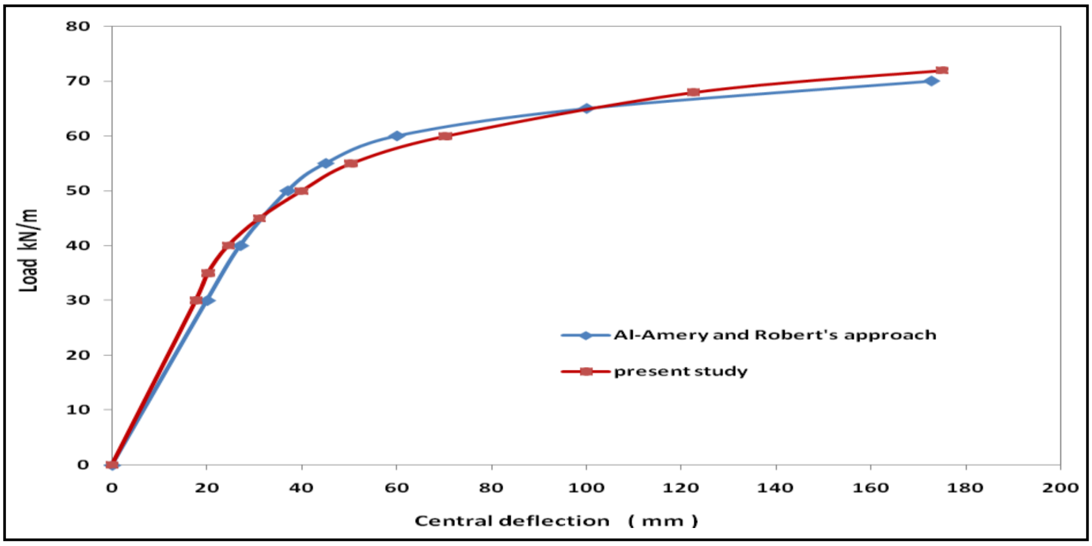

Figure 9 Load-deflection of the beams of example one.

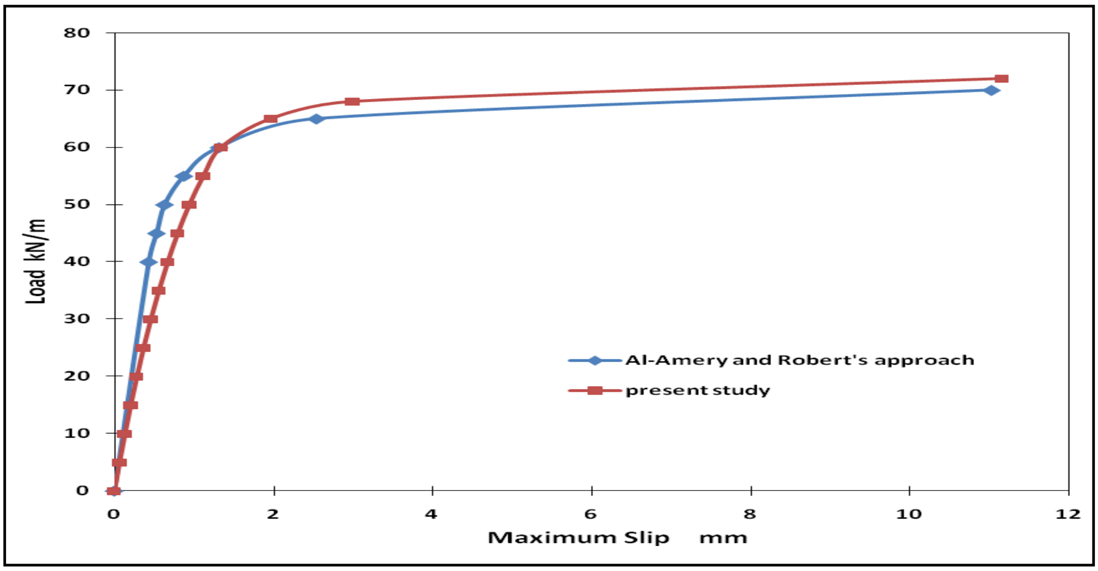

Figure 10 Load-slip curve of the beams of example one.

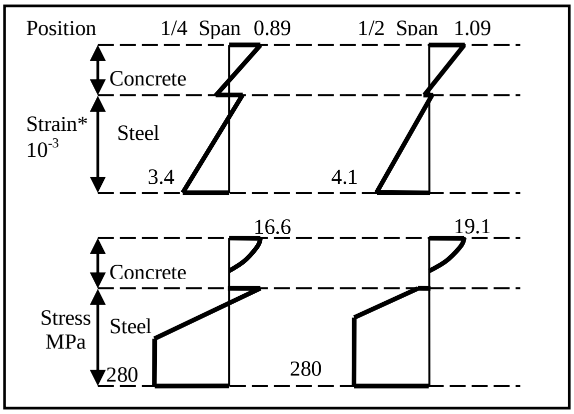

Figure 11 Strain and stress profiles at load levels 90% ρ_u.

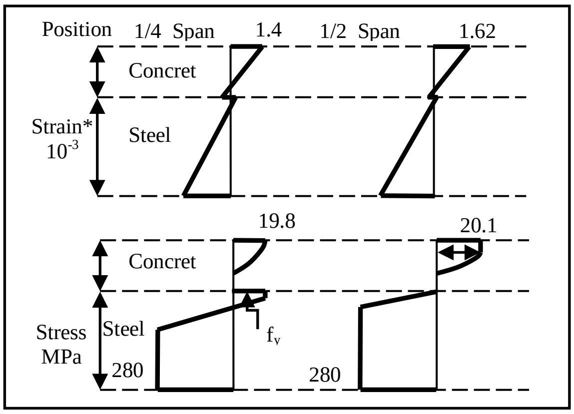

Figure 12 Strain and stress profiles at load levels 93% ρ_u.

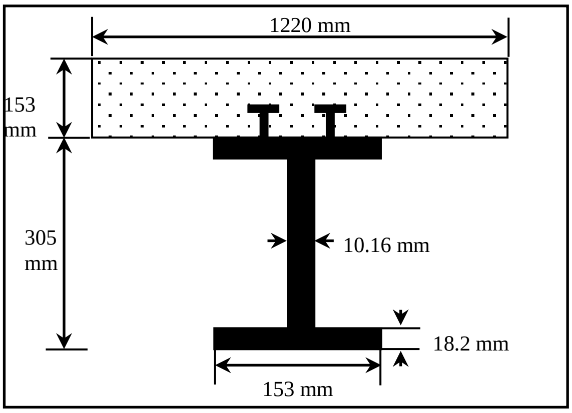

Figure 13 Composite beam cross-section dimensions of example two

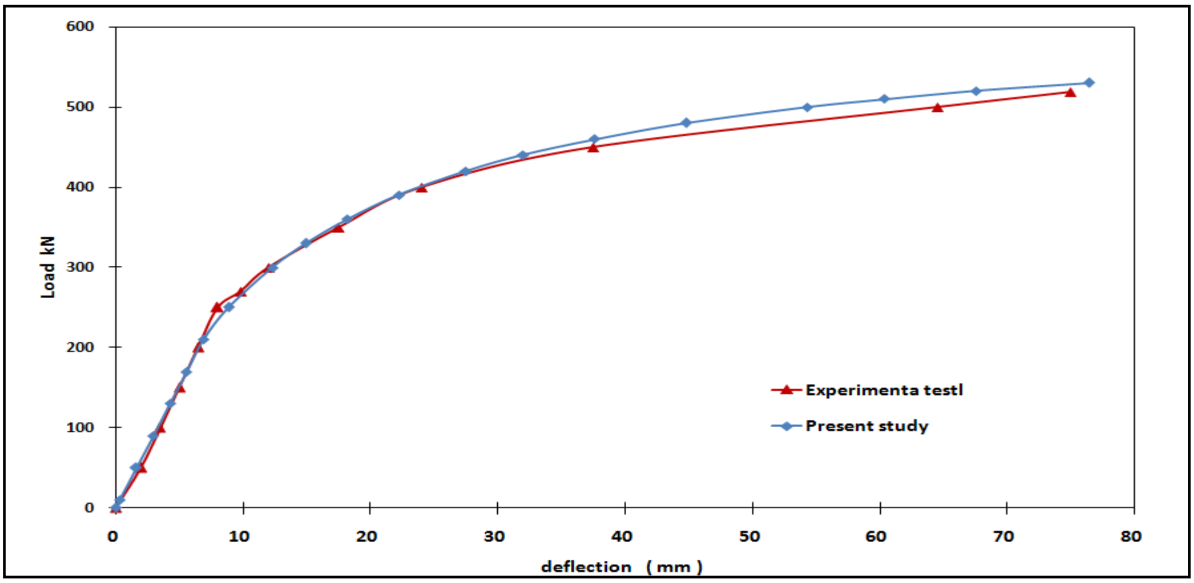

Figure 14 Load-deflection curve of beam EII **(Chapman and Balakrishnan, 1964)**.

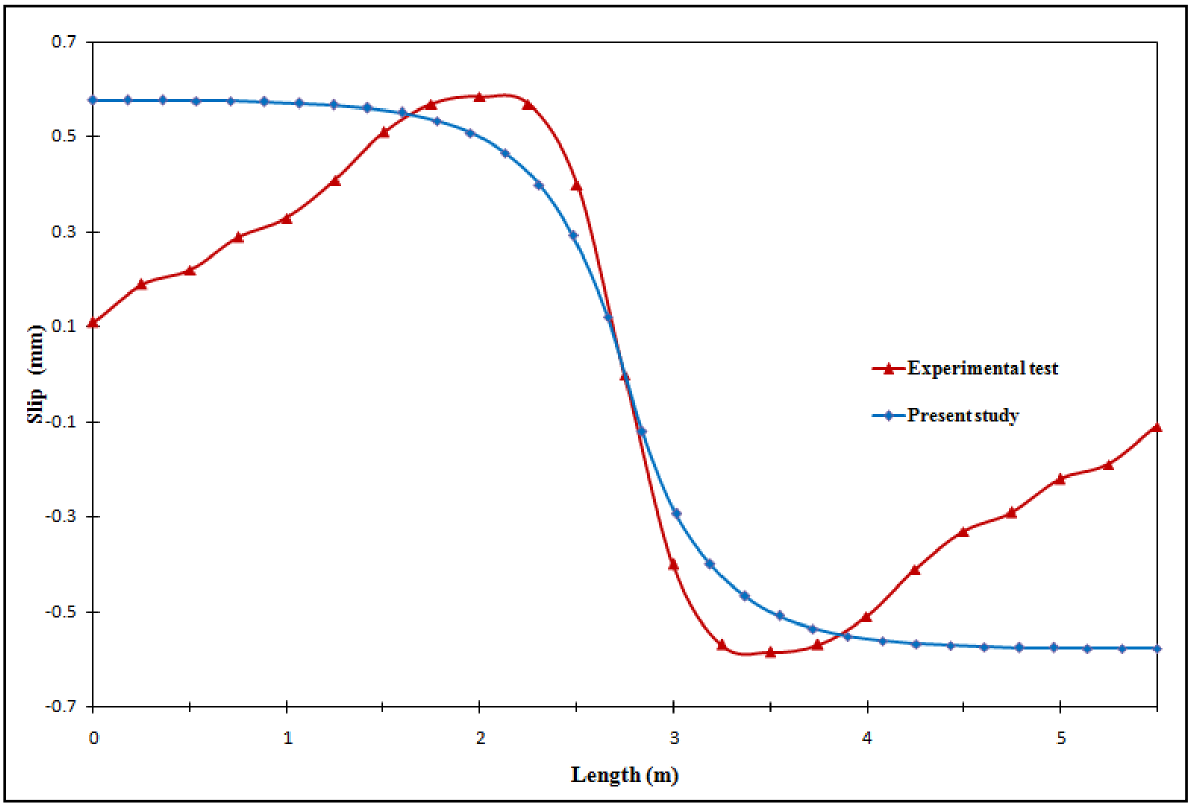

Figure 15 Slip along the span at load 450 kN for the second example.

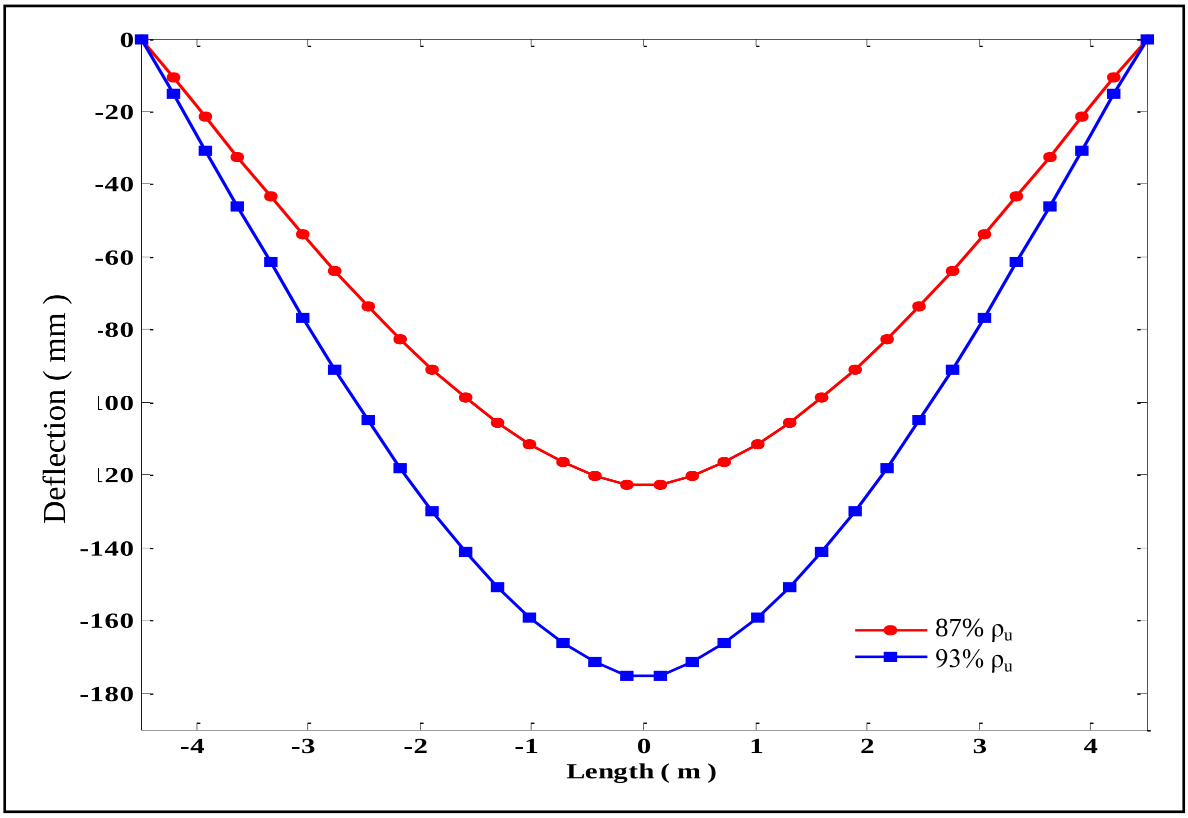

Figure 16 Deflection of example one along span at 87% ρu and at 93% ρu.

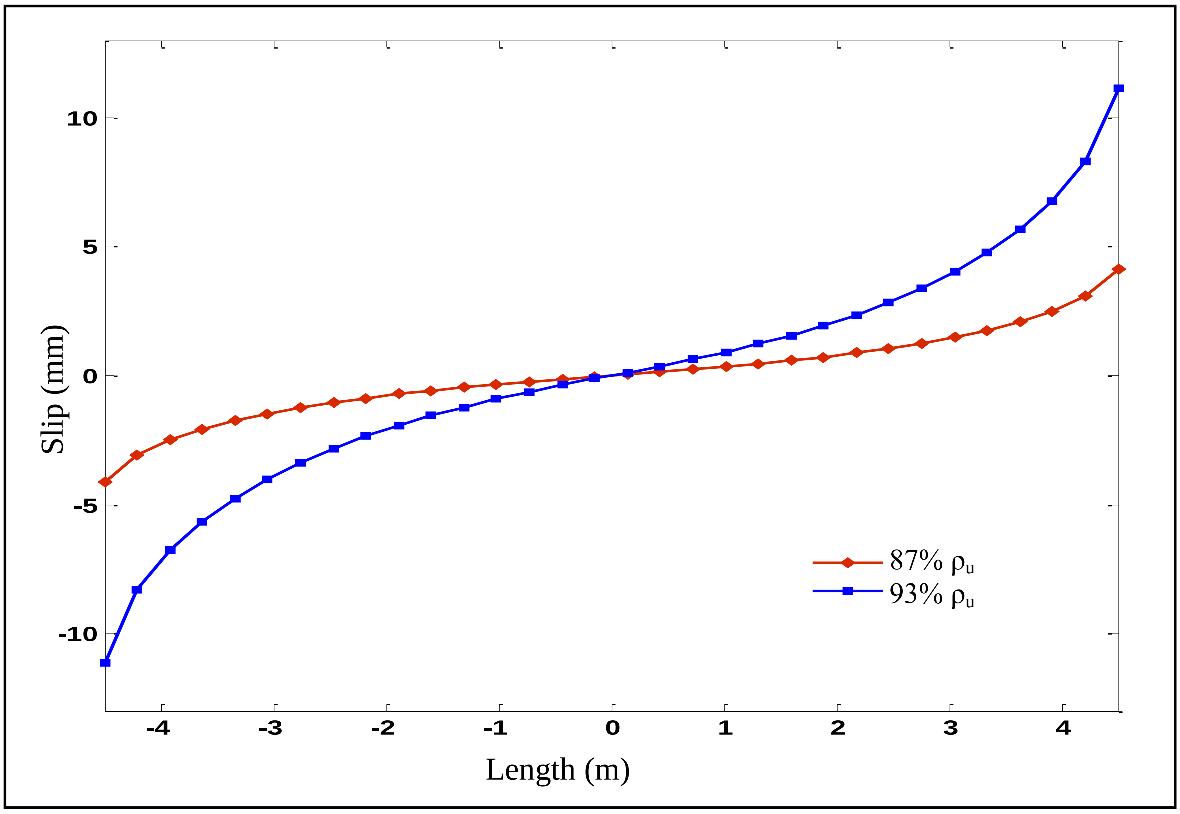

Figure17 Slip of example one along span at 87% ρu and at 93% ρu.

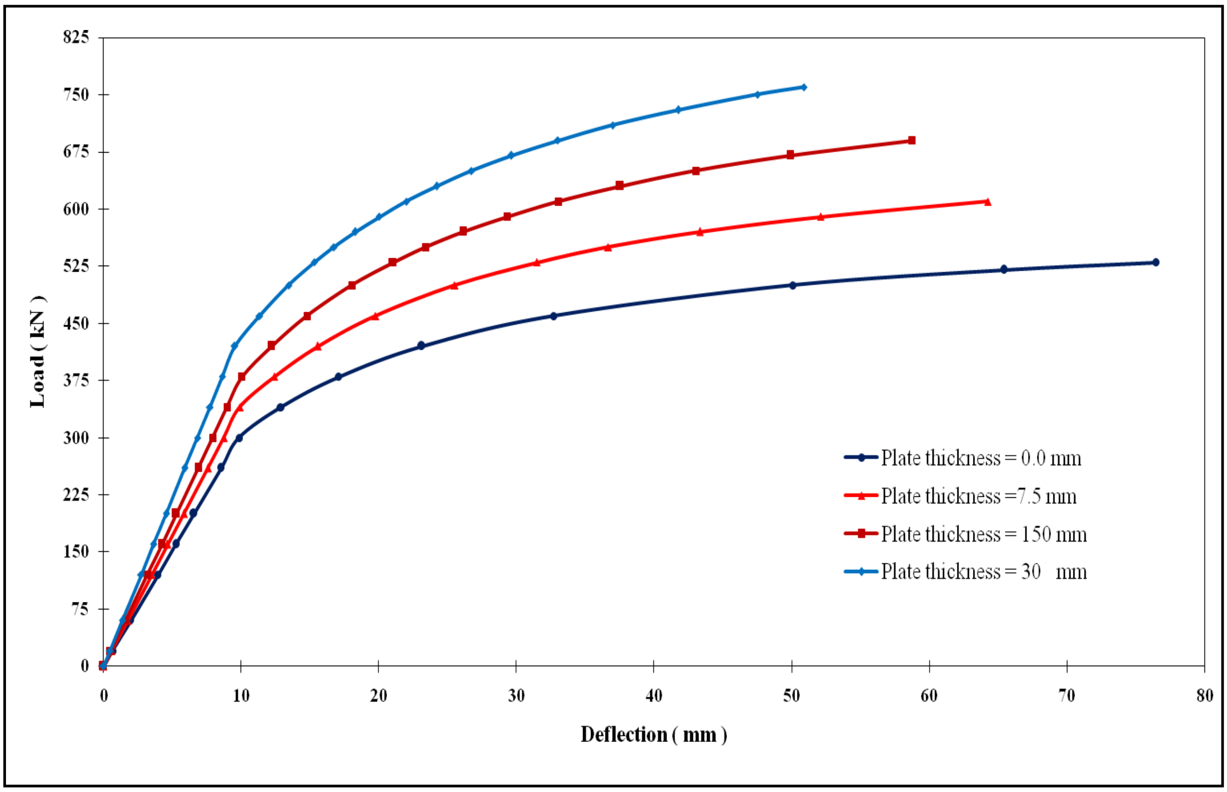

Figure 18 Load-deflection curve of strengthened composite beam of example two with various cover plates.

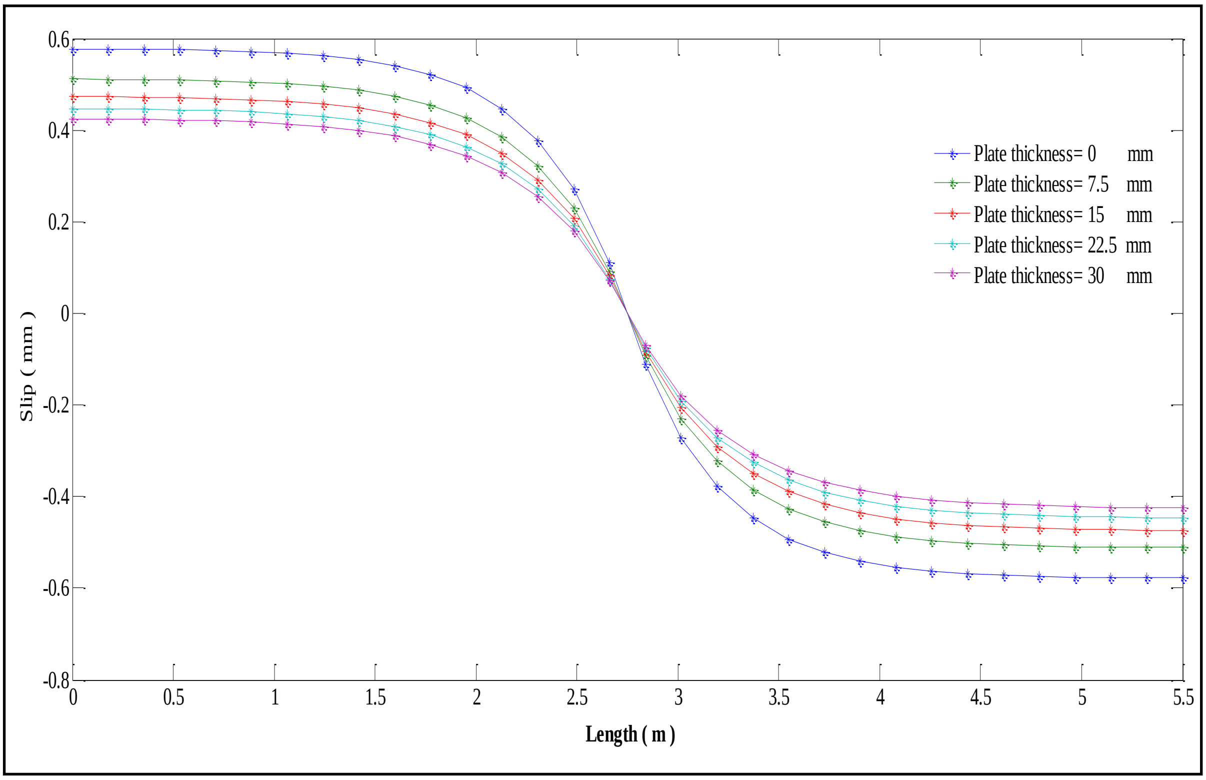

Figure 19 Slip distributions along beam of example two with various plate thicknesses at load 530 kN