



Behavior of Reinforced Concrete Deep Beams Strengthened with Carbon Fiber Reinforced Polymer Strips

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ABSTRACT

This research is concerned to investigate the behavior of reinforced concrete (RC) deep beams strengthened with carbon fiber reinforced polymer (CFRP) strips. The experimental part of this research is carried out by testing seven RC deep beams having the same dimensions and steel reinforcement which have been divided into two groups according to the strengthening schemes. Group one was consisted of three deep beams strengthened with vertical U-wrapped CFRP strips. While, Group two was consisted of three deep beams strengthened with inclined CFRP strips oriented by 45° with the longitudinal axis of the beam. The remaining beam is kept unstrengthening as a reference beam. For each group, the variable considered was the center to center spacing between strips (orthogonal spacing) which are (100 mm, 125 mm and 150 mm). Based on the experimental results it is found that the strengthening deep beams with CFRP strips by the two strengthening schemes, the mid-span deflection was decreased and both first cracking and ultimate loads capacities were increased compared to reference deep beam. For beams having the same spacing between strips, the enhancement occurred by using vertical U- wrapped scheme was somewhat better than using inclined scheme but it needs to use additional numbers of CFRP strips. The percentages increase in first cracking and ultimate loads were (50.0%, 46.0% and 20.5%) and (14.6%, 13.3% and 12.2%) respectively for beams strengthened with vertical U-wrapped scheme. While these percentages were changed to (36.5%, 18.0% and 12.5%) and (12.5%, 10.4% and 8.6%) for beams strengthened with inclined scheme. These results were obtained for center to center spacing between strips of (100 mm, 125 mm and 150 mm) respectively. The analytical part of this research was also adopted using the ACI 440 Code provisions to calculate the additional shear resistance carried by the CFRP strips. Good agreement was obtained between the experimental and analytical results.

Key words: carbon fiber reinforced polymer, strengthening, deep beams, strips

سلوك العتبات الخرسانية المسلحة العميقة المقواة بأشرطة الألياف الكربونية

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الخلاصة

يهتم هذا البحث بالتحري عن سلوك العتبات الخرسانية العميقة المقواة بأشرطة الألياف الكربونية. تضمن الجانب العملي من هذا البحث فحص سبعة عتبات عميقة لها نفس الأبعاد وحديد التسليح وتم تقسيمها الى مجموعتين حسب نمط التقوية. تألفت المجموعة الاولى من ثلاثة عتبات عميقة تم تقويتها بأشرطة الألياف الكربونية الشاقولية على شكل حرف U. بينما تألفت المجموعة الثانية من ثلاثة عتبات عميقة أيضا ولكن باستخدام اشرطة الألياف الكربونية المائلة بزواية 45° عن المحور الافقي للعتبة. اما العتبة المتبقية فتركزت بدون تقوية كعتبة مرجعية لأغراض المقارنة. المتغير الذي تم اعتماده لكل مجموعة هو المسافات المتعامدة بين مراكز اشرطة الألياف الكربونية التي كانت بواقع (100 ملم، 125 ملم و 150 ملم). وجد من النتائج العملية ان تقوية العتبات العميقة باستخدام اشرطة الألياف الكربونية لنمطي التقوية قد قلل من الهطول في منتصف العتبات

وكذلك ازداد حمل التشقق الاولي وقوة التحمل الاقصى للعتبات العميقة مقارنة مع العتبة المرجعية. كذلك وجد بانه لنفس المسافات المتعامدة بين اشربة الالياف الكربونية ان التحسن الحاصل باستخدام نمط التقوية باشربة الالياف الكربونية على شكل حرف U كان اكثر نوعا ما من التحسن الحاصل باستخدام نمط التقوية باشربة الالياف الكربونية المائلة ولكنها تحتاج الى زيادة في عدد اشربة الالياف الكربونية المستخدمة. كانت نسب الزيادة في حمل التشقق الاولي والحمل الاقصى (50.0%, 46.0% و 20.5%) و (14.6%, 13.3% و 12.2%) على التوالي عند استخدام نمط التقوية باشربة الالياف الكربونية على شكل حرف U. بينما تغيرت هذه النسب الى (36.5%, 18.0% و 12.5%) و (12.5%, 10.4% و 8.6%) على التوالي عند استخدام نمط التقوية باشربة الالياف الكربونية المائلة. تم حساب هذه النسب للمسافات المتعامدة بين مراكز اشربة الالياف الكربونية (100 ملم, 125 ملم و 150 ملم) على التوالي. تبنى الجانب التحليلي من هذا البحث استخدام علاقات المدونة الامريكية ACI 440 Code لحساب التعزيز الحاصل لقوه القص الناجم من التقوية باستخدام الالياف الكربونية حيث وجد ان هنالك تقارب جيد بين النتائج العملية والتحليلية.

الكلمات الرئيسية: الالياف الكربونية, تقوية, عتبات عميقة, اشربة

1. INTRODUCTON

Reinforced concrete (RC) deep beams are very useful members and are widely used in buildings, bridges and infrastructures. The deep beam is a beam having a large height (h) comparable with its span length (l). To consider a beam is deep, the span length to height ratio (l/h) should be less than a certain value. The **ACI-318 Code, 2014**, considers a beam is deep if (l/h) ratio is less than or equal to 4.

In the continuous development in science and technology, strengthening of RC structures may be needed due to additional loads that may be imposed or due to deterioration of RC structures as a result of steel corrosion or concrete cracking. There are many ways that may be used to improve this traditional concept in different structural members such as using fiber concrete instead of normal concrete, coating beams with bonded steel plates or fiber reinforced polymer (FRP) materials, **Shanafel, and Horn, 1985; Neale, 2000**.

Uses of FRP materials becomes an acceptable solution for strengthening and repairing in the field of civil engineering across the globe and are widely used for strengthening and retrofitting of RC structures because they have different properties such as light weight, resistance to chemicals and non-corrosive non-magnetic nature. Also, the formability of FRP produces their enforcement technique which is very simple to install, **Alkhrdaji, et al., 2000; Neale, 2000; Li, et al., 2002; Ludovico, 2002**.

Three types of FRP are mainly used for strengthening and rehabilitation of RC structures, Glass Fiber Reinforced Polymer (GFRP), Aramid Fiber Reinforced Polymer (AFRP) and Carbon Fiber Reinforced Polymer (CFRP). Among these three types, CFRP is found to be most efficient to increase the capacity of RC beams, **Li, et al., 2002, Abdel-Jaber, et al., 2003; Feng, and, Yuan, 2008**. Using CFRP sheets or strips is more suitable for applications that have complex geometrical arrangement like curved beams or beams having higher level of reinforcement. These are typically fattened with an epoxy resin in-situ, which likewise acts to bind the fibers to the structure. The comparatively high modulus of the carbon fiber materials makes them more suitable to promote the serviceability of steel structures, **Barros, and Dias, 2003; Mitali, and Gajjar, 2012; Liu, et al., 2012**.

2. EXPERIMENTAL PROGRAM

2.1 Introduction

The main purpose of this research is to investigate the behavior RC deep beams strengthened with CFRP strips. The primary variables considered are the strengthening schemes of CFRP strips and the center to center spacing between strips. Two schemes of strengthening using CFRP

strips were adopted. These schemes are vertical U-wrapped CFRP strips and inclined CFRP strips oriented by 45° with the longitudinal axis of the beam.

Standard tests were carried out to determine the properties of materials used in this research. In addition, instrumentation, experimental setup and testing procedures adopted throughout this investigation are presented.

2.2 Deep Beams Details

In this research work, seven simply supported RC deep beams were cast and tested up to failure by applying two-point loading. Six of these beams were strengthened with CFRP strips (strip width = 50 mm). While, the remaining deep beam (denoted as DB1) is kept unstrengthening as a reference beam. All tested beams having the same dimensions [total length = 1100 mm, width (b) = 150 mm and height (h) = 240 mm]. The center to center distance between supports (span = l) was 950 mm which results a span to height ratio (l/h) equals to 3.96 which lies within the ACI Code limits. All beams were designed to fail in shear. These deep beams were reinforced with $2\phi 16$ mm and $1\phi 12$ mm steel bars at the bottom side of beams. While, $\phi 6$ mm is used as stirrups spaced each 100 mm. In addition, a skin reinforcement of $1\phi 6$ mm was added at mid height of the beam cross section at both side faces. Also, $2\phi 6$ mm steel bars were located at the top of beams. **Fig. 1** shows full details of a typical tested beam.

The strengthened deep beams were divided into two groups according to the strengthening schemes. Group one was consisted of three deep beams denoted as DB2, DB3 and DB4 strengthened with vertical U-wrapped CFRP strips having spacing of (100, 125, and 150 mm) center to center between strips respectively. While, Group two was consisted of three deep beams denoted as DB5, DB6 and DB7 strengthened with inclined CFRP strips oriented by 45° with the longitudinal axis of beam having the same spacing above orthogonal to strips as shown in **Fig. 2**. Also, **Table 1** presents the description of tested beams conducted in this study.

2.3 Materials Used for Casting Beams

2.3.1 Cement

Ordinary Portland cement type (I) was used for concrete mix. This cement was tested chemically and physically according to the **Iraqi Specifications No. 5, 1984** for Portland cement.

2.3.2 Fine aggregate

Natural sand from Al-Akhaidher quarries was used throughout this study. This sand has a maximum particles size of (4.75mm). The sand washed with water and then dried and it was confirmed to the **Iraqi specification No.45, 1984**.

2.3.3 Coarse aggregate

Crushed gravel from Al-Sodor region with maximum size of 15 mm was used throughout this research. The crushed gravel was washed and it was conform to the **Iraqi specification No.45, 1984**.

2.3.4 Steel reinforcement

Three sizes of deformed steel bars were utilitied in the investigation having diameters of $\phi 6$ mm, $\phi 12$ mm and $\phi 16$ mm. These steel bars were tested according to the **ASTM standard A615, 2001**. The yield stress (f_y) for $\phi 6$ mm steel bars was 420 MPa. While for $\phi 12$ mm and $\phi 16$ mm steel bars, the yield stress was 460 MPa.

2.3.5 CFRP strips

CFRP type Sika Wrap Hex-230 and epoxy based impregnating resin of type Sikadur-330 were used in this study for externally strengthening the deep beams. The CFRP strips having width of 50 mm and thickness of 0.15 mm. Based on a data sheet given by the manufactured company, the ultimate tensile strength and the modulus of elasticity for the CFRP strips used throughout this research were 3400 MPa and 230000 MPa respectively.

2.4 Concrete Compressive Strength

In the present study, normal weight concrete was used for casting the specimens. After several trial mixes, the cylindrical compressive strength (f'_c) was 17.3 MPa. This value is slightly larger than the minimum recommended value that mentioned in the **ACI-318 Code, 2014**.

2.5 Preparation of Specimens

For casting the specimens, seven wooden molds were prepared and fabricated into the required beam dimensions. These molds were oiled and then the reinforcement cages were put into the required positions as shown in **Fig. 3**. After 28 days of curing, all specimens were white painted to recognize cracks during the test. Scraper machine was used to rough the surface of the beams at specified location of CFRP strips to make good cohesion between concrete surface and CFRP strips. The epoxy mix has been applied to the surface of concrete, and then the CFRP strips were pasted on the concrete surface as shown in **Fig. 4**.

2.6 Test Rig Components and Loading Procedure

All deep beams were tested up to failure by using a hydraulic testing machine available at the Civil Engineering Department Laboratory of the University of Bagdad. All beams were mounted on two steel rollers (supports) available at the laboratory and attached to the bottom face of the beams with distance of 950 mm measured center to center between supports. After mounting the specimens on the supports, a single dial gauge was fixed on the lower bed of the testing frame and attached to the bottom face of the beams at mid span.

A special system was achieved for applying two-point loading. This system was fabricated by welding two stiff steel angles with the bottom flange of IPE200 section. Then 2 ϕ 25 mm steel bars with distance of 300 mm center to center between them were also welded with these angles as shown in **Fig. 5**. A hydraulic jack having a capacity of 500 kN was put on the top flange of IPE200. Then a load cell with capacity of 200 kN was used with a hydraulic jack and the system was attached the testing frame at the top. Small amount of external load was applied until the specimen got stable on the supports. After that, reading of the load cell is reset and load was started to be applied gradually with a step of 5 kN. At each load step, mid span deflection was recorded. Also a magnifier was used to note the appearance of first crack for all tested beams. **Fig. 6** shows the setup of tested beams with devices used throughout the test.

3. EXPERIMENTAL RESULTS

3.1 Cracking and Ultimate Loads Test Results

All tested beams were characterized by the formation of diagonal shear cracks near supports. These diagonal cracks were propagated with an angle of 45° until they reached the top surface of beams then failure occurred. At stages of loading close to failure load, debonding of some CFRP strips where the cracks passed through occurred. **Fig. 7** shows all tested deep beams after failure.

Table 2 summarizes the experimental first cracking and ultimate loads results. From this table it can be noticed that strengthening deep beams with CFRP strips has a significant effect on increasing the first cracking load and slightly increases the ultimate load compared to reference

deep beam. For the first scheme of strengthening (vertical U-wrapped strips) which is represented by beams of group one, the maximum percentage increase in first cracking and ultimate loads were 50.0% and 14.6% respectively for deep beam DB2 which has been strengthened with 10-U wrapped strips spaced each 100 mm center to center of strips. While, for the second scheme of strengthening (inclined strips) which is represented by the beams of group two, the maximum percentage increase in first cracking and ultimate loads were 36.5% and 12.5% for deep beam DB2 which has been strengthened with 6-inclined strips spaced each 100 mm center to center orthogonal to strips. Also, from this table it can be noticed that as the spacing between strips is decreased, the percentages increasing in first cracking and ultimate loads are increased for both schemes of strengthening.

By comparing the results obtained from **Table 2** for both strengthening schemes, it may be noticed that for the same spacing between strips there is a small different in percentage increase in ultimate load compared to reference deep beam. Hence, for the same spacing between strips U-scheme of strengthening need more amount of CFRP strips than inclined scheme of strengthening. So that, the inclined scheme of strengthening may be more economic than the U-scheme of strengthening.

3.2 Load-Mid Span Deflection Response

Figs. 8 & 9 show the load-mid span deflection response for beams of Group one (deep beams strengthened with vertical U-wrapped CFRP strips) and beams of Group two (deep beams strengthened with inclined CFRP strips) respectively compared to reference deep beam (DB1). From these figures, it can be noticed that the deflection is decreased as the center to center spacing between strips is decreased for the entire range of loading of each group. This is because of increased stiffness for beams strengthened with CFRP strips when the spacing between strips was decreased. **Table 3** summarizes the experimental deflection values at mid-span of tested beams corresponding to a load level of (96 kN) which represents the ultimate load of the reference deep beam (DB1). From this table it can be noticed that the percentages decrease in deflections were 24.76%, 19.05% and 13.33% for beams strengthened with U-wrapped strips spaced at 100 mm, 125 mm and 150 mm respectively. While, the percentages decrease in deflection were 20.00%, 12.38% and 7.62% for beams strengthened with inclined strips having the same spacing above compared to reference deep beam.

Figs. 10, 11 & 12 were plotted to show the load-mid span deflection response for strengthened beams having the same spacing center to center between CFRP strips but differ in strengthening scheme (i.e U-wrapped and inclined strips). These figures correspond to center to center spacing between strips of 100 mm, 125 mm and 125 mm respectively. From these figures, it can be noticed that for beams strengthened with vertical U-wrapped CFRP strips had stiffer response than beams strengthened with inclined strips. This is because the U-wrapped strengthening scheme might made beams more confined than the inclined strengthening scheme.

4. ANALYTICAL INVESTIGATION BASED ON THE ACI-440 CODE PROVISIONS

The **ACI-440 Code, 2008** provides an expression to calculate the additional shear strength provided by the FRP strips (V_f) as given by Eq. (1). Also **Fig. 13** reveals the dimensional variables used in this equation.

$$V_f = \frac{A_{fv} \times f_{fe} \times (\sin\alpha + \cos\alpha) \times d_{fv}}{s_f} \quad (N)$$

where

1



A_{fv} = area of FRP strip within spacing (s_f) given by Eq. (2)

f_{fe} = effective stress in FRP strip given by Eq. (3)

α = angle of FRP strip orientation about x- axis

d_{fv} = effective depth of FRP strip (distance from center of flexural reinforcement to the extreme fiber of strips)

s_f = spacing center to center between strips in the horizontal direction

$$A_{fv} = 2 \times t_f \times w_f \quad (\text{mm}^2) \quad 2$$

where

t_f = FRP strip thickness (mm)

w_f = FRP strip width (mm)

$$f_{fe} = \varepsilon_{fe} \times E_f \quad (\text{MPa}) \quad 3$$

where

ε_{fe} = effective strain in FRP strip given by Eq. (4)

E_f = modulus of elasticity of FRP strip (MPa)

$$\varepsilon_{fe} = k_v \times \varepsilon_{fu} \leq 0.004 \quad 4$$

where

ε_{fu} = rupture strain of FRP strip given by Eq. (5)

k_v = bond dependent coefficient given by Eq. (6)

$$\varepsilon_{fu} = \frac{f_{fu}}{E_f} \quad 5$$

where

f_{fu} = ultimate tensile strength of FRP strip (MPa)

$$k_v = \frac{K_1 K_2 L_e}{11900 \varepsilon_{fu}} \quad 6$$

where

K_1 = modified concrete factor given by Eq. (7)

K_2 = modified FRP scheme factor given by Eq. (8)

L_e = active bond length of FRP strip given by Eq. (9)

$$K_1 = \left(\frac{f'_c}{27}\right)^{\frac{2}{3}} \quad 7$$

$$K_2 = \frac{d_{fv} - 2L_e}{d_{fv}} \quad \text{for two sides bonded} \quad 8\text{-a}$$

$$K_2 = \frac{d_{fv} - L_e}{d_{fv}} \quad \text{for U-wrapped bonded} \quad 8\text{-b}$$

$$L_e = \frac{23300}{(n_f \times t_f \times E_f)^{0.58}} \quad (\text{mm}) \quad 9$$

where

n_f = modular ratio of elasticity given by Eq. (10)

$$n_f = \frac{E_f}{E_c} \quad 10$$

where

E_c = modulus of elasticity of concrete given by Eq. (11)

$$E_c = 4700\sqrt{f'_c} \quad (\text{MPa}) \quad 11$$

Eq. (1) is applied to reference deep beam DB1 to compute the percentage increase in ultimate load for both U-wrapped and inclined strengthening schemes. A comparison between the experimental and the analytical results is listed in **Table 4**. From this table it can be noted that good agreement between the experimental and the theoretical results is obtained by using the ACI 440-Code provisions. Also, this table reveals that for both strengthening schemes having the same orthogonal spacing center to center between CFRP strips, the experimental and analytical results showed a convergence in the ultimate load results. As the U-wrapped strengthening scheme have more number of CFRP strips than the inclined strengthening scheme, so that it may be concluded that using inclined strips is more economic than using U-wrapped strips.

5. CONCLUSIONS

1. Based on experimental results it is found that strengthening deep beams by two schemes of strengthening using CFRP (vertical U-wrapped scheme and inclined scheme oriented by 45°), the mid-span deflection was decreased and both first cracking load and ultimate load capacities were increased compare to the reference deep beam.
2. For strengthening beams having the same spacing between strips, the enhancement occurred using vertical U- wrapped scheme was somewhat better than using inclined scheme but it needs to use more numbers of CFRP strips.
3. When using U-wrapped CFRP strengthening scheme, the percentages increase in first cracking loads were (50.0%, 46.0% and 20.5%) for deep beams having center to center spacing between strips of (100 mm, 125 mm and 150 mm) respectively compared to reference beam without strengthening. While when using inclined CFRP strengthening scheme, the percentages increases in first cracking loads were (36.5%, 18.0% and 12.5%) for deep beams having the same spacing above compared to reference beam.
4. The percentages increase in ultimate loads were (14.6%, 13.3% and 12.2%) and (12.5%, 10.4% and 8.6%) for U-wrapped strengthening scheme and inclined strengthening scheme respectively compared to reference beam without strengthening. These results were obtained for center to center spacing between strips of (100 mm, 125 mm and 150 mm) respectively.
5. Under a load level representing the ultimate load of reference deep beam DB1, the percentages decrease in deflections were 24.76%, 19.05% and 13.33% for beams strengthened with U-wrapped strips spaced at 100, 125 and 150 mm respectively. While, the percentages decrease in deflections were 20.00%, 12.38% and 7.62% for beams strengthened with inclined strips having the same spacing above compared to reference deep beam.
6. Based on theoretical analysis according to the ACI 440 Code provisions, good agreement was obtained between the experimental and theoretical results.
7. From the experimental and analytical results it is found for the same center to center spacing between strips there is a convergence in ultimate results for both styles of strengthening. Also, as



U-scheme of strengthening needs more amount of CFRP strips than inclined scheme of strengthening. Hence, the use of inclined scheme of strengthening may be more economic than using the vertical U-scheme of strengthening.

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NOMENCLATURE

- A_{fv} area of FRP strip within spacing (s_f)
- CFRP carbon fiber reinforced polymer
- d_{fv} effective depth of FRP strip
- E_c modulus of elasticity of concrete
- E_f modulus of elasticity of FRP strip
- f'_c cylindrical compressive strength of concrete
- f_{fe} effective stress in FRP strip
- f_{fu} ultimate tensile strength of FRP strip
- f_y yield stress of steel reinforcement
- h overall depth of the section
- K_1 modified concrete factor
- K_2 modified FRP scheme factor
- k_v bond dependent coefficient
- l span center to center of beam
- L_e active bond length of FRP strip
- n_f modular ratio of elasticity
- RC reinforced concrete
- s_f spacing center to center between strips in the horizontal direction
- t_f thickness of FRP strip
- V_f additional shear force carried by FRP strips
- W_f width of FRP strip
- ϵ_{fe} effective strain in FRP strip
- ϵ_{fu} rupture strain of FRP strip
- α angle of FRP orientation about x-axis

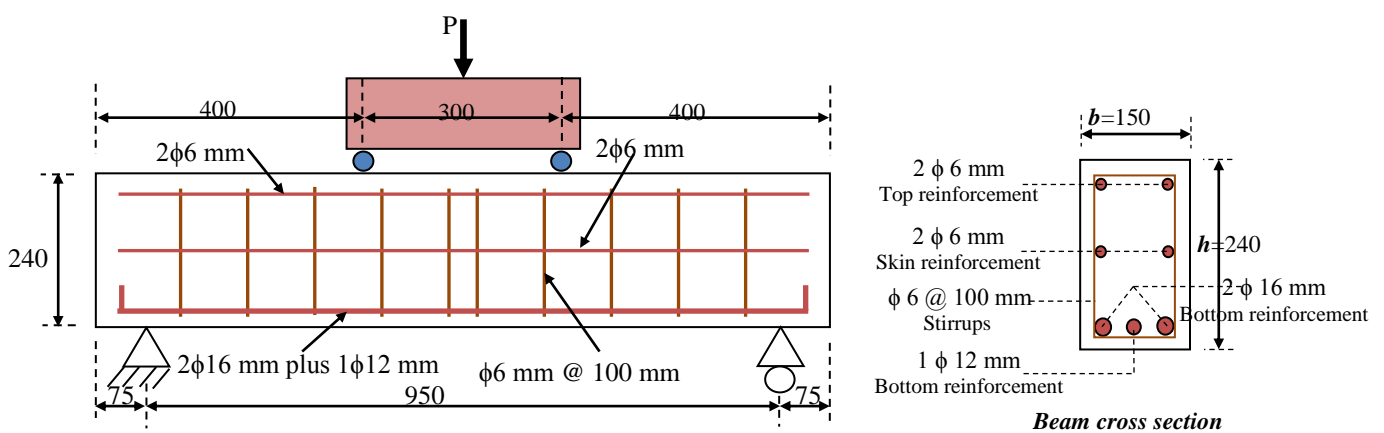


Figure 1. Layout of a typical tested deep beam (all dimensions are in mm).

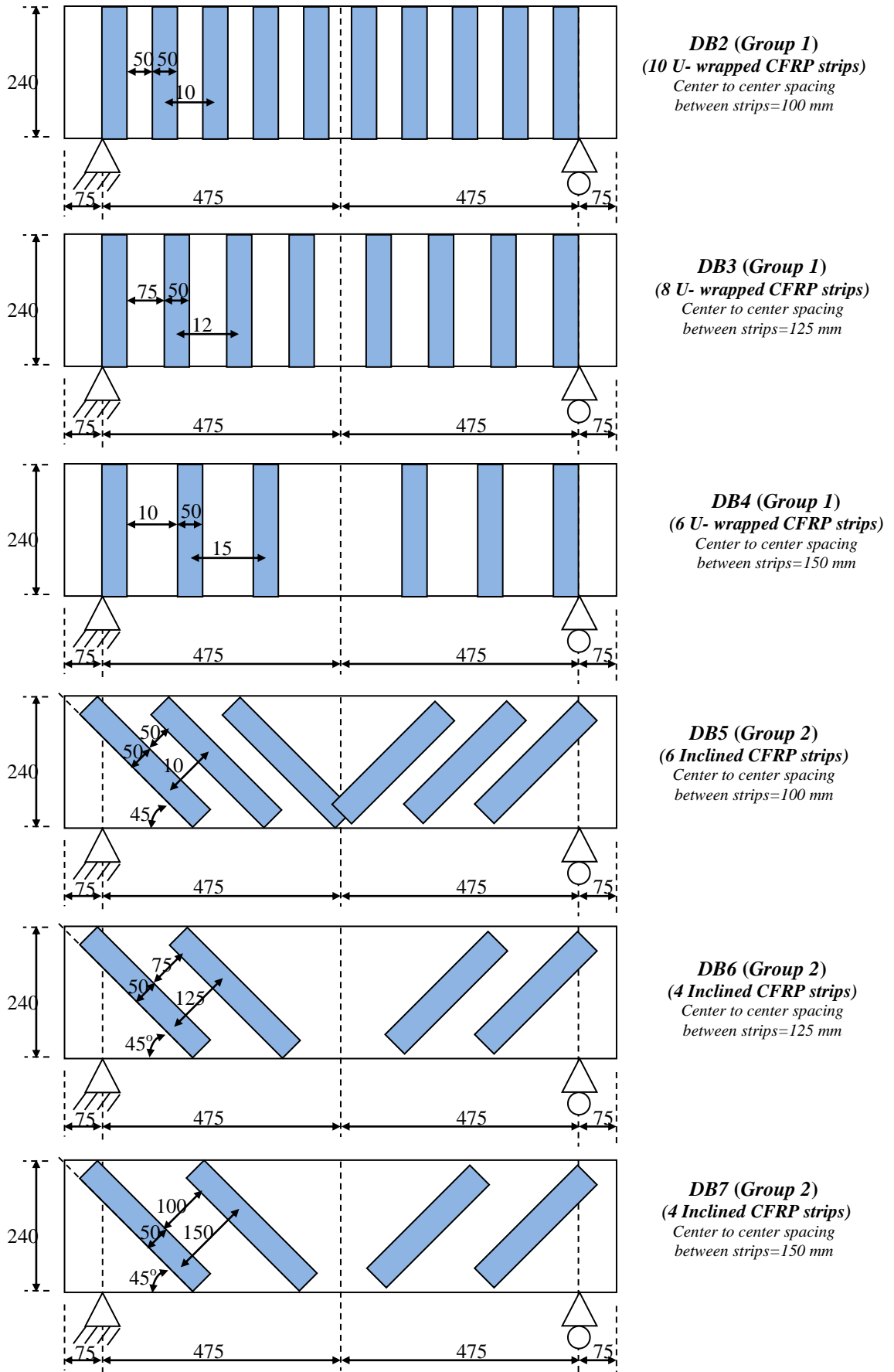


Figure 2. Strengthening schemes and tested beams designations (all dimensions are in mm).



Figure 3. Mold fabrication and reinforcement cage of a typical tested beam.



Figure 4. Roughing concrete surface and applying epoxy at CFRP strips location of a typical tested beam.

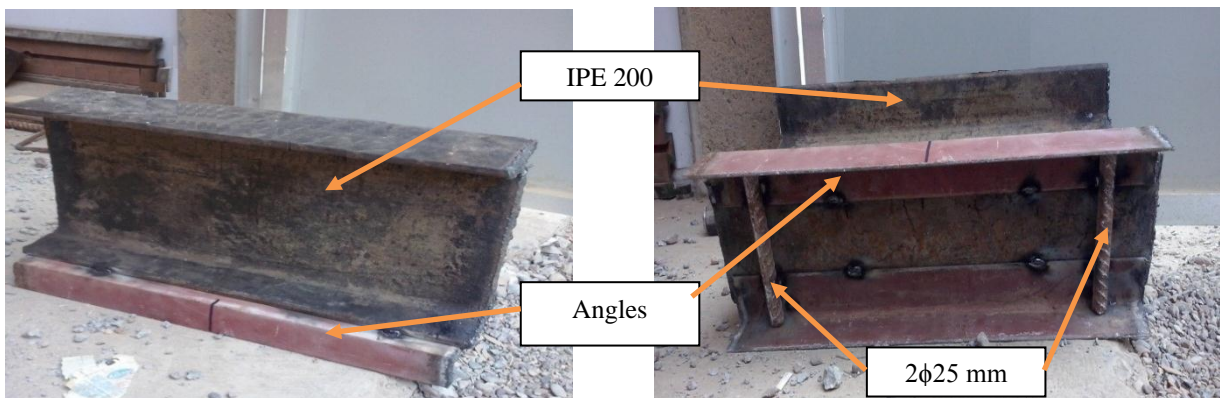


Figure 5. Fabricated system used for applying the load on tested beams.

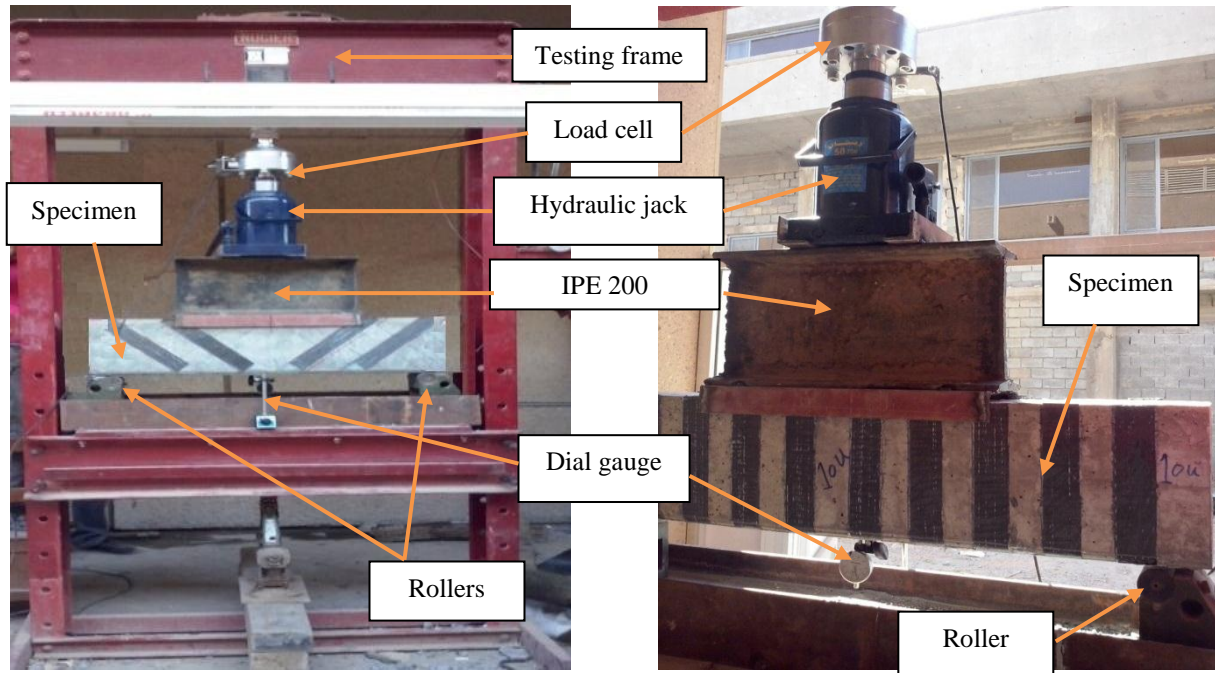


Figure 6. Setup of typical tested beams.

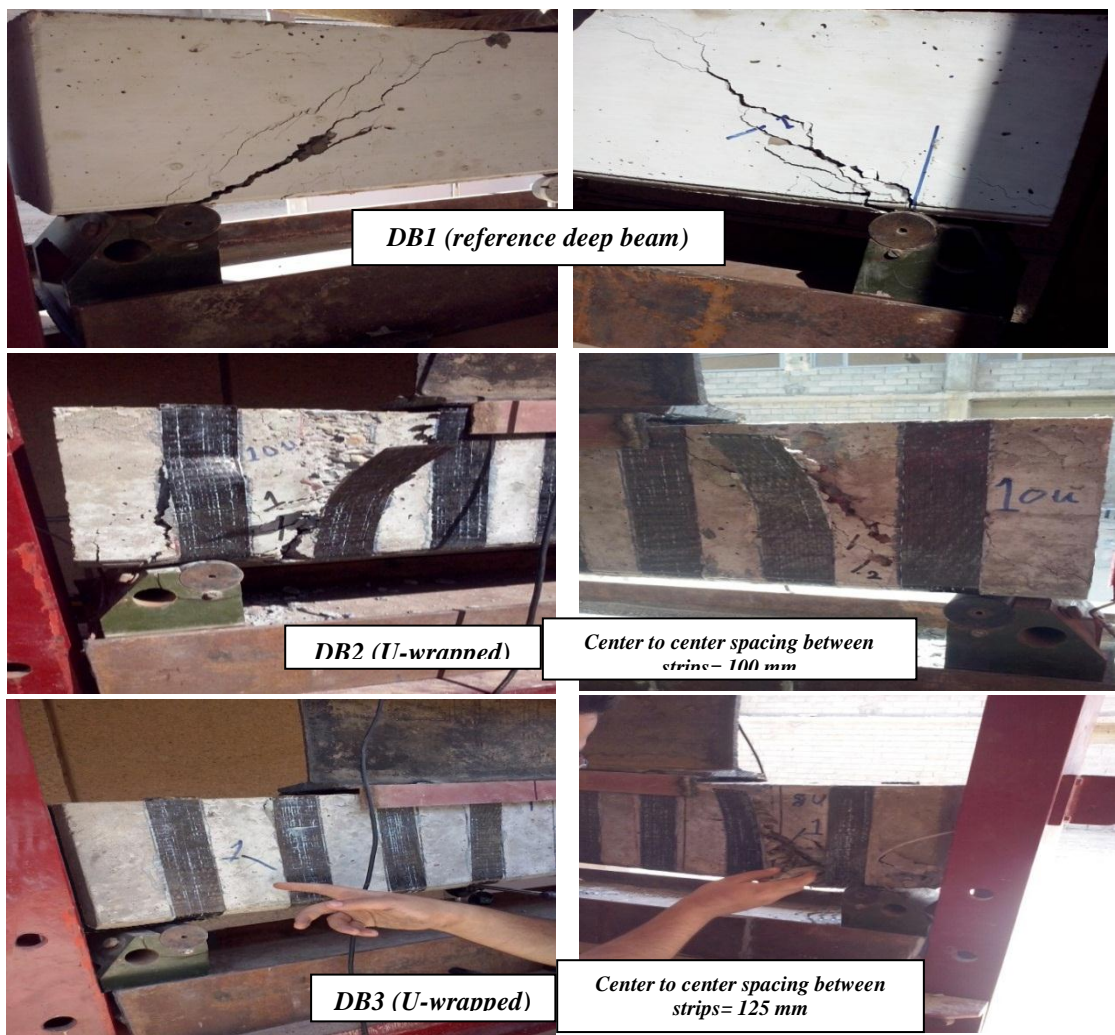


Figure 7. Tested beams after failure.



Figure 7. Continue.

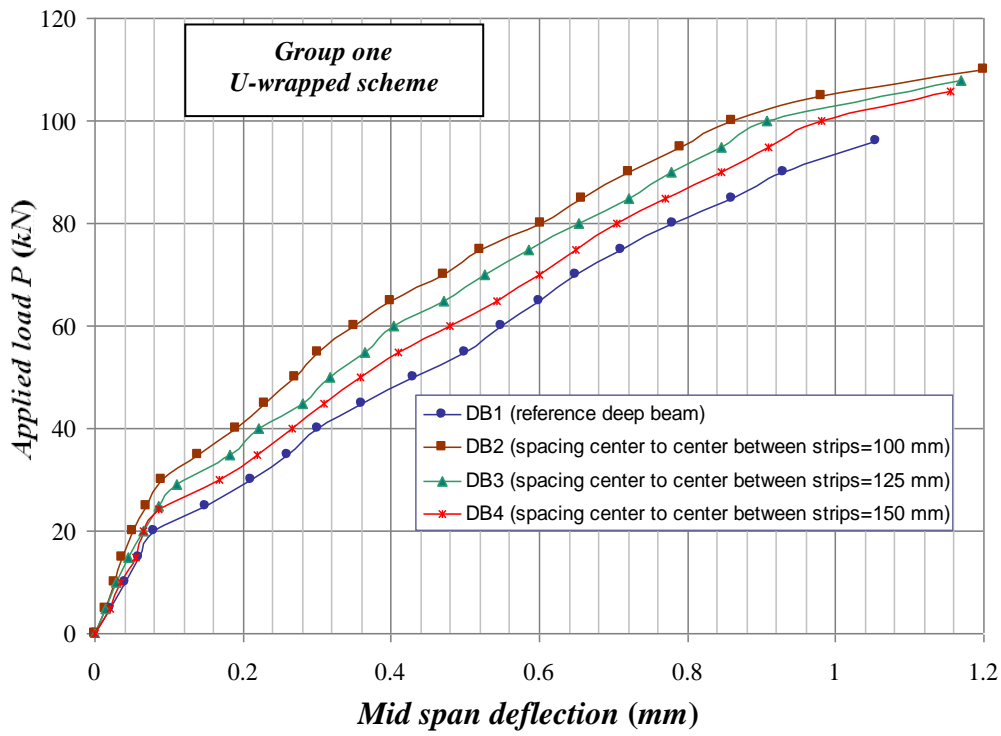


Figure 8. Load-mid span deflection for beams of Group one.

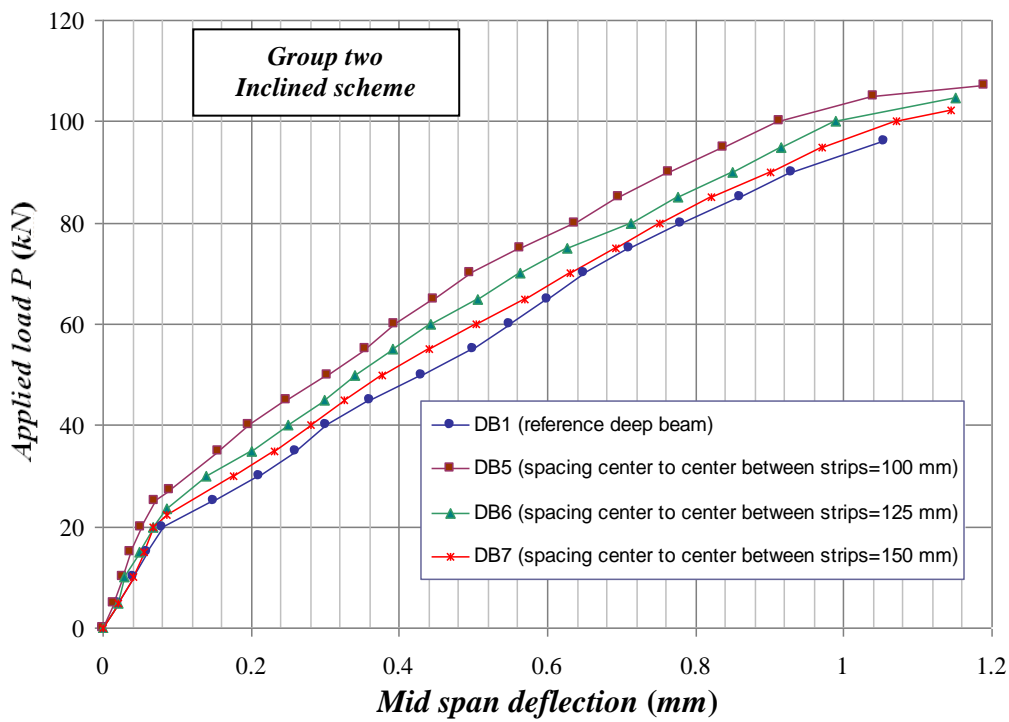


Figure 9. Load-mid span deflection for beams of Group two.

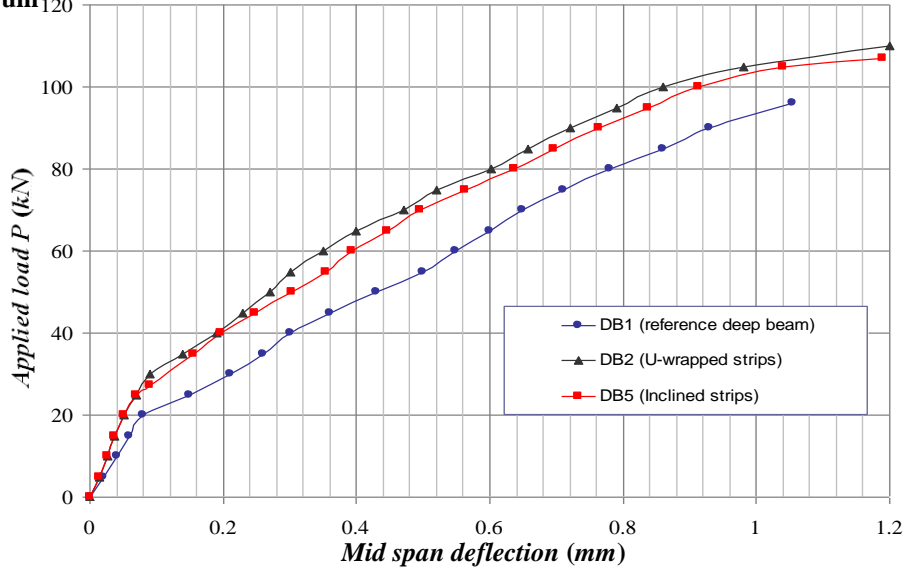


Figure 10. Load-mid span deflection for beams with center to center spacing between strips= 100 mm.

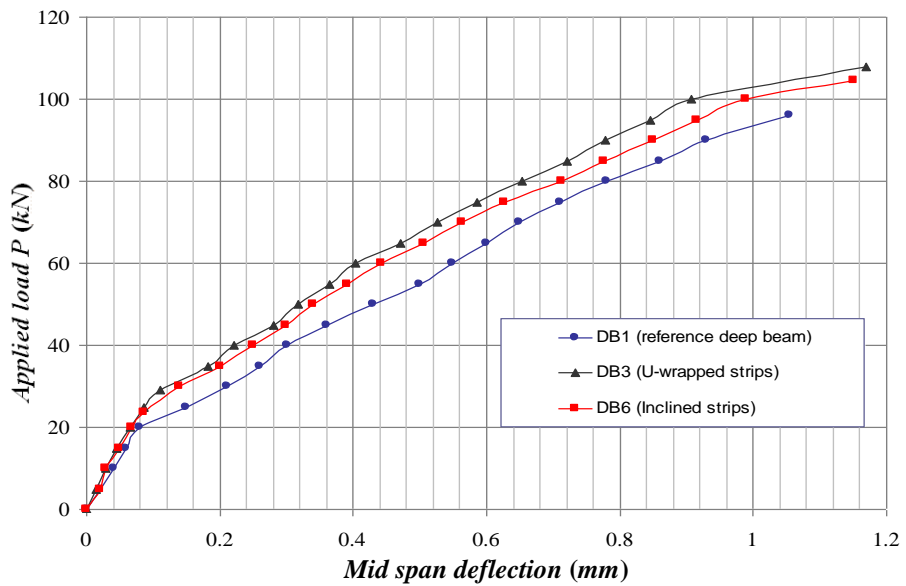


Figure 11. Load-mid span deflection for beams with center to center spacing between strips= 125 mm.

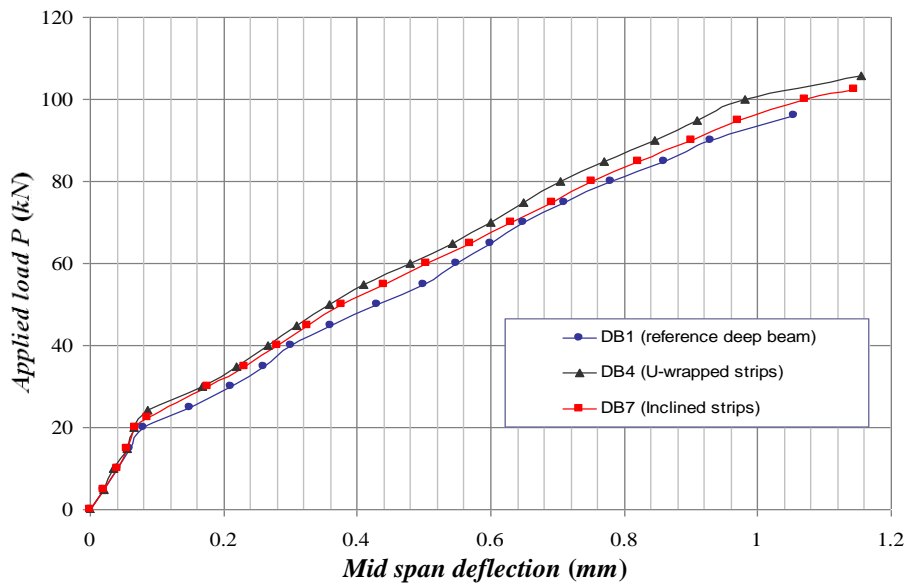


Figure 12. Load-mid span deflection for beams with center to center spacing between strips= 100 mm.

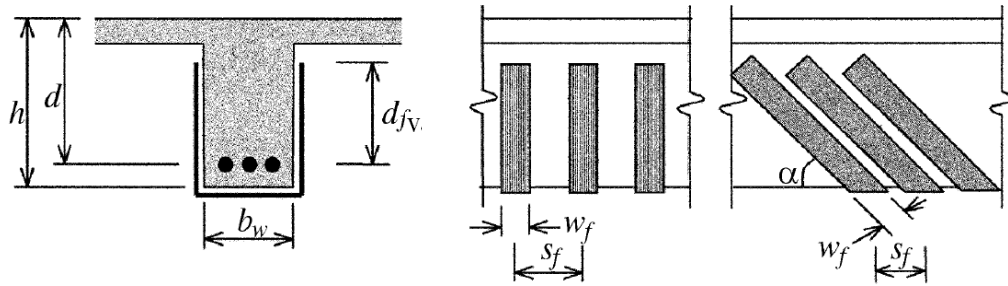


Figure 13. Illustration of the dimensional variables used for calculating the additional shear strength provided by FRP strips, **ACI 440 Code, 2008.**

Table 1. Tested beams designation and strengthening schemes.

Group No.	Beam designation	Beam type	CFRP Strengthening scheme	Center to center spacing between CFRP Strips (mm)
-	DB1	Reference	-	-
Group one	DB2	Strengthened	U- wrapped (10 strips)	100
	DB3	Strengthened	U- wrapped (8 strips)	125
	DB4	Strengthened	U- wrapped (6 strips)	150
Group two	DB5	Strengthened	Inclined with 45° (6 strips)	100 (orthogonal inclined)
	DB6	Strengthened	Inclined with 45° (4 strips)	125 (orthogonal inclined)
	DB7	Strengthened	Inclined with 45° (4 strips)	150 (orthogonal inclined)

Table 2. First cracking and ultimate loads of tested deep beams.

Beam designation		Center to center spacing between Strips (mm)	First cracking load P_{cr} (kN)	% Increase in first cracking load	Ultimate load P_u (kN)	% Increase in ultimate load
Reference	DB1	-	20.0	-	96.0	-
Group one (U-wrapped strips)	DB2	100	30.0	50.0	110.0	14.6
	DB3	125	29.2	46.0	107.8	13.3
	DB4	150	24.1	20.5	105.7	12.2
Group two (Inclined strips)	DB5	100 (orthogonal inclined)	27.3	36.5	107.1	12.5
	DB6	125 (orthogonal inclined)	23.6	18.0	104.6	10.4
	DB7	150 (orthogonal inclined)	22.5	12.5	102.3	8.6

$$\% \text{ Increase} = \frac{P(\text{strengthened}) - P(\text{reference})}{P(\text{reference})} \times 100$$

**Table 3.** Mid span deflection values at ultimate load level of tested deep beams.

Beam designation		Center to center spacing between Strips (mm)	Mid-span* deflection (mm)	% Decrease in mid-span deflection
Reference	DB1	-	1.05	-
Group one (U-wrapped strips)	DB2	100	0.79	24.76
	DB3	125	0.85	19.05
	DB4	150	0.91	13.33
Group two (Inclined strips)	DB5	100 (orthogonal inclined)	0.84	20.00
	DB6	125 (orthogonal inclined)	0.92	12.38
	DB7	150 (orthogonal inclined)	0.97	7.62

* Corresponding to a load level of (96 kN) which represents the ultimate load of the reference deep beam (DB1)

$$\% \text{ Decrease} = \frac{\text{deflection}(\text{strengthened}) - \text{deflection}(\text{reference})}{\text{deflection}(\text{reference})} \times 100$$

Table 4. Experimental and analytical percentages increase in ultimate load.

Beam designation		s^* (mm)	s_f^{**} (mm)	V_f^{***} (kN)	% Increase in [†] ultimate load (analytically)	Experimental ultimate load P_u (kN)	% Increase in ^{††} ultimate load (experimentally)
Reference	DB1	-	-	-	-	96.0	-
Group one (U-wrapped strips)	DB2	100	100	5.97	12.44	110.0	14.6
	DB3	125	125	4.78	9.96	107.8	13.3
	DB4	150	150	3.98	8.29	105.7	12.2
Group two (Inclined strips)	DB5	100	141	5.56	11.58	107.1	12.5
	DB6	125	177	4.43	9.23	104.6	10.4
	DB7	150	212	3.70	7.71	102.3	8.6

* Orthogonal spacing center to center between strips

** Horizontal spacing center to center between strips

*** By applying Eq. (1)

$$\dagger \% \text{ Increase in ultimate load (analytically)} = \frac{2 \times V_f}{P_u(\text{reference deep beam})} \times 100$$

$$\dagger\dagger \% \text{ Increase in ultimate load (experimentally)} = \frac{P_u(\text{strengthened}) - P_u(\text{reference})}{P_u(\text{reference})} \times 100$$