Experimental Evaluation of the Strut-and-Tie Model Applied to Deep Beam with Near-Load Openings

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ABSTRACT

It is commonly known that Euler-Bernoulli's thin beam theorem is not applicable whenever a nonlinear distribution of strain/stress occurs, such as in deep beams, or the stress distribution is discontinuous. In order to design the members experiencing such distorted stress regions, the Strut-and-Tie Model (STM) could be utilized. In this paper, experimental investigation of STM technique for three identical small-scale deep beams was conducted. The beams were simply supported and loaded statically with a concentrated load at the mid span of the beams. These deep beams had two symmetrical openings near the application point of loading. Both the deep beam, where the stress distribution cannot be assumed linear, and the existence of the openings, which causes stress discontinuity, make the use of Euler-Bernoulli's thin beam theorem not applicable. An idealized STM for the beam was first established and then experimental test was carried out to study the capability of STM to deal with the distortion of stress caused by the presence of near-load openings in addition to the nonlinear distribution of stress occurring in deep beam. The test results showed that the beam designed using STM was able to withstand a load higher than the designed ultimate load. The service load, in the other hand, was within the range of the estimated one. The outcome of this study can then be added to the relatively few available experimental studies related to STM technique to enhance the validation of STM to efficiently treat different structural configurations where the linear stress assumption cannot be applied.

Keywords: deep beam; strut-and-tie model; openings; nonlinear stress distribution; distorted stress regions.

دراسة عملية لنظرية الدعامة والشداد لاعتاب عميقة ذات فتحات قريبة من نقاط التحميل

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الخلاصة

نظرية اويلر-برنولي للاعتاب لا يمكن تطبيقها عندما يكون توزيع الاجهادات غير خطي اوغير مستمر على عمق المقطع الانشائي. لتصميم هذه العناصر، يمكن استخدام نظرية الدعامة والشداد. يقدم هذا البحث دراسة عملية لتطبيق نظرية الدعامة والشداد على ثلاثة نماذج متماثلة لاعتاب مصغرة. الاعتاب التي تم دراستها مسندة اسناد بسيط و تم تحميلها استاتيكيا بحمل مركز في منتصف العتبة. الاعتاب تحتوي على فتحتين متناظرتين قرب نقطة تحميل القوى. نظرية برنولي لايمكن استخدامها في هذه العتبات نتيجة كون العتبات عميقة والتي لايمكن فيها فرض توزيع الاجهادات خطي ونتيجة وجود الفتحات والتي تؤدي الى انقطاعات في توزيع الاجهادات. في هذا البحث، يتم اجراء دراسة عملية على نظرية الدعامة والشداد لمعرفة قدرتها على التعامل مع تشوه الاجهادات الناتج من وجود الفتحات قرب نقاط التحميل. نتائج البحث اظهرت ان الاعتاب المصممة بطريقة الدعامة والشداد كانت قادرة على مقاومة احمال قصوى اكثر من الاحمال المصممة لها. اما فيما يخص الاحمال الخدمية فكانت ضمن الحدود المتوقعة. النتائج المستخلصة من هذا البحث يمكن ان تكون اضافة للدراسات العملية القليله التي اجريت مسبقا لتأكيد قابلية نظرية الدعامة والشداد على التعامل مع حالات انشائية مختلفة تكون فيها نظرية برنولي غير قابلة للتطبيق.

الكلمات الرئيسية: عتب عميق، نظرية الدعامة والشداد، فتحات، توزيع الاجهادات اللاخطي، تشوه الاجهادات.

1. INTRODUCTION

Structural members come in a variety of configurations to comply with different structural demands. It is, therefore, hard to presume a unified assumption that can be applied precisely to all structural members. Euler-Bernoulli's thin beam theorem is one of those theories that cannot be applied to all structural components.

In general, structural elements can be grouped in two categories, **Wight** and **MacGregor 2012**,:

- B-regions (Beam regions) where the assumptions of Euler-Bernoulli's thin beam theorem can be applied including linear stress distribution assumption.
- D-regions (Disturbed or Discontinuous regions) where Euler-Bernoulli's thin beam theory cannot be used.

Strut-and–tie modeling approach is considered a valuable technique to design D-regions or nonlinear stress regions including deep beams. The strut-and–tie model was first introduced in the **ACI 318-02 code**, **2002**, in 2002 as an appendix. After that, it has been included within the code context as a main chapter including the latest **ACI code 318-14**, **2014**. The strut-and-tie model is a simple method that is basically derived from the truss analogy. The idea is to draw the flow of forces inside the structural member as struts and ties. The force in struts is resisted by concrete while reinforcement is designed and placed to resist the tension in ties. Several patterns of load paths may be constructed for the same loading conditions provided that the truss pattern and components satisfy the recommendations specified by the ACI code, as it is the reference code for this study, or the provisions specified by other codes of practice.

ACI 318-14 code, **2014**, defines the beam as a deep beam if a concentrated load acts within a distance less than twice the depth of the beam, or if the clear span between the beam's supports is less than fourth times the depth of the beam. Once the beam is identified as a deep beam, **ACI 318-14 code**, **2014**, recommends two methods to design the beam; either by taking into consideration the nonlinear distribution of the strains, without mentioning further details, or by using the strut-and-tie model.

One of the earliest and pioneering studies to address the design of deep beams with web openings was carried out by **Kong** and **Sharp**, **1977**. The study continued earlier pilot tests that had also been conducted by **Kong** and **Sharp**, **1973**, which focused on "Shear strength of lightweight reinforced concrete deep beams with web openings." The total beams tested in both studies were 56 deep beams with various beam and opening dimensions and different openings locations. At that time, no regulations within the codes of practice covered the design of deep beam with web opening. The researchers implicitly used the basics of the strut-and-tie model for the design. The final outcome made by **Kong** and **Sharp**, **1977**, was a modified shear strength formula and hints for the design of similar cases.

Following **Kong** and **sharp** several researches addressed the design of deep beams with web openings aiming to introduce a design method for such cases. Many of these researches used in some parts the load path method to bypass the existence of openings in the beam. The results of these assumptions proved that it was a good structural treatment for the presence of openings. After that, the load path treatment was developed to the strut-and-tie model approach as a simple yet a powerful design method. Among these researchers are **Kong et al., 1978**, **Kubik**, **1980**, **Mansur** and **Alwis**, **1984**, **Haque et al.**, **1986**, and **Schlaich et al.**, **1987**.

Several theoretical studies have been introduced later in order to address the use of strut-and-ties models in different structural members. However, a relatively few experimental verification tests have been made related to the implementation of the strut-and-ties models in various scenarios including tests carried out by **Maxwell** and **Breen**, **2000**, **Chen et al.**, **2002**, **Ley et al.**, **2007**, **Zhang** and **Tan**, **2007**, **Campione** and **Minafò, 2011**, **Arabzadeh et al.**, **2011**, **Nagrodzka-Godycka** and **Piotrkowski, 2012**, **He et al.**, **2012**, and **Tuchscherer et al.**, **2014**. Good amount of experimental verifications will support and encourage engineers to better utilize this simple method in many structural cases where the linear stress beam theory cannot be utilized. It was noted by reviewing the previous experimental tests that the web opening in most of these studies were placed relatively away from the loading source and closer to the supports or to the middle region of the beam. Therefore, it was intended herein to place two openings closer to loading area of beams tested to show that a good strut-and-tie model design can overcome the high stress distortions that occur between the loading point and the openings. The intent from this study is to introduce, test, and verify different untraditional structural problem to help increasing the number of experimental studies that ensure the capabilities of the strut-and–tie model to treat variety of structural scenarios.

2. SPECIMEN DETAILS

The test involved three identical small-scale deep beam specimens. The geometry of the beam is shown in **Fig. 1**. The specimen had a full length of 800mm and a depth of 250mm. The width of the beam was 100mm. The beam had two symmetrical 50mm by 50mm openings near the application of the load.

Regarding the size of the openings, there are no available limitations in the current codes of practice. Many researchers label the openings in their work as "small" or "large" openings without a clear definition of the size limits between both labels. **Mansur M. A., 1998,** related "small" and "large" openings definitions based on the beam response where the large openings result in Vierendeel action. Beams with large openings require special treatment.

The beam was simply supported with a span length of 600mm center-to-center of the support which makes the ratio of clear span to overall beam depth equals to 2.0. **ACI 318 code**, **2014**, classifies the beam as a deep beam if the aforementioned ratio is less than 4. Deep beams can either be designed by taking into consideration the nonlinearity of strain along the depth or by using the strut-and-tie model.

Steel bearing plates of 100mm by 100mm were provided at each support to provide the required bearing width and to simulate the bearing width provided by the columns or supporting girders. The beam was loaded at the mid span through a steel bearing plate of 100mm by 100mm.

When designed, the beam was intended to carry an ultimate concentrated load of 50kN at the middle of the beam. This load was supposed to be a combination of factored dead and live load according to the **ACI 318 code**, **2014**, with load factors equal to 1.2 for the dead load and 1.6 for the live load. The related service load could then be approximated by dividing the ultimate load by an average load factor of 1.4 for both dead and live load combined, **Maxwell and Breen**, **2000**, to obtain an equivalent service condition with load factor of 1.0 resulting in a related service load of 35.71 kN.

3. MATERIAL PROPERTIES

To accommodate the small-scale beam specimen, small reinforcing bar diameters and proper maximum size of coarse aggregate were adopted. Steel reinforcement and concrete used for this study were tested based on the test methods and procedures recommended by the **ASTM international** standards.

Two different reinforcing steel bars were used in this research;

- 4mm diameter deformed reinforcing bars with cross sectional area of 15.69 mm^2, and
- 6mm diameter deformed reinforcing bars with cross sectional area of 32.15 mm^2.

The reinforcing steel used in the study complies with **ASTM A615** specifications titled "*Standard Specification for Deformed and Plain Carbon Steel Bars for Concrete Reinforcement*." The results showed that Ø4mm deformed bars had an average yield strength of (457 MPa) and an ultimate strength of (606 MPa) while the average yield strength of Ø6mm deformed bars was (544 MPa) and the ultimate strength was (688 MPa).

Fine and coarse aggregate used in the study complies with **ASTM C33** titled *"Standard Specification for Concrete Aggregates*" including grading limits for both fine and coarse aggregate which was conducted in accordance with **ASTM C136** "*Standard Test Method for Sieve Analysis of Fine and Coarse Aggregates."*

Mix design was carried out based on the procedure provided by **ACI 211.1** titled *"Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete"* after gathering the required information from sieve analysis as well as mass densities of fine and coarse aggregates conducted in accordance with **ASTM C127** *"Standard Test Method for Density, Relative Density (Specific Gravity), and Absorption of Coarse Aggregate"* and **ASTM C128** *"Standard Test Method for Density, Relative Density (Specific Gravity), and Absorption of Fine Aggregate"* The cylinders that were taken at the day of casting the beams were tested according to **ASTM C39** *"Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens"* and showed that the concrete had an average specified compressive strength (f_c') of (29.6 MPa) at the age of 28 days resulting in modulus of elasticity equal to (25500 MPa) and modulus of rupture equal to (3.3 MPa).

4. STRUT-AND-TIE MODEL DESIGN

4.1 Concept

The design using the strut-and-tie model basically depends on visualizing the stress fields inside the structural member caused by the applied loads. These stress fields are then illustrated based on the truss analogy as a combination of struts, for compression stress fields, and ties, for tension stress fields. Struts and ties are assumed to be connected at nodes to form the complete geometry of the truss.

Different truss configurations can be drawn for the same loading conditions. The process of the design using the strut-and-tie model may involve some iterations to select the optimum truss for the given load. The optimum truss should be able to resist the designed factored load with a minimal use of reinforcement tie weight which ensures that the selected truss satisfies the strength requirement as well as the economic considerations. The final truss selected for design should not only satisfy the strength and economic requirements but also the safety needs regarding the failure type. In order to prevent brittle failure, a safe design can be accomplished by attempting to provide a design that allows the beam to deflect to a minimum deflection of (L/100) at failure, **Ley et al.**, **2007**. This ratio is usually considered among the structural engineers because it is generally thought to be close to the limit of deflection that is noticeable to the human eye.

4.2 Finite Element Implementation

As advised by **Schlaich et al.**, **1987**, finite elements model can be utilized to get a better understanding of the stress fields inside the unreinforced structural member. It has not been an obligation for the researchers to implement the use of finite element when designing using the strut-and-tie model as it is supposed to be an optional tool for a better design.

For this study, a two-dimensional finite element modeling was performed for the unreinforced specimen in order to get a better picture about the compression and tension field stresses in the beam which, in turns, could help configuring the outline of the truss. The two-dimensional finite element modeling was carried out using Abaqus FEA software.

Since it is a 2D FE Analysis, the element CPS4R was chosen to model the beam. CPS4R element is a 4-node bilinear plane stress quadrilateral, reduced integration, hourglass control**, Abaqus/CAE, 2011**. The beam was sketched with dimensions equal to 800mm in length and 250mm in depth. Two openings were drawn with dimensions of 50mm by 50mm and placed in its designated position in the tested beam shown in **Fig. 1**. As it is recommended by **Schlaich et al.**, **1987**, to model the unreinforced beam in the FE model to view the stress path in the concrete without the help of reinforcement, the only material that was fed to the program is concrete properties with a mass density of (2400 kg/m^3), Young's Modulus of (25500 MPa) and Poisson's Ratio of (0.15). A concentrated load was applied at the middle of the beam to simulate the loading case at the testing. The output of the major ranges of compression stress field and tension stress field are shown in **Figs. 2** and **3**.

4.3 Idealized Truss Model

After an extensive study and several iterations of different truss configurations, the idealized truss shown in **Fig. 4** was adopted. The selected truss complies with recommendations of the **ACI 318 code**, **2014**, found in Chapter 23 which is titled "The Strut-and-Tie Models."

The limitation of the **ACI 318 code**, **2014**, of providing a minimum angle of (25˚) between any strut and tie connecting at a single node was considered when designing the idealized truss layout. Also, the strengths of the ties, struts, and nodal zones of the idealized truss were checked to satisfy the following ACI criteria, **ACI 318 code**, **2014**.

$$\phi F_{ns} \geq F_{us} \quad (For\ Struts) \tag{1}$$

$$\phi F_{nt} \geq F_{ut} \quad (For\ Ties) \tag{2}$$

$$\phi F_{nn} \geq F_{us} \quad (For\ Nodal\ zones) \tag{3}$$

where F_{ns}, F_{nt}, and F_{nn} represent the nominal strength of struts, ties, and at nodal zones respectively while F_{us} and F_{ut} represent the factored compressive force in struts and tensile force in ties respectively. The strength reduction factor (ϕ) equals to 0.75 as specified in the **ACI 318 code**, **2014**.

The factored forces in each strut and tie were calculated after performing the analysis on the idealized truss as shown in **Fig. 4**. The nominal strengths of each strut, tie and node were calculated based on the criteria detailed in the **ACI 318 code**, **2014**. For each member of the truss, the inequalities of Eqs. (1) - (3) were checked and; hence, the designed truss should be able to transfer the concentrated load thought the truss to the supports.

4.4 Reınforcement Desıgn and Arrangement

Reinforcement layout and placing in the strut-and-tie model follow the idealized truss model. The reinforcement area is calculated according to the factored tie forces and placed according to the orientations and locations of the ties in the designed truss model.

The reinforcement was placed in three layers; a top layer that is located above the openings, a middle layer placed below the openings, and a bottom layer. According to the analysis performed on the idealized truss and based on the yield strength of the reinforcing steel used in this study, the following reinforcement was provided;

- For the top layer, four reinforcing bars of Ø4mm were provided.
- For the middle layer, three reinforcing bars of Ø4mm were provided.
- For the bottom layer, two reinforcing bars of Ø6mm were provided.

The reinforcement layout for the beam is shown in **Figs. 5(a)-(b)**. The anchorage lengths for the reinforcement were provided to ensure that the failure during the test would not occur due to the lack of anchorage of reinforcement.

It is important to mention that the design of deep beams using the strut-and-tie model does not require providing a minimum reinforcement ratio as per the recommendations of **ACI 318 code**, **2014**, for ordinary flexural members.

5. TEST ASSEMBLY

The test assembly was prepared as shown in **Fig. 6**. The test was carried out at the laboratories of College of Engineering / University of Baghdad. The test assembly consisted of:

- The examined deep beam specimen.
- Two supports; each connected to a bearing plate of 100x100mm to provide the required bearing width, that was checked throughout the design of the idealized truss model, and to simulate the bearing width provided by the columns or supporting girders.
- Hydraulic loading system equipped with a hydraulic shaft to deliver the desired load to the tested beam through a bearing plate with dimensions of 100x100mm.
- Load cell with a loading capacity of 20kN connected to a digital load indicator.
- Digital dial gauge to record the vertical deflections at the mid span of the beam specimens.
- Strain gauges and a digital strain indicator that is connected to a computer supplied with computer software that is designed to read and record strain readings from both concrete and steel reinforcement.

6. TEST RESULTS AND CRACK PATTERNS

The test performed on the three identical deep beams showed comparable results since they were identical in almost every detail. The slight difference between the results of the specimens is normal and due to the inherent randomness in material properties and testing environment for all three specimens.

The results of the test conducted on the beams showed that the beams designed using the strut-and –tie model were able to withstand concentrated loads higher than the designed ultimate load which was in the range of 50kN.

The vertical load-deflection curves of the three beam specimens show almost a linear relation between the applied vertical load and the vertical deflection at the mid span of the beam (see **Fig. 7**). The test carried out on the first sample showed that the beam resisted an ultimate load of 58.60kN before failure with an increase of 17% in the designed ultimate load. The second specimen was able to carry an ultimate load of 62.40kN before it failed resulting in an increase in the designed ultimate load of approximately 25%. The test on the third beam showed that this sample resisted before failure an ultimate load of 60.70kN with an increase of approximately 21% in the designed ultimate load. **Fig. 8** summarizes the percent of increase in the designed ultimate load for each specimen during the test. The average ultimate load obtained from the test of the three beam specimens was 60.57kN with an average increase of 21% in the designed ultimate load obtained from the strut-and-tie modeling technique.

For all specimens, the initial crack appeared at the lower center of the beam which indicates a flexural action at the service stage of loading. The initial crack at the three specimens occurred at a load approximately equal to 35kN which is close to the specified service load mentioned earlier.

As the load was being increased towards the ultimate load, diagonal cracks started to develop in the vicinity of the supports propagating from the left and right supports approaching the openings, which reflects shear action dominance at this level of loading. When the load was increased so that it became in the neighborhood of the ultimate load, the diagonal cracks kept propagating towards the openings causing the beam specimens to fail at loadings equal to 58.60kN, 62.40kN, and 60.70kN for beam specimens one, two, and three respectively. Initial crack location and general schematic crack pattern for the beam specimens are presented in **Figs. 9-10**.

The deflection recorded at the ultimate load for the first beam specimen was 5.78mm. For the second beam, the deflection measured at failure was 6.33mm. A deflection of 5.45mm was recorded for the third beam specimen. The average deflection of the three specimens was 5.85mm which is close to the deflection obtained from the ratio L/100 where the deflection is considered noticeable.

7. CONCLUSIONS AND RECOMMENDATIONS

The results of this study affirm that the strut-and-tie model is a plasticity method that is based on the lower bound theorem. Some previous experimental researches reported an increase in the ultimate strength of beams designed using the strut-and-tie method by ratios ranging between (0.09% -0.28%), **Chen et al.**, **2002**, while other researchers reported an increase in the ultimate strength up to 95% more than the designed ultimate load, **Maxwell** and **Breen**, **2000**. The average increase of the ultimate strength of the beams studied herein was 21% more than the designed ultimate load.

The strut-and-tie model has proven throughout this study and previous studies that it is a useful, safe, and simple tool to handle untraditional and complicated problems provided that all the limitations recommended by the **ACI-318, 2014**, code or the selected practice code are to be taken into consideration throughout the design process. Successful implementation of the strut-and-tie model also requires providing a good design for the idealized truss which may involve several iterations to achieve the best truss layout. It is worthwhile to mention, however, that the bending and placement of the reinforcement for members designed using the strut-and-tie method is usually more time-consuming than the traditional reinforcement constructions.

The good choice for the idealized truss model in this study successfully helped the beam overcome the diagonal shear forces until the beam reached satisfactory ultimate loads compared to the designed load. The existence of the inclined reinforcement above and below the openings restrained the propagations of the diagonal cracks and prevented the premature failure. The model was also able to deal with the distorted region existed due to the presence of two openings near the loading source which, in turns, prevented any concrete spalling in this region.

For the future studies, it may be useful to investigate the following cases:

- Perform and suggest different strut-and-ties models for the same loading and beam geometry. Then, conduct an experimental verification to ensure the ability of the strut-and-tie model to offer different adequate designs for the same scenario.
- Conduct a larger-scale experimental test for the same model in this research to investigate whether or not scaling the model will affect the percent of increase in the ultimate load-carrying capacity of the beams designed using the strut-and-tie model.

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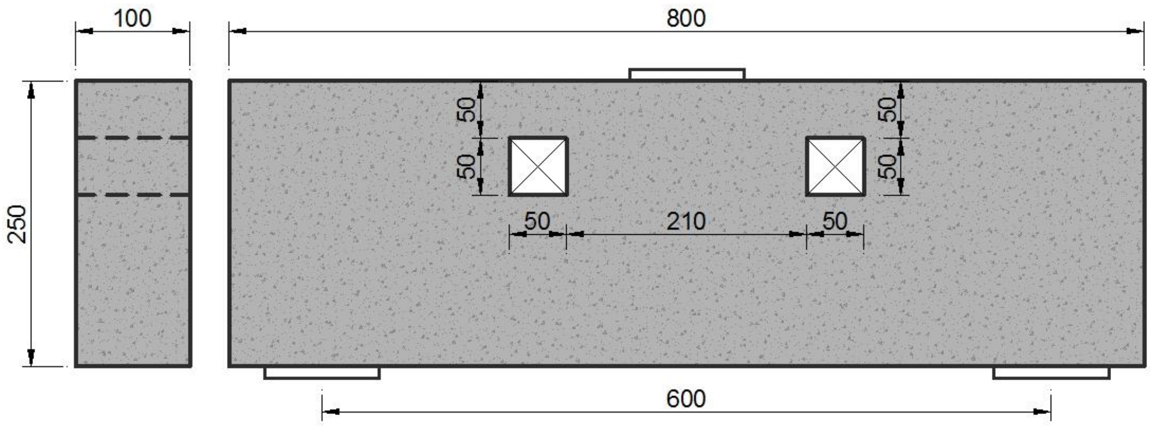


Figure 1. Beam geometry (dimensions are in mm).

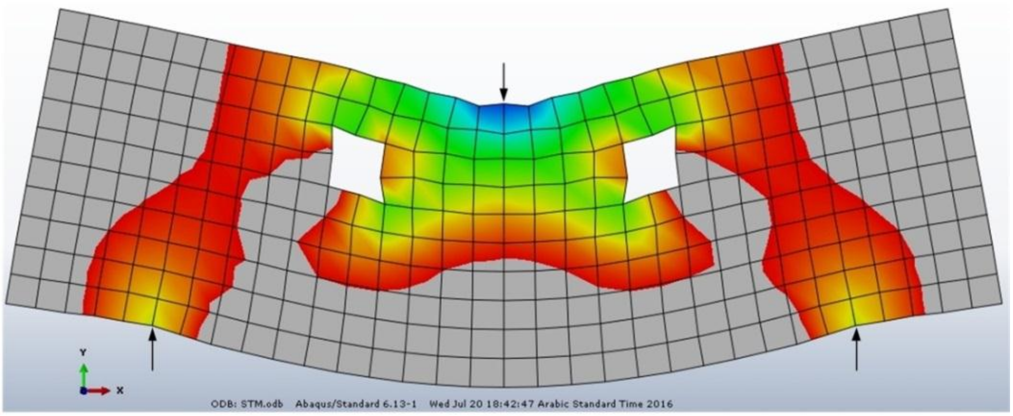


Figure 2. Compression stress field in FE model of the unreinforced beam specimen.

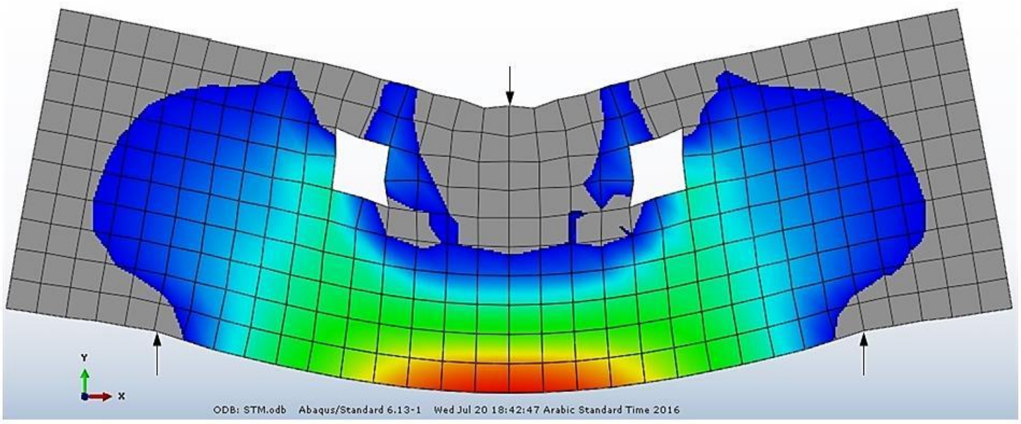


Figure 3. Tension stress field in FE model of the unreinforced beam specimen.

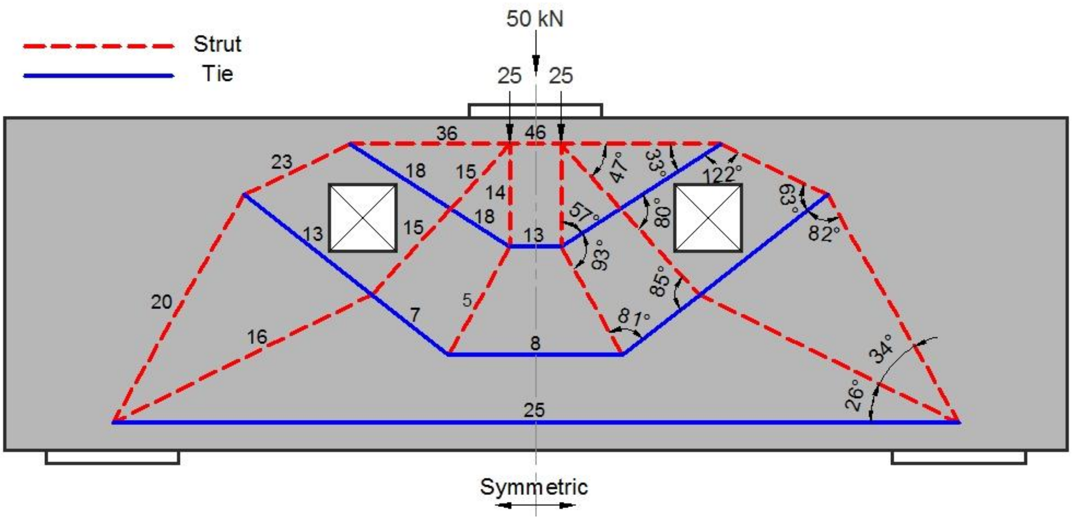
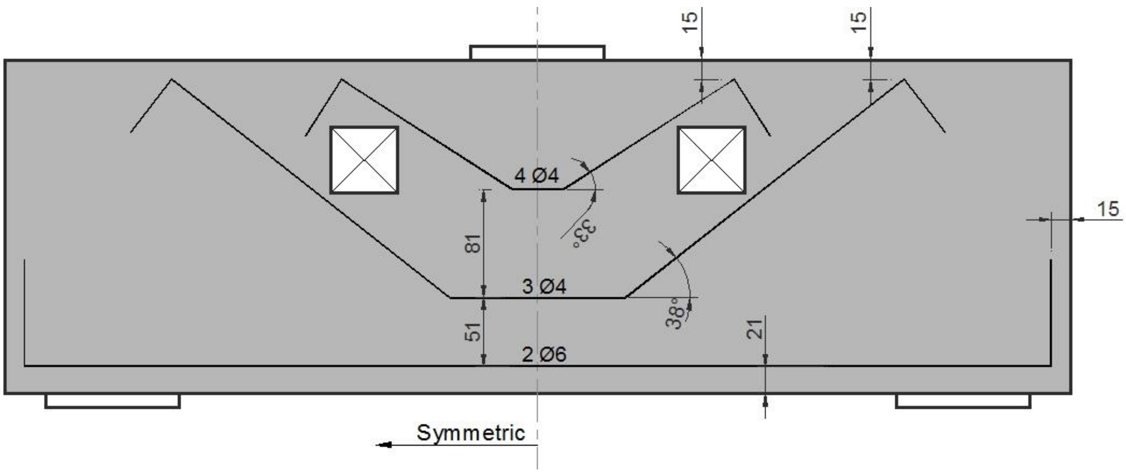
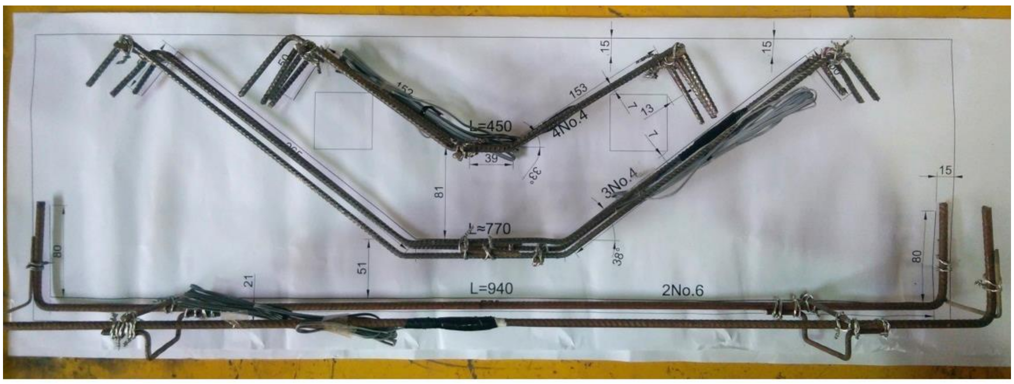


Figure 4. Idealized truss model and factored forces (forces are in kN).



(a) Reinforcement design



(b) Reinforcement construction

Figure 5. Reinforcement layout and construction (dimensions are in mm).

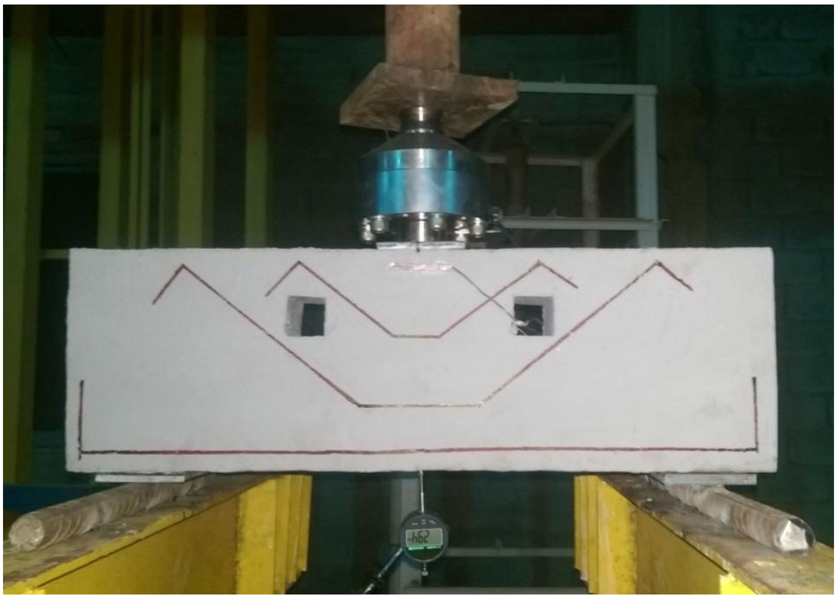


Figure 6. Test assembly.

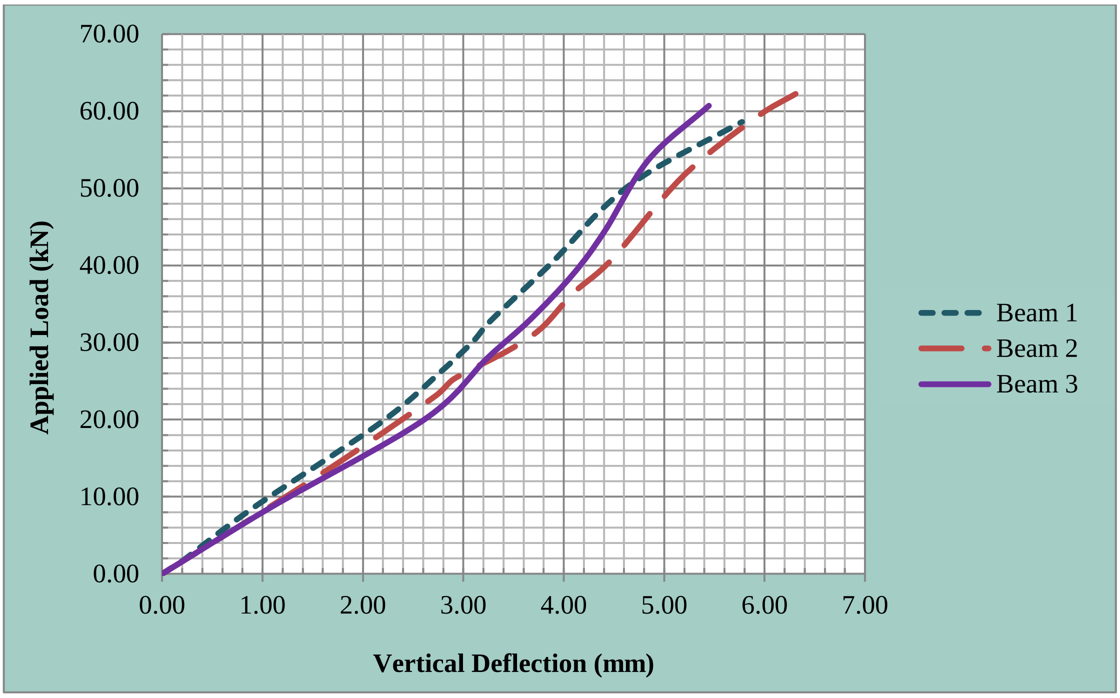


Figure 7. Vertical load-deflection for the tested beams.

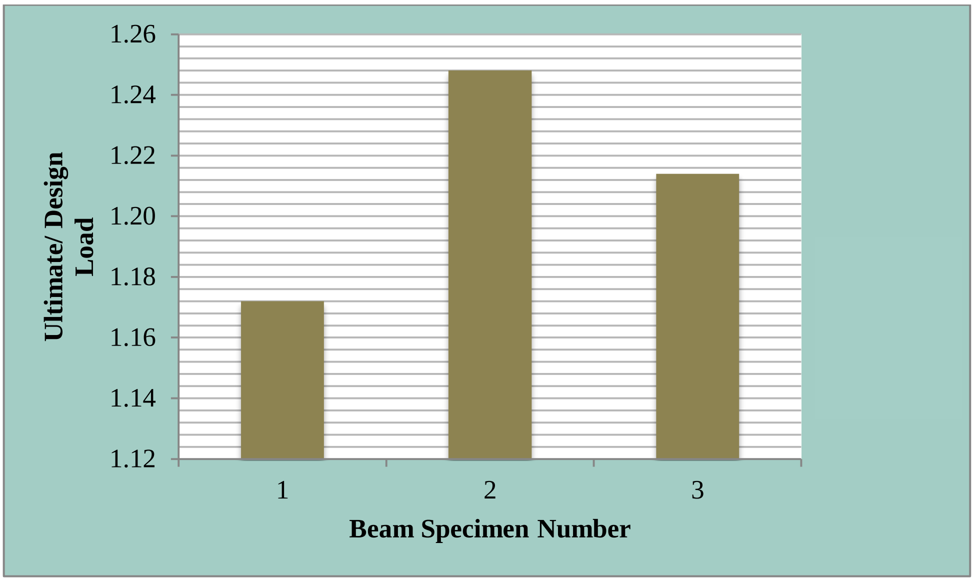


Figure 8. Percent of increase in the ultimate strength with respect to the design load.

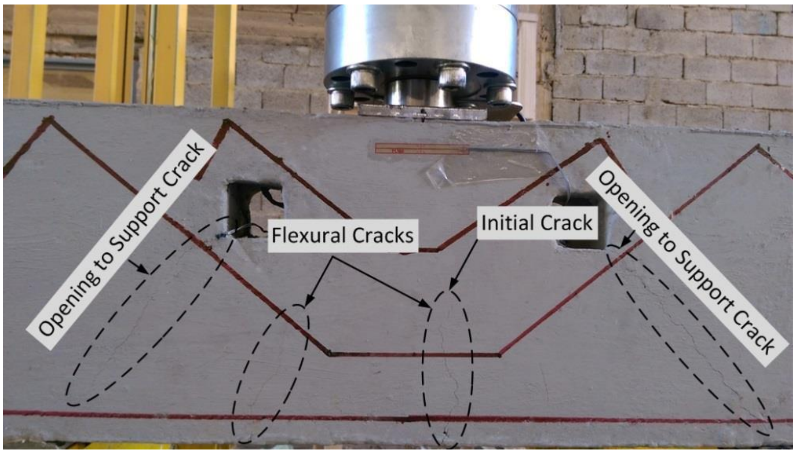


Figure 9. Initial and diagonal cracks captured during the test.

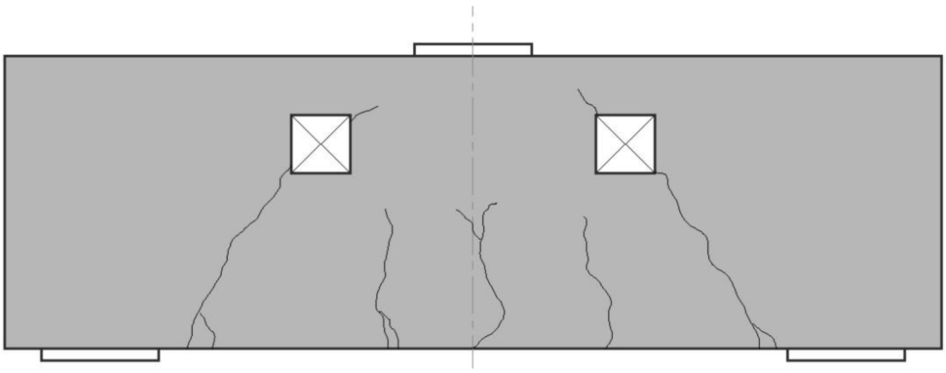


Figure 10. General crack pattern for the tested beams.