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Analysis of Traditional and Fuzzy Quality Control Charts to Improve Short-Run Production in the Manufacturing Industry

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ABSTRACT

Quality control charts are limited to controlling one characteristic of a production process, and it needs a large amount of data to determine control limits to control the process. Another limitation of the traditional control chart is that it doesn't deal with the vague data environment. The fuzzy control charts work with the uncertainty that exists in the data. Also, the fuzzy control charts investigate the random variations found between the samples. In modern industries, productivity is often of different designs and a small volume that depends on the market need for demand (short-run production) implemented in the same type of machines to the production units. In such cases, it is difficult to determine the control limits for the operations carried out on the same machines. This work aims to compare the traditional control charts and the fuzzy control charts for short-run production. In the traditional case, the data collected were processed using the (Minitab 21) software. It was found that the fuzzy control charts were more flexible and accurate in determining the control limits of the machine under study. The traditional deviation from nominal control charts showed false alarm of observation (15) as out-of-control, while the fuzzy (DNOM) showed that these observations were under control. Also, the standard deviation of the process was dropped from (σ =0.209041) to (σ =0.204401) after using the fuzzy control chart.

Keywords: Short run production, Traditional control charts, Statistical quality control, Fuzzy logic.

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تحليل مخططات مراقبة الجودة التقليدية والغامضة لتحسين الإنتاج على المدى القصير في الصناعات التحويلية

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الخلاصة

لوحات السيطرة النوعية تكون محدودة للسيطرة على خاصية واحدة من عملية انتاجية، وتحتاج الى حجم كبير من البيانات لغرض تحديد حدود الضبط للسيطرة على العملية. في الصناعات الحديثة غالبا ما يكون الانتاج ذو تصاميم مختلفة وحجم قليل يعتمد على حاجة السوق للطلب (دورة انتاج قصيرة) تنفذ على نفس النوع من المكائن للوحدات ألإنتاجية. في مثل هذه الحالات يصعب تحديد حدود الضبط للعمليات المنفذة على نفس المكائن. ومن المحددات الأخرى للوحات الضبط التقليدية انها لا تتعامل مع البيانات الغامضة. لوحات السيطرة الضبابية تتعامل مع عدم الدقة في البيانات. أيضا تتعامل لوحات الضبط الضبابية مع التغيرات العشوائية بين العينات. يهدف هذا العمل الى المقارنة بين لوحات الضبط التقليدية ولوحات الضبط الضبابية لدورة الانتاج القصير . البيانات التي تم جمعها عولجت باستخدام برنامج (Minitab 21) في الحالة التقليدية. بينما تم استخدام برنامج (excel 21) في تضبيب البيانات وتم استخدام برنامج (Minitab 21) في رسم لوحات السيطرة الضبابية لغرض المقارنة. ووجد ان لوحات الضبط الضبابية كانت اكثر مرونة واكثر دقة في حساب حدود الضبط للماكنة قيد الدراسة. أظهرت لوحات السيطرة التقليدية انذارا خاطئا بخروج العينة (15) خارج حدود الضبط. بينما اظهرت لوحات الضبط الضبابية ان هذه البيانات كانت تحت السيطرة. كما تم تخفيض الانحراف المعياري للعملية من (σ =0.209041) إلى (σ =0.204401) بعد استخدام لوحات السيطرة الضبابية.

الكلمات الرئيسية: أنتاج الدورة القصيرة، لوحات السيطرة التقليدية، ضبط الجودة الإحصائي، المنطق الضبابي.

1. INTRODUCTION

A control chart's primary purpose is to monitor a process and assess whether or not it is under control. When a process is "under control," it generates components with low variation and a target value that is relatively close to it. Conditions that are "out of control" occur when there is some identifiable reason, and as a result, the process produces products that are either too different from the target value, have too much variation, or both **(Fonseca et al., 2007)**. The control chart consists of three parallel lines: an inner line, known as the Center Line, and two outer lines, known as the Upper Control Limit (UCL) and Lower Control Limit (LCL) (CL). When the process is stable, the control limits are computed to have a high likelihood of containing the sample data between them, while the CL represents the average of the data **(Al-Khafaji et al., 2012)**. Operations research, control theory, management sciences, and other domains have all used the fuzzy sets theory **(Hassan et al., 2012)**. The control chart is one of the key methods used in the statistical control of a process **(Montgomery, 2013)**. To stabilize a process and increase capacity by minimizing variance, statistical quality control is a powerful set of effective problem-solving methods **(Al Obeidy et al., 2018).** The analysis of variance in repetitive operations can be effectively done with control charts **(Alwan et al., 2018).** Recently, fuzzy logic has been used in industrial statistics, specifically when the data are ambiguous. The standard control chart has a significant difficulty due to the data's confusion or uncertainty. In its capacity, fuzzy set theory handles ambiguity in data **(Akeem, 2018).**

Customers' needs drive market demand, which drives businesses to modify product quantity and design while maintaining the same manufacturing capacity. Elements have This flexibility in design is referred to as the flexible factory. Because of the variety of operations and the short-term, the batch production process in this factory is insufficient to provide enough data to establish a control chart. This kind of issue can be resolved by identifying an appropriate methodology for the design chart to modify the effectiveness of the monitoring process **(Alwan, 2018).** There are many control charts; the most widely used is the variable control charts **(Ahmed, 2019).** The performance of conventional control charts has been improved by using a fuzzy approach in control chart design. It has also made it possible to design control charts for linguistic variables with multinomial distributions using a straightforward method in univariate and multivariate cases. **(Razali et al., 2020)**. The company strives to provide goods that are free of defects and in accordance with requirements **(Thamer et al., 2021).** Fuzzy control charts can be produced by converting linguistic variables into fuzzy numbers or utilizing them straight without any transformation. It refers to a connected array of potential values rather than a single value; a fuzzy number is a generalization of a regular real number. (**Rodríguez-Alvarez et al., 2021)**. Research in sociology, medicine, engineering, economics, services, and management has frequently used fuzzy control charts. The fuzzy set theory can deal with fuzzy data systematically **(Razali et al., 2021)**. Short-run production, often known as "Low Volume Manufacturing," has become more common in the industrial sector in recent years. In such a scenario, the length of the production process is brief; typically, there are fewer than 50 productions. Hence, the short-run production process needs a control chart **(Qori'atunnadyah et al., 2021)**. Businesses today must develop new ways to produce and provide value to customers to survive the economy's fierce competition **(Mitlif, 2023)**.

The objectives of this work are to define fuzzy theory and fuzzy control charts, Define the deviation from the nominal dimension (DNOM) method, Construct a traditional DNOM $\bar{X}$-R control chart, Construct a fuzzy DNOM $\bar{X}$-R control chart, and Study the difference between the results of traditional and fuzzy control charts.

2. METHODS AND MATERIALS

One of the most essential methods for ensuring statistical quality is the quality control charts. **(Ahmed, 2019)**. This work presents a combination of two methods; the first is the deviation from the nominal dimension method that makes it possible to study different designs on the same machine. The second method is the fuzzy control chart, which deals with the uncertainty in the data environment and the random variations between the samples as it takes the left and right-hand sides for each observation.

2.1 Traditional $\bar{X}$ - R Control Chart

The formulation of traditional $\bar{X}$ control charts based on sample ranges are given as follows: **(Montgomery, 2013).**

$\overline{X}$ Control chart:

$UCL=\overline{\overline{X}}+A_2\overline{R}$ (1)

$CL=\overline{\overline{X}}$ (2)

$LCL=\overline{\overline{X}}-A_2\overline{R}$ (3)

R control chart:

$UCL=D_4\overline{R}$ (4)

$CL=\overline{R}$ (5)

$LCL=D_3\overline{R}$ (6)

where
$\overline{X}$ is the mean of the samples.
R is the range of the sample, and UCL is the upper control limit.
CL is the center line.
LCL is the lower control limit, A_2, D_3, D_4 is a control chart coefficient
$\overline{R}$ is the average of R_i's that are the ranges of samples **(Montgomery, 2013).**

2.2 Short Run Control Charts

One of the specific options for observing small-scale production, such as that found in lean manufacturing contexts, is the Deviation from the Nominal (DNOM) control chart. Since the DNOM control chart is simple, it is the most advised for tracking small batches **(Meiraa et al., 2022)**. Huge sample sizes are typical of mass production, and building a control chart is not hard. Smaller batch sizes or short production runs for flexible production employing a workshop method are the current trends in manufacturing. Consequently, some adjustments to standard control charts are necessary **(Alwan and Ahmed, 2018).** DNOM method can be represented by the target dimension chart, using: **(Alwan and Ahmed, 2018)**

$X_i=M_i-T_A$ (7)

where:
X_i is the Number of Deviations from Nominal.
M_i is The actual sample measurement.
T_A is the Target value of the Process.

2.3 Fuzzy $\overline{X}$ -R Control Chart:

Depending on the results of digital data that can be determined, the organization's production process is either under control or out of control, but in many instances, that limit cannot be correctly measured. As a result, the fuzzy-set theory was used to handle uncertainty and accuracy caused by human error, measurement errors, and environmental factors. In contrast to standard control charts, this was accomplished by transforming the numerical control boundaries to fuzzy control boundaries, utilizing fuzzy logic to make the

limits accurate and more flexible values to make accurate choices in the production process **(Hamada et al., 2020).**
Each triangular fuzzy number (a, b, c) The fuzzy control chart's sample or subgroup (n). The fuzzy triangle-shaped numbers are given by (X_a, X_b, X_c) for every fuzzily seen. Each finding was fuzzified as a triangular fuzzy number taking process variations into account, as in **Table 1. (Dilipkumar and Nanthakumar, 2019). Fig. 1** shows the graph for the sample's transformation from a crisp set to a fuzzy set.

Table 1. Fuzzification of data **(Dilipkumar and Nanthakumar, 2019)**

X_a (left)	X_b	X_c (Right)
X_b-(0-1.2) % ×X_b	X_b	X_c+(0-1) % ×X_b

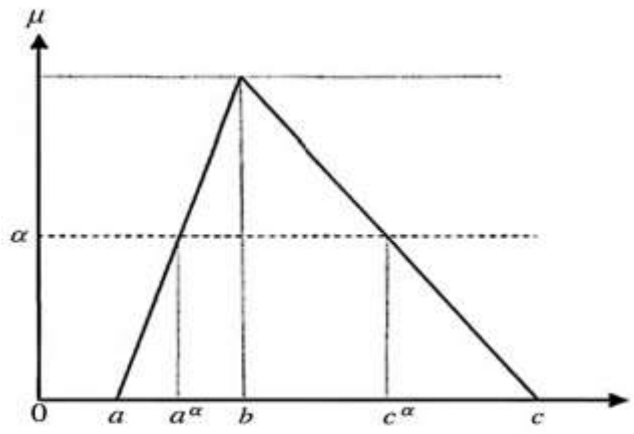

Figure 1. Graph Sample's Transformation Using Triangular Fuzzy Numbers **(Hamada et al., 2020)**

Fuzzy $\bar{X}$ -R control chart limits can be obtained in a similar way to traditional R control charts, but they are represented by fuzzy triangular numbers as follows **(Basri et al., 2016)**

$$\widetilde{UCL}_x = \widetilde{CL} + A_2 \bar{R} = (\bar{\bar{x}}_a, \bar{\bar{x}}_b, \bar{\bar{x}}_c) + A_2(\bar{R}_a, \bar{R}_b, \bar{R}_c) = \widetilde{UCL}_1, \widetilde{UCL}_2, \widetilde{UCL}_3 \quad (8)$$

$$\widetilde{CL}_x = (\bar{\bar{x}}_a, \bar{\bar{x}}_b, \bar{\bar{x}}_c) = \widetilde{CL}_1, \widetilde{CL}_2, \widetilde{CL}_3 \quad (9)$$

$$\widetilde{LCL}_x = \widetilde{CL} - A_2 \bar{R} = (\bar{\bar{x}}_a, \bar{\bar{x}}_b, \bar{\bar{x}}_c) - A_2(\bar{R}_a, \bar{R}_b, \bar{R}_c) = \widetilde{LCL}_1, \widetilde{LCL}_2, \widetilde{LCL}_3 \quad (10)$$

where $\bar{R}_a$, $\bar{R}_b$ and $\bar{R}_c$ are the arithmetic means of the least possible, most possible, and largest possible values, respectively. Firstly, R_{aj}, R_{bj}, R_{cj} are calculated as follows: **(Basri et al., 2016)**

$$R_{aj} = X_{max;aj} - X_{min;cj}, \quad (11)$$

$$R_{bj} = X_{max;bj}\, X_{min;bj} \quad (12)$$

and $R_{cj} = X_{max;cj}\, X_{min;aj}$; j = 1, 2, . . . , m. (13)

where $X_{max;aj}$; $X_{max;bj}$; $X_{max;cj}$ is the maximum fuzzy number in the sample, and $X_{min;aj}$; $X_{min;bj}$; $X_{min;cj}$ is the minimum fuzzy number in the sample. Then

$$\bar{R}_a=\sum R_{a,j}/m\ ,\ \bar{R}_b=\sum R_{b,j}/m\ ,\ \bar{R}_c=\sum R_{c,j}/m \quad (14)$$

For R chart, the control limits given by **(Basri et al., 2016):**

$$\widetilde{UCL}_R=D_4\bar{R}=D_4(\bar{R}_a, \bar{R}_b, \bar{R}_c) \quad (15)$$

$$\widetilde{CL}_R=(\bar{R}_a, \bar{R}_b, \bar{R}_c) \quad (16)$$

$$L\widetilde{CL}_R=D_3\bar{R}=D_3\overline{(R}_a, \bar{R}_b, \bar{R}_c) \quad (17)$$

2.4 α-cut Fuzzy Control Chart Formulation:

The α-cut level applies to all elements in non-fuzzy sets with membership greater than or equal to (α). Applying α-cuts of fuzzy sets, the values of $\overline{\overline{X_a^\alpha}}$ and $\bar{\bar{X}}_c^\alpha$ are determined as follows **(Basri et al., 2016).**

$$\overline{\overline{X_a^\alpha}}= \bar{\bar{x}}_a+\alpha\ (\bar{\bar{x}}_b - \bar{\bar{x}}_a) \quad (18)$$

$$\bar{\bar{X}}_c^\alpha = \bar{\bar{x}}_c +\alpha(\bar{\bar{x}}_c-\bar{\bar{x}}_b) \quad (19)$$

When applying the Fuzzy Cut Level on the control charts ($\bar{X}$), the main control limits according to the levels (UCL, CL, LCL) are as follows: **(Hamada et al., 2020)**

$$\widetilde{UCL}^\alpha =(\bar{\bar{X}}_a^\alpha, \bar{\bar{x}}_b, \bar{\bar{X}}_c^\alpha) +A_2(\bar{R}_a^\alpha,\bar{R}_b,\bar{R}_c^\alpha) = \widetilde{UCL}_1^\alpha, \widetilde{UCL}_2^\alpha, \widetilde{UCL}_3^\alpha \quad (20)$$

$$\widetilde{CL}^\alpha=(\bar{\bar{X}}_a^\alpha, \bar{\bar{x}}_b, \bar{\bar{X}}_c^\alpha) = \widetilde{CL}^\alpha_1, \widetilde{CL}^\alpha_2, \widetilde{CL}^\alpha_3 \quad (21)$$

$$L\widetilde{CL}^\alpha= (\bar{\bar{X}}_a^\alpha, \bar{\bar{x}}_b, \bar{\bar{X}}_c^\alpha) +A_2(\bar{R}_a^\alpha,\bar{R}_b,\bar{R}_c^\alpha) =L\widetilde{CL}^\alpha_1, L\widetilde{CL}^\alpha_2, L\widetilde{CL}^\alpha_3 \quad (22)$$

where:

$$\bar{R}_a^\alpha =\bar{R}_a+\alpha\ (\bar{R}_b-\bar{R}_a) \quad (23)$$

$$\bar{R}_c^\alpha =\bar{R}_c+\alpha(\bar{R}_c-\bar{R}_b) \quad (24)$$

Similar to the $\bar{x}$ control chart, an α-cut or control limits can be stated as follows: **(Basri et al., 2016)**

$$\widetilde{UCL}_R^\alpha=D_4\bar{R}^\alpha=D_4(\bar{R}_a^\alpha\ , \bar{R}_b^\alpha, \bar{R}_c^\alpha) \quad (25)$$

$$\widetilde{CL}_R^\alpha=(\bar{R}_a^\alpha\ , \bar{R}_b^\alpha, \bar{R}_c^\alpha) \quad (26)$$

$$L\widetilde{CL}_R^\alpha=D_3\bar{R}^\alpha=D_3\overline{(R}_a^\alpha\ , \bar{R}_b^\alpha, \bar{R}_c^\alpha) \quad (27)$$

2.5 Fuzzy Transformation Approach

Generally, there are four ways of fuzzy transformation, including α-level fuzzy midrange, fuzzy average, fuzzy median, and fuzzy mode, to characterize any given observation's average (central tendency). However, we'll employ the α -level fuzzy midrange in this study as a transformation technique, the fuzzy. $\bar{X}$–R control uses fuzzy triangular numbers **(Akeem, 2018).**

2.5.1 α-Level Fuzzy Midrange for α-cut Fuzzy $\bar{X}$ Control Chart Based on Ranges

One of the four transformation methods used to establish the fuzzy control limits is α-Level fuzzy midrange. These control limits determine if a process is in control or out of control. In this work, the fuzzy transformation method for calculating the control limits is α-level fuzzy midrange **(Dilipkumar, Nanthakumar, 2019).**

$$\widetilde{UCL}^{\alpha}_{mr,x}=\widetilde{CL}^{\alpha}_{mr,x}+A_2\left(\frac{\overline{Ra}\alpha+\bar{R}c\alpha}{2}\right) \quad (28)$$

$$\widetilde{CL}^{\alpha}_{mr,x}=\frac{\widetilde{CL}\alpha1+\widetilde{CL}\alpha3}{2} \quad (29)$$

$$\widetilde{LCL}^{\alpha}_{mr,x}=\widetilde{CL}^{\alpha}_{mr,x}-A_2\left(\frac{\overline{Ra}\alpha+\bar{R}c\alpha}{2}\right) \quad (30)$$

The value of the sample j's α-level fuzzy midpoint for the fuzzy $\bar{X}$ the control chart is **(Basri et al., 2016)**

$$S^{\alpha}_{mr\text{-}x,j}=\frac{(\overline{Xa}j+\overline{Xc}j)+\alpha((\overline{Xb}j+\overline{Xa}j)-(\overline{Xc}j+\overline{Xb}j))}{2} \quad (31)$$

2.5.2 α-Level Fuzzy Midrange for α-cut Fuzzy R Control Chart

Control limits of α-level fuzzy midrange for α-cut fuzzy R control chart can be calculated as follows: **(Basri et al., 2016)**

$$\widetilde{UCL}^{\alpha}_{mr,R}= D_4\,\widetilde{CL}^{\alpha}_{mr,R} \quad (32)$$

$$\widetilde{CL}^{\alpha}_{mr,R}=\frac{\widetilde{CL}\alpha1+\widetilde{CL}\alpha3}{2}=\frac{\overline{Ra}\alpha+\bar{R}c\alpha}{2} \quad (33)$$

$$\widetilde{LCL}^{\alpha}_{mr,R}= D_3\widetilde{CL}^{\alpha}_{mr,R} \quad (34)$$

The value of the sample j's α-level fuzzy midpoint for the fuzzy R control chart is: **(Basri et al., 2016)**

$$S^{\alpha}_{mr\text{-}R,j}=\frac{(\overline{Ra}j+\overline{Rc}j)+\alpha((\overline{Rb}j+\overline{Ra}j)-(\overline{Rc}j+\overline{Rb}j))}{2} \quad (35)$$

3. CASE STUDY

To apply the research methodology as well as achieve the aims, data were selected from previous research **(Alwan, 2018)**. The sample of a gas cylinder neck is shown in **Fig. 2**. This section is produced on a turning machine **(Alwan and Ahmed, 2018).** Four stages to manufacture this part are followed, these are:

Stage 1: Cutting Process
Cutting raw materials with a reciprocating saw to the length (28mm) is the first step in the technical path.

Stage 2: Drilling Process
The raw material is loaded following the cutting stage, and the manual lathe machine is set

up. The data collection for this diameter is shown in this stage's drilling diameter (22.5mm).

Stage 3: Face-Turning Process
Workpiece length decreases by a face from (28mm) to (26mm).

Stage 4: External Turning Process
In this phase, a workpiece was subjected to three overlapping operations to obtain the dimensions [diameter (45 mm), length (16mm)], and angle (this angle 8° was measured by a Profile Projector. **Table 2** includes the data collected from these four processes.

Figure 2 Gas cylinder neck sample **(Alwan and Ahmed, 2018)**

4. DATA REDUCTION

After collecting the data for the four processes, deviation from the nominal dimension was done using Eq. (7) by subtracting each process's nominal dimension from the characteristic's measured value. The range and mean of each sample are calculated and listed in **Table 3.** Traditional $\bar{X}$-R control chart was drawn using Minitab 21 software, as shown in **Fig. 3**. The control charts obtained by minitab21 showed that three observations were out of control. The samples (4, 14, and 15) refer to drilling and external turning diameter processes. The out-of-control samples are eliminated, and the final control limits are obtained, as presented in **Fig. 3**.

Table 2. Data collected for four processes

	.N	X1	X2	X3	X4	X5
Drilling process 22.5mm	1	22.05	22.45	22.27	22.43	22.39
	2	22.52	22.39	22.3	22.31	22.06
	3	22.14	22.08	22.48	22.09	22.43
	4	22.04	22.52	23.12	22.51	22.04
	5	22.25	22.07	22.02	22.14	22.08
	6	22.28	22.52	22.36	22.47	22.07
	7	22.1	22.02	22.45	22.19	22.15
Face turning to length 26 mm	8	25.9	25.1	25.98	26	25.55
	9	25.96	25.99	25.84	25.61	25.6
	10	25.55	25.88	25.27	25.52	25.41
	11	25.4	25.79	25.18	25.9	25.64
	12	25.9	25.88	25.52	25.76	25.86
	13	25.61	25.13	25.87	25.94	25.03

External turning to the diameter 45mm	14	44.08	44.92	45.43	44.79	45.02
	15	44.97	45.01	44.95	44.72	44.96
	16	44.93	44.95	45.01	45.09	44.77
	17	45.18	45.12	44.85	45.03	45.02
	18	44.98	44.74	44.84	44.93	44.97
	19	45.04	44.89	45.01	44.81	44.93
External turning to length 16mm	20	15.88	15.49	15.29	15.7	15.84
	21	16	15.9	15.8	15.71	15.98
	22	15.75	15.69	15.68	15.95	15.98
	23	15.9	15.54	15.55	15.88	16
	24	15.76	16	15.63	15.65	16.02
	25	15.82	16.05	15.22	15.48	16.01

Table 3. Calculations of short run

	S.N	X_1	X_2	X_3	X_4	X_5	$\overline{X}$	R
Drilling process 22.5mm	1	-0.45	-0.05	-0.23	-0.07	-0.11	-0.182	0.4
	2	0.02	-0.11	-0.2	-0.19	-0.44	-0.184	0.46
	3	-0.36	-0.42	-0.02	-0.41	-0.07	-0.256	0.4
	4	-0.46	0.02	0.62	0.01	-0.46	-0.054	1.08
	5	-0.25	-0.43	-0.48	-0.36	-0.42	-0.388	0.23
	6	-0.22	0.02	-0.14	-0.03	-0.43	-0.16	0.45
	7	-0.4	-0.48	-0.05	-0.31	-0.35	-0.318	0.43
Face turning to length 26 mm	8	-0.1	-0.9	-0.02	0	-0.45	-0.294	0.9
	9	-0.04	-0.01	-0.16	-0.39	-0.4	-0.2	0.39
	10	-0.45	-0.12	-0.73	-0.48	-0.59	-0.474	0.61
	11	-0.6	-0.21	-0.82	-0.1	-0.36	-0.418	0.72
	12	-0.1	-0.12	-0.48	-0.24	-0.14	-0.216	0.38
	13	-0.39	-0.87	-0.13	-0.06	-0.97	-0.484	0.91
External turning to the diameter 45mm	14	-0.92	-0.08	0.43	-0.21	0.02	-0.152	1.35
	15	-0.03	0.01	-0.05	-0.28	-0.04	-0.078	0.29
	16	-0.07	-0.05	0.01	0.09	-0.23	-0.05	0.32
	17	0.18	0.12	-0.15	0.03	0.02	0.04	0.33
	18	-0.02	-0.26	-0.16	-0.07	-0.03	-0.108	0.24
	19	0.04	-0.11	0.01	-0.19	-0.07	-0.064	0.23
External turning to length 16mm	20	-0.12	-0.51	-0.71	-0.3	-0.16	-0.36	0.59
	21	0	-0.1	-0.2	-0.29	-0.02	-0.122	0.29
	22	-0.25	-0.31	-0.32	-0.05	-0.02	-0.19	0.3
	23	-0.1	-0.46	-0.45	-0.12	0	-0.226	0.46
	24	-0.24	0	-0.37	-0.35	0.02	-0.188	0.39
	25	-0.18	0.05	-0.78	-0.52	0.01	-0.284	0.83
							$\overline{\overline{X}}$	$\overline{R}$
							0.2164	0.5192

Table 4. Deviation from nominal dimension for fuzzy numbers

		X_1			X_2			X_3			X_4			X_5		
	S.N	A	B	c	A	B	C	A	B	c	A	b	C	A	b	c
Drilling process to diameter (22.5 mm)	1	-0.671	-0.45	-0.229	-0.275	-0.05	0.175	-0.453	-0.23	-0.007	-0.294	-0.07	0.154	-0.334	-0.11	0.114
	2	-0.206	0.02	0.245	-0.333	-0.11	0.114	-0.423	-0.2	0.023	-0.413	-0.19	0.033	-0.661	-0.44	-0.219
	3	-0.581	-0.36	-0.139	-0.641	-0.42	-0.199	-0.245	-0.02	0.205	-0.631	-0.41	-0.189	-0.294	-0.07	0.154
	4	-0.680	-0.46	-0.239	-0.205	0.02	0.245	0.389	0.62	0.851	-0.215	0.01	0.235	-0.680	-0.46	-0.239
	5	-0.473	-0.25	-0.028	-0.651	-0.43	-0.209	-0.700	-0.48	-0.259	-0.582	-0.36	-0.139	-0.641	-0.42	-0.199
	6	-0.443	-0.22	0.003	-0.205	0.02	0.245	-0.364	-0.14	0.084	-0.255	-0.03	0.195	-0.651	-0.43	-0.209
	7	-0.621	-0.4	-0.179	-0.700	-0.48	-0.259	-0.275	-0.05	0.175	-0.532	-0.31	-0.088	-0.572	-0.35	-0.129
face turning to length (26 mm)	8	-0.359	-0.1	0.159	-1.151	-0.9	-0.649	-0.279	-0.02	0.239	-0.26	0	0.26	-0.706	-0.45	-0.195
	9	-0.299	-0.04	0.219	-0.269	-0.01	0.249	-0.418	-0.16	0.098	-0.646	-0.39	-0.134	-0.656	-0.4	-0.144
	10	-0.706	-0.45	-0.195	-0.379	-0.12	0.139	-0.983	-0.73	-0.477	-0.735	-0.48	-0.225	-0.844	-0.59	-0.336
	11	-0.854	-0.6	-0.346	-0.468	-0.21	0.048	-1.072	-0.82	-0.568	-0.359	-0.1	0.159	-0.616	-0.36	-0.104
	12	-0.359	-0.1	0.159	-0.379	-0.12	0.139	-0.735	-0.48	-0.225	-0.498	-0.24	0.018	-0.399	-0.14	0.119
	13	-0.646	-0.39	-0.134	-1.121	-0.87	-0.619	-0.389	-0.13	0.129	-0.319	-0.06	0.199	-1.220	-0.97	-0.719
external turning to diameter (45 mm)	14	-1.361	-0.92	-0.479	-0.529	-0.08	0.369	-0.024	0.43	0.884	-0.658	-0.21	0.238	-0.430	0.02	0.470
	15	-0.479	-0.03	0.419	-0.440	0.01	0.460	-0.499	-0.05	0.399	-0.727	-0.28	0.167	-0.489	-0.04	0.409
	16	-0.519	-0.07	0.379	-0.499	-0.05	0.399	-0.440	0.01	0.460	-0.361	0.09	0.541	-0.678	-0.23	0.218
	17	-0.272	0.18	0.632	-0.331	0.12	0.571	-0.599	-0.15	0.299	-0.420	0.03	0.480	-0.430	0.02	0.470
	18	-0.469	-0.02	0.429	-0.707	-0.26	0.187	-0.608	-0.16	0.288	-0.519	-0.07	0.379	-0.479	-0.03	0.419
	19	-0.410	0.04	0.490	-0.559	-0.11	0.339	-0.440	0.01	0.460	-0.638	-0.19	0.258	-0.519	-0.07	0.379
external turning to length (16 mm)	20	-0.279	-0.12	0.039	-0.665	-0.51	-0.355	-0.863	-0.71	-0.557	-0.457	-0.3	-0.143	-0.318	-0.16	-0.002
	21	-0.16	0	0.16	-0.259	-0.1	0.059	-0.358	-0.2	-0.042	-0.447	-0.29	-0.133	-0.179	-0.02	0.139
	22	-0.408	-0.25	-0.093	-0.467	-0.31	-0.153	-0.477	-0.32	-0.163	-0.209	-0.05	0.109	-0.179	-0.02	0.139
	23	-0.259	-0.1	0.059	-0.615	-0.46	-0.305	-0.606	-0.45	-0.295	-0.279	-0.12	0.039	-0.16	0	0.16
	24	-0.398	-0.24	-0.082	-0.16	0	0.16	-0.526	-0.37	-0.214	-0.507	-0.35	-0.194	-0.140	0.02	0.180
	25	-0.338	-0.18	-0.022	-0.111	0.05	0.211	-0.932	-0.78	-0.628	-0.675	-0.52	-0.365	-0.150	0.01	0.170

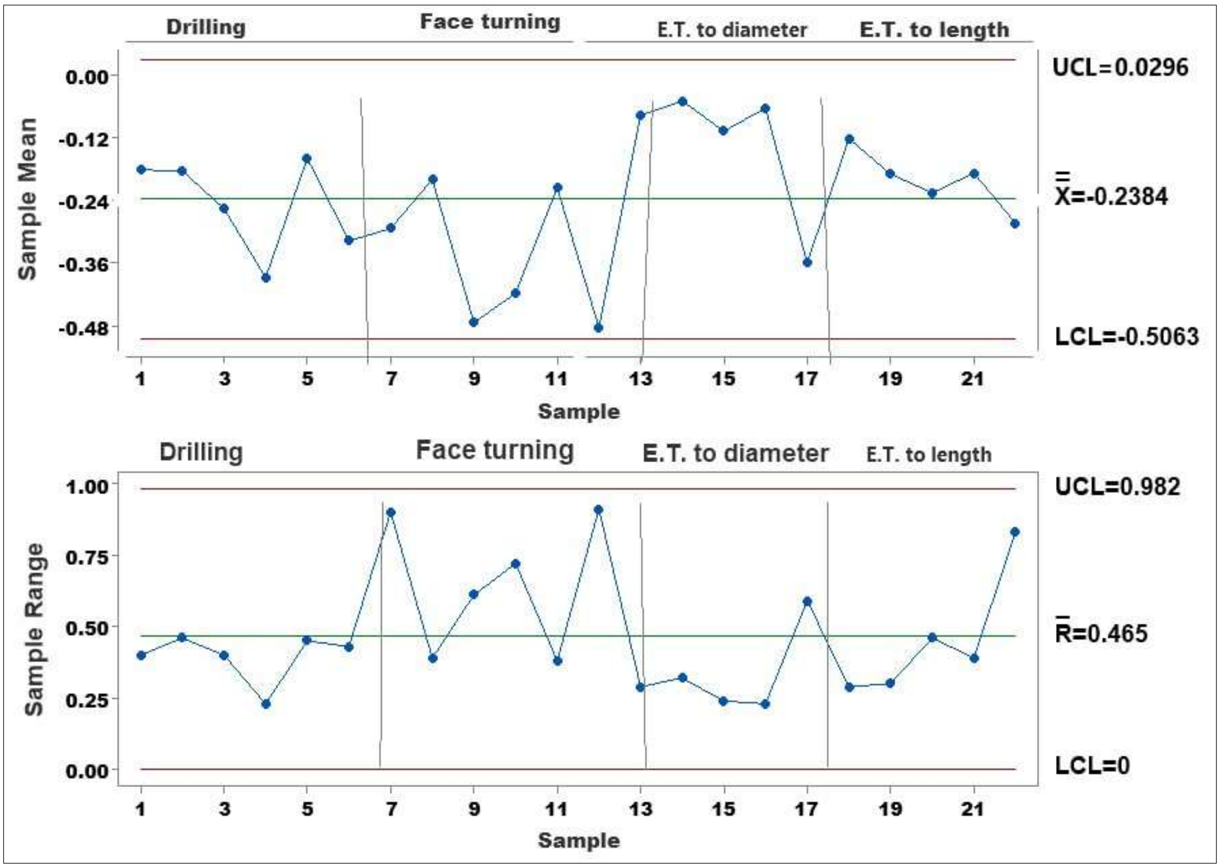

Figure 3. Approved Traditional DNOM $\bar{X}$-R Control Chart

A summary of the mean and range of the triangle fuzzy data are given in **Table** 5. They are calculated by Eqs. (11, 12, and 13). The next step was to calculate the control limits for fuzzy $\bar{X}$ and R control charts as in Eqs. (8, 9, 10, 15, 16, 17). Control limits are given in **Tables 6 and 7**.

Table 5. Mean and range of the data

	S.N.	$\bar{X}_a$	$\bar{X}_b$	$\bar{X}_c$	R_a	R_b	R_c
Drilling process to diameter (22.5 mm)	1	-0.40518	-0.182	0.04118	0.396	0.4	0.404
	2	-0.40716	-0.184	0.03916	0.4554	0.46	0.4646
	3	-0.47844	-0.256	-0.03356	0.396	0.4	0.404
	4	-0.27846	-0.054	0.17046	1.0692	1.08	1.0908
	5	-0.60912	-0.388	-0.16688	0.2277	0.23	0.2323
	6	-0.3834	-0.16	0.0634	0.4455	0.45	0.4545
	7	-0.53982	-0.318	-0.09618	0.4257	0.43	0.4343
face turning to length (26 mm)	8	-0.55106	-0.294	-0.03694	0.891	0.9	0.909
	9	-0.458	-0.2	0.058	0.3861	0.39	0.3939
	10	-0.72926	-0.474	-0.21874	0.6039	0.61	0.6161
	11	-0.67382	-0.418	-0.16218	0.7128	0.72	0.7272
	12	-0.47384	-0.216	0.04184	0.3762	0.38	0.3838
	13	-0.73916	-0.484	-0.22884	0.9009	0.91	0.9191

external is turning to diameter (45 mm)	14	-0.60048	-0.152	0.29648	1.3365	1.35	1.3635
	15	-0.52722	-0.078	0.37122	0.2871	0.29	0.2929
	16	-0.4995	-0.05	0.3995	0.3168	0.32	0.3232
	17	-0.4104	0.04	0.4904	0.3267	0.33	0.3333
	18	-0.55692	-0.108	0.34092	0.2376	0.24	0.2424
	19	-0.51336	-0.064	0.38536	0.2277	0.23	0.2323
external turning to length (16 mm)	20	-0.5164	-0.36	-0.2036	0.5841	0.59	0.5959
	21	-0.28078	-0.122	0.03678	0.2871	0.29	0.2929
	22	-0.3481	-0.19	-0.0319	0.297	0.3	0.303
	23	-0.38374	-0.226	-0.06826	0.4554	0.46	0.4646
	24	-0.34612	-0.188	-0.02988	0.3861	0.39	0.3939
	25	-0.44116	-0.284	-0.12684	0.8217	0.83	0.8383

Table 6. Control limits for fuzzy $\bar{\mathbf{X}}$ chart

Fuzzy X-bar control limits		
UCL	-0.32525	a
	0.083178	b
	0.637495	c
CL	-0.48604	a
	-0.2164	b
	0.053236	c
LCL	-0.64682	a
	-0.51598	b
	-0.53102	C

Table 7. Control limits for the fuzzy R chart

Fuzzy R control limits		
UCL	0.589366	a
	1.098108	b
	2.141607	c
CL	0.27866	a
	0.5192	b
	1.01258	c
LCL	0	a
	0	b
	0	c

The α –cut fuzzy midrange transformation approach was used to obtain the control limits. Also, it was used to calculate the sample midrange for each sample. Control limits were calculated by Eqs. (28, 29, 30, 32, 33, 34). The Eq. (31) calculated the sample midrange. The value of α= 0.65 was chosen according to the production process. The results are listed in **Tables 8, 9, and 1**0.

Table 8. α-Cut level of the X - bar chart

X-bar chart		
UCL	-0.05977	a
	0.083178	b
	0.250225	c
CL	-0.31077	a
	-0.2164	b
	-0.14899	c
LCL	-0.56177	a
	-0.51598	b
	-0.54821	c

Table 9. α-Cut level for R chart

Range chart		
UCL	0.920048	a
	1.098108	b
	1.463333	c
CL	0.435011	a
	0.5192	b
	0.691883	c
LCL	0	a
	0	b
	0	c

The Minitab 21 software was used to draw and analyze the fuzzy $\bar{X}$-R control charts using the data of sample midrange in **Table 10** as the input of the Minitab 21. **Fig. 4** presents the fuzzy $\bar{X}$-R control charts.

Table 10. Sample midrange

S.N.	$S_{mr-1}{}^{\alpha}$	$S_{mr-2}{}^{\alpha}$	$S_{mr-3}{}^{\alpha}$	$S_{mr-4}{}^{\alpha}$	$S_{mr-5}{}^{\alpha}$
1	-0.59333	-0.19593	-0.37476	-0.2158	-0.25554
2	-0.12638	-0.25554	-0.34495	-0.33502	-0.58339
3	-0.50391	-0.56352	-0.16612	-0.55359	-0.2158
4	-0.60326	-0.12638	0.46972	-0.13631	-0.60326
5	-0.39463	-0.57346	-0.62313	-0.50391	-0.56352
6	-0.36482	-0.12638	-0.28534	-0.17606	-0.57346
7	-0.54365	-0.62313	-0.19593	-0.45424	-0.49398
8	-0.26835	-1.06315	-0.18887	-0.169	-0.61608
9	-0.20874	-0.17894	-0.32796	-0.55647	-0.5664
10	-0.61608	-0.28822	-0.89426	-0.64588	-0.75517
11	-0.7651	-0.37764	-0.98367	-0.26835	-0.52666
12	-0.26835	-0.28822	-0.64588	-0.40744	-0.30809
13	-0.55647	-1.03335	-0.29816	-0.22861	-1.1327
14	-1.20652	-0.37198	0.134705	-0.50113	-0.27263
15	-0.32231	-0.28257	-0.34217	-0.57068	-0.33224
16	-0.36205	-0.34217	-0.28257	-0.20308	-0.521
17	-0.11367	-0.17328	-0.44153	-0.26269	-0.27263
18	-0.31237	-0.55081	-0.45146	-0.36205	-0.32231
19	-0.25276	-0.40179	-0.28257	-0.48126	-0.36205
20	-0.22322	-0.61068	-0.80939	-0.40205	-0.26296
21	-0.104	-0.20335	-0.3027	-0.39211	-0.12387
22	-0.35238	-0.41199	-0.42192	-0.15368	-0.12387
23	-0.20335	-0.56101	-0.55107	-0.22322	-0.104
24	-0.34244	-0.104	-0.47159	-0.45172	-0.08413
25	-0.28283	-0.05433	-0.87893	-0.62062	-0.09406

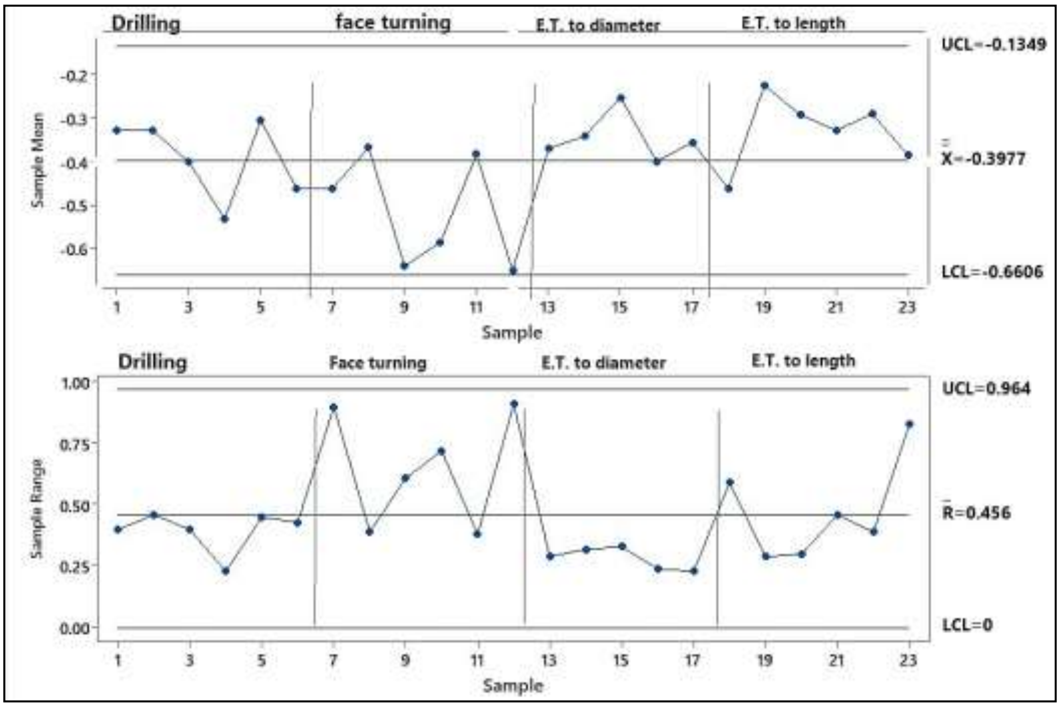

Figure 4. Fuzzy $\bar{X}$-R Control Chart

5. RESULTS AND DISCUSSION

The results above show that the traditional DNOM control chart gives a false alarm of an out-of-control state for some observations (15). The fuzzy DNOM control chart gives more flexibility to the results, as it considers the vagueness of data. The standard deviation of the process, which represent the dispersion of the data from its mean after fuzzification, was equal to (σ =0.204401), and it was less than the process's standard deviation before fuzzification, (σ =0.209041), a lower standard deviation means lower dispersion and this led to a better quality. The drop in the standard deviation of the process was because of the flexibility of the fuzzy control chart over the traditional control chart.

The fuzzy control limits in this process (Drilling to Diameter of 22.5 mm) have moved away by a standard deviation (σ =0.1992) from the specification limits. While the traditional control limits are (σ =0.2002) away from the specification limits. These results show that the process capability has improved by 1.2%, as the process capability was equal to Cp=0.83 in the traditional case and became equal to Cp=0.84 after using the fuzzy chart. We note that the two points (5 and 20) are close to the limits of the minimum specification. The normal and fuzzy control limits are close to the minimum specification because the process is a drilling process to obtain a diameter, and this helps increase the re-work of samples that are outside the control limits instead of destroying them. **Fig. 5.**

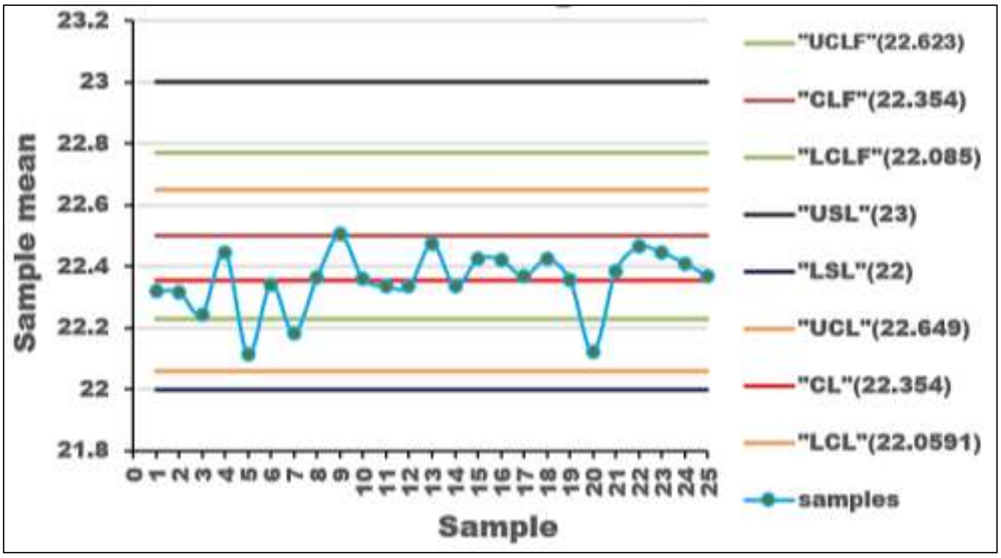

Figure 5. Drilling Process

5.1 Face Turning (26mm)

In this process, the standard deviation of the process before the fuzzification was (σ =0.24144), and after the fuzzification, it became (σ =0.2398), which means that the dispersion decreased by 0.016. The decrease in the standard deviation of the process led to an improvement in the capability of the process, as it was equal to 0.69 and became equal to 0.695 after using the fuzzy control chart. We note that the control limits are close to the upper limit of the specification because the process is face-turning, and this reduces the samples that are destroyed when they depart from the control limits, as shown in **Fig. 6.**

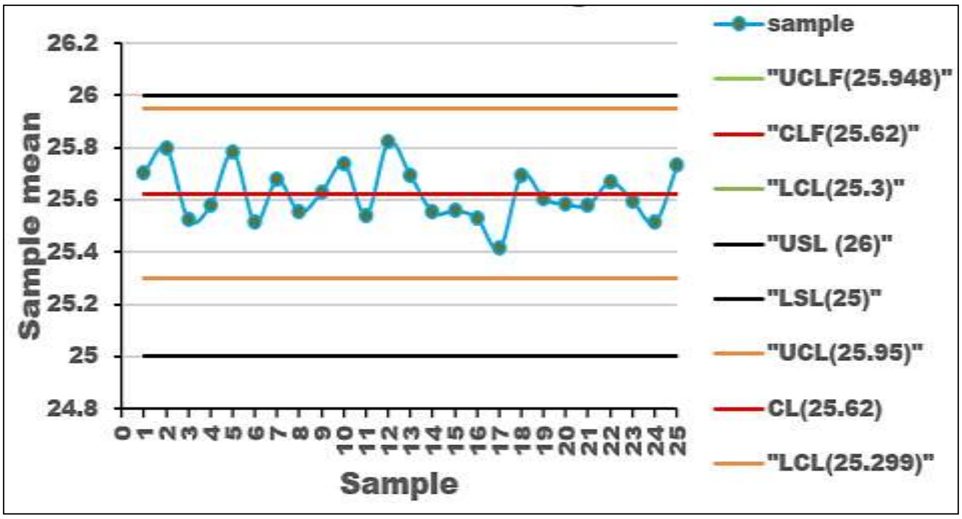

Figure 6. Face Turning Process

5.2 External Turning to Diameter (45mm)

The standard deviation of the process after the fuzzification became (σ =0.134972), while the standard deviation of the process before the fuzzification was (σ =0.135856). The traditional and fuzzy upper control limits are outside the upper specification limit for the process. The exit of the two samples (4 and 10) outside the upper specification limit of the process requires re-work on these two samples to obtain the required diameter. The traditional minimum control limit is outside the minimum specification for the process (refer to **Fig.** 7).

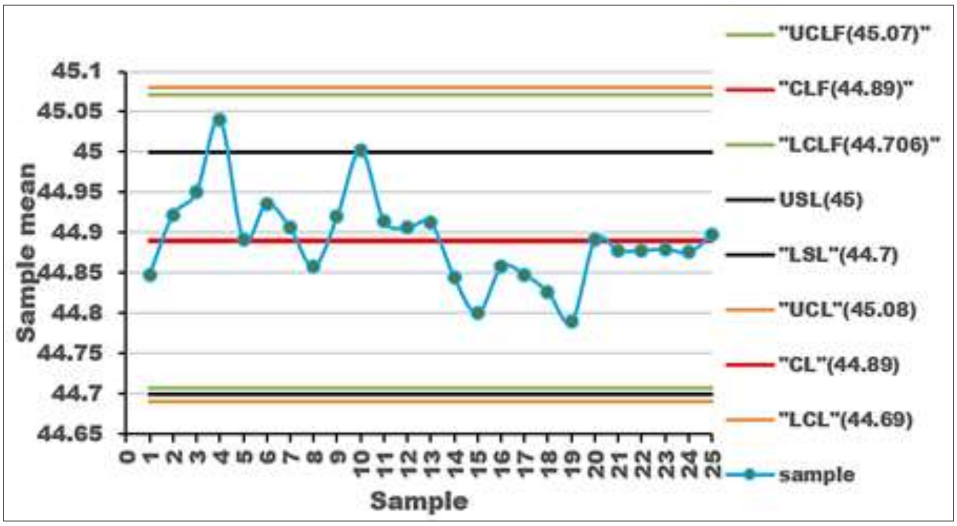

Figure 7. External Turning to Diameter

5.3 External Turning to Length (16mm)

The conformity of the upper limit of the specification with the upper traditional control limit. The fuzzy control limits are outside the upper and lower specification limits. The standard deviation of the process before the fuzzification is (0.274119), while that after the fuzzification was equal to (0.272337). Reducing dispersion improved the process's capability from (0.6080) to (0.6122) after fuzzification, as illustrated in **Fig.** 8.

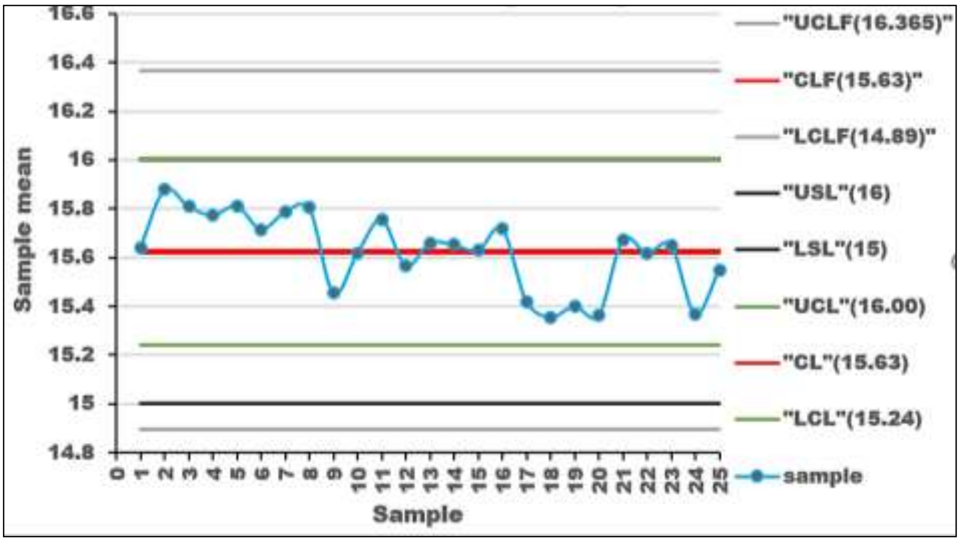

Figure 8. External Turning to Length

6. CONCLUSIONS

The objective of the work was to analyze and compare the traditional and fuzzy deviation from nominal dimension control charts for a process of a short-run nature. The conclusions can be extracted as follows:

1- The fuzzy control chart was applicable in short-run production.
2- The fuzzy control chart takes into consideration the vagueness of the data.
3- Fuzzy control charts take into account the insensible variations caused by the variations of the measurement instruments and the worker who measures them.
4- The fuzzy DNOM control chart was sensitive to indicate the out-of-control observations for the alpha cut of (0.65).
5- For observation, the traditional control chart can give a false alarm indicating the out-of-control state (15). While fuzzy control charts can be more flexible and accurate.
6- By decreasing the number of out-of-control data, the cost of inspecting new samples is lowered.
7- The process standard deviation is dropped using fuzzy control charts for (σ =0.209041) to (σ =0.204401).

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