Damage Detection and Assessment of Stiffness and Mass Matrices in Curved Simply Supported Beam Using Genetic Algorithm

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ABSTRACT

In this study, a genetic algorithm (GA) is used to detect damage in curved beam model, stiffness as well as mass matrices of the curved beam elements is formulated using Hamilton's principle. Each node of the curved beam element possesses seven degrees of freedom including the warping degree of freedom. The curved beam element had been derived based on the Kang and Yoo's thin-walled curved beam theory. The identification of damage is formulated as an optimization problem, binary and continuous genetic algorithms (BGA, CGA) are used to detect and locate the damage using two objective functions (change in natural frequencies, Modal Assurance Criterion MAC).

The results show the objective function based on change in natural frequency is the best objective and no error was recorded in prediction of location and small error in detecting damage value. Also the result show that the genetic algorithm method are efficient indicating and quantifying single and multiple damage with high precision, and the prediction error for the CGA are less than corresponding value for the BGA.

الخلاصة:

في هذا البحث تم استخدام الخوارزميه الجينيه لتقييم الضررفي العتبه المقوسه، تم صياغة مصفوفتي الجساءة والكتلة لعنصر العتبة المقوسة باستعمال مبدأ هاملتون. كل عقدة في العتبه المقوسه تحوي على سبع درجات من الحرية مع الاخذ بنظر الاعتبار الاعوجاج (warping).تم أشتقاق عنصر العتبة المقوسة بالإعتماد على نظرية كانك وياو للعتبة المقوسة ذات الجدران الرقيقة. استخدمت الاعداد الحقيقيه ونظام التشغيل الثنائي للخوارزميه الجينيه لتحديد كميه الضرر وموقعه باستخدام دالتين من دوال الهدف (objective function) (change in natural frequencies) و(Modal Assurance Criterion MAC)

النتائج اظهرت بأن الفرق في الترددات هي افضل داله موضوعيه و ليس هناك خطأ مسجل في التنبؤ بموقع الضرر وخطأ صغير في تحديد كميه الضرر.وكذلك اثبتت الخوارزميه الجينيه كفائتها في ايجاد كميه وموقع الضرر المفرد والمتعدد بدقه عاليه وان نسبه الخطا باستخدام الاعداد الحقيقيه اقل نسبيا عند استخدام نظام التشغيل الثنائي.

Keywords: curved beam; Genetic Algorithm; damage detection

INTRODUCTION

At the recent years, genetic algorithms have been recognized as promising intelligent search techniques for difficult optimization problems. Genetic algorithm method is very attractive in comparison with classical methods because it does not require a solution search within the whole solution space. Instead the algorithm starts from a small initial population of approximated solutions and converges rapidly from thereon **W.M.Ostachowicz** *et al.***1996. Mares and Surace 1996** employed a GA to identify damage in elastic structures. A modified version of residual force vectors in terms of the stiffness matrix of the damaged structure was chosen as an objective function to be minimized while stiffness reduction factors of all elements were chosen to be variables. **M. I. Friswell** *et al.* **1998** developed a technique, which is based on combined use of eigensensitivity and genetic algorithms to identify the location and magnitude of damage from measured vibration data. They employ a genetic algorithm to minimize a square-value of the frequency error. Structural damage is modeled by a reduction in Young's modulus, while the element number in the finite element model gives damage location. The objective is to identify the position of one or more damage sites in a structure, and to estimate the extent of the damage at these sites. The GA is used to optimize the discrete damage location variables. For a given damage location site or sites, a standard eigensensitivity method is used to optimize the damage extent. This two-level approach incorporates the advantage of both the GA and the eigensensitivity methods. Damage at one and two sites have been successfully located in the simulated example of a cantilever beam, also successfully location in an experimental cantilever plate. **J.H. Chou and J**. **Ghaboussi 2001** used a GA to solve an optimization problem formulated for detection and identification of structural damage. The "output error" indicating the difference between the measured and computed responses under static loading and the equation error indicating the residual force in the system of equilibrium equations are used to formulate the objective function to be optimized. The method proposed is capable of successfully detecting the location and magnitude of the damage as well as correctly determining the unmeasured nodal displacement, while avoiding the complete finite element analyses. **E. S. Sazonov** *et al.* **2002** used the GA to produce a sufficiently optimized amplitude characteristic filter to extract damage information from the strain energy mode shapes. A finite element model was used to generate training data set with the known location. The filter amplitude characteristic was encoded as a GA string where the pass coefficient for each harmonic of the Discrete Fourier Transform representation was a number between 0 and 1 in an 8 bit. The genetic optimization was performed based on the minimization of the signal- to- distortion ratio. The results obtained from the GA has confirmed the theoretical predictions and allowed improvements in the method's sensitivity to damages of lower magnitude.

In this study, it had been used a binary and continuous genetic algorithm for damage detection and location in (in and out-of-plane) curved beam by minimizing or maximizing the objective function which is based on frequency difference and modal assurance criterion MAC.

I. MODELING THE DAMAGED BEAM.

In this study the equation of motion for simply curved beam acquired from Kang and Yoo's theory of thin- walled curved beams to drive the element stiffness and mass matrices respectively. The curved beam element is shown in **Fig.1** in curvilinear coordinate system. Each node of the curved beam element possesses seven degrees of freedom including the warping degree of freedom. Using Hamilton's principle, the dynamic equilibrium can be expressed in the variation form as following **K. Young Yoon** *et al.* **2006.**

$$\int_{t_1}^{t_2}(\delta T + \delta U + \delta \forall)dt = 0 \qquad (1)$$

Where δT is the variation kinetic energy, δU is the variation strain energy, and $\delta \forall$ is the variation potential energy loss due to applied loads. The symbol (δ) means the first variation. For the linear elastic body, the variation of strain energy stored in the body is

$$\delta U = \int_V \tau_{ij}\, \delta\varepsilon_{ij} dV \qquad (2)$$

Where τ_{ij} refers to the components of the stress tensor and ε_{ij} to those of the strain tensor. The variation in kinetic energy of a thin-walled curved beam is

$$\delta T = \int_V \rho \frac{\partial^2 u_i}{\partial t^2} \delta u_i dV \qquad (3)$$

Where ρ is the mass density, u_i is the displacement components of the curved beam, and t is time. The variation potential energy loss due to applied loads with body forces neglected is

$$\delta \forall = -\int_l q_i\, \delta u_i dz \qquad (4)$$

Where q_i stands for distributed loads applied on the line of shear center and l is the length of the element.

A linear stiffness matrix and a consistent mass matrix are developed so that various analyses such as linear and free vibration analyses can be performed. Using shape functions, the dynamic equilibrium given in eq. (1) yields a set of simultaneous equations

$$\delta T + \delta U + \delta \forall = \delta d^T [\mathrm{M}\ddot{d} + \mathrm{K}d - \mathrm{f}] = 0 \qquad (5)$$

From which one obtains.

$$\mathrm{M}\ddot{d} + \mathrm{K}d - f = 0 \qquad (6)$$

Where K, M, d, and f are the linear stiffness matrix, the consistent mass matrix, the nodal displacement vector, and the applied force vector of a global structural system, respectively. The nodal forces and the corresponding nodal displacements are shown in **Fig.1** in the positive senses. The nodal forces are seven components $(F_z, M_x, M_y, B, T_T, V_x, \text{and } V_y)$.The corresponding nodal displacements are $(w_0, \gamma, -v_0^{'}, -\tau, \beta, u_0, and\ v_0)$ where γ and τ are defined as

$$\gamma = u_0^{'} + \frac{\omega_0}{R} \qquad (7a)$$

$$\tau = \beta^{'} + \frac{v_0^{'}}{R} \qquad (7b)$$

w_0, u_0, and γ describe the in-plane displacements whereas v_0, $-v_0^{'}$, β, and $-\tau$ are the out-of-plane displacements. These two parts of displacement fields are not coupled with each other and can be formulated separately. Then, the displacement fields can be expressed in terms of nodal displacements as following **K. Young Yoon** *et al.* **2005.**

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \beta \end{Bmatrix} = \begin{bmatrix} N_u & 0 & 0 & 0 \\ 0 & N_v & 0 & 0 \\ 0 & 0 & N_w & 0 \\ 0 & 0 & 0 & N_\beta \end{bmatrix} \begin{Bmatrix} d^u \\ d^v \\ d^w \\ d^\beta \end{Bmatrix} \qquad (8)$$

Where the shapes function, N is defined as.

$$N_u = \begin{bmatrix} 1-3\xi^2+2\xi^3 & l(\xi-2\xi^2+\xi^3) & 3\xi^2-2\xi^3 & l(-\xi^2+\xi^3) \end{bmatrix} \qquad (9a)$$

$$N_v = N_\beta = \begin{bmatrix} 1-3\xi^2+2\xi^3 & l(-\xi+2\xi^2-\xi^3) & 3\xi^2-2\xi^3 & l(-\xi^2+\xi^3) \end{bmatrix} \qquad (9b)$$

$$N_w = [1-\xi \quad \xi] \qquad (9c)$$

Where $\xi = z/l$

Where the nodal displacement, d is represented

$$d^u = [u_{oi} \quad \gamma_i \quad u_{oj} \quad \gamma_j]^T \qquad (10a)$$

$$d^v = [v_{oi} \quad -v_{oi}^{'} \quad v_{oj} \quad -v_{oj}^{'}]^T \qquad (10b)$$

$$d^w = [w_{oi} \quad w_{oj}]^T \qquad (10c)$$

$$d^\beta = [\beta_i \quad -\tau_i \quad \beta_j \quad -\tau_j]^T \qquad (10d)$$

From the variation of strain energy presented in eq. (2) and the shape function in equations (9a), (9b), and (9c) the element stiffness matrix for curved beam is derived as shown **K.Young Yoon** *et al.* **2005.**

$$[k_u] = \begin{bmatrix} EI_y K_a & 0 & 0 & 0 \\ 0 & EI_x K_b & 0 & 0 \\ 0 & 0 & EAK_c & 0 \\ 0 & 0 & 0 & EI_\omega K_d + GK_T K_e \end{bmatrix} \quad (11)$$

Where:

$$K_a = \int_l N_u''^T N_u'' dz = \frac{1}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ Sym. & & & 4l^2 \end{bmatrix},$$

$$K_b = \int_l N_v''^T N_v'' dz = \frac{1}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ Sym. & & & 4l^2 \end{bmatrix}$$

$$K_c = \int_l N_w'^T N_w' dz = \frac{1}{l}\begin{bmatrix} 1 & -1 \\ Sym. & 1 \end{bmatrix},$$

$$K_d = \int_l N_\beta''^T N_\beta'' dz = \frac{1}{l^3}\begin{bmatrix} 12 & -6l & -12 & -6l \\ & 4l^2 & 6l & 2l^2 \\ & & 12 & 6l \\ Sym. & & & 4l^2 \end{bmatrix}$$

$$K_e = \int_l N_\beta'^T N_\beta' dz = \frac{1}{30l}\begin{bmatrix} 36 & -3l & -36 & -3l \\ & 4l^2 & 6l & -l^2 \\ & & 36 & 3l \\ Sym. & & & 4l^2 \end{bmatrix}$$

From the variation kinetic energy presented in eq. (3) and following the similar procedure as used for the element stiffness matrix for curved beam formulation, the mass matrix is derived**.**

$$[m_u] = \rho \begin{bmatrix} AM_a + I_y M_e & 0 & 0 & 0 \\ 0 & AM_b + I_x M_b & 0 & 0 \\ 0 & 0 & AM_c & 0 \\ 0 & 0 & 0 & (I_x + I_y)M_a + I_\omega M_f \end{bmatrix} \quad (12)$$

Where:

$$M_a = \int_l N_u^T N_u dz = \frac{l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ & 4l^2 & 13l & -3l^2 \\ & & 156 & -22l \\ Sym. & & & 4l^2 \end{bmatrix},$$

$$M_b = \int_l N_v^T N_v dz = \frac{l}{420}\begin{bmatrix} 156 & -22l & 54 & 13l \\ & 4l^2 & -13l & -3l^2 \\ & & 156 & 22l \\ Sym. & & & 4l^2 \end{bmatrix},$$

$$M_c = \int_l N_w^T N_w dz = \frac{l}{6}\begin{bmatrix} 2 & 1 \\ Sym. & 2 \end{bmatrix}$$

$$M_e = \int_l N_\beta'^T N_\beta' dz = \frac{1}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ & 4l^2 & -3l & -l^2 \\ & & 36 & -3l \\ Sym. & & & 4l^2 \end{bmatrix},$$

$$M_f = \int_l N_v'^T N_v' dz = \frac{1}{30l}\begin{bmatrix} 36 & -3l & -36 & -3l \\ & 4l^2 & 3l & -l^2 \\ & & 36 & -3l \\ Sym. & & & 4l^2 \end{bmatrix}$$

II. APPLACATION OF A GENETIC ALGORITHM.

GA is a global probabilistic search algorithm inspired by Darwin's survival-of-the fittest theory. In this optimization method, information about a problem, such as variable parameters, is coded into a genetic string known as an individual (chromosome). Each of these individuals has an associated fitness value, which is usually determined by the objective function to be maximized or minimized. Genetic algorithms have been shown to be able to solve the optimization problem through mutation, crossover and selection operation applied to individuals in the population.

II.I Population

The initial population are created randomly by generating the required number of individuals but a new population developed from this initial population and to do this must apply the genetic operator. The initial populations are generated by the following equation **L. Randy Haupt**, **S. Ellen Haupt 2004:**

$$P = X_{LB} + rand\,(N_{pop}, N_{var})\,(X_{UB} - X_{LB}) \qquad (13)$$

Where:
(X_{UB}, X_{LB}) means the range of maximum and minimum values allowed for each variable respectively.
N_{pop} = The number of population.
N_{var} = The number of variable.

In this population, there are several individuals carrying different "genetic information" in their string or coding. When working with binary coded genetic algorithms each of the real parameters to be optimized is translated to binary codes.

- To transform the real values (b_i) to binary codes the following equation is used **H. M. Gomes and N. R. S. Silva (2007)**

$$s = bin_n\left\{round(2^{n\,bit} - 1)\left[\frac{b_i(k) - X_{LB}}{X_{UB} - X_{LB}}\right]\right\} \qquad (14)$$

Where bin_n indicates a binary traslation to a string **s** , and n bit means the number of bit.

- To transform the binary codes to real values (decoding) the following equation is used.

$$b_i(k) = X_{LB} + bin^{-1}(s)\left[\frac{X_{UB} - X_{LB}}{2^{n\,bit} - 1}\right] \qquad (15)$$

Where $bin^{-1}(s)$ is the nonnegative integer decoded from the base 2 binary representation, From this equation it is obviouse that the precision by the binary coding is $(X_{UB} - X_{LB}) / (2^{n\,bit} - 1)$

II.II Fitness Function

In order to determine the ability of an individual to search better solution, a fitness function is used to quantify how good the solution represented by a chromosome is. Depending on the problem characteristic, the fitness function can be any form of mathematical formulation, can be either a maximized or minimized function. This function generates an output from the set of input variables of a chromosome. The goal is to modify the output in some desirable fashion by finding the appropriate values of input variables.
In this work the two objective functions are used to assess the presence of damage in beam.

- Changes in Natural Frequencies.
- Modal Assurance Criterion.

Changes in Natural Frequencies

The natural frequency used as a diagnostic parameter in structural assessment procedures using vibration monitoring. One great advantage of using only eigenvalue in the damage assessment of structures is that they are cheaply acquired and the approach can give an inexpensive structural assessment technique. The objective function to be minimized is defined as follows **M. T. V. Baghmisheh *et al* 2008:**

$$\Delta\omega = \sum_i^n (\omega_i^m - \omega_i^a)^2 \quad (16)$$

Where:
i = Mode Number (i=1,2,3,….,n)
ω_i^m = Test natural frequencies
ω_i^a = Calculated natural frequencies.

The ω_i^m are the natural frequencies which are applied to our damage detection system as inputs. An objective value of zero indicates an exact match between the values of frequencies.

Modal Assurance Criterion.

The Modal Assurance Criterion MAC value indicates the degree of correlation between two modes and varies from 0 to 1, with 1 for full correlation, and 0 for no- correlation. The deviation from 1 can be interpreted as a damage indicator in structures. This index is based on comparisons between the changes in the mode shapes obtained both from tests and from calculations, the MAC is defined by **W. M. Ostachowicz** *et al.* **1996:**

$$MAC(\emptyset_i, \emptyset_j) = \frac{(\emptyset_i^T \emptyset_j)^2}{\emptyset_i^T \emptyset_i \emptyset_j^T \emptyset_j} \quad (17)$$

$\emptyset_i$ = Test mode shape vector.
$\emptyset_j$ = calculate mode shape vector.

II.III Selection (reproduction)

Reproduction is the first operator applied on a population. The first step in the reproduction is fitness assignment. Each individual receives a reproduction probability depending on the own objective (fitness) value and the objective value of all other individuals in the population. The evaluation of this objective function indicates which individuals will have more chances to procreate and to generate a large offspring.

There are various selection processes that are utilized in genetic algorithms such as roulette wheel selection, rank selection and tournament selection. A common processes and used in this work are the roulette wheel selection. This selection method was used to copy individuals according to their fitness values, individuals with higher fitness have a higher probability of contributing one or more offspring in the next generation. For each population individual a probability of being selected for copying is given by the following equation **D. E. Goldberg 1989**:

$$P_i = \frac{f_i}{\sum_{j=1}^{p_{size}} f_j}$$

$$i,j = 1,2,\ldots\ldots,P_{size} \quad (18)$$

Where f_j is the fitness of individual j, the sum is taken over all population members (P_{size}), and P_i is the probability of individual i with fitness f_j receiving an additional copy.

II.IV Recombination (Crossover)

Crossover is one of the recombination operators that is used for information exchange between any two individuals to create two offspring. Each pair of parents have a probability, P_c ,of producing offspring. Usually, a high crossover probability is used.

- Real value Recombination: The variable values of the offspring are chosen somewhere around and between the variable values of the parents. Offspring are produced according to the rule **H. Pohlheim 2007:**

$$Var_i^o = var_i^{p1} .\alpha_i + var_i^{p2}. (1-\alpha_i)$$
$$i \in (1,2,\ldots N_{var}) \quad (19)$$

Where α is a scaling factor chosen uniformly a random over an interval [-0.25 , 1.25] for each a new.

- Binary valued Recombination: The some of the crossover operators available in GA are single point crossover, two-point crossover and uniform crossover. In this work a single point crossover is applied, where one crossover position (n) a long the string is selected randomly between 1 and the string length less one. Two new strings are created by swapping all characters between the individuals about this point.

II.V Mutation

Mutation means a random change in the information of a chromosome, to add diversity to the genetic characteristics of the population. It is applied at a certain probability, P_m, to each gene of the offspring, the mutation probability also called mutation rate, is usually a small value, to ensure that good solutions are not distorted too much. Mutation of real variables means, that randomly created values are added to the variables selected. The mutation rule is:

$$C = P + rand\ (X_{UB} - X_{LB}) \qquad (20)$$

Where C is mean the child and P mean parent For binary mutation, randomly change a particular gene in a chromosome, thus, 1 may be changed to a 0 or vice versa.

II.VI Elitism

In the process of the crossover and mutation- taking place, there is high chance that the optimum solution could be lost. There is no guarantee that these operators will preserve the fittest string. To avoid this, the elitist models are often used. Elitism refers to the process of ensuring that the best chromosome (or few best chromosomes) of the current population survive to the next generation. The best individuals are copied to the new population without being mutated. Elitism can rapidly increase the performance of GA, because it prevents a loss of the best found solution **M. Obitko1998**

II.VII Termination

The GA may be terminated by using the convergence criterion in order to get an acceptable approximate solution, the terminate if there is no improvement over a number of consecutive generation, by monitoring the fitness of the best individual if there is no significant improvement over a time, GA is to stop. Or if the objective function value of the fittest individual is 0 or very small number, which means that the optimal solution has been found.

In the present work the chromosome has two variables, the damage location and the stiffness reduction. The objective function generates an output from the set of input variables of a chromosome. The goal is to modify the output in some desirable fashion by finding the appropriate values of input variables. **Fig.2** shows the flowchart of the method of damage detection using genetic algorithms.

III. NUMERICAL SIMULATION

The processes of damage detection are demonstrated using (in and out-of-plane) simply supported curved beam. The dimensions and material properties for the simply supported in and out-of-plane curved beam are shown in **Table 1** and **Table 2** respectively.

In and out-of-plane simply supported curved beam is divided into 30 finite elements of equal length, where the value of first natural frequency is used for convergent test for checking the stability of the results as shown in the **Fig. 3** and **Fig. 4** for in and out-of-plane respectively.

Six damage scenarios are investigated and are summarized in **Table 3**.In the first four cases for single damage, the scenarios were simulate by reducing the stiffness of an element near the beam's end and near the beam's mid-span. The remaining damage cases D5 and D6 in the same table correspond to a multiple damage scenario and were simulated by reducing the stiffness of assumed elements at two different locations. The following parameters of the GA have been used: size of the population is 40, probability of crossover P_c is 0.9, probability of mutation P_m is 0.05, number of elitism is 2 and number of bit is 20.

IV. RESULTS AND DISSCUSION

The frequency predictions from the FEM model of undamaged beam are validated by comparing with other researches as shown in **Tables 4** and **Table 5** for in and out-of-plane curved beam respectively.

IV.I Objective Function Based on Change in Natural Frequency.

The input first five natural frequencies of damage scenarios are shown in **Table 6** and **Table 7** for out-of-plane and in-plane curved beam respectively. A population of individuals is generated randomly then the natural frequencies and objective function are calculated for each individual. The GAs theory is used to find the optimal location and stiffness reduction by minimizing the eq. (16). For each scenario the algorithm is run from five different initial random population and the identified values for damage scenarios by using CGA and BGA are shown in **Table 8** for out-of-plane and **Table 9** for in-plane curved beam. In all scenarios there are no error recorded in prediction of damage element and the errors for the CGA are less than corresponding values for the BGA, because in the CGA deals with real values without using any encoding method.

Fig. 5 show the typical objective function curve for out-of-plane at D4 by using CGA, it is see that the objective function value tends to zero with the increasing number of generations and reach zero at around 21 generations. The **Fig. 6** shows the objective function curves at same damage scenario but using BGA, the convergence occurs at 28 generation.

IV.II Objective Function Based on Modal Assurance Criterion (MAC)

The mode shapes are calculated numerically using finite element model for the damaged scenarios, these used as test inputs for the GA operator. A population of individuals is generated randomly then the objective function is calculated for each individual and the GAs theory is applied. For each scenario the algorithm is run in five different initial randomly generated populations and the average results obtained by CGA and BGA listed in **Table 10** for out-of-plane and **Table 11** for in-plane curved beam. The errors for CGA are less than corresponding values for the BGA.

For out-of-plane curved beam the objective function with multi damage for D5 using CGA is shown in **Fig. 7** it can seen that convergence occurs at 15 generations.

V. CONCLUSIONS

The main conclusions from the present work may be stated as follows:

- The study shows that the genetic algorithm is effective in identifying positions and extents in single and multi damage.
- The results obtained from continuous genetic algorithms are more accurate then those obtained from binary genetic algorithms in damage assessment.
- The length of the run (in terms of generation number) and results depends on the initial randomly generated population and GA parameters and the test point.
- The objective function based on change in natural frequency is the best objective function, because the stiffness reduction has a relatively large

effect on the natural frequencies, as compared with mode shapes, it is insensitive of the modes to the damage.

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Notation

A Sectional area (m^2)
B_i Bimoment (N .m)
E Young modulus (N/m^2)
G Shear modulus (N/m^2)
GA Genetic Algorithm.
I_y Area moment of inertia about y-axis (m^4)
I_x Area moment of inertia about x-axis (m^4)
I_ω Warping moment of inertia (m^6)
J Area polar moment of inertia (m^4)
K_T St Venant constant of a straight member(m^4)
l Length of the finite element (m, cm)
M_x, M_y Moment about x- and y-axis (N.m)
MAC Modal Assurance Criterion
m_x, m_y, m_z Uniform distributed moments about x-, y-, and z-axis
m_ω Uniform distributed bimoment
N_{POP} Number of population
N_{var} Number of variable
P_{siz} Population size
q_x, q_y, q_z Uniform distributed forces about x-, y-, and z-directions
R Radius of initial curvature (m)
T Kinetic energy (N.m)
U Strain energy (N.m)
u_o, v_o Displacement components of the shear center in x- and y- directions, respectively
V Volume of body (m^3)
V_x, V_y Transverse shear forces (N)
w_o Average longitudinal displacement of cross-section
X_{UB} Maximum value of variable
X_{LB} Minimum value of variable

Greek letters

ρ Mass density (Kg/m^3)
β Rotation of the cross-section about z-axis
θ Subtended angle (degree)
ε_{ij} Components of strain tensor
δ Variation
γ, τ Nodal displacements
τ_{ij} Components of stress tensor

Table 1 Material properties of the in-plane curved beam

Area of cross section (A)	$4 \times 10^{-3} m^2$
Radius of the arch (R)	2.438 m
Mass density (ρ)	7850 kg/m^3
Subtended angle (θ)	97°
Modules of Elasticity (E)	200 GPa
Modules of Rigidity (G)	77 GPa
Moment of inertia (I)	$6.45 \times 10^{-6} m^4$

Table 2 Material properties of the out-of- plane curved beam

Area of cross section (A)	$9.3 \times 10^{-3} m^2$
Length (L)	10 m
Mass density (ρ)	7850 kg/m^3
Subtended angle (θ)	89°
Modules of Elasticity (E)	200 GPa
Modules of Rigidity (G)	77GPa
Moment of inertia (Ix)	$1.13 \times 10^{-4} m^4$
Moment of inertia (Iy)	$3.88 \times 10^{-5} m^4$
Warping moment of inertia (Iω)	$5.56 \times 10^{-7} m^6$
Venant constant (K_T)	$5.38 \times 10^{-7} m^4$

Table 3 Damage scenario for in and out-of-Plane curved beam

Damage Scenario	Damage Element	Stiffness reduction %
D1	8	25
D2	11	50
D3	16	25
D4	25	50
D5	6,15	25
D6	8,25	25

Table 4 Comparisons of modal frequencies for in-plane curved beam

Mode No.	Natural Frequency(rad/sec)		Error (%)
	[Ki. Young et al] Results	Present Numerical Results	
1	396.98	396.936	0.011
2	931.22	930.94	0.03
3	1797.31	1796.67	0.035

Table 5 First natural frequencies for the simply supported out-of- plane curved beam

Subtended Angle (degree)	Natural Frequency (rad/sec)			Error (%)
	Analytical Results[Ki-Young et al]	Numerical Results[Ki-Young et al]	Present Numerical Results	
0	53.3000	53.3000	53.266	0.06379
10	31.8648	31.8669	31.863	0.0056
20	19.9616	19.9614	19.9592	0.01202
30	13.9944	13.9931	13.9915	0.0207
40	10.5386	10.5372	10.5343	0.0408
50	8.2946	8.2888	8.28753	0.08523
60	6.7121	6.7012	6.70043	0.1739
70	5.5270	5.5090	5.50836	0.33725
80	4.5991	4.5707	4.57020	0.62838
90	3.8479	3.8048	3.87485	0.70038

Table 6 Natural frequencies for out-of-plane curved beam

Damage Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
D1	3.8583	45.456	168.575	381.32	661.097
D2	3.8452	45.113	168.525	373.46	647.75
D3	3.8664	45.532	168.139	381.2	658.47
D4	3.867	45.629	168.98	381.195	664.313
D5	3.8461	45.286	166.86	379.147	565.786
D6	3.8372	45.211	167.29	379.23	660.017

Table 7 Natural frequencies for in-plane curved beam

Damage Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
D1	392.685	926.68	1796.3	1980.6	2897.9
D2	389.175	929.03	1747.9	1963.6	2869.4
D3	396.876	921.37	1795.6	1980.4	2885.7
D4	386.261	905.81	1770.2	1960.9	2901.9
D5	386.182	878.912	1767.5	1923.3	2825.8
D6	374.657	893.524	1768.9	1926.9	2858.3

Table 8 Identified stiffness parameters for out-of-plane curved beam based on change in natural frequency

Test Element No.	Stiffness Parameters				
	Actual	Identified by CGA	Error %	Identified by BGA	Error %
8	0.75	0.7491003	0.11	0.749992	0.001
11	0.5	0.5002577	0.051	0.500381	0.076
16	0.75	0.7503839	0.051	0.7500715	0.01
25	0.5	0.4999999	0.00001	0.5000152	0.003
6,15	0.75	0.7555629	0.48	-	-
8,25	0.75	0.7334992	2.2	-	-

Table 9 Identified stiffness parameters for in-plane curved beam based on change in natural frequency

Test Element No.	Stiffness Parameters				
	Actual	Identified by CGA	Error %	Identified by BGA	Error %
8	0.75	0.7499999	0.00001	0.7500007	0.00009
11	0.5	0.5001478	0.03	0.4999847	0.003
16	0.75	0.7499999	0.00001	0.7499988	0.0001
25	0.5	0.5000117	0.002	0.5000152	0.003
6,15	0.75	0.7514097	0.18	-	-
8,25	0.75	0.7481410	0.24	-	-

Table 10 Identified stiffness parameters for out-of-plane curved beam based on MAC

Test Element No.	Stiffness Parameters				
	Actual	Identified by CGA	Error %	Identified by BGA	Error %
8	0.75	0.7505467	0.07	0.753685	0.5
11	0.5	0.5023270	0.46	0.5076601	1.5
16	0.75	0.7517007	0.22	0.7468184	0.42
25	0.5	0.5046008	0.92	0.5088810	1.77
6,15	0.75	0.7340752	2.2	-	-
8,25	0.75	0.7702559	2.7	-	-

Table 11 Identified stiffness parameters for in-plane curved beam based on MAC

Test Element No.	Stiffness Parameters				
	Actual	Identified by CGA	Error %	Identified by BGA	Error %
8	0.75	0.7430990	0.92	0.7757013	3.4
11	0.5	0.5044385	0.88	0.4923729	1.5
16	0.75	0.7547156	0.63	0.7459007	0.54
25	0.5	0.5123102	2.4	0.5155471	3.1
6,15	0.75	0.7256023	3.2	-	-
8,25	0.75	0.7829436	4.3	-	-

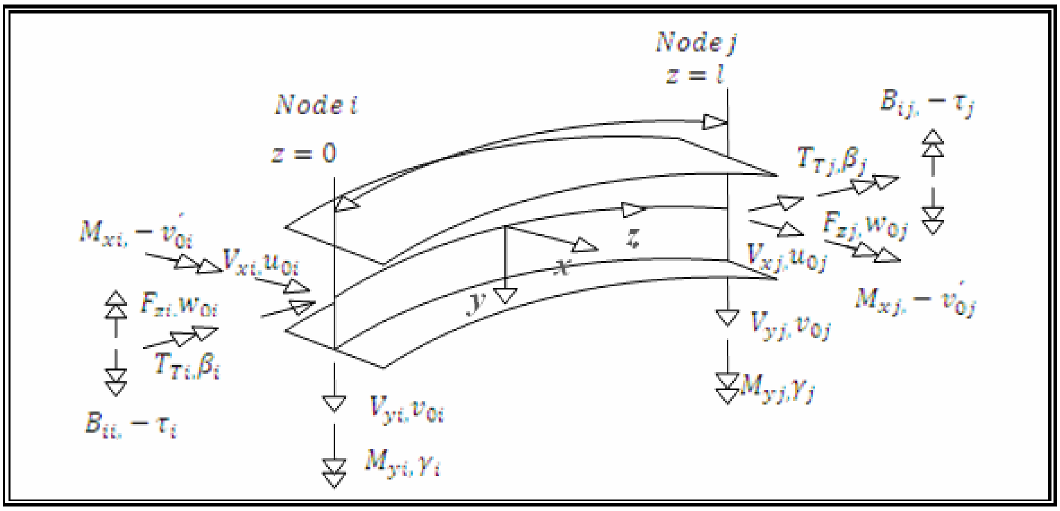

Fig. 1. Curved beam element

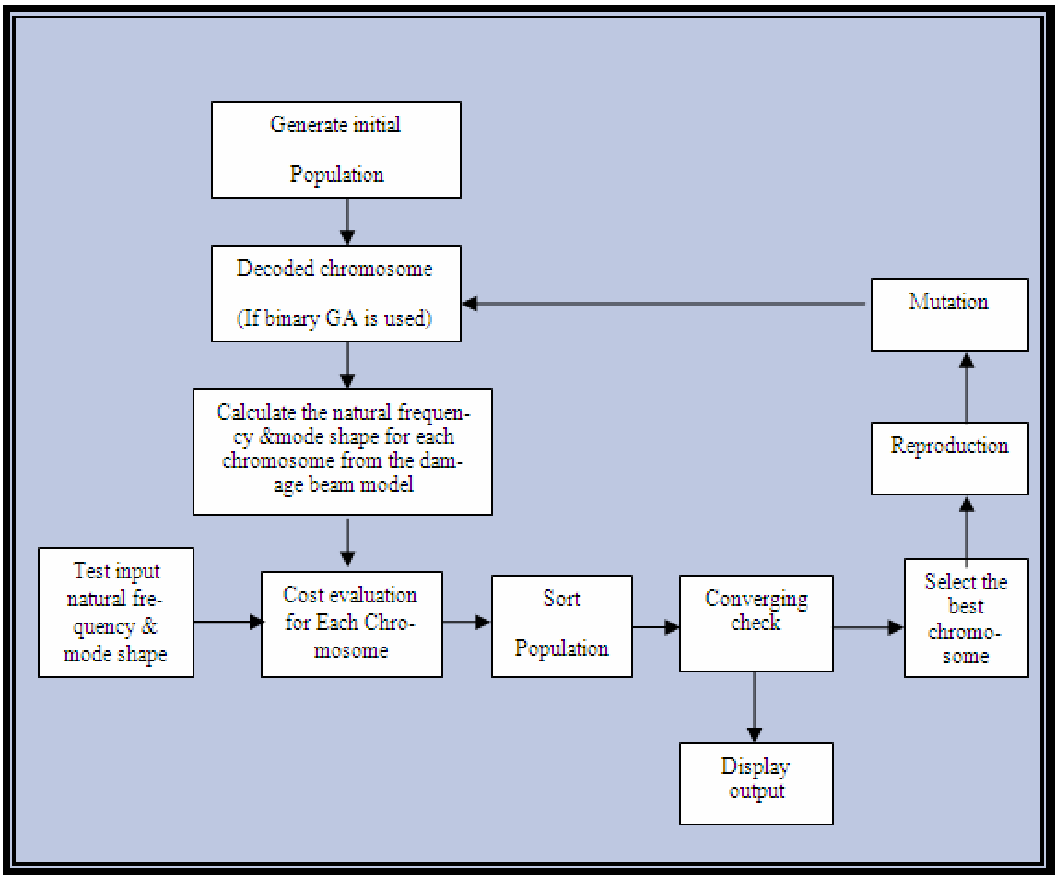

Fig. 2 Flowchart of suggested damage detection method using GAs

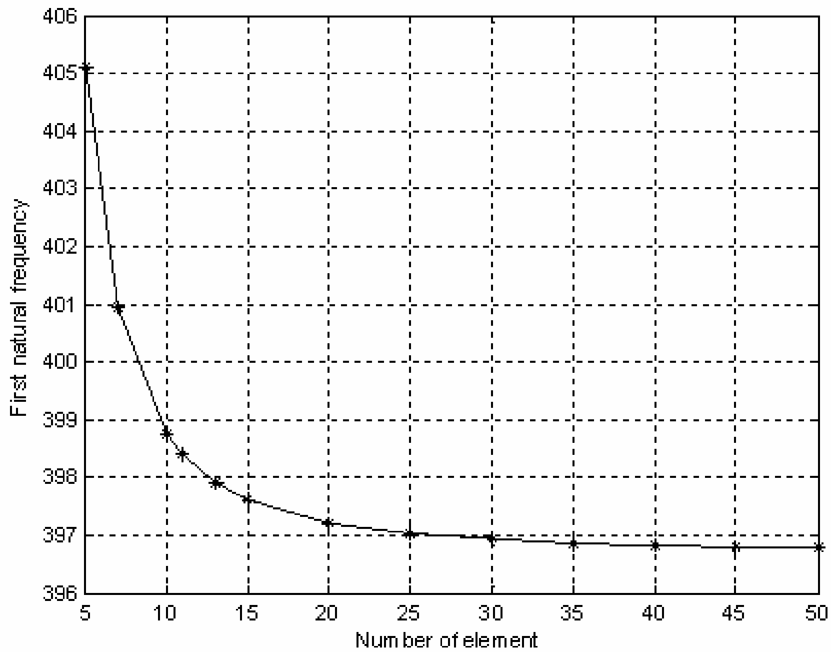

Fig. 3 Convergence test for in-plan curved beam

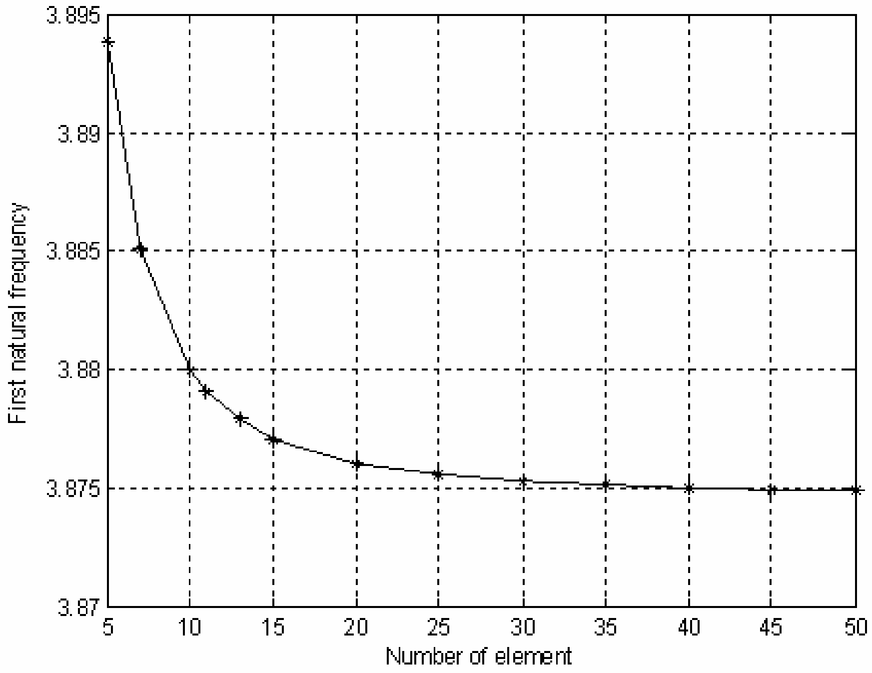

Fig. 4 Convergence test for out-of-plan curved beam

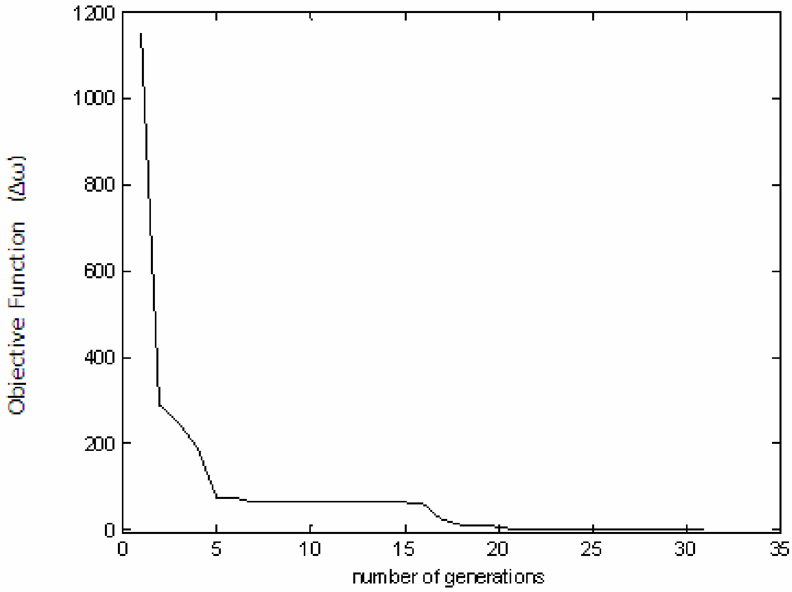

Fig. 5 A Typical objective function curve of CGA for out-of-plane curved beam

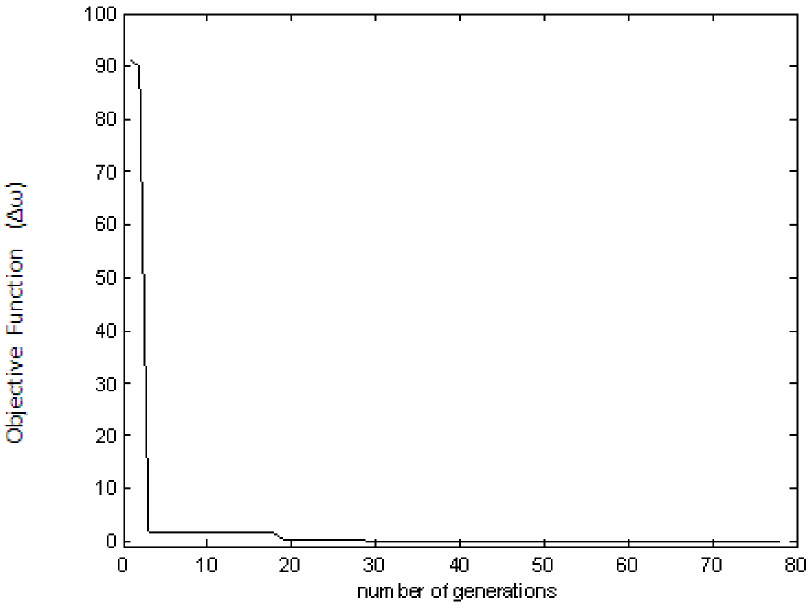

Fig. 6 A Typical objective function curve of BGA for out-of-plane curved beam

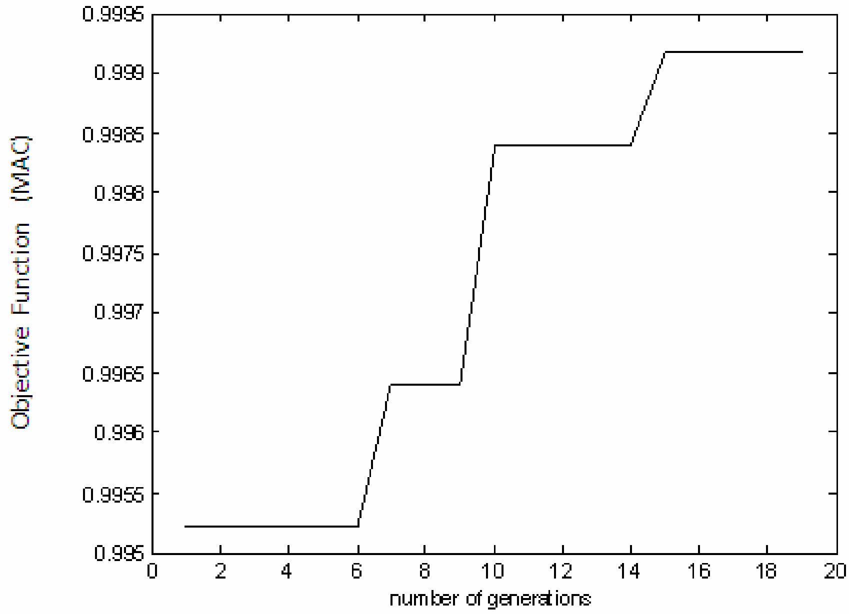

Fig. 7 A Typical objective function curve of CGA for out-of-plane curved beam