

## Cross Dipole Antennas Solution for Angle of Arrival Estimation

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### ABSTRACT:

The Multiple Signal Classification (MUSIC) algorithm is the most popular algorithm to estimate the Angle of Arrival (AOA) of the received signals. The analysis of this algorithm (MUSIC) with typical array antenna element ( $\lambda/2$  dipole) shows that there are two false direction indication in the plan aligned with the axis of the array.

In this paper a suggested modification on array system is proposed by using two perpendiculars crossed dipole array antenna in spite of one array antenna. The suggested modification does not affect the AOA estimation algorithm. The simulation and results shows that the proposed solution overcomes the MUSIC problem without any effect on the performance of the system.

**Keywords:** component; Angle of Arrival Estimation; MUSIC; Adaptive Array Antenna System.

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### الخلاصة:

ان خوارزمية تصنيف الاشارات المتعددة (MUSIC) من الخوارزميات الاكثر شعبية لتقدير زاوية وصول الاشارة الراديوية (AOA). ان التحليلات العملية لخوارزمية الـ (MUSIC) مع مجموعة الهوائيات ثنائية القطب ( $\lambda/2$ ) تظهر وجود دلالة لاتجاهيين خاطئين مع المحور المستلقي عليه مجموعة الهوائيات.

في هذا البحث قد تم اقتراح تعديل على مجموعة الهوائيات ويتم ذلك عن طريق استخدام هوائيين متعامدين من الهوائيات ثنائية القطب بدلاً من الهوائي الواحد. وان الحل المقترح لا يؤثر في خوارزمية اكتشاف زاوية وصول الاشارة الراديوية. حيث ان النتائج تظهر ان الحل المقترح يتغلب على مشكلة خوارزمية الـ (MUSIC) من دون اي تأثير في اداء النظام

### INTRODUCTION

Over the last decade, the smart antenna system was taken a great interest in the wireless communication industry which helps to improve the wireless communication system performance by increasing the channel capacity and spectrum efficiency, extending range coverage, steering the multiple beams to track any object (M. Chryssomailis, 2000).

The Angle of Arrival (AOA) estimator is one of the basic blocks that construct the smart antenna system, because, if there are several operating transmitters, it is important to estimate the (AOA) of all received signals in order to decide which transmitter are present and what are their angular directions.

The MUSIC algorithm is the most important and accurate algorithm which is used for estimating the (AOA) of received signals (F. Taga, 1997).

The adaptive antenna system shown in Fig. (1) is considering a smart antenna system.

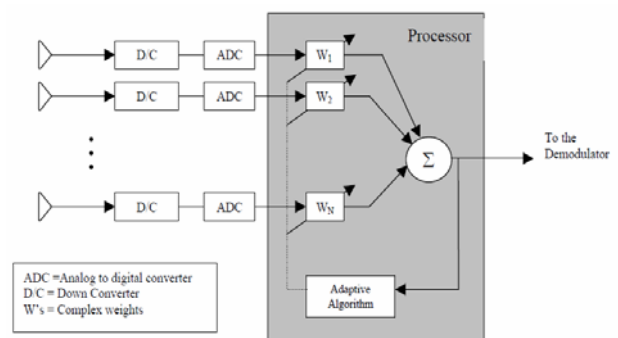


Figure (1) N weighted received signals

The covariance matrix of the received signals contains all the needed information to estimate the AOA of the received signals. To construct  $N \times N$  covariance matrix  $N$  elements array antenna is needed followed by  $N$  adapted weight vector as shown in Fig. (1).

The received signals vector  $\mathbf{X}(k)$ , where  $k$  is the number of received snapshot, is given by.

$$\mathbf{X}(k) = \mathbf{X}_d(k) + \mathbf{X}_i(k) + \mathbf{X}_n(k) \quad (1)$$

Where:  $\mathbf{X}_d(k)$ ,  $\mathbf{X}_i(k)$  and  $\mathbf{X}_n(k)$  are desired, interference and thermal noise ( $N \times 1$ ) vectors, respectively.

The adaptive array output signal can be written as

$$y(k) = \sum_{j=1}^N w_j x_j(k) \quad (2)$$

The vector form of Eq. (2) is

$$y(k) = \mathbf{W}^T \mathbf{X} = \mathbf{X}^T \mathbf{W} \quad (3)$$

Where the weight vector ( $\mathbf{W}$ ) and the received signals vector  $\mathbf{X}$  are given by

$$\mathbf{W} = [w_1, w_2, \dots, w_N]^T \quad (4)$$

$$\mathbf{X} = [x_1, x_2, \dots, x_N]^T \quad (5)$$

The covariance matrix of received signal vector of array antenna is defined as

$$\text{Cov}[\mathbf{X}\mathbf{X}] = E[(\mathbf{X} - E(\mathbf{X}))^* (\mathbf{X} - E(\mathbf{X}))^T] \quad (6)$$

Since  $\mathbf{X}(t)$  is zero mean stationary process, then

$$\text{Cov}[\mathbf{X}\mathbf{X}] = E[\mathbf{X}^* \mathbf{X}^T] = \mathbf{R}_{xx} \quad (7)$$

Where  $\mathbf{R}_{xx}$  is ( $N \times N$ ) autocorrelation matrix of a received signal vector  $\mathbf{X}(t)$  and for  $N$ -element array antenna it may be written in the following form

$$\mathbf{R}_{xx} = E[\mathbf{X}^* \mathbf{X}^T] = [\mathbf{X}^* \mathbf{X}^T] \quad (8)$$

The autocorrelation matrix  $\mathbf{R}_{xx}$  is Hermitian (i.e.  $\mathbf{R}_{xx} = \mathbf{R}_{xx}^{*T}$ ) (L.C. Godora, 1997).

## Eigen VECTOR DECOMPOSITION

From theory of matrices, a positive definite Hermitian matrix  $\mathbf{R}_{xx}$  can be diagonalized by a nonsingular orthonormal transformation matrix  $\mathbf{Q}$  which is formed by eigenvectors of  $\mathbf{R}_{xx}$  as follows:

$$\mathbf{Q}^* \mathbf{R}_{xx} \mathbf{Q}^T = \mathbf{A} \quad (9)$$

Where:  $\mathbf{A}$  Is ( $N \times N$ ) diagonal matrix, its diagonal elements are a real eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_N$ , and the corresponding eigenvectors is

$$\mathbf{Q} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N] \quad (10)$$

Where  $\mathbf{e}_i$  is ( $N \times 1$ ) eigenvector corresponds to eigenvalue  $\lambda_i$ .

The eigenvalues of  $\mathbf{R}_{xx}$  are given by the solutions of the equation

$$|\mathbf{R}_{xx} - \lambda_i \mathbf{I}| = 0 \text{ for } i = 1, 2, \dots, N \quad (11)$$

Where  $\mathbf{I}$  is ( $N \times N$ ) an identity matrix.

Corresponding to each eigenvalue there is an associated Eigenvector  $\mathbf{e}_i$  that satisfies

$$\mathbf{R}_{xx} \mathbf{e}_i = \lambda_i \mathbf{e}_i \quad (12)$$

Since,  $\mathbf{R}_{xx} = E\{\mathbf{X}^* \mathbf{X}^T\}$ , it follows that Eq.(8) may be written as

$$[\mathbf{Q}^* \mathbf{R}_{xx} \mathbf{Q}^T] = [\mathbf{Q}^* \mathbf{X}^* \mathbf{X}^T \mathbf{Q}^T] = [\mathbf{X}^{* \sim} \cdot \mathbf{X}^{T \sim}] \quad (13)$$

$$\text{then } x_i^{* \sim} = \mathbf{e}_i^\dagger \cdot \mathbf{X}^* \text{ for } i = 1, 2, \dots, N \quad (14)$$

where  $\dagger$  is dagger notation (i.e. tranjugate).

The array correlation matrix has  $N$  eigenvalues ( $\lambda_1, \lambda_2, \dots, \lambda_N$ ) along with  $N$  associated eigenvectors  $\mathbf{Q} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N]$  where  $N$  is the number of sensors.

If the eigenvalues are sorted from smallest to largest, matrix  $\mathbf{Q}$  can be divided into two sub space such that  $\mathbf{Q} = [\mathbf{Q}_n, \mathbf{Q}_s]$ .

The first subspace  $\mathbf{Q}_n$  is called the noise subspace and is composed of ( $N - D$ ) eigenvectors associated with the noise, the eigenvalues are given as  $\lambda_1 = \lambda_2 = \dots = \lambda_{N-D} = \sigma_n^2$  where  $D$  is the number of received signals.

The second subspace  $\mathbf{Q}_s$  is called the signal subspace and is composed of  $D$  Eigen vectors associated with the received signals (Md. Bakhar 2009).

## MUSIC ALGORITHM

The MUSIC algorithm is a simple, popular, high resolution and efficient Eigen structure method (Md. Bakhar 2009). The MUSIC spatial spectrum for the MUSIC algorithm can be expressed as follows

$$DF(\theta) = \frac{1}{\mathbf{c}^T(\theta) \mathbf{Q}_n \mathbf{Q}_n^\dagger \mathbf{c}^*(\theta)} \quad (15)$$

Where  $C(\theta)$  is a spatial vector given by

$$C(\theta) = [g_z(\theta) e^{-jz\beta d \cos(\theta)}]^T \quad z = 0, 1, \dots, N-1 \quad (16)$$

So  $g_z(\theta)$  is the element pattern for the array and  $\theta$  is a spatial angle from 0 to  $2\pi$

The MUSIC technique flow graph is illustrated in Fig (2)

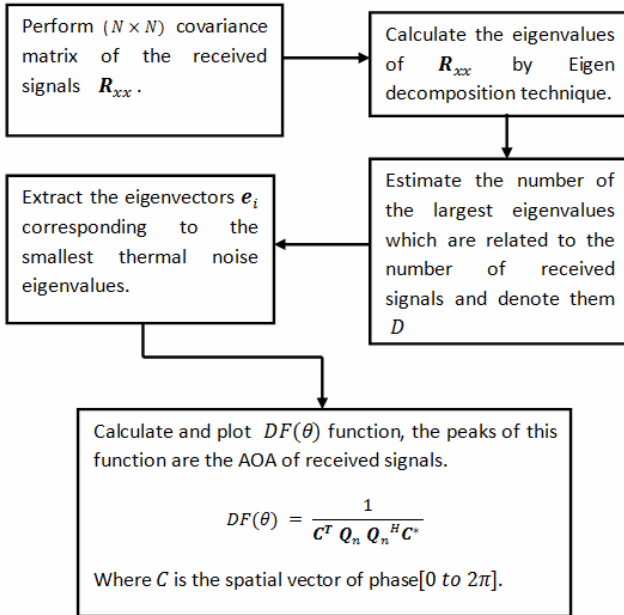


Figure (2) the flow graph of MUSIC technique.

### Cross Dipole Array Elements

In order to make the resultant array antenna pattern approximates the isotropic array pattern (i.e. spherical pattern) two perpendicular crossed dipoles in the same plane are proposed as shown in Fig.(3) and Fig(4).

The pattern for a single  $\lambda/2$  dipole in the (z-y) plane laying along z axis is given by (Stutzman, 1981).

$$g_1(\theta) = \frac{\cos[\frac{\pi}{2} + \cos\theta]}{\sin\theta} * \sin\varphi \quad (17)$$

Since the elevation angle  $\varphi = 90^\circ$  for the received signal and that leads to  $\sin\varphi = 1$ .

And for a dipole in the same (z-y) plane laying along y axis is given by

$$g_2(\theta) = \frac{\cos[\frac{\pi}{2} + \cos(\theta - \frac{\pi}{2})]}{\sin(\theta - \frac{\pi}{2})} * \sin\varphi \quad (18)$$

The resultant pattern of cross dipole  $g(\theta)$  is given by

$$g(\theta) = g_1(\theta) + g_2(\theta)$$

$$= \frac{\cos[\frac{\pi}{2} + \cos\theta]}{\sin\theta} + \frac{\cos[\frac{\pi}{2} + \cos(\theta - \frac{\pi}{2})]}{\sin(\theta - \frac{\pi}{2})} \quad (19)$$

The array factor for circular isotropic array elements given by (Liu Jin, 2008)

$$AF_c(\theta, \varphi) = \sum_{i=0}^{N-1} e^{-j(\beta r \cos(\theta - \varphi_i))} \quad (20)$$

Where  $\varphi_i$  is the array antenna distribution angle as shown in Fig.(4) and it is given by

$$\varphi_i = \frac{2\pi(i-1)}{N} \quad \text{for } i = 1, 2, \dots, N \quad (21)$$

and r is array circle radius with respect to wavelength  $\lambda$  and given by

$$r = N * d * \lambda / 2\pi \quad (22)$$

Then the total circular array pattern array with cross dipole element can be written as

$$G_c(\theta) = g(\theta) * AF_c(\theta, \varphi) \quad (23)$$

Where  $g(\theta)$  is the array element pattern of the crossed dipole. The array pattern in Eq.(23) is considered as a basic block for generating the covariance matrix of the received signals which in turn used in the calculating the eigenvectors of noise channel in the DF function of the MUSIC see Fig(2). The effects of using crossed dipole will reflect directly on the performance of MUSIC technique when a  $\lambda/2$  dipole element is used in the antenna system.

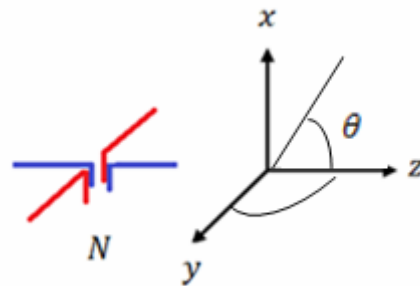


Figure (3) cross dipole element

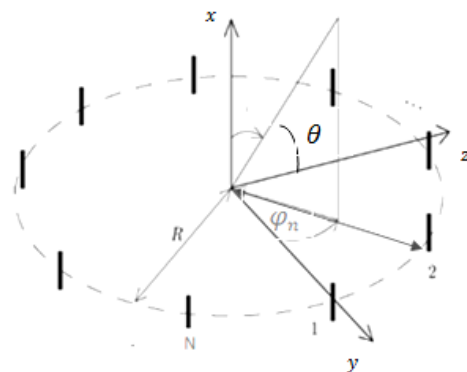


Figure (4) cross dipole circular array

**Simulation Results**

All the simulation programs were written by MATLAB 7.1 and the following assumption are used

- a) The array number of elements is six (N=6) which they are distributed on the circumference of a circle.
- b) The inter element spacing equal to  $0.5 \lambda$
- c) The input **SNR = 0dB**
- d) The radius of the circle r according to the E.(30) equal to  $0.4774 \lambda$ .

Three type of antenna element were tested in this simulation.

**A. Isotropic elements:**

Six isotropic elements with  $0.5 \lambda$  inter element spacing distributed on a circle with radius  $0.4774 \lambda$ .

Figs.(5) shows the final MUSIC DF plot for one received signal from broadside ( $\theta = 90^\circ$ ) and Fig.(6) shows four received signals from ( $0^\circ, 30^\circ, 120^\circ, 180^\circ$ ) it can be shown that the system does not suffer any problem to track a multiple sources at the same time.

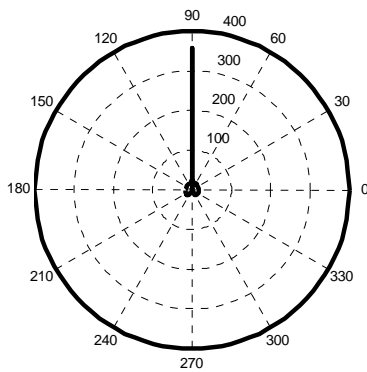


Figure (5) DF function plots for MUSIC Technique with six isotropic elements array for incoming signal from  $90^\circ$

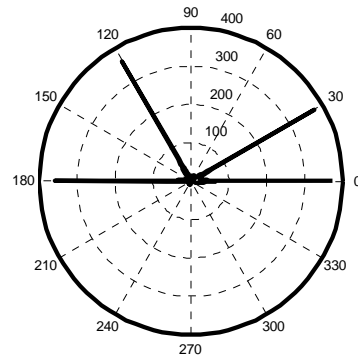


Figure (6) DF function plots for MUSIC Technique with six isotropic elements array for incoming signals from ( $0^\circ, 30^\circ, 120^\circ, 180^\circ$ )

**B-  $\lambda/2$  Dipole Elements:**

Six  $\lambda/2$  dipoles array elements with  $0.5 \lambda$  inter element spacing distributed on a circle with radius  $0.4774 \lambda$ .

Fig.(7) shows that there are three sources from angles  $0^\circ, 90^\circ, 180^\circ$ , while there is only one actual source from  $90^\circ$  which means that the MUSIC DF function gives a false indication from angles  $0^\circ, 180^\circ$  in addition to the real source. These two false DF direction are due to the existence of nulls in the element pattern from these direction as shown in Fig.(8). According to the mathematical form of the MUSIC DF function these nulls will be interpreted as a real source from these directions.

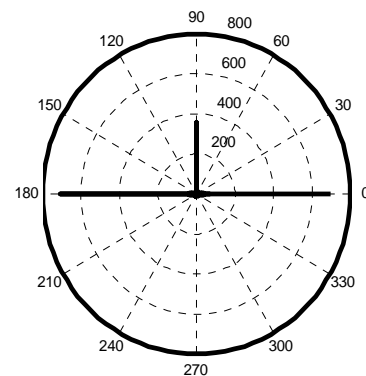


Figure (7) DF function plots for MUSIC Technique with six  $\lambda/2$  dipole elements array for incoming signal from  $90^\circ$

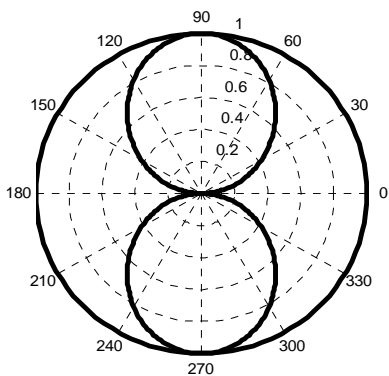


Figure (8) Element pattern for  $\lambda/2$  dipole

**C- Perpendicular Crossed Dipole Elements:**

For the same assumption of case B the crossed  $\lambda/2$  dipole antenna are used in spite of a single  $\lambda/2$  dipole antenna as elements of the array antenna.

Fig.(9) shows that the resultant pattern of the crossed dipole is approximate the isotropic pattern which leads to the cancellation of nulls appears in single dipole .

Fig.(10) shows that the final DF pattern shows only the single source from  $\theta = 90^\circ$  and nothing from  $(0^\circ, 180^\circ)$ .

To insure that the suggested solution does not affect the performance of the system four signals coming from  $(0^\circ, 30^\circ, 120^\circ, 180^\circ)$  is considered, Fig.(11) shows that there are a clear indication from these directions and the suggested solution doesn't cancel the reading for the incoming signal from  $(0^\circ, 180^\circ)$

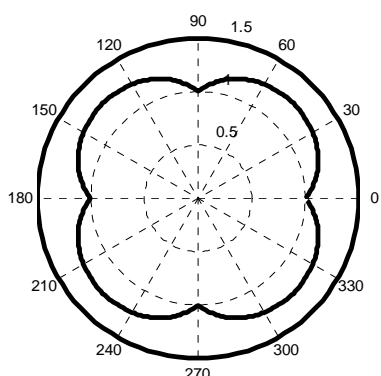


Figure (9) Element pattern for cross  $\lambda/2$  dipole

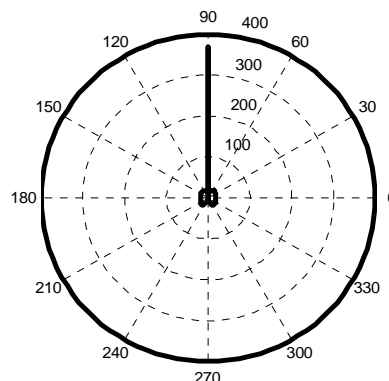


Figure (10) DF function plots for MUSIC Technique with six crossed dipole elements array for incoming signal from  $90^\circ$

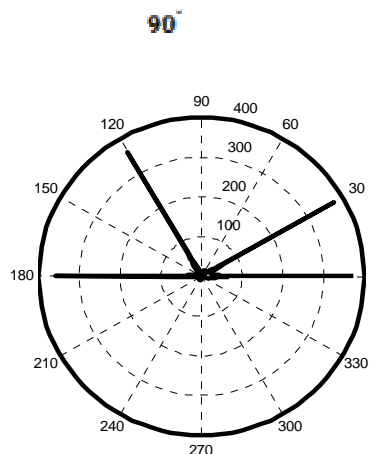


Figure (11) DF function plots for MUSIC Technique with six crossed dipole elements array for incoming signal from  $(0^\circ, 30^\circ, 120^\circ, 180^\circ)$ .

**Conclusions**

Angle of arrival (AOA) estimation based on MUSIC algorithm with isotropic,  $\lambda/2$  dipole and crossed  $\lambda/2$  dipole is investigated.

The MUSIC AOA estimation results show the following:-

- 1- With isotropic array elements the MUSIC DF can simultaneously read the direction to the multiple source even from the sources placed at  $0^\circ$  or  $180^\circ$ .
- 2- When  $\lambda/2$  dipole is used as an array element, it has been found that the MUSIC DF system gives a false direction from angles  $0^\circ, 180^\circ$  which coincides with the nulls in the element pattern.

- 3- The suggested perpendicular crossed  $\lambda/2$  dipole gives a satisfied solution for the false direction referred in (2), without degradation in the performance of the system when dealing with real source placed at angles  $0^\circ, 180^\circ$
- 4- Finally the suggested solution will increase the complexity of the antenna system by increasing the number of the elements.

## REFERENCES

F. Taga "Smart MUSIC Algorithm for DOA Estimation" ELECTRONIC LETTERS Vol.33 No.3 30<sup>th</sup> January 1997.

L.C. Godora "Application of Antenna Arrays to Mobile Communications, Part II: Beam-Forming and Direction-of-Arrival Considerations" proceeding of IEEE, VOL. 85, NO. 8 , pages 1195-1245, AUGUST 1997.

Liu Jin, Li li, Huazhi Wang "Investigation of Different Types of Array Structures for Smart Antennas" IEEE. ICMMT Proceedings. 2008.

M. Chryssomailis "Smart Antennas" IEEE antenna and propagation magazine, volume 42, No.3, June 2000, pages 129-136.

Md. Bakhar, Dr. Vani. R.M and Dr. P.V. Hunagund "Eigen Structure Based Direction of Arrival Estimation Algorithms for Smart Antenna Systems" IJCSNS International Journal of Computer Science and Network Security, VOL.9 No.11, November 2009.

Stutzman "Antenna Theory and Design" ISBN 0-471-04458-x, year 1981.