Natural Convection Heat Transfer from a Plane Wall to Thermally Stratified Environment

Prof. Dr. Ihsan Y. Hussain
Department of Mechanical Engineering
University of Baghdad
E-mail: dr.ihsanyahya1@yahoo.com

Naseem K. Ali
Department of Mechanical Engineering
University of Baghdad
E-mail: alokaili@yahoo.com

The effect of linear thermal stratification in stable stationary ambient fluid on free convective flow of a viscous incompressible fluid along a plane wall is numerically investigated in the present work. The governing equations of continuity, momentum and energy are solved numerically using finite difference method with Alternating Direct implicit Scheme. The velocity, temperature distributions and the Nusselt number are discussed numerically for various values of physical parameters and presented through graphs. ANSYS program also used to solve the problem. The results show that the effect of stratification parameter is marginalized with the increase in Prandtl number, and the increase in Grashof number does not practically vary the effect of stratification parameter.

Key words: Natural convection, Thermal Stratification, Linear, Boundary Layer,

Introduction

Convective heat transfer in thermally stratified ambient fluid occurs in many industrial applications and is an important aspect in the study of heat transfer. If stratification occurs, the fluid temperature is function of distance. Convection in such environment exists in lakes, oceans, nuclear reactors. The problem had been investigated by many researcher analytically and numerically, see for example (Cheesewright 1967), (Yang et al 1972), (Jaluria and Himasekhar 1983), (Kulkarni et al 1987), (Angirasa and Srinivasan 1992), (Pantokratoras 2003), (Saha and Hossain 2004), (Ahmed 2005), (Ishak et al 2008), (Deka and Neog 2009), (Singh et al 2010). Experimental works also had been reported; see (Tanny and Cohen 1998).Theoretical and experimental work had been investigated by (Chen and Eichhorn 1976). The present work investigates the problem numerically with wide range of stratification parameter for different kinds of fluids (air, water, and oil), different Grashof number and different inclination angle. To support the numerical solution the problem was solved also using ANSYS Program.

Formulation of the Problem

Consider the two dimensional thermal boundary layer flows natural convection heat transfer of an incompressible fluid along a plane wall immersed in a stable thermally stratified fluid. The coordinates system and the flow configuration are shown in figure 1. Using Boussinesq approximations, the following continuity, momentum and energy equations in nondimensional form for laminar flow adjacent to a plane wall are obtained;

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$\frac{\partial U}{\partial t^*}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\theta \qquad (2)$$

$$\frac{\partial V}{\partial t^*}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\theta\tan\varphi \qquad (3)$$

$$\frac{\partial \theta}{\partial t^*}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)-SU \qquad (4)$$

where (Angirasa and Srinivasan 1992);

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{u_c},\quad V=\frac{v}{u_c},$$

$$t^*=\frac{t\cdot u_c}{L},\quad Pr=\frac{\mu c_p}{k},$$

$$u_c=\sqrt{g\cdot\cos\varphi\cdot\beta\Delta T_0\cdot L},$$

$$\theta=\frac{T-T_{\infty,x}}{T_w-T_{\infty,0}}=\frac{\Delta T_x}{\Delta T_0},\quad Gr=\frac{g\cdot\beta\cdot\cos\varphi\cdot L^3\cdot\Delta T_0}{\vartheta^2},$$

$$\beta=-\left(\frac{1}{\rho}\right)\cdot\left(\frac{\partial\rho}{\partial T}\right)_P,\quad S=\frac{1}{\Delta T_0}\cdot\left(\frac{dT_{\infty,X}}{dx}\right)$$

The initial condition can be written in nondimensional form as follows:

$$U=0,\ V=0,\ \theta=0 \quad \text{for all } X, Y \qquad (5)$$

The boundary conditions in nondimensional form are:

$$U=0,\ V=0,\ \theta=\theta_w \quad \text{at } Y=0 \text{ for all } X \qquad (6)$$

$$U=0,\ V=0,\ \theta=0 \quad \text{at } Y\rightarrow\infty \text{ for all } X \qquad (7)$$

$U = 0, V = 0, \Theta = 0 \qquad at\ Y \rightarrow \infty\ for\ X = 0$ (8)

The local rate of heat transfer in term of the local Nusselt number at the plate is given by;

$$Nu_X = -\frac{X}{\theta_w} \cdot \left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \qquad (9)$$

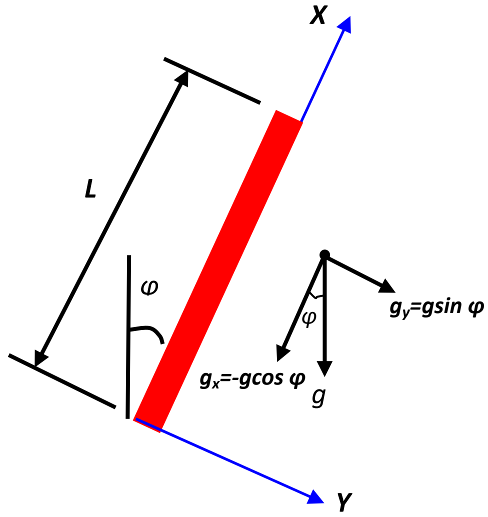

Figure1. Physical Model

Numerical Solution

Finite Difference Method is considered as efficient technique to solve the thermal problems; therefore it has been used in the present study. The Momentum and Energy equation are solved by Alternating Direction Implicit Scheme (ADI). Numerical results were first obtained to check for grid dependency. The results showed that no considerable difference in the results of suggested grid size after (51x51) and showed that no considerable different in the results of suggested transverse distance after (0.5). Therefore in the present study the grid size of (51x51) and transverse distance of (0.5) was used. The convergence of the solution to the steady state result for large time was obtained with a convergence criterion of (1×10^{-4}). This criterion was chosen after varying it over a wide range so that the steady state results were essentially independent of the chosen value. The mathematical model was solved by computer program which was written by Visual basic language to solve the momentum and energy equations and to calculate Nusselt number. The Tridiagonal system of equation was used to solve the matrix of dependent variables. Heat transfer process by natural convection in stratified media was also solved by Mechanical ANSYS Parametric Design Language (APDL). The FLUID 141 element is used which can solve model of transient or steady state fluid/thermal systems that involve fluid and/or non-fluid regions.

Results and Discussion

Theoretical investigation are done for three working fluids, air (Pr = 0.7), water (Pr = 6) and oil (Pr = 6400), three Grashof numbers (1E4, 1E5 and 1E6) and three angle (-30, 0 and 30) for wide range of thermal stratification (S = 0, 0.5, 1, 1.5, 2, 3, 4). Figure 2 shows the temperature profile for the different values of the stratification level at mid high wall plane (X = 0.5). The temperature profile decreases with increasing the stratification parameter. For the higher value of stratification, the ambient temperature exceeds the wall temperatures, which lead to negative temperature profile. The figures also show that the temperature profile equal to zero at (S = 2) because of the equalization between the wall and ambient temperature. A comparative study of figures 2 to 4 indicates that the effect of stratification parameter is marginalized with the increase in Prandtl number, as the separateness among the temperature profile reduces. Also for given value of Prandtl number the velocity and thermal boundary layer thickness are almost the same while with the increase in Prandtl number the boundary layer thickness reduces. Figure 5 shows that the temperature profile decreases with the increase in Prandtl number. In addition, the reversal of temperature was found to be stronger at high Prandtl numbers and weaker at low numbers. It can be suitably remarked that the increase in Grashof number does not practically vary the effect of stratification factor on temperature profiles. Figures 6 show that with increase in Grashof number the fluid temperature deceases. This must happen because buoyancy force assists the flow by increasing fluid velocity and hence the heat is convected readily thereby reducing fluid temperature. Figure 7 illustrates the influence of the inclination angle (φ) on temperature profile for stratified media (S = 2), where observed that in addition to the influence of thermal stratification the temperature profile will be less effected by the inclination angle of the wall, this is considerably noted for high levels of thermal stratification, therefore the orientation marginalized the effect of the stratification parameter. Figure 8 shows that the velocity profile decreases with increasing the stratification parameter. It is understood, since the factor ($T_w - T_{\infty,x}$) reduces with the increase in stratification factor, thus buoyancy effect very close to the plate is marginalized thereby reducing the fluid velocity. A comparative study of figures 8 to 10 indicates that the effect of stratification parameter is marginalized with the increase in Prandtl number, as the separateness among the velocity profile reduces. Figure 11 shows that the velocity profile decreases with the increase in Prandtl number. At high Prandtl numbers there is a small reversal of flow while for low Prandtl numbers the flow reversal is much stronger. It can be suitably remarked that the increase in Grashof number does not practically vary the effect of stratification factor on velocity profiles. Figure 12 shows that with increase in Grashof number the fluid velocity increases. This is because the buoyancy force assists the flow by increasing fluid velocity. Figure 13 shows that the velocity profile decreases with increasing the Grashof number. This phenomenon is clear at high Prandtl number which lowers fluid velocity. Figure 14 illustrates the influence of the inclination angle (φ) on velocity profile for stratified level (S = 2), where observed that in addition to the influence of thermal stratification the velocity profile will be less effected by the inclination angle of the wall, this is considerably noted for high levels of thermal stratification, therefore the orientation marginalized the effect of the stratification parameter. The effect of the stratification parameter is to reduce the Nusselt number. Nusselt number is equal to zero at any location of the plane wall when the wall temperature equal to ambient temperature. This equalization is result of stratification level. The figure 15 shows the deceases in Nusselt number with the stratification parameter because of the Nusselt number dependence on the temperature profile which decreased with increase in stratification parameter as mentioned above. As the Prandtl number increases the Nusselt number first deceases, then increases. An increase in Prandtl number is found to cause a decrease in thermal boundary layer thickness and an increase in the absolute value of the temperature gradient at the surface. In unstratified media the local Nusselt number for air is higher than for water and less than for oil as shown in figure 16. In stratified media, the Nusselt number has the same behavior of the unstratified environment. Nusselt number is dependent on many variables, one of these variables Grashof numbers which affects the heat transfer behavior from fluid to another. The fluids which have small Prandtl number, Nusselt number decreases with increasing the Grashof number**.** The reverse behavior is for fluids which have large Prandtl number where the Nusselt number increases with increasing Grashof number as shown in figure 17.The Nusselt number has the same behavior in stratified and unstratified environment. Consider an inclined hot plate that makes an angle (φ = 30) from vertical wall plane. The difference between the buoyancy and gravity force acting on a unit volume of fluid in the boundary layer is always in the vertical direction. In the case of inclined plate, this force can be resolved into two components, the parallel force drive the flow along the plate and the normal force on the wall plane. The force that drives the motion is reduced; therefore the convection currents to be weaker and the rate of heat transfer to be lower relative to the vertical plane case. In the case (φ = -30) the opposite behavior is observed. The reason for this behavior is that the normal force component initiates upward motion in addition to the parallel motion along the wall plane, and thus the boundary layer breaks up and forms plumes. As result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation. In the stratified media, the Nusselt number has the same behaviors of the unstratified media until temperature defect occurs, then the Nusselt number has the opposite behaviors as shown in figure 18.

ANSYS Analysis

Figures 19 to 21 show the temperature distribution for different stratification parameter. It is clear that in the thermal stratified environment, the fluid temperature increases with height and with stratification parameter. The domain have region with no heat transfer because of equalization between wall and fluid temperatures, the fluid above the equalization region have temperature more than wall temperature, therefore the temperature defect happens. Figures 22 to 24 show the velocity decreases with increase in stratification parameter and the reverse flow was happened in the media which have stratification parameter more than one. The figures 25 and 26 show visualization to fluid flow in thermal stratified media. The figures 27 to 29 show the heat transfer coefficient decrease with increase in stratification level.

Verification

To verify the results obtained for the present study, a comparison is made with the results achieved by previous studies. Temperature profile (figure 1) agrees with the results of Ahmed (2005) (numerical study) shown in figure 30 and Tanny and Cohen (1998) (experimental study) shown in figure 31. Velocity Profile (figure 8) agrees with the results of Angirasa and Srinivasan (1992) (numerical study) shown in figure 32 and Cheesewright (1967) (analytical study) shown in figure 33. The effect of Prandlt number on temperature Profile (figure 5) agrees with results of Singh et al (2010) (numerical study) shown in figure 34. Nusselt number (figure 15) agrees with the result of Ahmed (2005) shown in figure 35.

Conclusions

1. For constant wall temperature when the stratification parameter increases, the temperature profile steepens near the surface, the buoyancy level decrease and the maximum velocity decreases because of the decrease in buoyancy.
2. For constant wall temperature when the values of stratifications more than one, the local temperatures adjacent to the wall exceed the wall temperature in regions of

the top portion of the wall, which receives heat from the fluid, and a reverse flow will exist.

3. The effect of stratification parameter is marginalized with the increase in Prandtl number.
4. The reversal of temperature is strong at high Prandtl numbers and weaker at low numbers and the reversal of flow velocity is strong at low Prandtl numbers and weaker at high numbers.
5. The increase in Grashof number does not practically vary the effect of stratification on temperature and velocity profiles.
6. The local Nusselt number decreases when the stratification increases.

7. As Prandtl number increases the Nusselt number first decrease, then increase.

Nomenclature

Gr	Grashof number
g	gravitational Acceleration
L	characteristic length of the plane wall
Nu	Nusselt number
Pr	Prandtl number
S	thermal stratification parameter
T	temperature
t	time
t*	non-dimensional time
u	velocity in x-direction
u_c	characteristic velocity
U	non-dimensional velocity in X-direction
v	velocity in y-direction
V	non-dimensional velocity in Y-direction
X	non-dimensional downstream coordinate
x	downstream coordinate
Y	non-dimensional horizontal space coordinate
y	horizontal space coordinate

Greek letters

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
ϑ	kinematic viscosity
ρ	density
φ	angle of inclination
Θ	non-dimensional temperature

Superscript

∞	location away from the wall outside the boundary layer
∞, 0	location away from the wall at x = 0
∞, x	location away from the wall at any x
w	wall

References

Ahmed, T.E. (2005) "Transient natural convection heat transfer from a plane wall to a thermally stratified media" MSc. Thesis, University of Technology, Baghdad.

Angirasa, D. and Srinivasan, J. (1992) "Natural convection heat transfer from an isothermal vertical surface to a stable thermal stratified fluid" Journal of Heat Transfer Vol.114/917

Cheesewright, R. (1967) "Natural convection from a plane vertical surface in non-isothermal surroundings" Int. J. Heat and Mass Trans. Volume 10, Issue 12, Pages 1847-1859.

Chen, C.C and Eichhorn, R. (1976) "Natural convection from a vertical surface to a thermally stratified fluid" ASME Journal of Heat Transfer Vol.98, PP.446-451.

Deka, Rudra Kt and Neog, Bhaben Ch. (2009)" Unsteady natural convection flow past an accelerated vertical plate in a thermally stratified fluid" Theoret. Appl. Mech., Vol.36, No.4, PP. 261-274, Belgrade.

Isak, Anuar, Nazar, Roslinda and Pop, Ioan. (2008) "Mixed convection boundary layer flow adjacent to a vertical surface embedded in a stratified medium" Int. J. Heat and Mass Trans.vol. 51, PP. 3693-3695.

Jaluria, Y. (1980) "Natural convection heat and mass transfer" Pergamon Press.

Jaluria, Y. and Himasekhar, K. (1983) "buoyancy-induced two-dimensional vertical flows in a thermally stratified environment" Computers & Fluids Vol. 11, Issue 1, Page 39-49.

Kulkarni, A. K., Jacobs, H. R. and Hwang, J. J. (1987) " Similarity solution for natural convection flow over an isothermal vertical wall immersed in thermally stratified medium " Int. J. Heat and Mass Transfer Volume 30, Issue 4, PP 691-698

Pantokratoras, A. (2003) "A note on the Nusselt number adjacent to a vertical isothermal plate immersed in thermally stratified water at low temperatures" Int. J. Heat and Fluid Flow Vol. 32, Issue 2, Pages 278-281.

Saha, S.C, Hossain, M.A. (2004) " Natural convection flow with combined buoyancy effects due to thermal and mass diffusion in a thermally stratified media " Nonlinear Analysis: Modeling and Control, Vol. 9, No. 1, PP. 89–102.

Singh, Gurminder, Sharma, P.R and Chamkha, A.J. (2010) "Effect of thermally stratified ambient fluid on MHD convective flow along a moving non-isothermal vertical plate" international of Physical Science Vol.5 (3), PP.208-215.

Tanny, J. and Cohen, J. (1998) "The mean temperature field of a buoyancy induced boundary layer adjacent to a vertical plate immersed in a stratified medium" Int. J. Heat and Mass Transfer Volume 41, Issue 14, PP. 2125-2130.

Yang, K.T, Novotny, J.L and Cheng, Y.S. (1972) "Laminar free convection from a non-isothermal plate immersed in a temperature stratified medium" Int. j. Heat and Mass Transfer, Vol. 15, PP. 1097–1109.

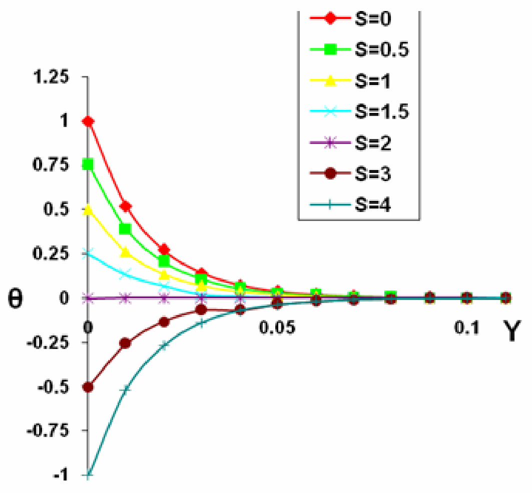

Figure 2. Temperature profile for Pr = 0.7 and Gr =1E5

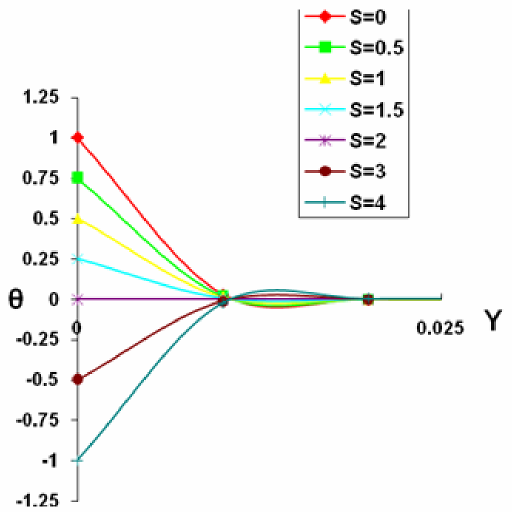

Figure 4. Temperature profile for Pr = 6400 and Gr =1E5

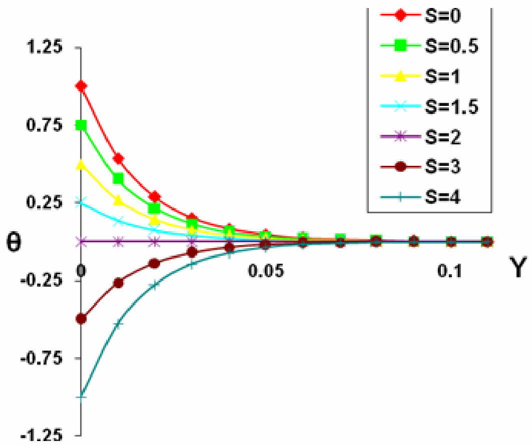

Figure 3. Temperature profile for Pr = 6 and Gr =1E5

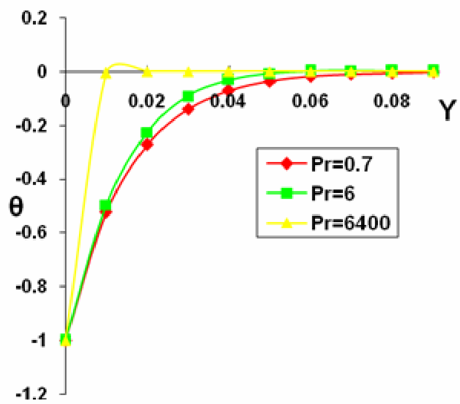

Figure 5. Temperature profile for S = 4 and Gr =1E6

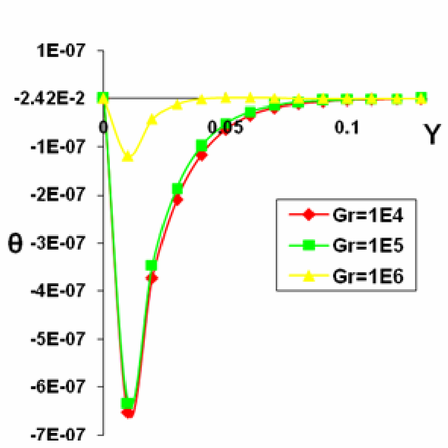

Figure 6. Temperature profile for Pr = 6400 and S = 2

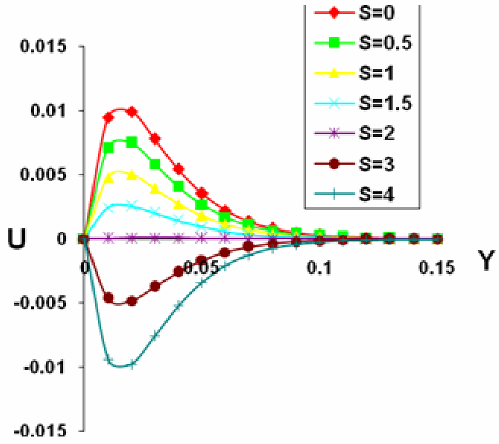

Figure 8. Velocity profile for Pr = 0.7 and Gr =1E5

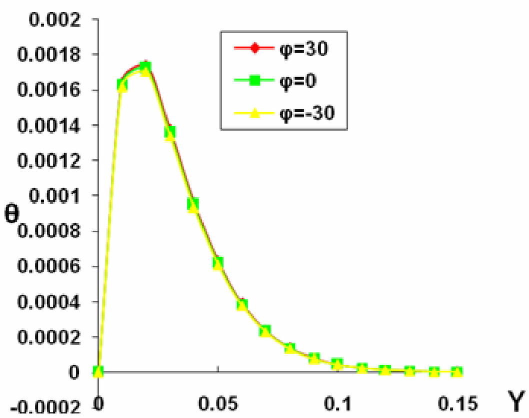

Figure 7. Temperature profile for Pr = 6 , Gr =1E5 and S = 2

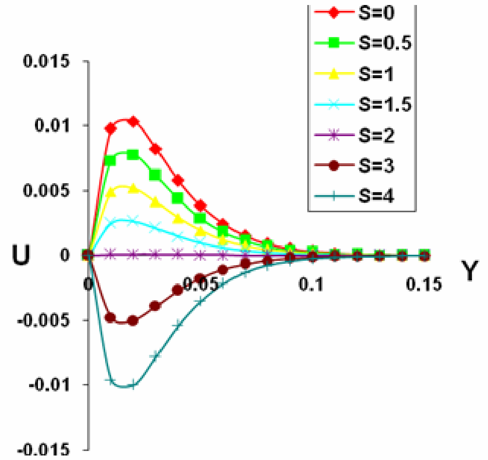

Figure 9. Velocity profile for Pr = 6 and Gr =1E5

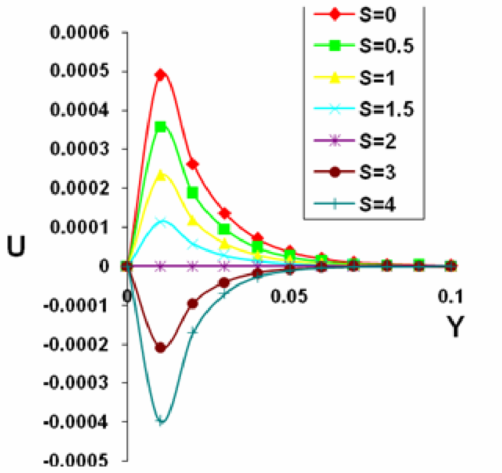

Figure 10. Velocity profile for Pr = 6400 and Gr =1E5

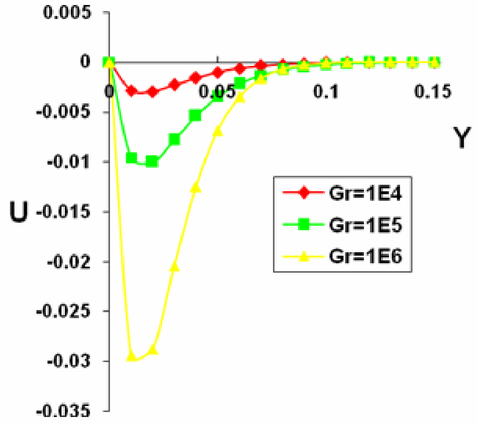

Figure 12. Velocity profile for Pr = 6 and S = 4

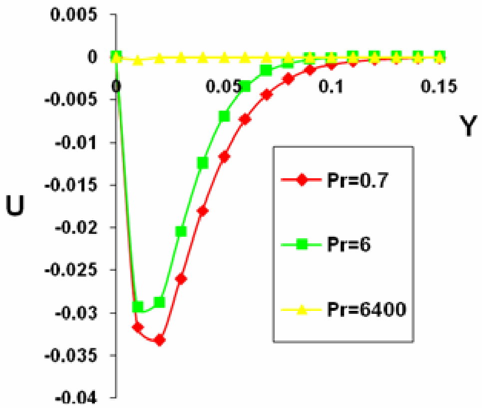

Figure 11. Velocity profile for S = 4 and Gr = 1E6

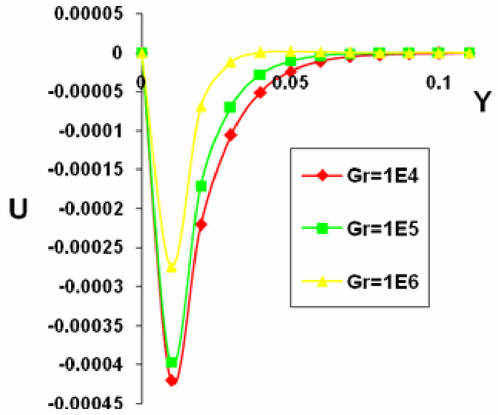

Figure 13. Velocity profile for Pr = 6400 and S = 4

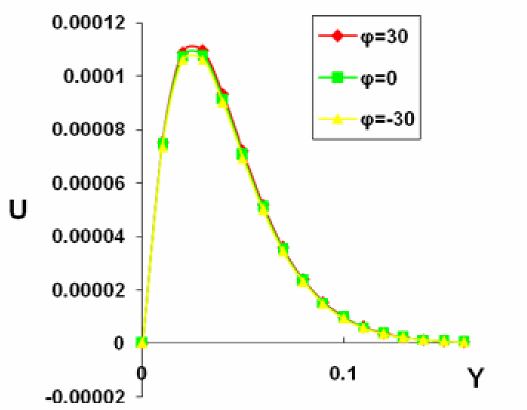

Figure 14. Velocity profile for Pr = 6, S = 2 and Gr = 1E5

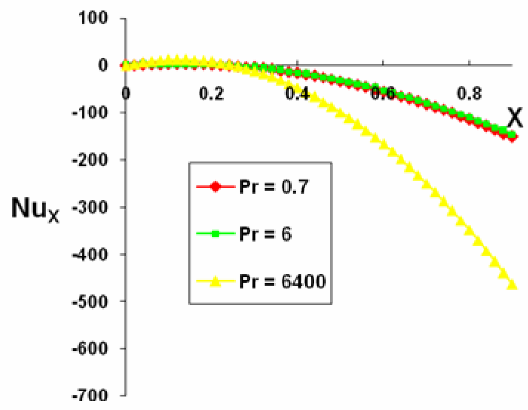

Figure 16. Local Nusselt number for S = 4 and Gr = 1E5

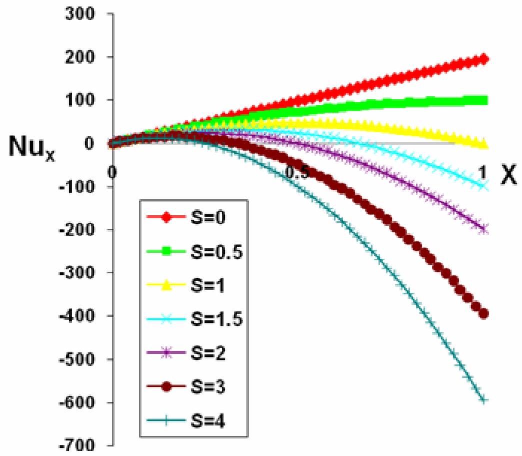

Figure 15. Local Nusselt number for Pr = 6400 and Gr =1E5

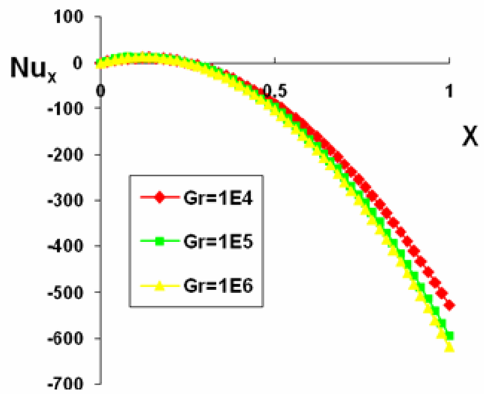

Figure 17. Local Nusselt number for Pr = 6400 and S = 4

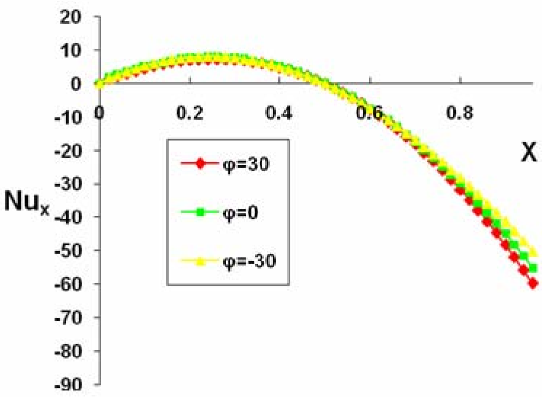

Figure 18. Local Nusselt number for Pr = 6, S = 2 and Gr =1E5

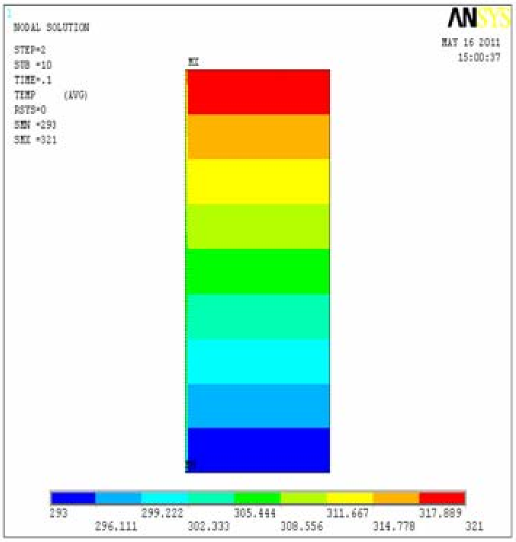

Figure 20. Temperature distribution for Pr = 0.7 and S = 2

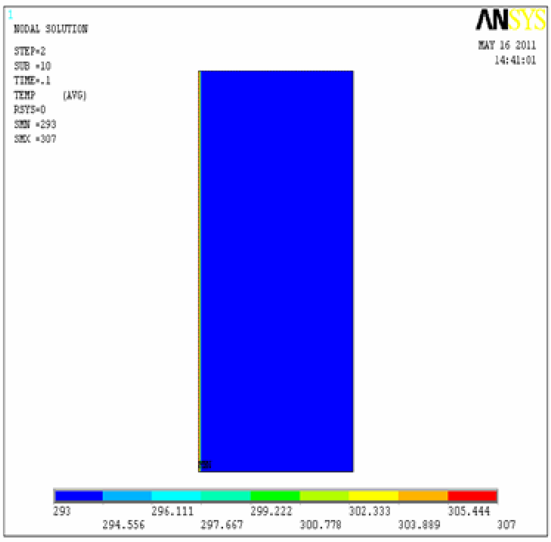

Figure 19. Temperature distribution for Pr = 0.7 and S = 0

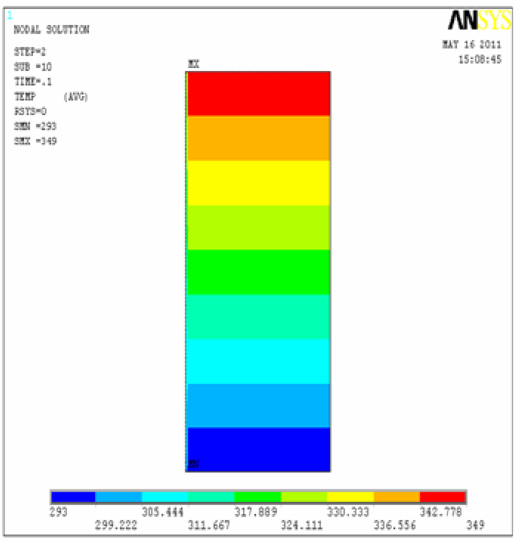

Figure 21. Temperature distribution for Pr = 0.7 and S = 4

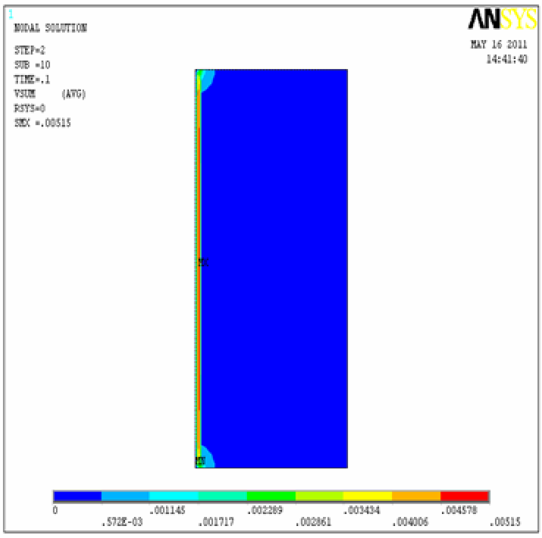

Figure 22. Velocity distribution for Pr = 0.7 and S = 0

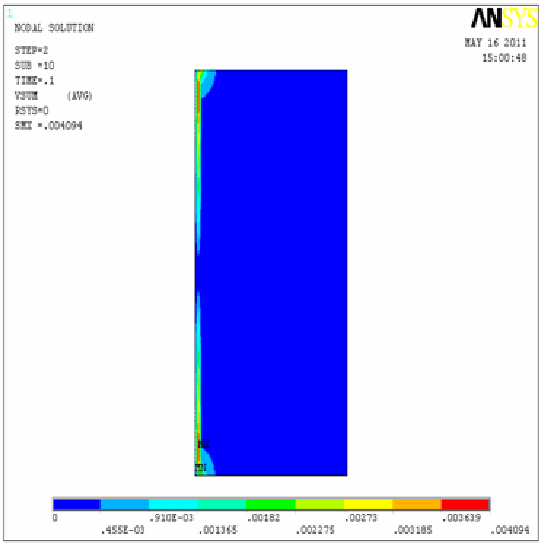

Figure 23. Velocity distribution for Pr = 0.7 and S = 2

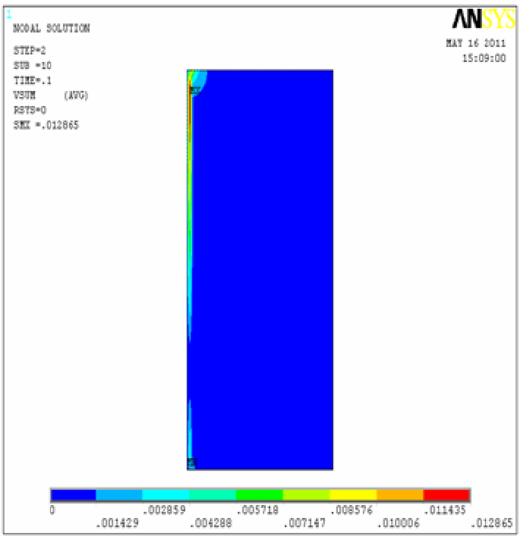

Figure 24. Velocity distribution for Pr = 0.7 and S = 4

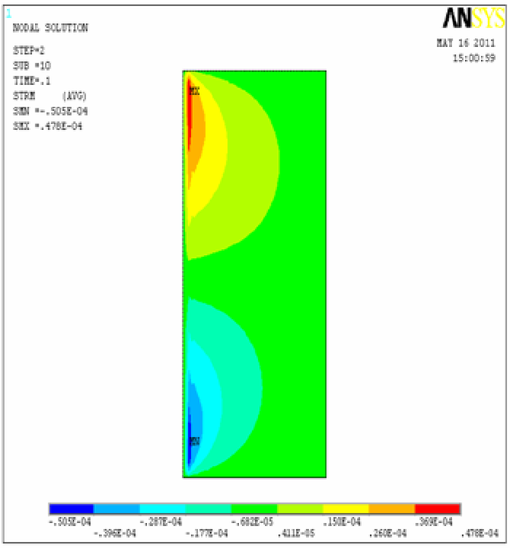

Figure 25. Stream function for Pr = 0.7 and S = 2

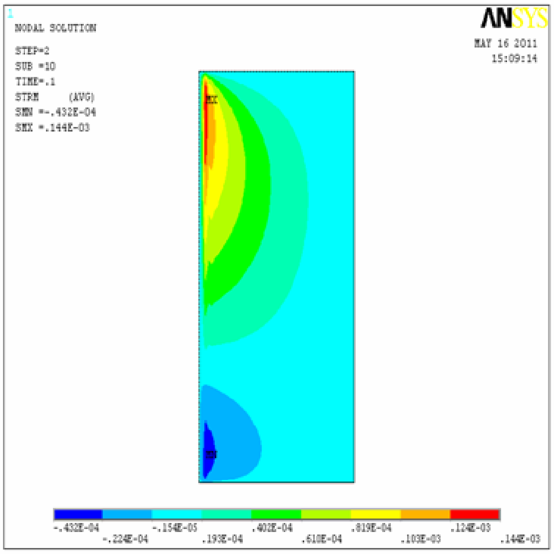

Figure 26. Stream function for Pr = 0.7 and S = 4

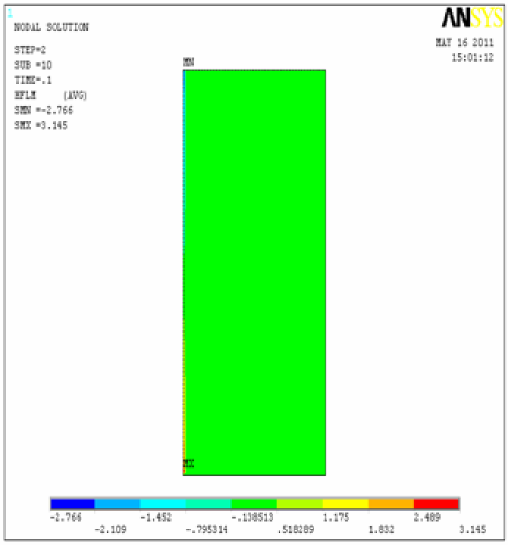

Figure 28. Heat transfer Coefficient for Pr = 0.7 and S =2

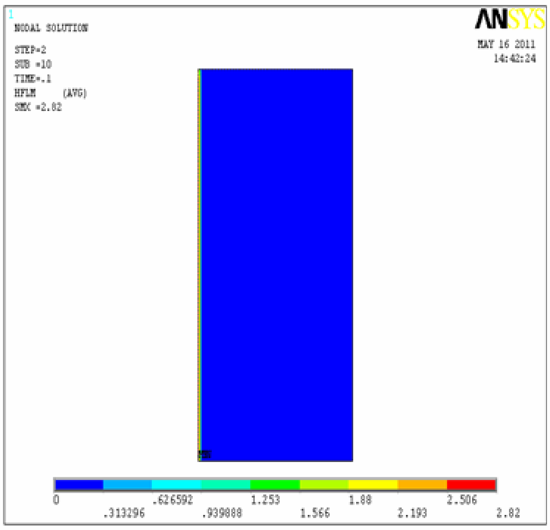

Figure 27. Heat transfer coefficient for Pr = 0.7 and S = 0

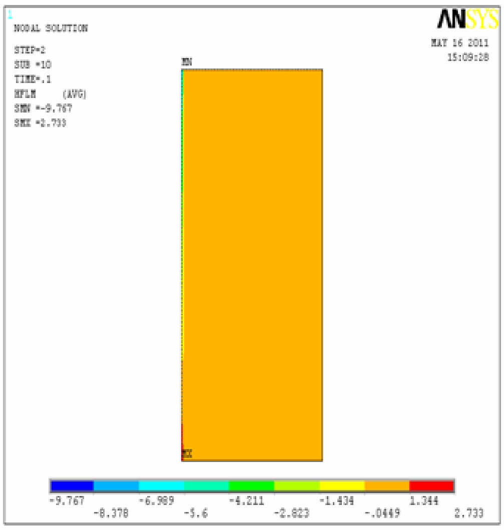

Figure 29. Heat transfer Coefficient for Pr = 0.7 and S = 4

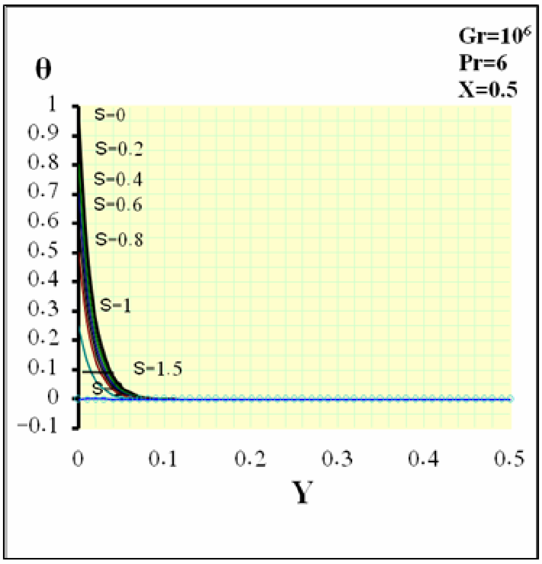

Figure 30. Temperature profile (Ahmed 2005)

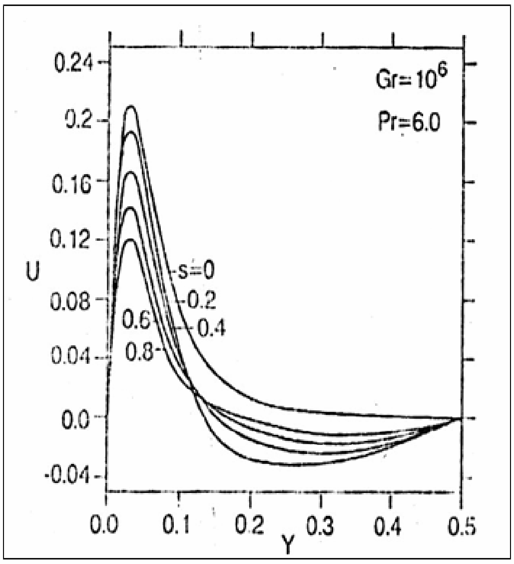

Figure 32. Velocity profile (Angirasa and Srinivasan 1992)

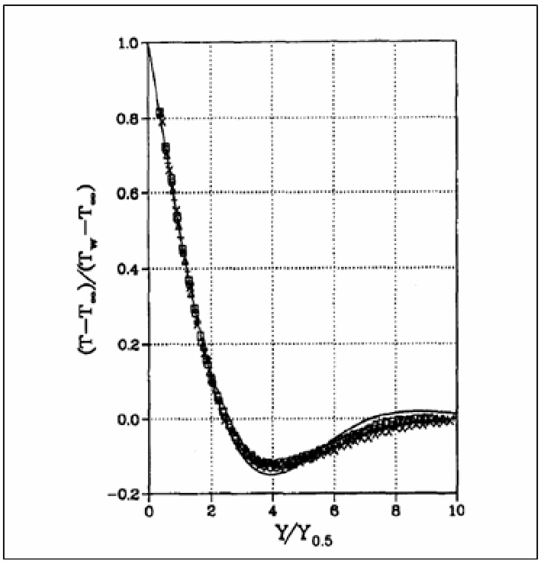

Figure 31. Temperature profile (Tanny and Cohen 1998)

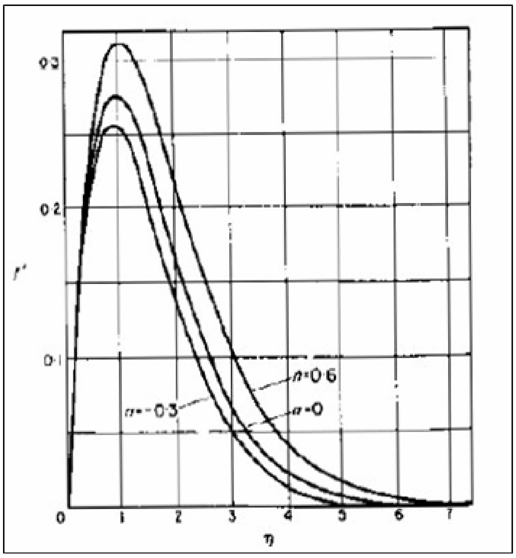

Figure 33. Velocity profile (Cheesewright 1967)

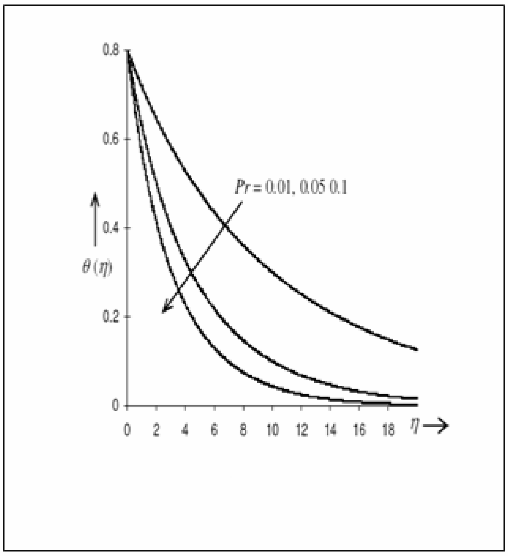

Figure 34. Effect Prandtl number (Singh et al 2010)

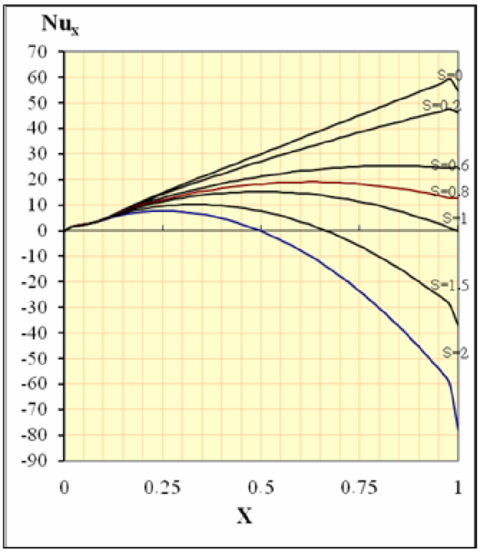

Figure 35. Nusselt number (Ahmed 2005)