System Identification Algorithm for Systems with Interval Coefficients

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Abstract

In this research a new system identification algorithm is presented for obtaining an optimal set of mathematical models for system with perturbed coefficients, then this algorithm is applied practically by an "On Line System Identification Circuit", based on real time speed response data of a permanent magnet DC motor. Such set of mathematical models represents the physical plant against all variation which may exist in its parameters, and forms a strong mathematical foundation for stability and performance analysis in control theory problems.

:الخلاصة

في هذا البحث تم تقديم طريقة جديدة لتعريف الأنظمة من أجل الحصول على المجموعة المثلى للنماذج الرياضية   للأنظمة التي تحتوي على معاملات غير ثابتة. تم تطبيق هذه الطريقة عمليا عن طريق منظومة لتعريف الأنظمة تعتمد على القراءات العملية للاستجابة الزمنية لسرعة المحرك التيار المستمر ذي المغناطيس الثابت. هذه المجموعة من النماذج الرياضية سوف تقوم بتمثيل الأنظمة الفيزياوية على الرغم من جميع التغييرات التي يمكن ان تحصل في معاملاتها العددية، و سوف تقوم بتكوين قاعدة رياضية قوية لمعالجة الأستقرارية و الأداء في منظومات السيطرة.

Keywords: system identification, interval coefficients, Sensitivity, uncertainty, transfer function.

1. Introduction:

In the controller design problem for any system, the basic goal is that the performance and stability characteristics meet certain specifications set by the designer; as a result this needs an accurate mathematical model for the system.

In the physical world, the process of obtaining accurate model is impossible, because input-output information and physical laws have never been complete. For example, ohm's law describes the relationship between current and voltage in the resistors, but it doesn't describe the effect of temperature, gravitational fields, or magnetic fields [1].

On the other hand all experimental data represent nonlinear functions between inputs and outputs, which in control analysis is not desirable because of the computational cost, so mostly these functions are linearized, and this linearization affect the accuracy of the model.

The general trend in the system identification theory is to model system with inaccuracy as a set of mathematical models, which represent all possible aspects of physical plants, so inaccuracy appears as sets of bounded range parameters. A system identification technique is presented in [2]; it is based on taking input-output data in frequency domain obtained from experimental test signals, and fitting these data to interval transfer function that contains complete frequency behavior with respect to frequency. In this paper alternative algorithm is presented to deal with input-output data obtained in time domain, this algorithm uses the sensitivity of functions to its parameters to estimate the range and weight of perturbation of each parameter.

2. Theory:

The problem of interval system identification can be formulated as follows: For any experimental set of data:

$D(t_i)$ for i = 1,2, ..., N.

N: No. of data.

let $G_I(t)$ be the nominal function that represents these data, such that:

$$G_I(t) = a_1 f_1(t) + a_2 f_2(t) + \ldots + a_n f_n(t) \quad (1)$$

n: number of parameters.

a_j: constant coeficients.

$f_j(t)$: function of time

for $j = 1,2, ..., n$.

there is family of mathematical models:

$$\mathbf{G}(t) := \left\{ G(t) : \hat{a}_j \in \left[a_j - \omega_j \epsilon_j^-, a_j + \omega_j \epsilon_j^+ \right] \right\} (2)$$

$$\bar{\omega} = [\omega_1 \quad \omega_2 \quad \omega_3 \ldots \ldots \ldots \omega_n]_{1 \times n}$$
$$\bar{\epsilon}^+ = [\epsilon_1^+ \quad \epsilon_2^+ \quad \epsilon_3^+ \ldots \ldots \ldots \epsilon_n^+]_{1 \times n}$$
$$\bar{\epsilon}^- = [\epsilon_1^- \quad \epsilon_2^- \quad \epsilon_3^- \ldots \ldots \ldots \epsilon_n^-]_{1 \times n}$$

ω_i : wight of perturbation.

ϵ_j^- : lower limit of perturbation.

ϵ_j^+ : upper limit of perturbation.

for $j = 1,2, ..., n$.

That can represents all perturbation possibilities that occur for parameters. In any interval system identification, the basic requirements are [2]:

1. $\mathbf{G}(t)$ represents all experimental data.
2. $\|\bar{\epsilon}^+\|$ and $\|\bar{\epsilon}^-\|$ must be as small as possible.
3. $\bar{\omega}$ must be choosen so that the interval is tightly

   bounded around the parameter.

In this paper the procedure is divided into three parts. The first part include the identification of the nominal function $G_I(t)$, and this can be done by using Least Square Fit algorithm, then by using the sensitivity of function to parameters, the weight of perturbations for each parameter ω_i is calculated, finally the deflection of nominal function $G_I(t)$ from the experimental data is obtained as sets of bounded interval around parameters.

a) Nominal System Identification.
   For this task, the Least Square Fit algorithm is used, a brief description for this algorithm, that is the process of curve fitting for data set that contains a significant amount of noise and this can be done by minimizing the following function [3]:

$$S(a_1, a_2, ..., a_n) = \sum_{i=1}^{N} W_i^2 [D(t_i) - G_I(t_i)] (3)$$

where W_i is the weight of experimental data, optimal set of parametrs canbe obtained by solving the following equation:

$$\frac{\partial S}{\partial a_j} = 0 \quad j = 1,2,\dots,n \quad (4)$$

The above notation implies that we already have mathematical form of $G_I(t)$, usually from the theory related to experimental data, so the fitting process is exclusively for the parameters.

b) Weight Selection.

According to mathematical definition in (2), range of perturbation for each parameter is mainly dependant on $\bar{\omega}$, $\bar{\varepsilon}^-$ & $\bar{\varepsilon}^+$, so the problem of finding appropriate weight is considered in this part. Wight selection is extremely important for minimizing the family of models, by eliminating unnecessary members. Each parameter in the nominal function has its particular weight, which is defined as the average value of deflections which occur in nominal function at each time of experimental data, caused by small variation in that parameter.

First let define Δ_i, which represents the error between nominal function and experimental data.

$$\Delta_i = D(t_i) - G_I(t_i) \quad (5)$$
for $i = 1,2,\dots,N$.

Sensitivity of nominal function to particular parameter can be defined as:

$$S_{aj} = \frac{\partial G_I(t)}{\partial a_j} \quad (6)$$
for $j = 1,2,\dots,n$.

Now the participation of each parameter in the error between the nominal function and experimental data can be calculated from:

$$\Delta_{aij} = \Delta_i * \hat{S}_{aj} \quad (7)$$

$$\hat{S}_{aj} = \frac{S_{aj}}{\sum_{j=1}^{n} S_{aj}}$$

for $j = 1,2,\dots,n$.
for $i = 1,2,\dots,N$.
Δ_{aij}: Deflection in nominal function caused by variation in parameter (j) at each experimental data (i).

This relationship can be proved by the following simple argument:

Assume n dimensions space, where $G_I(t)$ of n parameters. The change in $G_I(t)$ can be approximated in term of the change in the n planes as follows:

$$\Delta = \sum_{j=1}^{n} \Delta_{aj} \quad (8)$$

Δ: change in $G_I(t)$.
Δ_{aj}: change in the jth plane.

lets assume that eq. (7) is valid and substitute it in eq. (8).

$$\Delta = \sum_{j=1}^{n} \Delta * \frac{S_{aj}}{\sum_{j=1}^{n} S_{aj}} \quad (9)$$

multiply both sides by $\left(\sum_{j=1}^{n} S_{aj} \right)$

$$\Delta * \sum_{j=1}^{n} S_{aj} = \Delta * \sum_{j=1}^{n} S_{aj}$$

since both sides are equal then eq. (7) is valid.

For mathematical simplicity, matrix notation will be used in calculation of weights, so first we construct $\hat{S}_a$ and $\bar{\Delta}$ which is respectively:

$$\hat{S}_a = \begin{bmatrix} S_{a11} & \cdots & S_{a1n} \\ \vdots & \ddots & \vdots \\ S_{aN1} & \cdots & S_{aNn} \end{bmatrix}_{N \times n}$$

$$\bar{\Delta} = [\ \Delta_1 \quad \Delta_2 \quad \dots \quad \Delta_{N-1} \quad \Delta_N \ ]_{1 \times N}$$

The perturbation in nominal function which is defined as:

$$\hat{\Delta}_a = [\hat{\Delta}_{a1} \quad \hat{\Delta}_{a2} \quad \hat{\Delta}_{a3} \dots \dots \hat{\Delta}_{an}]_{1 \times n}$$

can be calculated as follows:
$$\hat{\Delta}_a = \bar{\Delta} * \hat{S}_a \quad (10)$$

Finally the weight of perturbation is:

$$\bar{\omega} = \frac{1}{N} * \hat{\Delta}_a \quad (11)$$

$$= [\omega_1 \quad \omega_2 \quad \omega_3 \ldots \ldots \omega_n]_{1 \times n}$$

c) Parameter Interval Identification.

In this part the range of parameter perturbation is calculated, and this can be done by solving the following equation for the variable ϵ_j at each time of experimental data:

$$D(t_i) = (a_1 + \omega_1 \epsilon_1) f_1(t_i) + (a_2 + \omega_2 \epsilon_2) f_2(t_i) + \cdots + (a_n + \omega_n \epsilon_n) f_n(t_i) \quad (12)$$

The solution of above n variable can be estimated according to the following:

Lets define the ℓth range of perturbation in $\bar{\epsilon}$:

$$\bar{\epsilon} = \begin{cases} 0, & j \neq \ell \\ \hat{\epsilon}_\ell, & j = \ell \end{cases} \quad (13)$$

by substituting eq. (13) in (12),
this yields the following equation:

$$D(t_i) = a_1 f_1(t_i) + \cdots + a_{\ell-1} f_{\ell-1}(t_i) + (a_\ell + \omega_\ell \hat{\epsilon}_\ell) f_\ell + a_{\ell+1} f_{\ell+1} + \cdots + a_n f_n(t_i) \quad (14)$$

Solving eq. (14) yields $\hat{\epsilon}_\ell$, which represents the maximum deflection in j^{th} parameter caused by the difference between experimental data and nominal function.

The actual value of deflection can be calculated by using the following equation:

$$\epsilon_\ell = \hat{\epsilon}_\ell * \frac{S_{a\ell}}{\sum_{j=1}^{n} S_{aj}} \quad (15)$$

This equation can be proved according to the following argument:
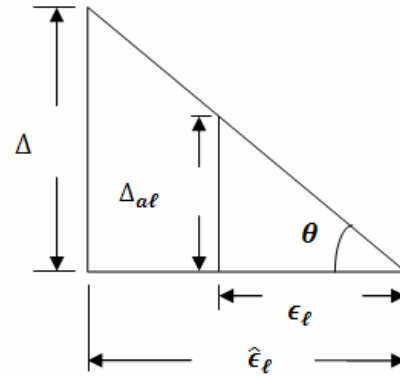Consider the following triangle:

Fig. (1)

This triangle without losing generality represents the relationship between deflection in function and deflection in ℓ^{th} parameter, where:

$$\theta = \tan^{-1} S_{a\ell} \quad (16)$$

According to the triangle similarity:

$$\frac{\epsilon_\ell}{\hat{\epsilon}_\ell} = \frac{\Delta_{a\ell}}{\Delta} \quad (17)$$

and according to eq. (7):

$$\frac{\Delta_{a\ell}}{\Delta} = \frac{S_{a\ell}}{\sum_{j=1}^{n} S_{aj}} \quad (18)$$

By substituting eq. (17) in (18), it yields:

$$\epsilon_\ell = \hat{\epsilon}_\ell * \frac{S_{a\ell}}{\sum_{j=1}^{n} S_{aj}}$$

Now, for $(\ell = 1,2,\ldots,n)$, we will have :

$$\bar{\epsilon} = [\epsilon_1 \quad \epsilon_2 \quad \epsilon_3 \ldots \ldots \epsilon_n]_{1 \times n}$$

for upper and lower range of perturbation
for parameters respectively

$$\epsilon_j^+ = \max_i \{0, \epsilon_i\}$$

$$\epsilon_j^- = \min_i \{0, \epsilon_i\}$$

3. Practical Application of Interval System Identification for DC motor

3.1. Mathematical Model of DC Motor [4,5,6]

The model of the motor shown in **Fig**. [2] based on the ideal permanent magnet DC-motor
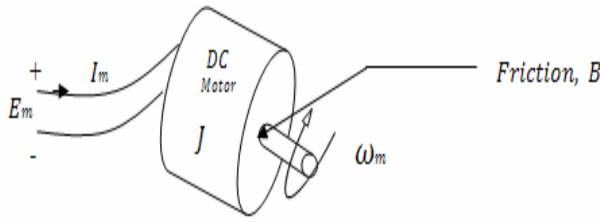

Fig. [2], Permanent Magnet DC Motor

From **Fig** [2], J is the inertia of the rotor, B is the damping coefficient, ω_m is the motor speed (rpm), and Em & Im is the supply voltages and current respectively.

Now the torque generated by the motor T_m is:

$$T_m = K\,I_m \quad (19)$$

also

$$E_m = R_o \quad (20)$$

from Newton second law of motion

$$J\frac{d\omega_m}{dt} + B\,\omega_m = T_m \quad (21)$$

substitute eq. s (19) & (20) into eq. (21)

$$J\frac{d\omega_m}{dt} + B\,\omega_m = \frac{K\,E_m}{R_o} \quad (22)$$

by taking Laplace Transformation for eq. (22):

$$\frac{\omega_m(s)}{E_m(s)} = \frac{\frac{K}{R_o J}}{s + \frac{B}{J}} \quad (23)$$

Eq. (23) represents the transfer function for DC motor. By assuming that:

$$G(s) = \frac{\omega_m(s)}{E_m(s)}$$

and taking $\mathcal{L}^{-1}$ for both sides of eq(23) :

$$G(t) = a_1\,e^{-a_2 t} \quad (24)$$
$$a_1 = K/R_o J$$
$$a_2 = B/J$$

According to eq. (24), in order to find the response of motor speed as a function of time, there are some parameters needed to be found first, like K (motor constant), $B, J,$ and R_o (motor's coil resistance), so motor static gain and time constant must be calculated, but this may be possible if the response of the motor speed can be found practically.

3.2. On Line System Identification Circuit

On Line System Identification Circuit is specially designed in this paper to collect data about DC motor speed response from initial condition till steady state, passing through transient state, this circuit is illustrated in the block diagram in **Fig**. [2].
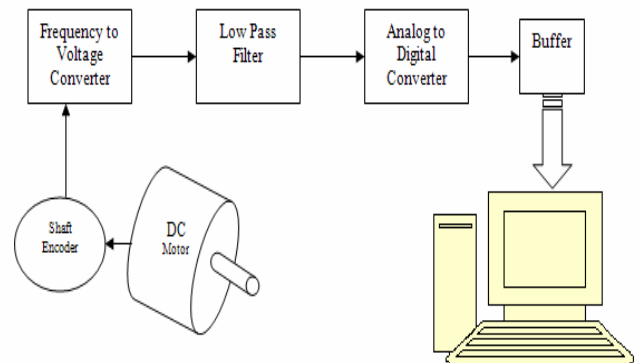

Fig. [3], On Line System Identification Circuit for DC Motor

According to the above block diagram, by using shaft encoder angular velocity first is converted to train of pulses, whose frequency is proportional directly to the angular velocity of motor; this train

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of pulses then is converted to analogue voltage using Frequency to Voltage Converter Circuit, which implements LM331. The output voltage then is filtered by low pass filter, and converted to binary equivalent number by using ADC0804. The equivalent binary number is fed to the computer through parallel port LPT1 by a tri-state buffer 74LS245. As we can see from above description the process of reading velocities of DC motor by computer consists of successive sub processes, the synchronization and enabling of these sub processes is done by C++ program in that computer, after reading the data, the same program directly starts processing the algorithm presented in this paper and display the final result, which is the mathematical model of the motor plus the range and weight of perturbations of its parameters.

4. Results

For Hitachi permanent magnet DC motor (type D04A321E), the following results is obtained according to the three algorithm steps:

A) Nominal System Identification.

By using the Least Square Fit procedure, the transfer function can be obtained, and this transfer function can be verified by calculating it with an alternative way, since the static gain defined as the steady-state speed value relative to the supplied voltage value, and the time constant is the time by which the output of the system will reach 63% of its final value, then these two values can be found by matching the diagram in **Fig**.[4] to the practical response of system in **Fig**. [5], and the both approaches should have the same results which are:

$$G(S) = \frac{138.508386}{(S + 23.079689)}$$
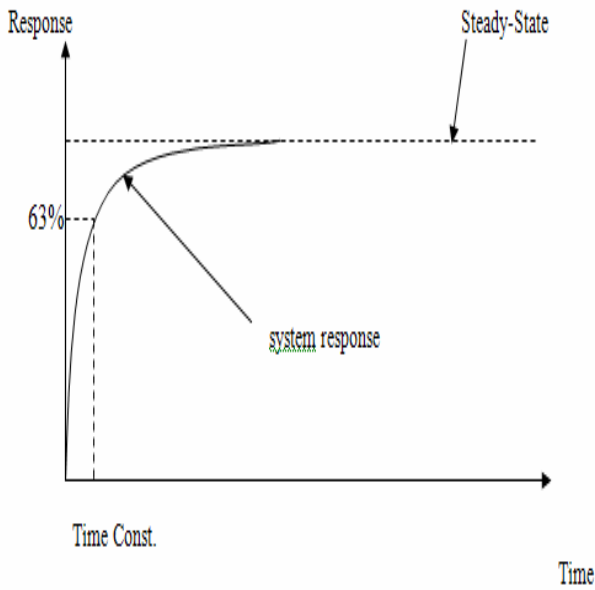
$$a_1 = 138.508386$$

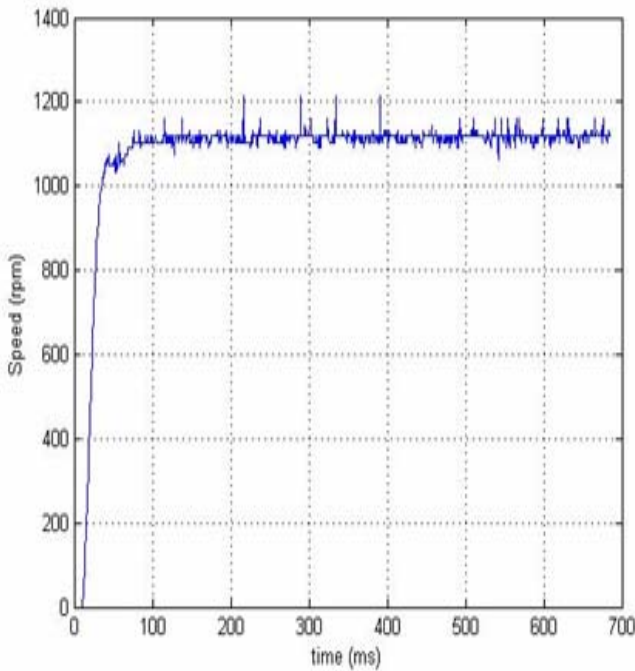$$a_2 = 23.079689$$

Fig. [4], Response of First Order System

Fig [5], Practical Response for the DC Motor.

B) Weight Selection.

$$\bar{\omega} = [\quad \omega_{a1} \qquad \qquad \omega_{a2}]$$

$$= [0.115734 \quad 0.710374]$$

C) Parameter Interval Identification.

$$\bar{\epsilon}^+ = [\quad \epsilon_{a1}{}^+ \qquad \qquad \epsilon_{a2}{}^+]$$

$$= [11.305207 \quad 1.777578]$$

$$\bar{\epsilon}^- = [\quad \epsilon_{a1}{}^- \qquad \qquad \epsilon_{a2}{}^-]$$

$$= [17.177452 \quad 3.033393]$$

The flowchart for the program that was used to obtain the above result is shown in **Fig.** (6).

5. Conclusions

MagnetSynchronous Motor Drive System", University Of Puerto Rico Mayagüez Campus 2006.

1. The nominal function for the DC motor was derived using Least Square Fit method, which is an optimization technique, so the ranges of perturbation for parameters represent deviation between nominal function and experimental data, which are mainly caused by this optimization technique.

2. The assumption of linearity is also an important reason in forming the perturbation ranges.

3. The experiment for the DC motor speed response was carried without load, in case loaded motor a new ranges of perturbation will result, according to the amount of load.

According to above, all unexpected condition or incorrect assumption in practical field can be interpreted in term of ranges of perturbation, which will result a family of models. Members in the resulted family don't have the same stability and performance conditions, so this variety in characteristics must be taken in consideration.

References:

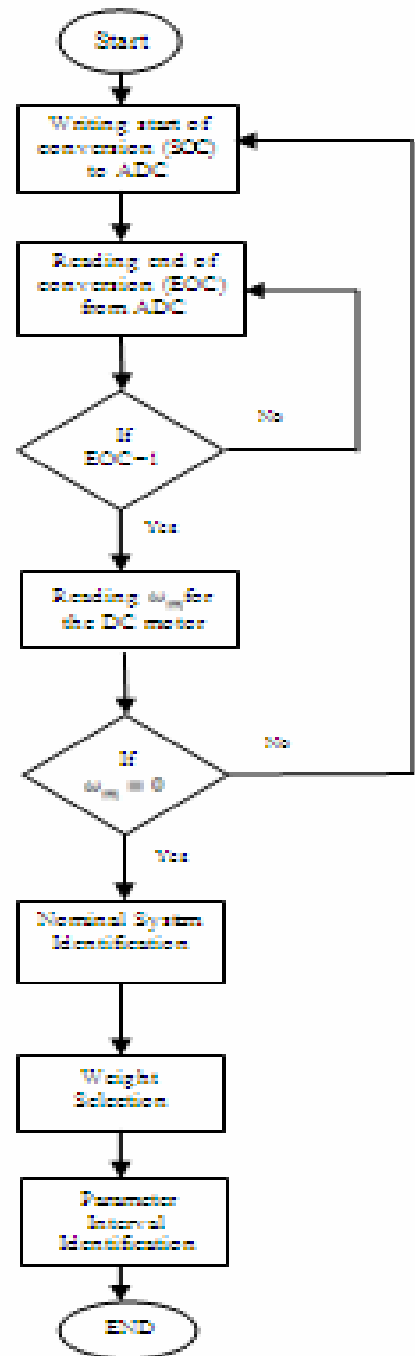[1] R.S. Sanchez, "*Robust System Theory and Application*", John Wiley & sons (1998).

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Fig. (6) Flowchart of the C++program.