FREE VIBRATION ANALYSIS OF COMPOSITE LAMINATED PLATES USING HOST 12

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ABSTRACTE

This paper presents an application of a Higher Order Shear Deformation Theory (HOST 12) to problem of free vibration of simply supported symmetric and antisymmetric angle-ply composite laminated plates. The theoretical model HOST12 presented incorporates laminate deformations which account for the effects of transverse shear deformation, transverse normal strain/stress and a nonlinear variation of in-plane displacements with respect to the thickness coordinate – thus modeling the warping of transverse cross-sections more accurately and eliminating the need for shear correction coefficients. Solutions are obtained in closed-form using Navier's technique by solving the eigenvalue equation. Plates with varying number of layers, degrees of anisotropy and slenderness ratios are considered for analysis. The results compared with those from exact analysis and various theories from references.

الخلاصة

هذا البحث يقدم التطبيق لنظرية ((Higher Order Shear Deformation Theory (HOST 12) لمسألة الأهتزاز الحر للصفائح الطبقية المركبة المتماثلة والغير متماثلة (ذات الألياف الغير متعامدة). النظرية المقدّمة HOST 12 تدمج تشويهات الطبقات التي تفسّر تأثيرات تشويه القصّ المستعرض، إجهاد طبيعي مستعرض / إجهاد و التوزيع اللاخطّي لأزاحات المستوي بالنسبة للسمك - هكذا تشكّل تشويه المقاطع العرضية المستعرضة بدقة أكثر وتزيل الحاجة لمعاملات تصحيح القصّ. الحلول للمعادلات تضمنت الحل المضبوط للصفائح الطبقية المركبة (Navier Solution). أخذ بالأعتبار تأثير تنويع مواصفات الصفائح كعدد الطبقات ودرجة الأنتروبي ونسبة السمك الى العرض. النتائج قورنت مع حلول مضبوطة ونظريات متنوعة من مصادر متعددة.

Keywords: Free vibration; Higher order theory; Shear deformation; Angle-ply plates; Analytical solutions.

INTRODUCTION

Laminated composite plates and shells are finding extensive usage in the aeronautical and aerospace industries as well as in other fields of modern technology. It has been observed that the strength and deformation characteristics of such structural elements depend upon the fiber orientation, stacking sequence and the fiber content in addition to the strength and rigidities of the fiber and matrix material. Though symmetric and antisymmetric laminates are simple to analyze and design, some specific application of composite laminates requires the use of symmetric and antisymmetric laminates to fulfill certain design requirements. Symmetric and antisymmetric angle-ply laminates are the special form of symmetric and antisymmetric laminates and the associated theory offers some simplification in the analysis. The Classical Laminate Plate Theory (Reissner E. and Stavsky Y., 1961) which ignores the effect of transverse shear deformation becomes inadequate for the analysis of multilayer composites. The First Order Shear Deformation Theories (FSDTs) based on (Reissner E., 1945) and (Mindlin RD., 1951) assume linear in-plane stresses and displacements respectively through the laminate thickness. Since FSDTs account for layerwise constant states of transverse shear stress, shear correction coefficients are needed to rectify the unrealistic variation of the shear strain/stress through the thickness. In order to overcome the limitations of FSDTs, higher order shear deformation theories (HSDTs) that involve higher order terms in the Taylor's expansions of the displacement in the thickness coordinate were developed. (Hildebrand et al., 1949) were the first to introduce this approach to derive improved theories of plates and shells. Using the higher order theory of (Reddy, 1984) free vibration analysis of isotropic, orthotropic and laminated plates was carried out by (Reddy and Phan, 1985). A generalized Levy-type solution in conjunction with the closed form solution was developed for the bending, buckling and vibration of antisymmetric angle-ply laminated plates by A. (Khdeir A., 1989). The exact solutions were obtained for the classical Kirchhoff theory and the numerical results were compared with their counterparts using the first order transverse shear deformation theory. The comparisons showed that the results obtained within the classical laminated theory could be significantly inaccurate. A selective review of the various analytical and numerical methods used for the stress analysis of laminated composite and sandwich plates was presented by (Kant and Swaminathan, 2001). Using the higher order refined theories already reported in the literature by (Kant, 1982), (Pandya and Kant, 1988) and (Kant and Manjunatha, 1988), analytical formulations, solutions and comparison of numerical results for the buckling, free vibration and stress analyses of cross-ply composite and sandwich plates were presented by (Kant and Swaminathan, 2002).

Recently the theoretical formulations and solutions for the static analysis of antisymmetric angle-ply laminated composite and sandwich plates using various higher order refined computational models were presented by (Swaminathan and Ragounadin, 2004), (Swaminathan et al., 2006) and (Swaminathan and Patil, 2008).

THEORETICAL FORMULATION

Higher Order Shear Deformation Theory (HSDT 12)

For the first time, derived the equation of motion based on higher-order shear deformation theory (HOST 12) in the present study.

The assumptions of a higher order plate theory can also be used within equivalent single layer formulation from (Swaminathan and Patil, 2008):

$$\begin{aligned} u(x,y,z,t) &= u_o(x,y,t) + z\theta_x(x,y,t) \\ &\quad + z^2 u_o^*(x,y,t) + z^3\theta_x^*(x,y,t) \\ v(x,y,z,t) &= v_o(x,y,t) + z\theta_y(x,y,t) \\ &\quad + z^2 v_o^*(x,y,t) + z^3\theta_y^*(x,y,t) \\ w(x,y,z,t) &= w_o(x,y,t) + z\theta_z(x,y,t) \\ &\quad + z^2 w_o^*(x,y,t) + z^3\theta_z^*(x,y,t) \end{aligned} \qquad (1)$$

1. The plate may be moderately thick.
2. The in-plane displacement u (x, y, z, t) and v (x, y, z, t) are cubic functions of z.
3. The transverse displacement w (x, y, z, t)of any point (x, y) cubic functions of z.
4. The transverse shear stress σ_{XZ} , σ_{YZ} are parabolic in z.
5. The in-plane stresses σ_X , σ_Y and τ_{XY} are cubic functions of z.
6. The normal to the mid-surface before deformation are straight, but not necessarily remaining normal to the mid-surface after deformation.
7. The transverse normal strain σ_Z is not zero.

The parameters u_o, v_o are the in-plane displacements and w_o is the transverse displacement of a point (x, y) on the middle plane. The functions θ_x, θ_y are rotations of the normal to the middle plane about *y* and *x* axes respectively. The parameters u_o^*, v_o^*, w_o^*, $\theta_x^*, \theta_y^*, \theta_z^*$ and θ_z are the higher-order terms in the Taylor's series expansion and they represent higher-order transverse cross sectional deformation modes. This is done by taking into account the parabolic variation of transverse shear stresses through the thickness of the plate (Swaminathan and Patil, 2008).

The strain components will be derived, based on the displacement, as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} + z^2\begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} + z^3\begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} \qquad (2)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varphi_y \\ \varphi_x \end{Bmatrix} + z\begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} + z^2\begin{Bmatrix} \varphi_y^* \\ \varphi_x^* \end{Bmatrix} + z^3\begin{Bmatrix} \kappa_{yz}^* \\ \kappa_{xz}^* \end{Bmatrix}$$

where:

$$\left\{\begin{matrix}\varepsilon_{xo}\\ \varepsilon_{yo}\\ \varepsilon_{xyo}\end{matrix}\right\} = \left\{\begin{matrix}\frac{\partial u_o}{\partial x}\\ \frac{\partial v_o}{\partial y}\\ \frac{\partial u_o}{\partial y}+\frac{\partial v_o}{\partial x}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\varepsilon^*_{xo}\\ \varepsilon^*_{yo}\\ \varepsilon^*_{xyo}\end{matrix}\right\} = \left\{\begin{matrix}\frac{\partial u^*_o}{\partial x}\\ \frac{\partial v^*_o}{\partial y}\\ \frac{\partial u^*_o}{\partial y}+\frac{\partial v^*_o}{\partial x}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\varepsilon_{zo}\\ \varepsilon^*_{zo}\end{matrix}\right\} = \left\{\begin{matrix}\theta_z\\ 3\theta^*_z\end{matrix}\right\}$$

$$\left\{\begin{matrix}\kappa_x\\ \kappa_y\\ \kappa^*_z\\ \kappa_{xy}\end{matrix}\right\} = \left\{\begin{matrix}\frac{\partial \theta_x}{\partial x}\\ \frac{\partial \theta_y}{\partial y}\\ 2w^*_o\\ \frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}\end{matrix}\right\} \qquad (3)$$

$$\left\{\begin{matrix}\kappa^*_x\\ \kappa^*_y\\ \kappa^*_{xy}\end{matrix}\right\} = \left\{\begin{matrix}\frac{\partial \theta^*_x}{\partial x}\\ \frac{\partial \theta^*_y}{\partial y}\\ \frac{\partial \theta^*_x}{\partial y}+\frac{\partial \theta^*_y}{\partial x}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\kappa_{xz}\\ \kappa_{yz}\end{matrix}\right\} = \left\{\begin{matrix}2u^*_o+\frac{\partial \theta_z}{\partial x}\\ 2v^*_o+\frac{\partial \theta_z}{\partial y}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\kappa^*_{xz}\\ \kappa^*_{yz}\end{matrix}\right\} = \left\{\begin{matrix}\frac{\partial \theta^*_z}{\partial x}\\ \frac{\partial \theta^*_z}{\partial y}\end{matrix}\right\}$$

$$\left\{\begin{matrix}\phi_x\\ \phi^*_x\\ \phi_y\\ \phi^*_y\end{matrix}\right\} = \left\{\begin{matrix}\theta_x+\frac{\partial w_o}{\partial x}\\ 3\theta^*_x+\frac{\partial w^*_o}{\partial x}\\ \theta_y+\frac{\partial w_o}{\partial y}\\ 3\theta^*_y+\frac{\partial w^*_o}{\partial y}\end{matrix}\right\}$$

Substituting eq. (2) in the stress- strain relation of the lamina, the constitutive relations for any layer in the (x, y) can be expressed in the form:

$$\left\{\begin{matrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{matrix}\right\}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & \bar{Q}_{14} & 0 & 0\\ & \bar{Q}_{22} & \bar{Q}_{23} & \bar{Q}_{24} & 0 & 0\\ & & \bar{Q}_{33} & \bar{Q}_{34} & 0 & 0\\ & & & \bar{Q}_{44} & 0 & 0\\ & Symmetric & & & \bar{Q}_{55} & \bar{Q}_{56}\\ & & & & & \bar{Q}_{66}\end{bmatrix}_k \left\{\left\{\begin{matrix}\varepsilon_{xo}\\ \varepsilon_{yo}\\ \varepsilon_{zo}\\ \varepsilon_{xyo}\\ \phi_y\\ \phi_x\end{matrix}\right\}+z\left\{\begin{matrix}\kappa_x\\ \kappa_y\\ \kappa^*_z\\ \kappa_{xy}\\ \kappa_{yz}\\ \kappa_{xz}\end{matrix}\right\}+z^2\left\{\begin{matrix}\varepsilon^*_{xo}\\ \varepsilon^*_{yo}\\ \varepsilon^*_{zo}\\ \varepsilon^*_{xyo}\\ \phi^*_y\\ \phi^*_x\end{matrix}\right\}+z^3\left\{\begin{matrix}\kappa^*_x\\ \kappa^*_y\\ 0\\ \kappa^*_{xy}\\ \kappa^*_{yz}\\ \kappa^*_{xz}\end{matrix}\right\}\right\} \qquad (4)$$

where $[\bar{Q}]$ from equ. $[\bar{Q}]=[T]^T[Q][T]$,

[Q] given by :

$$\begin{aligned}
Q_{11}&=E_1(1-\nu_{23}\nu_{32})/\Delta\\
Q_{12}&=E_1(\nu_{12}-\nu_{31}\nu_{23})/\Delta\\
Q_{13}&=E_1(\nu_{31}-\nu_{21}\nu_{32})/\Delta\\
Q_{23}&=E_2(\nu_{32}-\nu_{12}\nu_{31})/\Delta\\
Q_{33}&=E_3(1-\nu_{31}\nu_{23})/\Delta\\
Q_{44}&=G_{12}\\
Q_{55}&=G_{23}\\
Q_{66}&=G_{13}
\end{aligned} \qquad (5a)$$

where:

$$\Delta= (1-\nu_{12}\nu_{21} -\nu_{23}\nu_{32} -\nu_{31}\nu_{13} -2\nu_{12}\nu_{23}\nu_{31}) \qquad (5b)$$

And the transformation matrix [T] is given by the transformation equations:

$$\bar{Q}_{11}= Q_{11}c^4 + 2(Q_{12} + 2Q_{33})s^2c^2+ Q_{22}s^4 \qquad (5c)$$

$\overline{Q}_{12}$= (Q_{11} + Q_{22} - 4Q_{33})s^2c^2 + Q_{12}(s^4 + c^4)

$\overline{Q}_{13}$= $Q_{13}c^2$ + $Q_{23}s^2$

$\overline{Q}_{14}$= (Q_{11} - Q_{12}- 2Q_{44}) sc^3 +(Q_{12} - Q_{22} +2Q_{44})cs^3

$\overline{Q}_{22}$= $Q_{11}s^4$+2(Q_{12} + 2Q_{33})s^2c^2 + $Q_{22}c^4$

$\overline{Q}_{23}$= $Q_{13}s^2$ + $Q_{23}c^2$

$\overline{Q}_{24}$= (Q_{11} - Q_{12} - 2Q_{44})cs^3 +(Q_{12} - Q_{22} + 2Q_{44})sc^3

$\overline{Q}_{33}$= Q_{33}

$\overline{Q}_{34}$= (Q_{31}- Q_{32})cs

$\overline{Q}_{44}$= (Q_{11} - 2Q_{12} + Q_{22} - 2Q_{44})c^2 s^2 + Q_{44} (c^4 + s^4) (5d)

$\overline{Q}_{55}$= $Q_{55}s^2$ + $Q_{66}c^2$

$\overline{Q}_{56}$= (Q_{66} - Q_{55})cs

$\overline{Q}_{66}$= $Q_{55}s^2$+$Q_{66}c^2$

All other elements of [Q_{ij}] and [$\overline{Q}_{ij}$] are zero.

The entire collection of forces and moments resultants for N-layered laminated are defined as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_z \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix}_k dz \tag{6a}$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & \overline{Q}_{14} \\ & \overline{Q}_{22} & \overline{Q}_{23} & \overline{Q}_{24} \\ & symmetric & \overline{Q}_{33} & \overline{Q}_{34} \\ & & & \overline{Q}_{44} \end{bmatrix}_k$$

$$\left\{ \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} z\, dz \right.$$

$$\left. + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} z^2\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} z^3 dz \right\}$$

$$\begin{Bmatrix} N_x^* \\ N_y^* \\ N_z^* \\ N_{xy}^* \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix}_k z^2 dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & \overline{Q}_{14} \\ & \overline{Q}_{22} & \overline{Q}_{23} & \overline{Q}_{24} \\ & symmetric & \overline{Q}_{33} & \overline{Q}_{34} \\ & & & \overline{Q}_{44} \end{bmatrix}_k \tag{6b}$$

$$\left\{ \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} z^2\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} z^3\, dz \right.$$

$$\left. + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} z^4\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} z^5 dz \right\}$$

$$\begin{Bmatrix} M_x \\ M_x \\ M_z^* \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z\, dz_k$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & \overline{Q}_{14} \\ & \overline{Q}_{22} & \overline{Q}_{23} & \overline{Q}_{24} \\ & symmetric & \overline{Q}_{33} & \overline{Q}_{34} \\ & & & \overline{Q}_{44} \end{bmatrix}_k \tag{6c}$$

$$\left\{ \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} z\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} z^2\, dz \right.$$

$$\left. + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} z^3\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} z^4 dz \right\}$$

$$\begin{Bmatrix} M_x^* \\ M_y^* \\ 0 \\ M_{xy}^* \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix}_k z^3 dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & \overline{Q}_{14} \\ & \overline{Q}_{22} & \overline{Q}_{23} & \overline{Q}_{24} \\ & symmetric & \overline{Q}_{33} & \overline{Q}_{34} \\ & & & \overline{Q}_{44} \end{bmatrix}_k \tag{6d}$$

$$\left\{ \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} z^3\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} z^4\, dz \right.$$

$$\left. + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} z^5\, dz + \int_{Z_{k-1}}^{Z_k} \begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} z^6 dz \right\}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{56} \\ \overline{Q}_{56} & \overline{Q}_{66} \end{bmatrix}_k \tag{6e}$$

$$\left\{ \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} z dz \right.$$

$$\left. + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y^* \\ \phi_x^* \end{Bmatrix} z^2 dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz}^* \\ \kappa_{xy}^* \end{Bmatrix} z^3 dz \right\}$$

$$\begin{Bmatrix} Q_y^* \\ Q_x^* \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k z^2 dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{56} \\ \overline{Q}_{56} & \overline{Q}_{66} \end{bmatrix}_k \tag{6f}$$

$$\left\{ \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} z^2 dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} z^3 dz \right.$$

$$\left. + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y^* \\ \phi_x^* \end{Bmatrix} z^4 dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz}^* \\ \kappa_{xy}^* \end{Bmatrix} z^5 dz \right\}$$

$$\begin{Bmatrix} S_y \\ S_x \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k z dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{56} \\ \overline{Q}_{56} & \overline{Q}_{66} \end{bmatrix}_k$$

$$\left\{ \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} z dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} z^2 dz \right. \tag{6g}$$

$$+ \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y^* \\ \phi_x^* \end{Bmatrix} z^3 dz$$

$$\left. + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz}^* \\ \kappa_{xy}^* \end{Bmatrix} z^4 dz \right\}$$

$$\begin{Bmatrix} S_y^* \\ S_x^* \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_k z^3 dz$$

$$= \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{56} \\ \overline{Q}_{56} & \overline{Q}_{66} \end{bmatrix}_k \tag{6h}$$

$$\left\{ \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} z^3 dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} z^4 dz \right.$$

$$\left. + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \phi_y^* \\ \phi_x^* \end{Bmatrix} z^5 dz + \int_{Z_{K-1}}^{Z_K} \begin{Bmatrix} \kappa_{yz}^* \\ \kappa_{xy}^* \end{Bmatrix} z^6 dz \right\}$$

Laminate Constitutive Equations

$$
\left\{\begin{matrix} \begin{Bmatrix} N_x \\ N_y \\ N_z \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} N_x^* \\ N_y^* \\ N_z^* \\ N_{xy}^* \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_z^* \\ M_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x^* \\ M_y^* \\ 0 \\ M_{xy}^* \end{Bmatrix} \end{matrix}\right\} =
\begin{bmatrix}
\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ & A_{22} & A_{23} & A_{24} \\ & & A_{33} & A_{34} \\ SYM & & & A_{44} \end{bmatrix} &
\begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ & B_{22} & B_{23} & B_{24} \\ & & B_{33} & B_{34} \\ SYM & & & B_{44} \end{bmatrix} &
\begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ & D_{22} & D_{23} & D_{24} \\ & & D_{33} & D_{34} \\ SYM & & & D_{44} \end{bmatrix} &
\begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} \\ & E_{22} & E_{23} & E_{24} \\ & & E_{33} & E_{34} \\ SYM & & & E_{44} \end{bmatrix} \\
 &
\begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ & D_{22} & D_{23} & D_{24} \\ & & D_{33} & D_{34} \\ SYM & & & D_{44} \end{bmatrix} &
\begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} \\ & E_{22} & E_{23} & E_{24} \\ & & E_{33} & E_{34} \\ SYM & & & E_{44} \end{bmatrix} &
\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ & F_{22} & F_{23} & F_{24} \\ & & F_{33} & F_{34} \\ SYM & & & F_{44} \end{bmatrix} \\
 & &
\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ & F_{22} & F_{23} & F_{24} \\ & & F_{33} & F_{34} \\ SYM & & & F_{44} \end{bmatrix} &
\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ & G_{22} & G_{23} & G_{24} \\ & & G_{33} & G_{34} \\ SYM & & & G_{44} \end{bmatrix} \\
 & SYM & &
\begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ & H_{22} & H_{23} & H_{24} \\ & & H_{33} & H_{34} \\ SYM & & & H_{44} \end{bmatrix}
\end{bmatrix}
\left\{\begin{matrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{zo} \\ \varepsilon_{xyo} \end{Bmatrix} \\ \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z^* \\ \kappa_{xy} \end{Bmatrix} \\ \begin{Bmatrix} \varepsilon_{xo}^* \\ \varepsilon_{yo}^* \\ \varepsilon_{zo}^* \\ \varepsilon_{xyo}^* \end{Bmatrix} \\ \begin{Bmatrix} \kappa_x^* \\ \kappa_y^* \\ 0 \\ \kappa_{xy}^* \end{Bmatrix} \end{matrix}\right\} \qquad (7)
$$

$$
\begin{Bmatrix} Q_y \\ Q_x \\ Q_y^* \\ Q_x^* \\ S_y \\ S_x \\ S_y^* \\ S_x^* \end{Bmatrix} =
\begin{bmatrix}
\begin{bmatrix} A_{55} & A_{56} \\ A_{56} & A_{66} \end{bmatrix} & \begin{bmatrix} B_{55} & B_{56} \\ B_{56} & B_{66} \end{bmatrix} & \begin{bmatrix} D_{55} & D_{56} \\ D_{56} & D_{66} \end{bmatrix} & \begin{bmatrix} E_{55} & E_{56} \\ E_{56} & E_{66} \end{bmatrix} \\
 & \begin{bmatrix} D_{55} & D_{56} \\ D_{56} & D_{66} \end{bmatrix} & \begin{bmatrix} E_{55} & E_{56} \\ E_{56} & E_{66} \end{bmatrix} & \begin{bmatrix} F_{55} & F_{56} \\ F_{56} & F_{66} \end{bmatrix} \\
 & & \begin{bmatrix} F_{55} & F_{56} \\ F_{56} & F_{66} \end{bmatrix} & \begin{bmatrix} G_{55} & G_{56} \\ G_{56} & G_{66} \end{bmatrix} \\
SYM & & & \begin{bmatrix} H_{55} & H_{56} \\ H_{56} & H_{66} \end{bmatrix}
\end{bmatrix}
\begin{Bmatrix} \phi_y \\ \phi_x \\ \kappa_{yz} \\ \kappa_{xz} \\ \phi_y^* \\ \phi_x^* \\ \kappa_{yz}^* \\ \kappa_{xz}^* \end{Bmatrix}
$$

where the overall laminate stiffnesses A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij} and H_{ij} are:

$$
\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}^{(k)}\left(1, z, z^2, z^3, z^4, z^5, z^6\right) dz \qquad (8)
$$

$$
i, j = 1, 2, 3, 4, 5, 6, 7)
$$

If A_{ij}, B_{ij}, etc, are written in terms of the ply stiffness $\overline{Q}_{ij}^{(k)}$ and the ply coordinates z_k and z_{k-1}, the following is obtained:

$$
\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\right) = \tfrac{1}{n} \sum_{k=1}^{N} \overline{Q}_{ij}^{(k)}\left(z_{k+1}^{n} - z_{k}^{n}\right) \qquad (9)
$$

$$
(n = 1, 2, 3, 4, 5, 6, 7)
$$

Differential Equations of Equilibrium of Laminated Plates

The equilibrium differential equations in terms of the moments and forces resultants for a plate are (Swaminathan and Patil, 2008):

$$
\begin{aligned}
N_{x,x}+N_{xy,y} &= I_1\ddot{u}_0+I_2\ddot{\theta}_x +I_3\ddot{u}_0^*+I_4\ddot{\theta}_x^* \\
N_{y,y}+N_{xy,x} &= I_1\ddot{v}_0+I_2\ddot{\theta}_y +I_3\ddot{v}_0^*+I_4\ddot{\theta}_y^* \\
Q_{x,x}+Q_{y,y}+P_z &= I_1\ddot{w}_0+I_2\ddot{\theta}_z +I_3\ddot{w}_0^*+I_4\ddot{\theta}_z^* \\
M_{x,x}+M_{xy,y}-Q_x &= I_2\ddot{u}_0 +I_3\ddot{\theta}_x+I_4\ddot{u}_0^*+I_5\ddot{\theta}_x^* \\
M_{y,y}+M_{xy,x}-Q_x &= I_2\ddot{v}_0 +I_3\ddot{\theta}_y+I_4\ddot{v}_0^*+I_5\ddot{\theta}_y^* \\
S_{x,x}+S_{y,y}-N_z+\frac{h}{2}(P_z) &= I_2\ddot{w}_0 +I_3\ddot{\theta}_z+I_4\ddot{w}_0^*+I_5\ddot{\theta}_z^* \\
N_{x,x}^*+N_{xy,y}^*-2S_x &= I_3\ddot{u}_0 +I_4\ddot{\theta}_x+I_5\ddot{u}_0^*+I_6\ddot{\theta}_x^* \\
N_{y,y}^*+N_{xy,x}^*-2S_y &= I_3\ddot{v}_0 +I_4\ddot{\theta}_y+I_5\ddot{v}_0^*+I_6\ddot{\theta}_y^* \\
Q_{x,x}^*+Q_{xy,y}^*-2M_2^*+\frac{h^2}{4}(P_z) &= I_3\ddot{w}_0 +I_4\ddot{\theta}_z+I_5\ddot{w}_0^*+I_6\ddot{\theta}_z^* \\
M_{x,x}^*+M_{xy,y}^*-3Q_x^* &= I_4\ddot{u}_0 +I_5\ddot{\theta}_x+I_6\ddot{u}_0^*+I_7\ddot{\theta}_x^* \\
M_{y,y}^*+M_{xy,x}^*-3Q_y^* &= I_4\ddot{v}_0+I_5\ddot{\theta}_y +I_6\ddot{v}_0^*+I_7\ddot{\theta}_y^* \\
S_{x,x}^*+S_{y,y}^*-3N_z^*+\frac{h^3}{8}(P_z) &= I_4\ddot{w}_0+I_5\ddot{\theta}_z +I_6\ddot{w}_0^*+I_7\ddot{\theta}_z^*
\end{aligned}
\qquad (10)
$$

The following plate inertia can be introduced:

$$
\left(I_1,I_2,I_3,I_4,I_5,I_6,I_7\right)= \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\rho^{(k)}\left(1,z,z^2,z^3,z^4,z^5,z^6\right)dz \qquad (11)
$$

If the material for all the layer is identical, that is if the density $\rho^{(k)}$ is the same for all k, then

$$I_2=I_4=I_6=0 \qquad (12)$$

Exact Solution for Simply Supported Rectangular Plates

The exact analytical solution of the differential eq. (1) ***(HOST 12)*** for a general laminate plate under arbitrary boundary conditions are impossible task. However, closed-form solution for 'simply-supported' rectangular plates is to be considered.

The following simply supported boundary conditions are assumed (see fig. 1).

B. C. of cross-ply laminated plate associated SS-1 **:**

At edges $x=0$ and $x=a$:

$$
\begin{aligned}
&v_0=0,\quad w_0=0,\quad \theta_y=0,\quad \theta_z=0,\quad M_x=0,\\
&v_0^*=0,\quad w_0^*=0,\quad \theta_y^*=0,\quad \theta_z^*=0,\quad M_x^*=0,\\
&N_x=0,\quad N_x^*=0.
\end{aligned}
$$

At edges $y=0$ and $y=b$: (13a)

$$
\begin{aligned}
&u_0=0,\quad w_0=0,\quad \theta_x=0,\quad \theta_z=0,\quad M_y=0,\\
&u_0^*=0,\quad w_0^*=0,\quad \theta_x^*=0,\quad \theta_z^*=0,\quad M_y^*=0,\\
&N_y=0,\quad N_y^*=0.
\end{aligned}
$$

B. C. of angle-ply laminated plate associated SS-2 **:**

At edges $x=0$ and $x=a$;

$$
\begin{aligned}
&u_o=0;\quad w_o=0;\quad \theta_y=0;\quad \theta_z=0;\quad M_x=0;\quad N_{xy}=0;\\
&u_o^*=0;\quad w_o^*=0;\quad \theta_y^*=0;\quad \theta_z^*=0;\quad M_x^*=0;\quad N_{xy}^*=0.
\end{aligned}
$$

At edges $y=0$ and $y=b$; (13b)

$$
\begin{aligned}
&v_o=0;\quad w_o=0;\quad \theta_x=0;\quad \theta_z=0;\quad M_y=0;\quad N_{xy}=0;\\
&v_o^*=0;\quad w_o^*=0;\quad \theta_x^*=0;\quad \theta_z^*=0;\quad M_y^*=0;\quad N_{xy}^*=0
\end{aligned}
$$

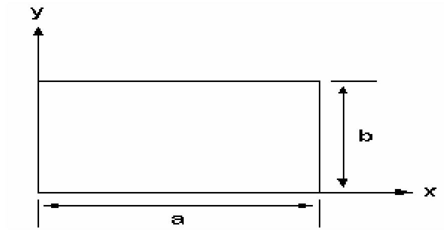

Fig. 1 Geometry and the co-ordinate system of a rectangular plate of thickness h

Equation of Motion in Terms of Displacements HOST12

For the first time, the equations of motion to the HOST12 eq. (1) can be expressed in terms of displacements $\left(u_0, v_0, w_0, \theta_x, \theta_y, \theta_z, u_0^*, v_0^*, w_0^*, \theta_x^*, \theta_y^*, \theta_z^*\right)$ by Substituting eq. (6) into eq. (10) as in (Salam Ahmed A., 2008).

Free Vibration Solution by HOST12

The following form of solution satisfies the differential the equations of motion and the boundary condition eq. (13), when the applied load q (x, y, t) on the right hand side of the equations of motion is set to zero.

$$\begin{aligned}
w_0 &= \sum_{m,n=1}^{\infty} w_{0mn} \sin\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
w_0^* &= \sum_{m,n=1}^{\infty} w_{0mn}^* \sin\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
\theta_x &= \sum_{m,n=1}^{\infty} \theta_{x\,mn} \cos\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
\theta_x^* &= \sum_{m,n=1}^{\infty} \theta_{x\,mn}^* \cos\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
\theta_y &= \sum_{m,n=1}^{\infty} \theta_{y\,mn}^* \sin\alpha x \cos\beta y e^{i\omega_{mn}t}, \\
\theta_y^* &= \sum_{m,n=1}^{\infty} \theta_{y\,mn}^* \sin\alpha x \cos\beta y e^{i\omega_{mn}t}, \\
\theta_z &= \sum_{m,n=1}^{\infty} \theta_{z\,mn} \sin\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
\theta_z^* &= \sum_{m,n=1}^{\infty} \theta_{z\,mn}^* \sin\alpha x \sin\beta y e^{i\omega_{mn}t}
\end{aligned} \tag{14a}$$

For antisymmetric cross-ply laminates:

$$\begin{aligned}
u_0 &= \sum_{m,n=1}^{\infty} u_{0mn} \cos\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
u_0^* &= \sum_{m,n=1}^{\infty} u_{0mn}^* \cos\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
v_0 &= \sum_{m,n=1}^{\infty} v_{0mn} \sin\alpha x \cos\beta y e^{i\omega_{mn}t}, \\
v_0^* &= \sum_{m,n=1}^{\infty} v_{0mn}^* \sin\alpha x \cos\beta y e^{i\omega_{mn}t}
\end{aligned} \tag{14b}$$

For antisymmetric angle-ply laminates:

$$\begin{aligned}
u_0 &= \sum_{m,n=1}^{\infty} u_{0mn} \sin\alpha x \cos\beta y e^{i\omega_{mn}t}, \\
u_0^* &= \sum_{m,n=1}^{\infty} u_{0mn}^* \sin\alpha x \cos\beta y e^{i\omega_{mn}t}, \\
v_0 &= \sum_{m,n=1}^{\infty} v_{0mn} \cos\alpha x \sin\beta y e^{i\omega_{mn}t}, \\
v_0^* &= \sum_{m,n=1}^{\infty} v_{0mn}^* \cos\alpha x \sin\beta y e^{i\omega_{mn}t},
\end{aligned} \tag{14 c}$$

By substituting eq. (14) into the equations of motion and expressing the a result in matrix form the following is obtained:

$$\left[[K] - w_{mn}^2 [M]\right]\{\Delta\} = \{0\} \tag{15}$$

The elements of the matrix [K] (*Stiffness Matrix*) [M] (*Mass Matrix*) are given in (Salam Ahmed A., 2008).

RESULT AND DISCUSSION

In the following, it is assumed that the material is fiber-reinforced and remains in the elastic range. The boundary conditions are SSSS, and the analytical procedure (HOST 12) is used in this work.

The material properties are :-

$E_2 = 6.92 \times 10^9$ N/m^2, $E_1 = 40E_2$,

$G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.6E_2$, $v_{12} = 0.25$

Dimensions of plate:

a=1 m , b=1 m , h=0.02 m

Table 1 Effect of degree of orthotropy of individual layers on the fundamental frequency of simply supported symmetric square laminates: a/h=5, $\bar{\omega} = 10 x \omega (\rho h^2 / E_2)^{1/2}$

No. of layers	Source	E1/E2				
		3	10	20	30	40
3	Exact	2.6474	3.2841	3.8241	4.1089	4.3006
	GTTR	2.6286	3.2679	3.7011	3.9456	4.1150
	Present HOST 12% error*	2.5271 4.54%	3.2197 1.96%	3.6834 4.99%	3.9442 4.00%	4.1198 4.20%
5	Exact	2.6587	3.4089	3.9792	4.3140	4.5374
	GTTR	2.6416	3.3802	3.9439	4.2809	4.5106
	Present HOST 12% error	2.5009 5.93%	3.2913 3.44%	3.8905 2.22%	4.2476 1.53%	4.4901 1.04%
7	Exact	2.6640	3.4432	4.0547	4.4210	4.6679
	GTTR	2.6460	3.4202	4.0310	4.4008	4.6533
	Present HOST 12% error	2.4508 8.00%	3.2729 4.94%	3.9204 3.31%	4.3120 2.46%	4.5780 1.92%

*Values in parenthesis the give percentage error for natural frequency with respect to exact solution mentioned above (Bose P. & Reddy J. N., 1998). GTTR (General Third Order Theory of Reddy) (Bose P. & Reddy J. N., 1998).

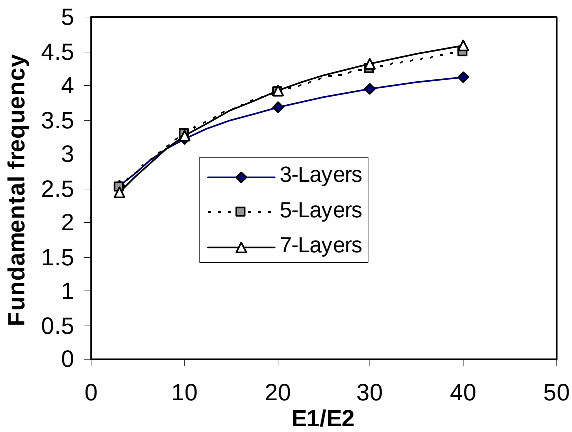

Fig. 2 Effect of degree of orthotropy of individual layers on the fundamental frequency of simply supported symmetric square laminates using (HOST 12): a/h=5, $\bar{\omega} = 10 x \omega (\rho h^2 / E_2)^{1/2}$

Table 2 shows analytical solutions of the variation of natural frequencies with respect to side-to-thickness ratio a/h for different E1/E2 ratio for two and four layered a simply supported antisymmetric angle-ply (45/-45/…) square laminated plate E1/E2 = open, E2 = E3, G12 = G13 = 0.6E2, G23 = 0.5E2, υ12 = υ13 = υ23 = 0.25.

Table 2 Analytical effect of degree of orthotropy and (a/h) ratio of individual layers on the fundamental frequency $\overline{\omega} = (\omega a^2 / h) x (\rho / E_2)^{1/2}$

No. of layers	E_1/E_2	Source	a/h			
			2	4	10	100
2	3	GTTR	4.5312	6.1223	7.1056	7.3666
		HOST12	4.6159	6.2572	7.2879	7.5013
		Error %	1.87 %	2.20 %	2.56 %	1.82 %
	10	GTTR	4.9742	7.2647	8.9893	9.5123
		HOST12	5.0312	7.4041	9.1163	9.6453
		Error %	1.15 %	1.91 %	1.41 %	1.39 %
	20	GTTR	5.1817	8.0490	10.6412	11.5385
		HOST12	5.2725	8.1604	10.7724	11.6738
		Error %	1.75 %	1.38 %	1.23 %	1.17 %
	40	GTTR	5.3325	8.8426	12.9115	14.6668
		HOST12	5.3817	8.8933	13.0762	14.7414
		Error %	0.92 %	0.57 %	1.27 %	0.50 %
4	3	GTTR	4.6498	6.4597	7.6339	7.9545
		HOST12	4.7628	6.6133	7.8131	8.1412
		Error %	2.43 %	2.37 %	2.34 %	2.35 %
	10	GTTR	5.2061	8.3447	11.4116	12.5351
		HOST12	5.3307	8.4752	11.5777	12.6353
		Error %	2.40 %	1.56 %	1.45 %	0.79 %
	20	GTTR	5.4140	9.3306	14.4735	16.9927
		HOST12	5.5308	9.4927	14.6128	17.1034
		Error %	2.16 %	1.73 %	0.96 %	0.65 %
	40	GTTR	5.5674	10.0731	17.8773	23.4499
		HOST12	5.6147	10.1535	17.9268	23.5912
		Error %	0.85 %	0.80 %	0.30 %	0.60 %

*Values in parenthesis the give percentage error for natural frequency with respect to exact solution mentioned above (Swaminathan and Patil, 2008). GTTR (General Third Order Theory of Reddy) (Swaminathan and Patil, 2008).

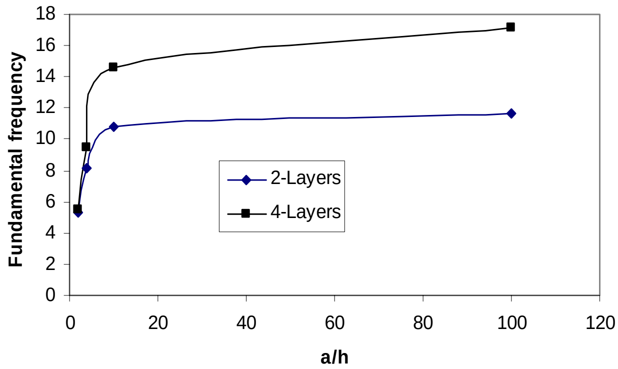

Fig. 3 Analytical effect of degree of (a/h) ratio of individual layers on the fundamental frequency using (HOST 12): $E_1/E_2 = 20$, $\overline{\omega} = (\omega a^2 / h) x (\rho / E_2)^{1/2}$

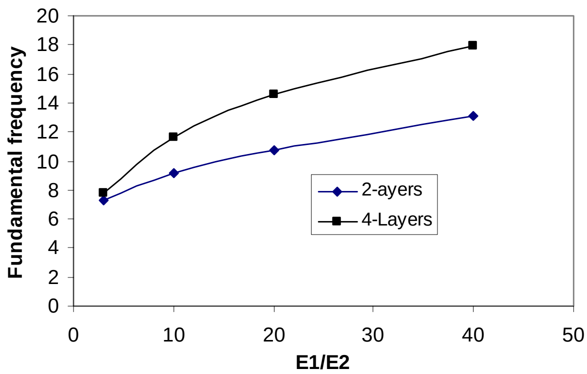

Fig. 4 Analytical effect of degree of orthotropy of individual layers on the fundamental frequency using (HOST 12): a/h=10, $\overline{\omega} = (\omega a^2 / h) x (\rho / E_2)^{1/2}$

It is also demonstrated that increasing the fundamental frequency with increases the degrees of orthotropy (E_1/E_2) for laminate plate due to the increase plate stiffness. The number of layers has different effects in laminated plates. As the span-to-thickness ratio (a/h) increases, the fundamental frequency decreases, due to the decrease in the stiffness of the plate, but the factor of nondimensional is gives opposite relation.

CONCLUSIONS

1. Analytical formulations and solutions to the natural frequency analysis of simply supported antisymmetric angle-ply composite and sandwich plates hitherto not reported in the literature based on a higher order refined theory which takes in to account the effects of both transverse shear and transverse normal deformations are presented. The accuracy of the present computational model with 12 degrees of freedom in comparison to other higher order model with five degrees of freedom has been established.
2. The effect of degree of (*a/h*) ratio becomes more pronounced as the number of layers increases. Increasing the ratio (*a/h*) from (2 to 20) the natural frequency very increases and from (20 to 100) remains stable roughly for (E_1/E_2= 20).
3. The effect of degree of orthotropy (E1/E2) becomes more pronounced as the number of layers increases (for the same laminate thickness). Increasing the ratio (E1/E2) from (10 to 40) increases the natural frequency for (a/h = 5) and (a/h=10).

It has been concluded that for all the parameters considered Reddy's theory very much over predicts the natural frequency values both for the composite and sandwich plates.

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