Spectral Technique for Baud Time Estimation

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ABSTRACT

A new approach for baud time (or baud rate) estimation of a random binary signal is presented. This approach utilizes the spectrum of the signal after nonlinear processing in a way that the estimation error can be reduced by simply increasing the number of the processed samples instead of increasing the sampling rate. The spectrum of the new signal is shown to give an accurate estimate about the baud time when there is no *apriory* information or any restricting preassumptions. The performance of the estimator for random binary square waves perturbed by white Gaussian noise and ISI is evaluated and compared with that of the conventional estimator of the zero crossing detector.

Key words- Bit time estimation, baud time estimation, bit rate estimation, baud rate estimation.

الخلاصة

تم اعتماد اسلوب جديد لتقدير زمن الترميز (او معدل الترميز). هذا الاسلوب يعتمد على طيف الاشارة بعد معالجتها بصورة غير خطية بحيث يمكن تقليل خطأ التقدير عن طريق زيادة عدد العينات المعالجة بدلاً من زيادة معدل اخذ العينات. ان طيف الاشارة الجديدة اظهر تقدير دقيق لمعدل الترميز من دون معلومات او فرضيات مسبقة. أظهر التقييم لأداء لعملية التقدير لاشارة ثنائية عشوائية مضافاً اليها الضوضاء الكاوسي والتداخل الرمزي وُأجريت مقارنة مع طريقة كشف التقاطع الصفري.

INTRODUCTION

Many researches have been made for identifying the type of modulation of the unknown signal, now the interest is moved one step ahead to the estimation of the baud rate (or baud time) of the unknown signal. The estimation of the baud rate can be used with information obtained from the modulation identification to provide better knowledge about the characteristics of the signal, this sometimes referred to as signal identification [1, 2]. Besides that the knowledge of the baud rate is needed if the information contained in the unknown signal is to be extracted, since the baud rate is required for the timing recovery block of the demodulator. If the technique used for the clock recovery is PLL based, the baud rate is used to specify the center frequency of the VCO or if the spectral line technique is used, the baud rate is used to determine the center frequency of the BPF [5]. The quality of the baud time estimator is generally measured by the following criteria [1]:

1- The amount of available *apriory* information. The less information available the better the system is.
2- Time taken for the baud rate estimation or it can be measured as the amount of computation.
3- The accuracy of the estimation.

Several techniques have been developed for estimating the baud time, most of the techniques presented in the literature analyze the zero-crossing of the baseband signal. Wegener [1] presented detailed description of different approaches that rely on the zero crossings of the signal and no performance evaluation was presented. Gaby [2] presented an algorithm that relies both on zero-crossing and bit pattern analysis and he used second derivative of the filtered modulating signal to locate the baud transitions which is more general than the zero crossing because it works for the multilevel signaling. The second derivative helps locating the inflection point of smoothed (filtered) signal which is actually the baud transition point. Azzouz [3] used wavelet transform in his work and he also relied on the zero-

crossings of the signal and he used the derivative if the signal to enhance the part of the signal where the zero-crossing occurs. Scheets [4] have used adaptive filtering of the random binary signal and monitored the coefficients of the adaptive filter, which gave the estimation of the bit time of the binary signal. His work was compared to the zero crossing method for estimating the bit time. Sills [5] developed an algorithm based on histogram of baud transition. Boulinguez [6] approach is based on time-frequency representation of the modulated signal combined with periodicity analysis using Kalman filter.
These works although differ in the details of their approaches but they all share the same limitation of having their analysis in the time domain, such approach will limit the accuracy of the estimation by the choice of the sampling frequency as will be explained later. If high accuracy is needed the ratio of sampling frequency to the baud rate should by be high enough. This indicates the need of having *apriory* information available for estimating the baud rate. Also if filtering is used, the choice of the cutoff frequency will raise the same issue.

In the proposed approach it is assumed that no *apriory* information available and the approach is developed to depend on the spectrum of a non-linear processed version of the input signal. This nonlinear processing introduces impulses at multiples of the baud rate in the spectrum, and by identifying the position of these impulses the baud rate or baud time is estimated. Very high estimation accuracy can be obtained about the value of baud time T_b when high resolution of the spectrum is available which is achieved by simply increasing the amount of the processed samples for the same sampling frequency. The quality of the spectral estimator is compared to that of the conventional estimator based on generating histogram (clusters) of the zero crossings [1], [2]. The analysis will be presented for a two levels (M=2) and four levels (M=4) signals. For simplicity of explanation of the approach, the two level signaling will be considered and the terms bit time and bit rate will be used rather than baud time and baud rate. The algorithm that will be presented works perfectly on the multilevel signals (M>2) without any modifications.

Effect of Sampling Frequency on Bit Time Estimation

Before starting with the details of the spectral estimator, it is useful to elaborate about the sampled signals bit time. In sampled data domain, the time is represented by samples rather than seconds because the time is quantized to sampling intervals or sample time T_s given by

$$T_s = \frac{1}{f_s} \qquad (1)$$

Where f_s is the sampling frequency in samples/sec. Therefore any time-related information, like the bit time, is given in terms of number of samples. In other words the bit time will be represented as multiple of T_s. According to above, the bit time in the sampled data domain can be found by [1]

$$T = \frac{T_b}{T_s} \qquad (2)$$

Where,
- T_b is the bit time in seconds
- T_s is the sample time in sec/sample
- T is the bit time in samples

Or

$$T = \frac{f_s}{R_b} \qquad (3)$$

Where, R_b is the bit rate. According to eq. (3) T may not be an integer only if the sampling rate is an integer multiple of the bit rate and since the bit rate is unknown, actually the problem here is to estimate the bit rate, we cannot assume T an integer.
As mentioned earlier, previous works relied on time domain processing of the signal. The disadvantage of a time domain approach is that the need of high sampling frequency compared to the bit rate. The reason of that is illustrated in Fig. 1, where in Fig. 1a a sampled binary signal is shown, its bit time T=12 samples. In Fig. 1b the same signal is sampled with a half the sampling frequency of the one in Fig. 1a which made the bit time T=6 samples. In case (b) the quantization of time is not as fine as in case (a), therefore the estimation of the bit time in case (b) will be less precise than case (a).
In Fig. 1c another binary signal is shown with bit rate twice as much as in case (a) and sampled with the same sampling frequency, which made the bit time T=6 also. So, to have the same estimation precision as in case (a) the sampling frequency needs to be doubled. In summary the estimation of bit time in time domain raises the problem of having the estimation accuracy dependant on the sampling frequency. It will be explained that using the frequency domain feature of the signal does not make high values of the sampling frequency a requirement for the estimation precision.

Spectral Properties of Random Binary Signal

In this work the estimation of the bit time is based on the frequency domain as a feature of the signal. The spectrum of the signal gives an idea about the bit time because it is directly affected by the bit time. For example the power spectral density (psd) of a random binary (non-return to zero) signal x(t) is in the form of the well known sinc function [8].

$$S_x(\omega) = A^2 T_b \left[\frac{\sin(\omega T_b/2)}{\omega T_b/2} \right]^2 \qquad (4)$$

It is clear from eq. (1) that the bit time T_b shrinks or spreads the spectrum according to its value. Moreover this spectrum contains nulls (zero values) at multiples of the bit rate (the reciprocal of the bit time). But unfortunately, this form of spectrum can only give a rough estimate about the bit time, mainly because of the difficulty of locating the nulls in the presence of noise.

In Fig. 2a and 2b the spectrum of a random binary signal with SNR equals to 100 dB and 10 dB is shown. It is possible to estimate the bit rate (and hence the bit time) if the first null is located, but due to the presence of noise, the exact location of the null is lost. This is expected since the noise mostly affects the small values of the spectrum.

To tackle this problem, the proposed approach is to introduce impulses (or spectral line) at the locations of the nulls, because the impulses are easier to locate and more immune to noise. But the spectrum of random binary signals of most formats do not contain these impulses, actually the impulses in the spectrum is a characteristic of a periodic signal [8,9]. To introduce these impulses into the spectrum of the binary signal we first need to look at the spectrum of the general random binary signal f(t) with the following properties [8]

1. Each pulse is of T_b duration.
2. The two possible states in each interval are represented by the waveforms $f_1(t)$ and $f_2(t)$ with corresponding Fourier transforms $F_1(\omega)$ and $F_2(\omega)$.
3. The probability that $f_1(t)$ is selected in any interval is p and the probability that $f_2(t)$ is selected is q=(1-p).
4. The choice of $f_1(t)$ and $f_2(t)$ in any interval is statistically independent of that in any other interval.

The psd of such signal is given by

$$S_f(\omega) = p(1-p)\frac{1}{T_b}\left|F_1(\omega) - F_2(\omega)\right|^2 + \frac{2\pi}{T_b^2}\sum_{n=-\infty}^{\infty}\left|pF_1(n\omega_b) + (1-p)F_2(n\omega_b)\right|\delta(\omega - n\omega_b) \qquad (5)$$

Where, $\omega_b=2\pi/T_b$. The second term of eq. (5) can be used to estimate the bit rate (and hence bit time) because it is in the form of impulses at multiples of bit rate that can be easily detected even in the presence of noise. To have this term produced when p=q=0.5, two conditions should be satisfied

$$F_1(\omega) \neq -F_2(\omega) \qquad (6a)$$
$$F_1(n\omega_b) \neq 0 \text{ and } F_2(n\omega_b) \neq 0 \qquad (6b)$$

The first condition to make $F_1(\omega)$ and $F_2(\omega)$ do not cancel each other, and the second condition is to make the weight of the impulses nonzero.
Now if we consider the usual case of the binary signal where $f_1(t)$= A and $f_2(t)$= –A and the both cases are equally likely to occur (p=0.5). It is clear that the first condition is not satisfied (and it can be shown that the second is also not satisfied). Therefore, the impulses term vanishes in the spectrum of such signal and strictly speaking, for this case eq. (5) will be reduced to eq. (4).

Proposed Spectral Approach for Bit Time Estimation

To introduce the impulses term of eq. (5) we need to modify the signal f(t) in such a way that the two conditions given by eq. (6) are satisfied. There is no unique way to modify the signal f(t) in order to satisfy these conditions. The approach introduced in this paper is to differentiate the signals and taking the absolute value (a different approach can use squaring instead of absolute value). The new signal g(t) will have the conditions of eq. (6).

$$g(t) = \left|\frac{df(t)}{dt}\right| \qquad (7)$$

g(t) is now in the form of positive impulses at every bit change of f(t) and zero elsewhere, as shown in Fig. 3.
It can be seen that g(t) has all the above mentioned properties with

$$g_1(t) = 2A\delta(t) \qquad (8a)$$
$$g_2(t) = 0 \qquad (8a)$$

with Fourier Transforms

$G_1(\omega) = 2A$ (9a)
$G_2(\omega) = 0$ (9b)

It can be seen that $G_1(\omega)$ and $G_2(\omega)$ satisfy the two conditions in eq. (6). From eq. (5) and eq. (9) the psd of g(t) is

$$S_g(\omega) = \frac{A^2}{T_b} + \frac{2\pi}{T_b} A^2 \sum_{n=-\infty}^{\infty} \delta(\omega - \omega_b) \qquad (10)$$

This proposed approached has made the new signal g(t) in a way that the value of the spectral component at the bit rate frequency as high as possible (impulse), instead of zero as it was in the case of f(t) before processing. The impulses of the spectrum of g(t) can now be detected even in the presence of noise (as shown in Fig. 4), because the effect of noise on the spectral component of the bit rate of g(t) will be much less than the noise effect on the zero (null) value for the same spectral component of f(t). This will make it much easier to detect this impulse, and from its location the bit time can be estimated

Development of the Estimator for the Sampled Signals

As explained earlier, in the sampled data domain the bit duration is represented by the number of samples per bit T, where T can be obtained by eq. (2) and eq. (3), note that T may not be an integer.
Now the signal in question f(t) will be represented after sampling as f(n). From this sampled signal the formula of eq. (7) will be in the form

$$g(n) = \left| f(n) - f(n-1) \right| \qquad (11)$$

In sampled data domain the differentiation is replaced with difference [9, 10]. This processing is adequate at high values of signal-to-noise ratio, but at low values of signal-to-noise ratio the time domain impulses created at the bit changes may not be so distinct because they come as the difference of two samples only. An alternative and more practical modifying formula is

$$g(n) = \left| \sum_{i=0}^{L} \left(f(n-i) - f(n-i-L-1) \right) \right| \qquad (12)$$

Note that eq. (11) is a special case of eq. (12) for L=0. This formula takes a window of 2(L+1) samples and sums a group of (L+1) consecutive samples and the group of the next (L+1) consecutive samples and takes the difference between the two sums. This produces some averaging of the noise and hence reducing its effect. Besides the time-domain impulses will have a non-zero width which in turn makes the frequency-domain impulses have a decreasing weight [8]. This makes it easier to detect the impulse at $\omega=\omega_b$ (which will be referred to as the bit rate impulse) as the largest impulse other than that at $\omega=0$.
Now the spectrum of g(n) can be obtained using Fast Fourier Transform (FFT) and it will be denoted by G(k) where k is the frequency index. Since G(k) is defined over discrete values of the spectrum and the bit rate (or bit time) has a continuous range of values, the bit rate impulse would not appear exactly at one of these discrete values except for the special case where[9]

$$\frac{NR_b}{f_s} = k \qquad (13)$$

where N is the number of FFT points and k is any integer. If this expression is not satisfied, which is usually the case, the impulse will be split into two impulses at successive values of the frequency index k. Let these two values of k be k_b and k_b+1, the estimate of the impulse location $\bar{k}_b$ can be obtained as the weighted average of k_b and k_b +1 with G(k) and G(k+1) as their weights respectively, i.e.

$$\bar{k}_b = \frac{|G(k_b)|(k_b) + |G(k_b+1)|(k_b+1)}{|G(k_b)| + |G(k_b+1)|} \qquad (14)$$

The estimate of the number of samples per bit is obtained by

$$\bar{T} = \frac{N}{\bar{k}_b} \qquad (15)$$

This equation, despite its simplicity, has a very interesting feature. As has been explained earlier in section 1, the bit time T cannot necessarily be an integer value, and that the precision of time domain approach for the bit time estimation can only be enhanced by increasing the sampling frequency. Equation (15) is telling us that the estimate of the bit time is the ration of two numbers, one is the number of the processed samples N and the other is the frequency index $\bar{k}_b$. It is clear that the value of a non integer number is better approximated by the ratio of large integers, which is the case for eq. (15) when N is increased. This is when compared with the time domain approach is a very big advantage because to have a better estimation precision in the frequency domain approach, it only requires to increase the number of the processed samples as will be seen in the results presented in the next section, which is a much easier condition to satisfy than increasing the sampling rate, which usually a hardware requirement.

Performance Evaluation against Awgn and Isi

The estimator performance against band limited channel and AWGN is to be investigated now. A random binary signal of 15 kb/s bit rate is simulated and sampled with a sampling frequency of 100 ksample/s, this makes T=6.667 samples/bit. This choice of values was to show that in presented approach there is no need to have high values of T in order to have good estimation accuracy unlike other works were the choice was T=30 in [4] and T=96 in [3]. Also our choice of is T is a non-integer value to avoid any loss of generality. Channel and noise model in continuous time is in the form

$$f(t) = \int h(\tau)s(t-\tau)d\tau + n(t) \qquad (16)$$

Where s(t) is the baseband transmitted signal that is assumed to be a rectangular NRZ signal since this format is the most widely used format, h(t) is the combined impulse response of the transmitter filter, channel filter and the receiver filter, which is responsible for introducing the intersymbol interference (ISI) into the received signal. The term n(t) is the AWGN, and the signal-to-noise ratios is defined in terms of SNR per bit [4] where

$$SNR = \frac{P_s f_s}{P_n R} = \frac{P_s T}{P_n} \qquad (17)$$

P_s and P_n are variances of the signal and noise samples, respectively. Equation (17) reflects more accurately the noise power inside the signal bandwidth than the standard SNR=P_s/P_n.

The effect of transmission over a band limited channel is important in the evaluation of this work not only because it is a practical factor that affects most of the communication systems, but it also affects the shape of the transmitted pulse. And since our work depends mainly on the wave shaping, it is expected that the performance would be degraded by the presence of the band limiting (or ISI) because it is expected to soften the sharp edges of the transmitted rectangular pulse. In simulation, eq. (16) needs to be transformed into sampled data format

$$f(n) = \sum_{k=0}^{K} h(k)s(n-k) + n(n) \qquad (18)$$

Where n is the time index and K is the total number of samples of the combined impulse response h(n) which is referred to as the channel memory. The summation in eq. (18) represents the weighted sum of the delayed versions of the transmitted signal s(n), it should be noted that in some literatures, the discrete nature of eq. (18) comes from sampling the signal with sampling rate equals the symbol rate, i.e., one sample per symbol, this means that the delay of s(n) only occurs at multiple of the symbol time, which is generally not the case. In our work the summation of (18) is of the samples of the signal due to the sampling rate mentioned earlier which is greater than the symbol rate, i.e., more than one sample per symbol and that the delays occur at fractions of the symbol time. This representation of the ISI is more practical and will affect the value of the transmitted symbol as well as the shape of the wave of the symbol, unlike the one sample per symbol model that only affects the value of the transmitted symbol only.

In regards to the amount of the ISI introduced by the channel, the value of the channel memory K is sometimes used, but this is not an accurate measure because the values of h(k) are not considered. A more informative parameter is the channel bandwidth, since the channel effect is band limiting and our signal s(n) is a baseband signal h(n) is chosen as an FIR LPF and its bandwidth will be the measure adopted here for the amount of ISI in our simulation.

The number of taps K of the FIR LPF is chosen to be 101, and considering the value of T=6.667, this is equivalent to about 15 symbols interference. Hanning window is chosen and the results were obtained for different values of the cutoff frequency which is expressed as a relative frequency or digital frequency given by

$$r = \frac{f}{f_s} \qquad (19)$$

Note that comparing the above equation with eq. (2), it defines the digital frequency corresponding to the baud rate as 1/T which for our case is 0.15. Fig. 5 shows an example of a signal in three conditions; the clean signal, signal with ISI and the signal with ISI and noise.

The spectrum of the signal g(n) is estimated using the periodogram averaging method [10]. It was found that the number of 10 periodograms which is a convenient number to avoid increasing the number of calculations and it was found sufficient to produce very acceptable results

The results are obtained for standard deviation or root mean square (RMS) of the estimation error given by

$$E_{RMS} = \sqrt{\frac{\sum_{1}^{Ns} (T - \overline{T})^2}{Ns}} \qquad (20)$$

Where Ns is the number of signal segments and it is chosen 100 signal segments for this work. The results are presented as graphs of the error root mean square against the number of samples N of each signal segment and different graphs are made for different values of SNR, this will show how the estimation error is decreased with the increase of the number of samples N, which is the basic idea of this paper.

The performance of both the spectral estimator and the conventional zero-crossing estimator are examined over a range of signal-to-noise ratio for segments of different numbers of samples. The root mean square of the estimation error E_{RMS} is plotted versus number of processed sample N in Fig. 6 and Fig. 7, each graph is for a different value of SNR, and it is obtained for 100 different signal segments. Two curves are used for the spectral estimator one with L=1 and the other with L=2. These graphs show clearly the superiority of the spectral estimator over the zero-crossing estimator which is a time domain based estimator. From these figures, it can be seen that the estimation error can have large values at low values of SNR as in Fig. (6a). This is because the bit rate impulse is not distinct and it can be mistaken by any other spectral component which makes the error value very high.

On the other hand, if the bit rate impulse is correctly located, the error will be very small and it is mainly due to the discrete nature of the FFT. The most important observation is that, the error of the spectral estimator decreases significantly with the increase of the number of FFT points N. On the other hand the error of the zero-crossing estimator showed a slight decrease with the increase of N. This is an expected result, since for the spectral estimator, the increase of N increases the resolution of the spectrum and hence more accuracy is achieved about the bit rate impulse location, and for the zero-crossing estimator no increase in the sampling rate means no improvement in the estimation error.

The comparison between the L=1 and the L=2 curves shows that the L=2 curve has a better results only at lower values of SNR, and this is the point of using the modified formula of eq. (12), which is for better performance against noise. While for higher values of SNR the difference between L=1 and L=2 curves is not distinct because in both cases the location of the bit rate impulse k_b is detected correctly, and the only cause of error is the discrete nature of the FFT which depends on the number of samples N. The performance of the spectral estimator is also evaluated for the multilevel signaling and the results are shown in Fig. 7 and no modifications were made in the algorithm. The conventional zero-crossing do not work in this case and a more general symbol transition detection approach can be used like the one introduced in [2]. In regards to the performance against ISI, it was found that the lowest value of channel bandwidth at witch the baud time can be estimated is at r=0.08. At lower values of r the estimator was unable to produce acceptable results, this is because lowering the value of r means that more spectral components of the received signal is lying in the high attenuation spectral region of the channel which produce high amount of ISI. It must be noted that this value of r is directly related to the chosen value of T which means that if the value of T increased (by lowering the baud rate) and hence the spectrum of the signal is shrunk inside low attenuation spectral region of the channel, the amount of ISI will be lower and acceptable estimation results can be obtained. As has been explained by eq. (19) that the equivalent digital frequency of the baud rate is1/T=0.15, this indicate that when r=0.08, about half of the spectrum of the signal is in the stop band of the equivalent LPF of the channel.

Another advantage of this method is that the amount of computations is not very large, because the main part of this algorithm is the FFT, although it takes more computations than that of the zero crossing technique which is the best in this side. It is known that the amount of computations of the FFT is usually measured as the number of complex multiplications which is given by $N\log_2 N$, where N is the number of samples On the other hand, special techniques can be used for efficient calculation of FFT of real data which reduces the amount of calculations to the half [10].

Conclusions

A new approach of baud time estimation, which depends on frequency domain, has been presented. This approach has the main advantages over the time domain approaches, which is to overcome the limitation imposed by the value of sampling frequency on the estimation accuracy. In addition to that the approach is very simple and flexible and it used to binary signals and multilevel signals.

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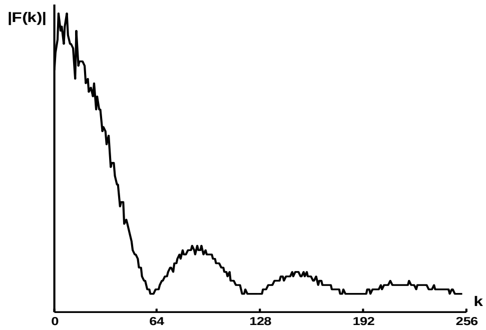
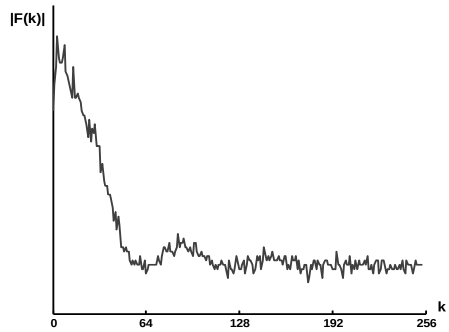


(a)



(b)

Fig. 2 The FFT of a random binary signal f(t) with 1200 bps samples with 10k samples/s.
(a) SNR=100 signal, (b) SNR=10 dB

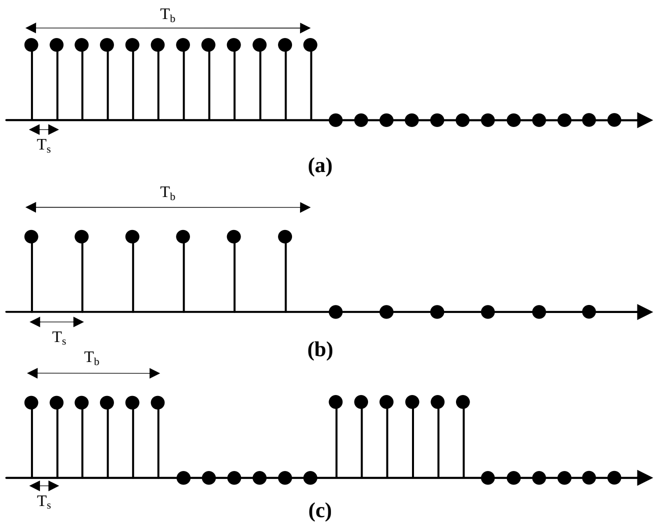
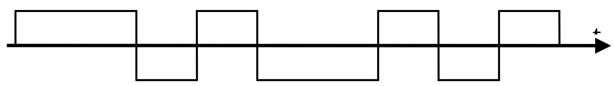
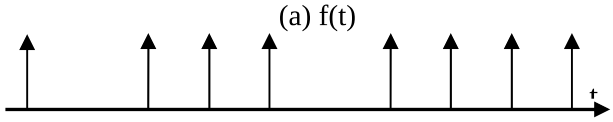


Fig. 1 Example showing the effect of changing sampling frequency and bit rate on the sampled binary signal.



(a) f(t)



(b) g(t)

Fig. 3 f(t) random binary signal and g(t) its processed version.

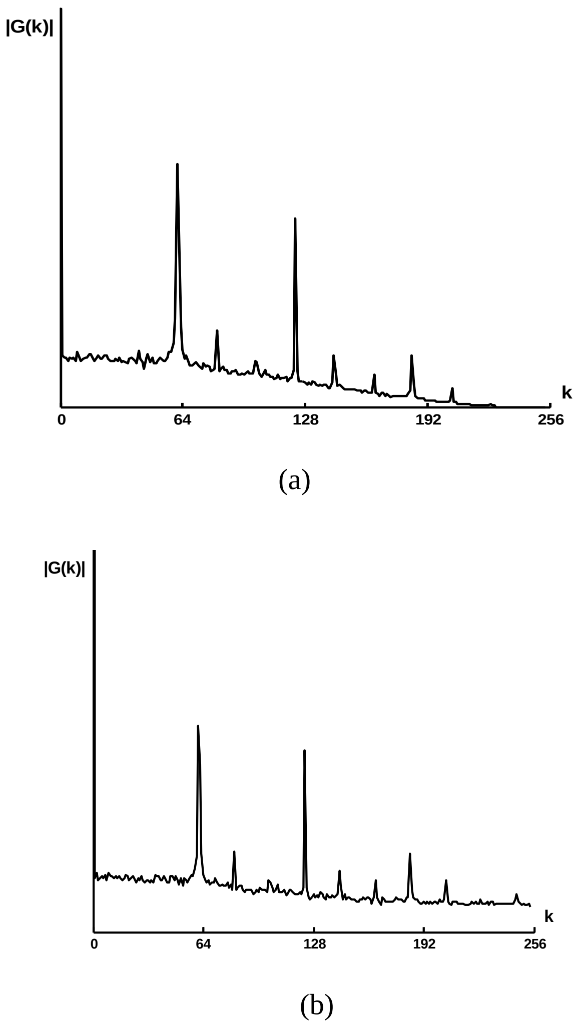


Fig. 4 The FFT of the modified random binary signal g(t). (a) noise free signal, (b) signal with noise

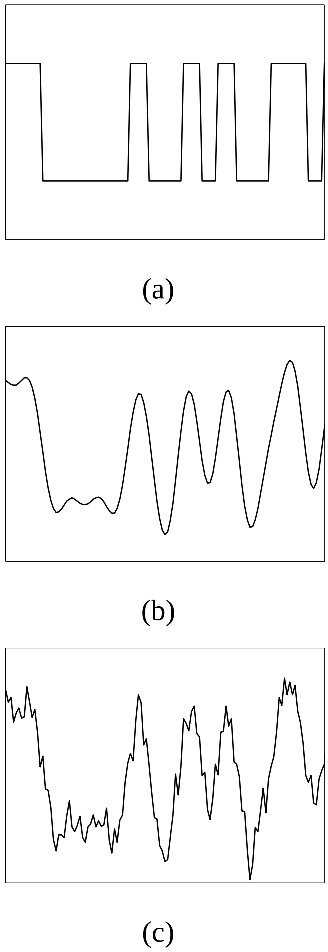
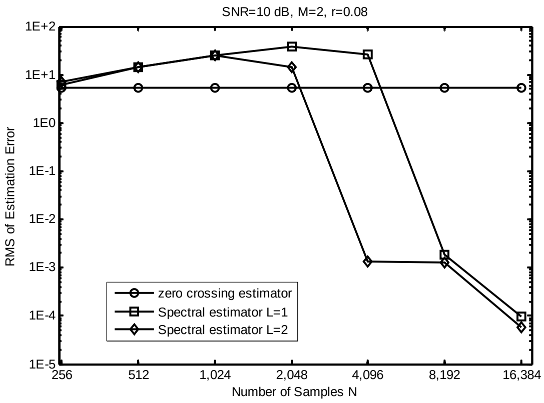
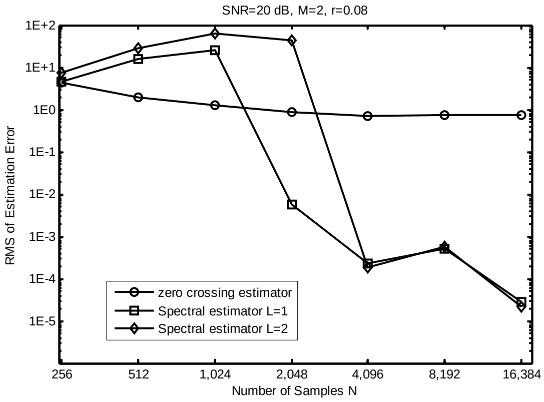


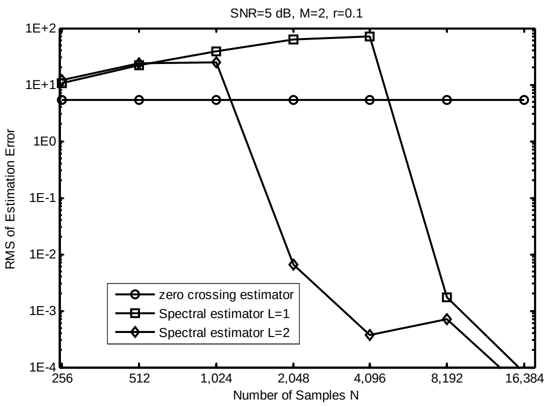
Fig. 5 a binary signal before and after adding the perturbing factors. (a) clean signal, (b) signal with ISI (r=0.08), (c) signal with ISI and noise (r=0.08, SNR=20dB).



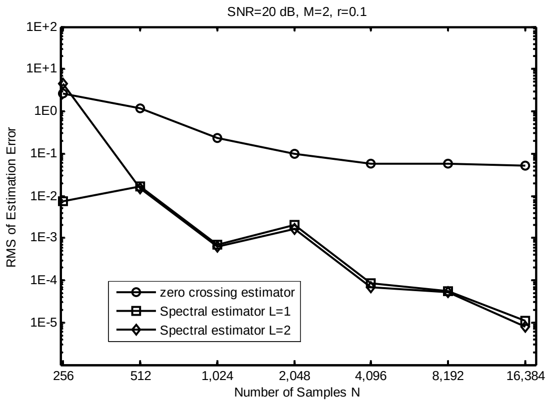
(a)



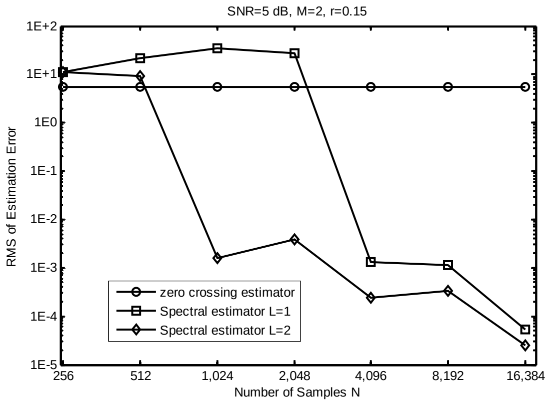
(b)



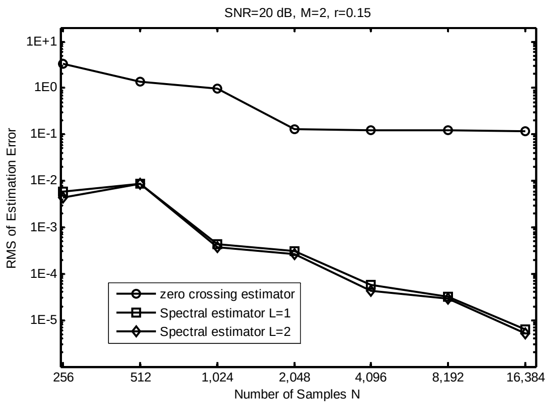
(c)



(d)

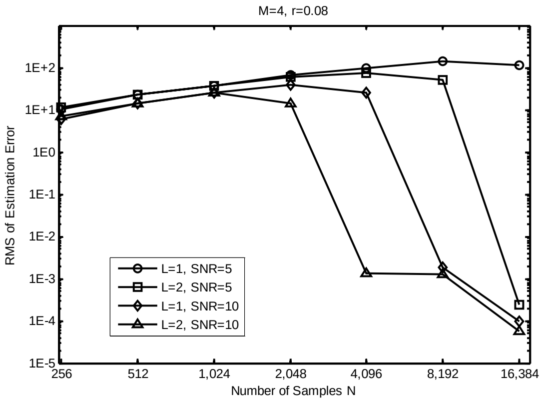


(e)

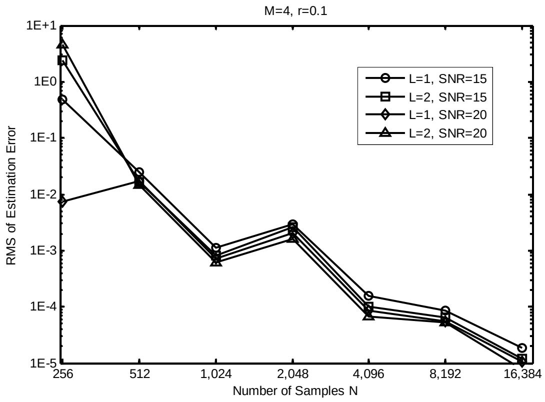


(f)

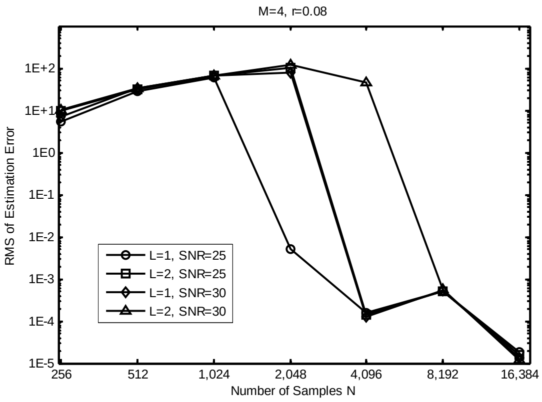
Fig. 6 The RMS of the estimation error for M=2 and different values of SNR and r.



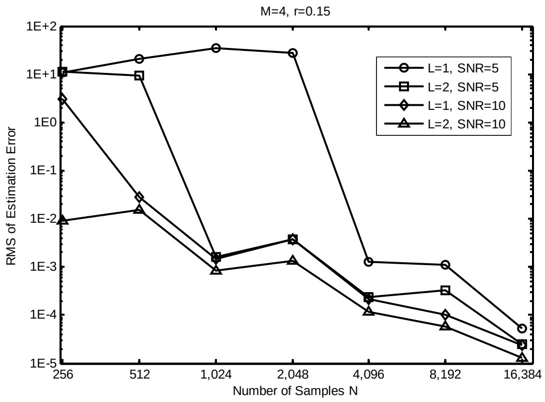
(a)



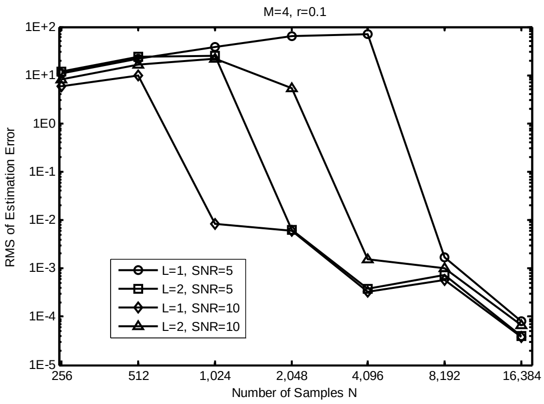
(d)



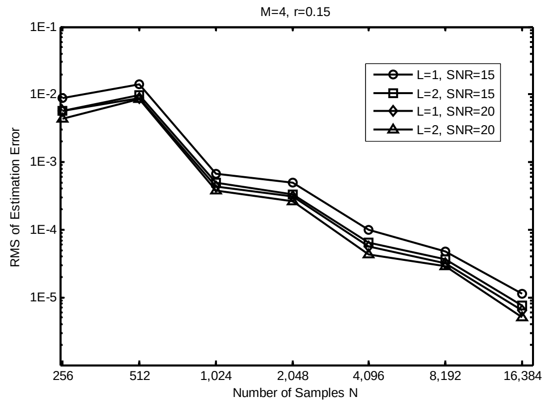
(b)



(e)



(c)



(f)

Fig. 7 The RMS of the estimation error for M=4 and different values of SNR and r