# The Effect of Variable Load on Dynamic Behavior of Thin Pipe by Hamilton Principle and Cfx-Ansys

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## Abstract

This paper presents the study and analysis, analytically and numerical of circular cylindrical shell pipe model, under variable loads, transmit fluid at the high velocity state (fresh water). The analytical analysis depended on the energy observation principle (*Hamilton Principle),* where divided all energy in the model to three parts , strain energy, kinetic energy and transmitted energy between flow and solid (kinetic to potential energy). Also derive all important equations for this state and approach to final equation of motion, free and force vibration also derived. the relations between the displacement of model function of velocity of flow, length of model, pipe thickness, density of flowed with location coordinate x-axis and angle are derived.

In numerical analysis the models are created by using ANSYS Workpench-12 program, where build two models one for fluid, and another for pipe (solid). Depended on CFX-ANSYS package, can transfer all parameters in the fluid (temp., presser, energy) to solid model. The result show a good agreement and low of percentage error between the analytically and numerical result. Also shows the effects of length and flow velocity on the behaviour of pipe.

**KEYWORD: Flow Solid Interfaces; cfx-Ansys, Vibration, Turbulent flow, thin pipe**

# تأثير الاحمال المتغيرة على السلوك الديناميكي على انبوب رقيق باستخدام مبدأ هيملتون و الحل العددي داخل سي اف اكس-انسيز

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**الخلاصة**

**تم في هذ البحث اجراء دراسة وتحليل نظري وعددي لنموذج انبوب رقيق تحت تاثير احمال مختلفة نتيجة نقل مائع بسرع مختلفة (ماء عذب). اعتمد التحليل النظري على مبدأ حفظ الطاقة والاعتماد على مبدأ هيميلتون للطاقة, حيث تم تجز الطاقة الكلية في النموذج الى ثلاث مجاميع هي طاقة الانفعال, والطاقة الحركية, واخيرا الطاقة المنتقلة من المائع الى الهيكل الصلب, تم اشتقاق المعادلات الخاصة بالاهتزاز الحر والاهتزاز القسري من معادلة هيملتون الى الصيغة النهائية للحركة للانبوب كعلاقة بين الاحداثيات الطولية والدورانية للانبوب مع سرعة الجريان وطول العينة, وسمك الانبوب وخواص السائل. في التحليل العددي تم بناءنموذج متكامل داخل برنامجAnsys workpench-12 النموذج مكون من جزئين احدهما يمثل المائع والاخر يمثل الهيكل الصلب , وبالاعتمادعلى ملفات برنامج AnsysCFXالموجودة داخل بيئة workbench الذي يتيح عمل علاقة عددية بين المائع والهيكل الصلب والحصول على خواص المائع وتاثيرها على الهيكل منها(الحرارة, الضغط, والطبقة المتاخمة). اظهرت النتائج تقارب جيد ومقبول بين النتائج النظرية والعددية, وظهور تاثير واضح للسرعة وطول الانبوب والمادة المنقولة داخل الانبوب على سلوك الانبوب.**

# 1- Introduction

The study of internal flow-induced vibration of thin pipes are problems that treatment an interaction between the solid vibration and fluid flow. Flow-induced vibration can be divided into three categories: turbulence-induced vibration – as seen in fluttering pipes, vortices shedding –induced vibration – the phenomena that destroyed the Tacoma Narrows Bridge, and fluid elastic instability – a unique form of flow-induced vibration that is most commonly seem in nuclear heat exchangers after the tube velocity reaches a critical value(Matthem,2003). It is of great practical importance to a number of fields, especially so whenever this flow-induced vibration leads to structural fatigue or excessive noise (Birgersson,2004).The topic of flow-induced vibration has been studied widely and development in recent researches for,

Stephani R. et al, [Stephani, 2009]. They presented analytical and numerical analysis of microscale resonators containing internal flow, modeled here as microfabricated pipe conveying fluid, and investigates the effect of flow velocity on damping, stability, and frequency shift. The analysis was performed within the context of classical continuum mechanics and numerical results. The results show that the slender elastomeric pipe can become unstable by divergence and flutter at flow velocity of 10 m/s.

Michael S., Hans Irschik. (Michael, 2005) Presented the analytically and numerical study for nonlinear dynamics of elastic pipe conveying fluid at arbitrary flow rates. They derived the nonlinear equation of motion by using a unified form of the Lagrange equation for non-material volumes with different boundary conditions and compared the existing results obtained by using different formulations. Show that good agreement of result obtained with the numerical and different formulation.

M. Sekavcnik et al (Sekavenik, 2006). Presented a study of unsteady flow of viscous-compressible fluid through a pipe system induced by a transient dynamic disturbance at the pipe intake. The flow through the pipe is calculated using a system of RANS equations combined with various turbulence models. Time-domain results are transformed into the frequency-domain in order to determine the frequency content of the dynamic response for each simulation. They proposed a diagram of such contain different parameters, which allows convenient evaluation of the performance of various pipe systems.

Matthew T., Jonathan D. (Matthem, 2003). They presented a focuses on the development of a numerical, fluid-structure interaction (GSI) model that will help define the relationship between pipe wall vibration and the physical characteristics of turbulent flow. Also analysis of large eddy simulation (LES) flow models that compute the instantaneous fluctuation in turbulent flow. Show that a near quadratic relationship between the standard deviation of pressure field on the pipe wall and flow rate.

F. Birgersson et al, (Birgersson, 2004). They studied a vibration of pipes by using the Arnold–Warburton theory for thin shells and a simplified theory valid in a lower frequency regime. Also the vibrational response is described numerically with the spectral finite element method (SFEM), and compares results with wind tunnel measurements. They comparison between a simplified cylinder theory and the Arnold-Warburton theory proved the usefulness of the simplified theory in lower frequency regime. Also may be applied to all the other thin circular cylindrical shell theories.

# 2- Theoretical Modeling Analyses

## 2.1 Pressure Turbulent Flow

The transport of a fluid (liquid or gas) in a closed conduit (commonly called a *pipe* if it is of round cross section or a *duct* if it is not round) is extremely important in our daily operations. A brief consideration of the world around us will indicate that there is a wide variety of applications of pipe flow. Such applications range from the large, pipeline that carries crude oil, to the more complex natural systems of “pipes” that carry blood throughout our body and air into and out of our lungs. Other examples include the water pipes in our homes and the distribution system that delivers the water from the city well to the house. Numerous hoses and pipes carry hydraulic fluid or other fluids to various components of vehicles and machines (Bruce, 2002).

The flow of a fluid in a pipe classified depended on Reynolds number eq. (1), into three types, *laminar flow* Re<2100, *transitional flow* 2100<Re<4000, and *turbulent flow* Re>4000[10]. In this paper a treatment of pipe under fully developed turbulent flow see **Table 1**. That is, the velocity profile is the same at any cross section of the pipe.

$$(1) Re = \frac{\rho D v}{\mu}$$

The properties of Fresh water is using in this pipe at ($20^oC$) are; dynamic viscosity=1.00E-3 ($N.s/m^2$), velocity of sound in water =1481.328 (m/s), density= 998.3298($kg/m^3$), specific weight=9.783($KN/m^3$) (Eugene, 1996)

Depended on the result As shown by **Sgard** and **Atalla** (Durant, 2000), show when the Mach number (M) eq. (2), less than roughly 0.5, there is a little influence of the mean flow velocity on the natural vibration characteristics.

$$M = U/C \qquad (2)$$

Where U is a typical measure of flow speed and c is the speed of sound in medium (Pijush, 2002), compared the result of Mach number in **Table 1,** with results of reference (Durant, 2000). the internal presser $(p_i)$ can be calculated without the fluctuation waves of turbulent. Hint: The Mach number range used in this study is M= (0.003-0.06) < 0.5.

## 2.2 Vibration Analysis Modeling

A vibration system is a dynamic system for which the variables such as the excitations (inputs) and responses (outputs) are time-dependent, the analysis of a force vibration system **Fig. 1** usually involves derivation of the governing equations, solution of the equations, and interpretation of the results (Singiresu, 2005).

### 2.2.1 Governing Equations of Modeling

A derive the governing equation of motion for a thin pipe, assume, isotropic, and homogeneous shells of constant thickness have neutral surfaces, just as beams in transverse deflection have neutral fibers (Werner, 2004), **Fig.1,** its content uncompressible fluid (fresh water) in fully developed turbulent flow, to derive the equation of motion can by using the energy method (Hamilton's principle) eq. (3), (Werner, 2004).

$$\delta \int_{t_0}^{t_1} (K - U + W_{in})\, dt = 0 \qquad (3)$$

Or in other simple form eq. (4):

$$\delta \int_{t_1}^{t_2} K\, dt - \delta \int_{t_1}^{t_2} U\, dt + \delta \int_{t_1}^{t_2} W_{in} dt = 0 \qquad (4)$$

Where **K** kinetic energy, **U** strain energy **and** $\mathbf{W}_{in}$ are all energy input to the system, $\boldsymbol{\delta}$ is variation, operationally equivalent to a total differential.

For simplification analysis harmonic motion of model can by assumed the Eqs. (5, 6):

$$u_1(x,\theta,t) = U_1(x,\theta)e^{i\omega t}$$
$$u_2(x,\theta,t) = U_2(x,\theta)e^{i\omega t} \qquad (5)$$
$$u_3(x,\theta,t) = U_3(x,\theta)e^{i\omega t}$$

Where

$$U_1(x,\theta) = U\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\theta}{2}\right)$$

$$U_2(x,\theta) = V\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\theta}{2}\right) \qquad (6)$$

$$U_3(x,\theta) = W\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\theta}{2}\right)$$

And Boundary condition eq. (7):

$$u_1(0,\theta,t) = U_1(0,\theta) = 0$$
$$u_2(0,\theta,t) = U_2(0,\theta) = 0$$
$$u_3(0,\theta,t) = U_3(0,\theta) = 0$$
$$u_1(L,\theta,t) = U_1(L,\theta) = 0 \qquad (7)$$
$$u_2(L,\theta,t) = U_2(L,\theta) = 0$$
$$u_3(L,\theta,t) = U_3(L,\theta) = 0$$

Now eq. (4), content three parts represented all energy components on any system, also the pipe model content a three parts:

**Part one**: The strain energy of general neutral shell surface **Fig.2,** can by calculated from eq. (5) (Werner, 2004), **Fig.2** shows the Cartesian coordinates system (x, y, z), and curvilinear surface coordinates system for small element $(\alpha_1, \alpha_2, \alpha_3)$.

For model of circular cylindrical shell shown in **Fig.1**, the curvilinear coordinates are $\alpha_1 = x, and\ \alpha_2 = \theta$, the lines of principal curvature are in this model parallel to the axis of revaluation, then $R_x(R_1) = \infty\ , and\ R_\theta(R_2) = a$, also $A = 1, and\ B = a$ . For another detail see reference (Werner, 2004). The strain energy stored in one infinitesimal element that is acted on by stresses and strains is eq. (8), (Eduard, 2001).

$$U = \frac{1}{2}\int_0^L\int_0^{2\pi}[N_1\varepsilon_1 + N_2\varepsilon_2 + S\gamma_{12} + M_1X_1 + M_2X_2 + 2HX_{12}]A\,B\,d\theta\,dx \qquad (8)$$

Also

$$\delta\int_{t_1}^{t_2}U\,dt = \delta\frac{1}{2}\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}[N_1\varepsilon_1 + N_2\varepsilon_2 + S\gamma_{12} + M_1X_1 + M_2X_2 + 2HX_{12}]A\,B\,d\theta\,dx\,dt \qquad (9)$$

Where:

$$\varepsilon_1 = \frac{1}{A}\frac{\partial u1}{\partial x} + \frac{u2}{AB}\frac{\partial A}{\partial\theta} - \frac{u3}{R_1},\ \ \varepsilon_2 = \frac{1}{B}\frac{\partial u2}{\partial\theta} + \frac{u1}{AB}\frac{\partial B}{\partial x} - \frac{u3}{R_2},$$

$$\gamma_{12} = \frac{B}{A}\frac{\partial}{\partial x}\left(\frac{u2}{B}\right) + \frac{A}{B}\frac{\partial}{\partial\theta}\left(\frac{u1}{A}\right)$$

$$X_1 = -\left[\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{u1}{R_1} + \frac{1}{A}\frac{\partial u3}{\partial x}\right) + \frac{1}{AB}\frac{\partial A}{\partial\theta}\left(\frac{u2}{R_2} + \frac{1}{B}\frac{\partial u3}{\partial\theta}\right)\right]$$

$$X_2 = -\left[\frac{1}{B}\frac{\partial}{\partial\theta}\left(\frac{u2}{R_2} + \frac{1}{B}\frac{\partial u3}{\partial\theta}\right) + \frac{1}{AB}\frac{\partial B}{\partial x}\left(\frac{u1}{R_1} + \frac{1}{A}\frac{\partial u3}{\partial x}\right)\right]$$

$$X_{12} = -\left[\frac{1}{AB}\left(-\frac{1}{A}\frac{\partial A}{\partial\theta}\frac{\partial u3}{\partial x} - \frac{1}{B}\frac{\partial B}{\partial x}\frac{\partial u3}{\partial\theta} + \frac{\partial^2 u3}{\partial x\partial\theta}\right) + \frac{1}{R_1}\frac{A}{B}\frac{\partial}{\partial\theta}\left(\frac{u1}{A}\right) + \frac{1}{R_2}\frac{B}{A}\frac{\partial}{\partial x}\left(\frac{u2}{B}\right)\right]$$

$$N_1 = \frac{Eh}{1-v^2}(\varepsilon_1 + v\varepsilon_2)\,, N_2 = \frac{Eh}{1-v^2}(\varepsilon_2 + v\varepsilon_1)\ ,$$

$$M_1 = D(X_1 + vX_2)\ ,\quad M_2 = D(X_2 + vX_1)\ ,$$

$$S = \frac{Eh}{2(1+v)}\gamma_{12}\,, H = D(1-v)X_{12}, D = \frac{Eh^3}{12(1-v^2)}$$

The solution of strain energy (part one) eq. (9), is:

Let

$$\delta\int_{t_1}^{t_2}U\,dt = \frac{1}{2}\delta\left[\int_{t_1}^{t_2}U1\,dt + \int_{t_1}^{t_2}U2\,dt + \int_{t_1}^{t_2}U3\,dt + \int_{t_1}^{t_2}U4\,dt + \int_{t_1}^{t_2}U5\,dt + \int_{t_1}^{t_2}U6\,dt\right] \qquad (10)$$

Where

$$\delta\int_{t_1}^{t_2}U1\ dt = \delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{aEh}{1-v^2}\left(\frac{\partial u1}{\partial x}\right)^2 d\theta\,dx\,dt$$
$$+\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{v}{a}\frac{aEh}{1-v^2}\left(\frac{\partial u2}{\partial\theta}\frac{\partial u1}{\partial x}\right)d\theta\,dx\,dt - \delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{v}{a}\frac{aEh}{1-v^2}\left(\frac{u3}{a}\frac{\partial u1}{\partial x}\right)d\theta\,dx\,dt \qquad (11)$$

$$\delta\int_{t_1}^{t_2}U2\ dt = \frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{1}{a}\frac{\partial u2}{\partial\theta}\right)^2 d\theta\,dx\,dt$$
$$-\frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a}\frac{\partial u2}{\partial\theta}\frac{u3}{a}d\theta\,dx\,dt - \frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}u3\frac{1}{a}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt + \frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}u3\frac{u3}{a}d\theta\,dx\,dt$$
$$+\frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}v\frac{1}{a}\frac{\partial u1}{\partial x}\frac{\partial u2}{\partial\theta}d\theta\,dx\,d - \frac{aEh}{1-v^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}v\frac{u3}{a}\frac{\partial u1}{\partial x}d\theta\,dx\,dt \qquad (12)$$

$$\delta\int_{t_1}^{t_2}U3\ dt = \frac{aEh}{2(1+v)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial u2}{\partial x}\right)^2 d\theta\,dx\,dt$$
$$+\frac{aEh}{2(1+v)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}2\frac{\partial u2}{\partial x}\frac{1}{a}\frac{\partial u1}{\partial\theta}d\theta\,dx\,dt + \frac{aEh}{2(1+v)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{1}{a}\frac{\partial u1}{\partial\theta}\right)^2 d\theta\,dx\,dt \qquad (13)$$

$$\delta\int_{t_1}^{t_2}U4\ dt = \frac{aEh^3}{12(1-v^2)}\ \delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial^2 u3}{\partial x^2}\right)^2 d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{1}{a^2}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial x^2}\right)d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{1}{a^2}\frac{\partial^4 u3}{\partial\theta^2\partial x^2}\right)d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(v\frac{1}{a^2}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial x^2}\right)d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(v\frac{1}{a^2}\frac{\partial^4 u3}{\partial\theta^2\partial x^2}\right)d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}v\left(\frac{\partial^2 u3}{\partial x^2}\right)^2 d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^2}\frac{\partial^2 u3}{\partial x^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\left(\frac{\partial u2}{\partial\theta}\right)^2 d\theta\,dx\,dt$$
$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\frac{\partial^2 u3}{\partial\theta^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\left(\frac{\partial u2}{\partial\theta}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\frac{\partial^2 u3}{\partial\theta^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^2}\frac{\partial^2 u3}{\partial x^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^2}\frac{\partial^4 u3}{\partial x^2\partial\theta^2}d\theta\,dx\,dt+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)^2 d\theta\,dx\,dt+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^2}\frac{\partial^4 u3}{\partial x^2\partial\theta^2}d\theta\,dx\,dt \qquad (14)$$

$$\delta\int_{t_1}^{t_2} U5\;dt=\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\left(\frac{\partial u2}{\partial\theta}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^2}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial x^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^2}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial x^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\left(\frac{\partial u2}{\partial\theta}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^4}\frac{\partial u2}{\partial\theta}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\frac{\partial^2 u3}{\partial\theta^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a^4}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{\partial^2 u3}{\partial x^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{\partial^2 u3}{\partial x^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a^2}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a}\frac{\partial^2 u3}{\partial x^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{1}{a}\frac{\partial^2 u3}{\partial x^2}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} a\left(\frac{\partial^2 u3}{\partial x^2}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} va\left(\frac{\partial^2 u3}{\partial x^2}\right)^2 d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a}\frac{\partial^2 u3}{\partial x^2}\frac{\partial u2}{\partial\theta}d\theta\,dx\,dt$$

$$+\frac{aEh^3}{12(1-v^2)}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} v\frac{1}{a}\frac{\partial^2 u3}{\partial x^2}\frac{\partial^2 u3}{\partial\theta^2}d\theta\,dx\,dt \qquad (15)$$

$$\int_{t_1}^{t_2} U6\;dt=\left(\frac{aEh^3}{6(1-v^2)}(1-v)\right)\frac{1}{a^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial^2 u3}{\partial\theta\partial x}\right)^2 d\theta\,dx\,dt$$

$$+2\left(\frac{aEh^3}{6(1-v^2)}(1-v)\right)\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{1}{a^2}\frac{\partial u2}{\partial x}\frac{\partial^2 u3}{\partial\theta\partial x}\right)d\theta\,dx\,dt$$

$$+\left(\frac{aEh^3}{6(1-v^2)}(1-v)\right)\frac{1}{a^2}\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial u2}{\partial x}\right)^2 d\theta\,dx\,dt \qquad (16)$$

**Part two**: Return to eq. (4) , The Kinetic energy of one infinitesimal element is eq. (17):

$$K=\frac{1}{2}\rho h\int_0^L\int_0^{2\pi}\left(\frac{\partial u1}{\partial t}\right)^2 d\theta\,dx+\frac{1}{2}\rho h\int_0^L\int_0^{2\pi}\left(\frac{\partial u2}{\partial t}\right)^2 d\theta\,dx+\frac{1}{2}\rho h\int_0^L\int_0^{2\pi}\left(\frac{\partial u3}{\partial t}\right)^2 d\theta\,dx \qquad (17)$$

With Applying Hamilton's principle again gives eq. (18):

$$\delta\int_{t_1}^{t_2} K\,dt=\frac{1}{2}\rho h\,\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial u1}{\partial t}\right)^2 d\theta\,dx\,dt+\frac{1}{2}\rho h\,\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial u2}{\partial t}\right)^2 d\theta\,dx\,dt+\frac{1}{2}\rho h\,\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left(\frac{\partial u3}{\partial t}\right)^2 d\theta\,dx\,dt \qquad (18)$$

We examine the double integral of eq. (18) by parts with boundary condition eq. (7), therefore becomes eq. (19):

$$\delta\int_{t_1}^{t_2} K\,dt=-\left(\frac{1}{2}\rho h\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{\partial^2 u1}{\partial t^2}(\delta u1)\,d\theta\,dx\,dt+\frac{1}{2}\rho h\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{\partial^2 u2}{\partial t^2}(\delta u2)\,d\theta\,dx\,dt\right.$$
$$\left.+\frac{1}{2}\rho h\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\frac{\partial^2 u3}{\partial t^2}(\delta u3)\,d\theta\,dx\,dt\right) \qquad (19)$$

**Part three** is divided to two components: firstly the variation of energy introduced into the shell by distributed load components in the $\alpha_1,\alpha_2, and\ \alpha_3$ directions, namely are $q_1, q_2, and\ q_3$ $(\frac{N}{m^2})$ , eq. (20), see **Fig. 3**.

$$W_{in1}=\int_0^L\int_0^{2\pi}(q_1u_1+q_2u_2+q_3u_3)\,A_1A_2d\theta\,dx \qquad (20)$$

For a cylindrical model with internal load, $q_1=q_2=0, and\ q_3=P$, eq. (20) become, eq. (21)

$$W_{in1}=\int_0^L\int_0^{2\pi}(q_3u_3)\,A_1A_2d\theta\,dx \qquad (21)$$

If the pressure is variable along the cylindrical coordinate, depended on the $\alpha_1$-axis eq. (22),

$$q_3 = P_{\alpha 1} = P_{\alpha 1=0} - \Delta P \tag{22}$$

And

$$\Delta P = f\,\frac{x\,\rho\,Velocity^2}{2D} \quad \text{See Table 1.} \tag{23}$$

Secondly the potential energy is added to the wall elements are from fluid flow. The water molecules adjacent to the pipe wall do no move; in other wards they have no velocity, and consequently no kinetic energy. However, as the molecules approach the wall, they do have kinetic energy (Matthem, 2003). When the particle of water draws near the wall, converted all kinetic energy to another form of energy, which converted to heat and potential energy add to the pipe wall (Pijush, 2002). For simple solutions assume the heat energy equals to zero. Then all energy from the flow through passing in pipe is converted to potential energy (potential energy of flow $W_{in2}$).eq. (24),

$$W_{in2} = \int_0^L \gamma\, f\, \frac{\text{velocity}^2}{2D\rho_{fluid}}\,\Phi\, dx \tag{24}$$

Sub. eq. (22) into eq. (21), and sum with eq. (24) (over all internal energy transfer from flowed to solid) eq. (25):

$$W_{in} = \int_0^L\int_0^{2\pi}\left(\left(P_{\alpha 1=0} + f\,\frac{\alpha_1\rho\, v^2}{2D}\right)u_3\right)A_1A_2\,d\theta\, dx + \int_0^L \gamma\, f\,\frac{(\text{velocity})^2}{2D\rho_{fluid}}\,\Phi\, dx \tag{25}$$

With Applying Hamilton’s principle gives eq. (26):

$$\delta\int_{t_1}^{t_2} W_{in}\,dt = P_{(\alpha 1=0)}\,\delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} u3\, a\, d\theta\, dx\, dt + \delta\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi} f\,\frac{\alpha_1\rho\, v^2}{2D}\,u_2 a\, d\theta\, dx\, dt + \delta\int_{t_1}^{t_2}\int_0^L \gamma f\,\frac{(\text{velocity})^2}{2D\rho_{fluid}}\,\Phi\, dx\, dt \tag{26}$$

Where

$\gamma$: $spisifec\ wight\ of\ fluid$

$f$: $factor\ from\ moduy\ chart\ (stephani, 2009)$

$\epsilon = 0.002\ mm(stainless - new)(jamal, 2002)$

velocity : $averag\ velocity\ of\ flow$

$\rho_{fluid}$: $density\ of\ fluid\ flow$

$\Phi$: $flow\ rati\ of\ fluid\ flow$

The equation of motion must be solved to find the response of system $U_1, U_2, U_3$ : Sub. eqs. (11, 12, 13, 14, 15, 16) into eq. (10). And by Solving eq. (19 and 26) and substitute all into eq. (4), gives:

$$\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\left[\left(\frac{1}{2}\rho h\left(\frac{\partial^2 u1}{\partial t^2}\right)\right) - \left(f\,\frac{a\,\alpha_1\rho\, v^2}{2D}\right) + \left(\gamma\, f\,\frac{(\text{velocity})^2}{2D\rho_{fluid}}\,\Phi\right) - \frac{Eh}{(1+v)}\left[\frac{1}{(1-v)}\left\{a\,\frac{\partial}{\partial x}\left(\frac{\partial u1}{\partial x}\right) - v\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial x}\right) - v\frac{\partial u3}{\partial x}\right\} + \left\{\frac{1}{2}\frac{\partial}{\partial\theta}\left(\frac{\partial u1}{\partial\theta}\right) + \frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial x}\right)\right\}\right]\right]\delta u3\, d\theta\, dx\, dt = 0 \tag{27}$$

$$\begin{aligned}
&\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\Bigg[\left(-\frac{1}{2}\rho h\left(\frac{\partial^2 u2}{\partial t^2}\right)\right) + \frac{Eh}{(1-v^2)}\Bigg\{\left(-v\,\frac{\partial}{\partial x}\left(\frac{\partial u1}{\partial\theta}\right)\right)\\
&-\left(\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial\theta}\right)\right) - \left(\frac{1}{a}\left(\frac{\partial u3}{\partial\theta}\right)\right) + \left(\frac{h^2}{12a}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right) + \left(\frac{vh^2}{12a}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)\\
&+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial\theta}\right)\right) + \left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right) + \left(\frac{vh^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial\theta}\right)\right)\\
&+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right) + \left(\frac{vh^2}{12a}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right) + \left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right)\\
&+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right) + \left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial\theta}\right)\right) + \left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right)\\
&+\left(\frac{h^2}{12a}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right) + \left(\frac{vh^2}{12a}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right) + \left(\frac{vh^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u2}{\partial\theta}\right)\right)\\
&+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right) + \left(\frac{h^2}{12a^2}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial\theta^2}\right)\right) + \frac{vh^2}{12a}\left(\frac{\partial u2}{\partial\theta}\right)\\
&+\left(\frac{h^2}{12}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right) + \left(\frac{vh^2}{12}\frac{\partial}{\partial\theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)\Bigg\} + \frac{2Eh}{(1+v)}\\
&\left(\frac{a}{4}\frac{\partial}{\partial x}\left(\frac{\partial u2}{\partial x}\right)\right) + \left(\left(\frac{h^2}{6a}\right)\frac{\partial}{\partial x}\left(\frac{\partial^2 u3}{\partial\theta\partial x}\right)\right) + \left(\left(\frac{h^2}{12a}\right)\frac{\partial}{\partial x}\left(\frac{\partial u2}{\partial x}\right)\right)\Bigg]\\
&\delta u2\, d\theta\, dx\, dt = 0
\end{aligned} \tag{28}$$

$$\int_{t_1}^{t_2}\int_0^L\int_0^{2\pi}\Bigg[\left(\frac{v}{a}\frac{Eh}{(1-v^2)}\left(\frac{\partial u1}{\partial x}\right)\right)-\left(\frac{Eh}{1-v^2}\left(\frac{\partial u2}{\partial \theta}\right)\right)-\left(\frac{Eh}{1-v^2}u3\right)+\left(\frac{aEh^3}{12(1-v^2)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{Eh^3}{12a(1-v^2)}\frac{\partial}{\partial \theta}\left(\frac{\partial^3 u3}{\partial \theta\ \partial x^2}\right)\right)$$
$$+\left(\frac{vEh^3}{12a(1-v^2)}\frac{\partial}{\partial \theta}\left(\frac{\partial^3 u3}{\partial \theta\ \partial x^2}\right)\right)+\left(\frac{avEh^3}{12(1-v^2)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{Eh^3}{12a(1-v^2)}\frac{\partial}{\partial x}\left(\frac{\partial^3 u3}{\partial x\ \partial \theta^2}\right)\right)$$
$$+\left(\frac{Eh^3}{12a^3(1-v^2)}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vEh^3}{12a^3(1-v^2)}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vEh^3}{12a(1-v^2)}\frac{\partial}{\partial x}\left(\frac{\partial^3 u3}{\partial x\ \partial \theta^2}\right)\right)$$
$$+\left(\frac{Eh^3}{12a^3(1-v^2)}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{aEh^3}{12(1-v^2)}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{avEh^3}{12(1-v^2)}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{vEh^3}{12a(1-v^2)}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)$$
$$+\left(\frac{Eh^3}{12(1-v^2)}\frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{a^2Eh^3}{12(1-v^2)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{va^2Eh^3}{12(1-v^2)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{vEh^3}{12(1-v^2)}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)$$
$$+\left(\left(\frac{aEh^3}{6(1-v^2)}(1-v)\right)\frac{1}{a^2}\frac{\partial^2}{\partial \theta \partial x}\left(\frac{\partial^2 u3}{\partial \theta \partial x}\right)\right)+P_{(z1=0)}\,a\Bigg]\delta u3\,d\theta\,dx\,dt=0 \quad (29)$$

Since $\delta ui$ is arbitrary, eqs. (27, 28, and 29), can be satisfied only if the coefficient of $\delta ui$ is zero and Therefore and rearrangements its Equations give:

$$(\rho h\,\ddot{u}1)-\frac{2Eh}{(1+v)}\left[\frac{1}{(1-v)}\left\{a\left(\frac{\partial^2 u1}{\partial x^2}\right)+\frac{(1-v)}{2}\left(\frac{\partial^2 u1}{\partial \theta^2}\right)-v\frac{\partial u3}{\partial x}\right\}+\left\{\frac{\partial^2 u2}{\partial \theta\,\partial x}\left(1-\frac{v}{(1-v)}\right)\right\}\right]=\left(f\,\frac{a\,\alpha_1\rho\,v^2}{D}\right)-\left(\gamma f\,\frac{(\text{velocity})^2}{D\rho_{fluid}}\mp\right) \quad (30)$$

$$(\rho h\,\ddot{u}2)-\frac{2Eh}{(1-v^2)}\Bigg\{+\left(\frac{h^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{vh^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{h^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vh^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)$$
$$+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{h^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{vh^2}{12a}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)$$
$$+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{h^2}{12}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\left(\frac{vh^2}{12}\frac{\partial}{\partial \theta}\left(\frac{\partial^2 u3}{\partial x^2}\right)\right)+\frac{2(1-v^2)}{(1+v)}\left(\left(\frac{h^2}{6a}\right)\frac{\partial}{\partial x}\left(\frac{\partial^2 u3}{\partial \theta \partial x}\right)\right)$$
$$-\left(v\frac{\partial}{\partial x}\left(\frac{\partial u1}{\partial \theta}\right)\right)-\left(\frac{\partial}{\partial \theta}\left(\frac{\partial u2}{\partial \theta}\right)\right)-\left(\frac{1}{a}\left(\frac{\partial u3}{\partial \theta}\right)\right)+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial u2}{\partial \theta}\right)\right)+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial u2}{\partial \theta}\right)\right)+\left(\frac{h^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial u2}{\partial \theta}\right)\right)+\left(\frac{vh^2}{12a^2}\frac{\partial}{\partial \theta}\left(\frac{\partial u2}{\partial \theta}\right)\right)$$
$$+\frac{vh^2}{12a}\left(\frac{\partial u2}{\partial \theta}\right)+\frac{2(1-v^2)}{(1+v)}\left(\frac{a}{4}\frac{\partial}{\partial x}\left(\frac{\partial u2}{\partial x}\right)\right)+\frac{2(1-v^2)}{(1+v)}\left(\left(\frac{h^2}{12a}\right)\frac{\partial}{\partial x}\left(\frac{\partial u2}{\partial x}\right)\right)\Bigg\}=0 \quad (31)$$

$$(\rho h\ddot{u}3)-\frac{2Eh}{(1-v^2)}\Bigg[\frac{ah^2}{12}\Bigg[\left(\frac{\partial^4 u3}{\partial x^4}\right)+\left(\frac{1}{a^2}\left(\frac{\partial^4 u3}{\partial \theta^2\,\partial x^2}\right)\right)+\left(\frac{v}{a^2}\left(\frac{\partial^4 u3}{\partial \theta^2\,\partial x^2}\right)\right)+\left(v\left(\frac{\partial^4 u3}{\partial x^4}\right)\right)+\left(\frac{1}{a^2}\left(\frac{\partial^4 u3}{\partial x^2\,\partial \theta^2}\right)\right)+\left(\frac{1}{a^4}\left(\frac{\partial^4 u3}{\partial \theta^4}\right)\right)+\left(\frac{v}{a^4}\left(\frac{\partial^4 u3}{\partial \theta^4}\right)\right)$$
$$+\left(\frac{v}{a^2}\left(\frac{\partial^4 u3}{\partial x^2\,\partial \theta^2}\right)\right)+\left(\frac{1}{a^4}\left(\frac{\partial^4 u3}{\partial \theta^4}\right)\right)+\left(\frac{\partial^2 u3}{\partial x^2}\right)+v\left(\frac{\partial^2 u3}{\partial x^2}\right)+\left(\frac{v}{a^2}\left(\frac{\partial^2 u3}{\partial \theta^2}\right)\right)+\left(\frac{1}{a}\left(\frac{\partial^4 u3}{\partial \theta^2\,\partial x^2}\right)\right)+\left(a\left(\frac{\partial^4 u3}{\partial x^4}\right)\right)$$
$$+\left(va\left(\frac{\partial^4 u3}{\partial x^4}\right)\right)+\left(\frac{v}{a}\left(\frac{\partial^4 u3}{\partial x^2\,\partial \theta^2}\right)\right)+\left(\left(\frac{2}{a^2}(1-v)\right)\left(\frac{\partial^4 u3}{\partial \theta^2\,\partial x^2}\right)\right)\Bigg]+\frac{v}{a}\left(\frac{\partial u1}{\partial x}\right)-\left(\frac{\partial u2}{\partial \theta}\right)-(u3)\Bigg]=2a\,P_{(z1=0)} \quad (32)$$

These are the general equations of motion of thin pipes in three dimensions. To obtain the frequency and mode of the model, use eqs. (5 and 6), to solve equations (30, 31, and 32) and simplification form gives eqs. (33, 34, and 35):

$$\left(\frac{(1+v)\rho h}{2Eh}\,\omega^2+K_{11}\right)U+K_{12}\,V+K_{13}\,W=\underline{F}1 \quad (33)$$

$$K_{21}U-\left(\frac{(1-v^2)\rho h}{2Eh}\,\omega^2+K_{22}\right)V-K_{23}W=0 \quad (34)$$

$$K_{31}\,U-K_{32}V+\left((\rho h\,\omega^2)-K_{33}\right)W=\underline{F2} \quad (35)$$

Where

$$\underline{F1} = \frac{(1+v)}{2Eh}\left\{\left(f\,\frac{a\,\alpha_1\rho\,v^2}{D}\right) - \left(\gamma\, f\,\frac{(\text{velocity})^2}{D\rho_{fluid}}\,\Phi\right)\right\}$$

or

$$\underline{F1} = F3\;F4 - F3\;F5 * (\omega * a)^2$$

$$F3 = \frac{(1+v)}{2Eh};\; F4 = \left(f\,\frac{a\,\alpha_1\rho\,v^2}{D}\right);\; F5 = \left(\gamma\, f\,\frac{1}{D\rho_{fluid}}\,\Phi\right)$$

$$K_{11} = -\frac{1}{(1-v)}\frac{a\,n^2\pi^2}{L^2} + \frac{n^2}{8}$$

$$K_{12} = \frac{n^2\pi}{L}\left(1 - \frac{v}{(1-v)}\right)$$

$$K_{13} = \frac{1}{(1-v)}\frac{v\,n\,\pi}{L}$$

$$B_1 = \left(\frac{\pi^2}{3\,aL^2}(1-v) + \frac{1}{24a^3}\left(2 + v(2+\theta)\right)\right)$$

$$B_2 = \frac{L^3}{4a^2\pi^2}\left(4 + 3v - \frac{48\,a^2}{h^2\,n^2} + \frac{4a^3\pi^2 v\,\theta}{n^3 x L^2}\right)$$

$$K_{22} = \frac{n^2}{2}\left(\frac{a\pi^2}{L^2}(1-v) + h^2 B_1 - \frac{1}{2}\right)$$

$$K_{21} = \left(v\,\frac{n^2\pi}{2L}\right)$$

$$K_{23} = \frac{n^3\pi^2 h^2}{24a\,L^3}\left(3(1+v) + a + 4(1-v) + B_2\right)$$

Assume

$$C_1 = \frac{\pi^4}{L^2}\,(1+a)$$

$$C_2 = \frac{1\pi^2}{4\,a^2}\left((2+a) + 2\,\frac{(1-v)}{(1+v)}\right)$$

$$C_3 = \frac{L^2}{16\,a^4}\,\frac{(2+v)}{(1+v)} - \frac{\pi^2}{n^2} - \frac{vL^2}{a^2 n^2\,4}$$

$$C_4 = \frac{L^2(1-v^2)}{n^4\,2Eh(1+v)}$$

$$K_{31} = \frac{2vn\,\pi Eh}{aL(1-v^2)}$$

$$K_{32} = \frac{nEh}{(1-v^2)}$$

$$K_{33} = \frac{2Eh}{(1-v^2)}\,\frac{ah^2}{12}\left[(1+v)\frac{n^4}{L^2}(C_1 + C_2 + C_3 - C_4)\right]$$

$$\underline{F2} = 2a\,P_{(\alpha 1=0)}$$

Rewriting eqs. (33, 34, 35) in general matrix form eq. (36):

$$\begin{bmatrix} \frac{(1+v)\rho h}{2Eh}\,\omega^2 + K_{11} & K_{12} & K_{13} \\ K_{21} & \frac{(1-v^2)\rho h}{2Eh}\,\omega^2 + K_{22} & K_{23} \\ K_{31} & K_{32} & (\rho h\,\omega^2) - K_{33} \end{bmatrix}\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} \underline{F1} \\ 0 \\ \underline{F2} \end{Bmatrix} \quad (36)$$

To obtain the natural frequency and modes, we set $\underline{F1} = \underline{F2} = 0$,gives eq. (37):

$$[Z(\omega)] = 0 = \begin{vmatrix} \frac{(1+v)\rho h}{2Eh}\,\omega^2 + K_{11} & K_{12} & K_{13} \\ K_{21} & \frac{(1-v^2)\rho h}{2Eh}\,\omega^2 + K_{22} & K_{23} \\ K_{31} & K_{32} & (\rho h\,\omega^2) - K_{33} \end{vmatrix} \quad (37)$$

And determine eq. (37), gives:

$$\left(\frac{\rho^2 h(v+1)(1-v^2)}{4e^2}\right)\omega^6$$
$$+\left(\frac{\rho^2 h(v+1)}{2e}K_{22} - \frac{\rho^2(v+1)(1-v^2)}{4e^2}\,K_{33} + \frac{\rho^2 h(1-v^2)}{2e}\,k_{11}\right)\omega^4$$
$$+\left(\rho\,h\,k_{11}K_{22} - \frac{\rho(v+1)}{2e}K_{22}K_{33} - \frac{\rho(1-v^2)}{2e}k_{11}k_{33} - \frac{\rho(1-v^2)}{2e}K_{13}K_{31}\right.$$
$$\left.-\frac{\rho(v+1)}{2e}K_{23}K_{32} - \rho\,h\,k_{12}\,k_{21}\right)\omega^2$$
$$-K_{22}k_{11}k_{33} - K_{22}K_{13}K_{31} - K_{11}K_{23}K_{32}$$
$$+K_{33}K_{12}K_{21} + K_{12}K_{23}K_{31} + K_{13}K_{21}K_{32} = 0 \quad (38)$$

Or

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$$

To calculate the frequencies use the mathematic method in reference (Murray, 1968):

Where:

$$\lambda_i = \omega_i^2 \quad i = 1,2,3$$

$$C_1 = \left(\frac{B}{A}\right) = \left[\frac{\left(\frac{\rho^2 h(v+1)}{2e}K_{22} - \frac{\rho^2(v+1)(1-v^2)}{4e^2}K_{22} + \frac{\rho^2 h(1-v^2)}{2e}k_{11}\right)}{\left(\frac{\rho^2 h(v+1)(1-v^2)}{4e^2}\right)}\right] \quad (39)$$

$$C_2 = \left(\frac{C}{A}\right) = \left[\frac{\left(\rho\, h\, k_{11}K_{22} - \frac{\rho(v+1)}{2e}K_{22}K_{22} - \frac{\rho(1-v^2)}{2e}k_{11}k_{22} - \frac{\rho(1-v^2)}{2e}K_{12}K_{21} - \frac{\rho(v+1)}{2e}K_{22}K_{22} - \rho\, h\, k_{12}\, k_{21}\right)}{\left(\frac{\rho^2 h(v+1)(1-v^2)}{4e^2}\right)}\right] \quad (40)$$

$$C_3 = \left(\frac{D}{A}\right) = \left[\frac{-K_{22}K_{11}K_{22} - K_{22}K_{12}K_{21} - K_{11}K_{22}K_{22} + K_{22}K_{12}K_{21} + K_{12}K_{22}K_{21} + K_{12}K_{21}K_{22}}{\left(\frac{\rho^2 h(v+1)(1-v^2)}{4e^2}\right)}\right] \quad (41)$$

$$\therefore \omega_{in} = \sqrt{\left(2\sqrt{\frac{C_1^2}{9} - \frac{C_2}{3}}\right) cos\left(\frac{1}{3}\cos^{-1}(-\bar{\varphi}) + \frac{\pi\,(i*2-2)}{3}\right) - \frac{C_1}{3}} \quad (42)$$

Where

$$\bar{\varphi} = \frac{(2C_1^3 - 9C_1C_2 + 27C_3)}{54\left(\sqrt{-\left(\frac{1}{3}(3C_2 - C_1^2)\right)^3/27}\right)}$$

The natural mode component amplitudes are obtained by Solving ($U$ ) and ($V$)in terms of ($W$), gives:

$$\begin{bmatrix} \left(\frac{(1+v)\rho}{2e}\,\omega_{ni}^2\right) + K_{11} & K_{12} \\ K_{21} & \left(\frac{(1-v^2)\rho}{2e}\,\omega_{ni}^2\right) + K_{22} \end{bmatrix} \begin{Bmatrix} U_i \\ V_i \end{Bmatrix} = \{W_i\} \begin{bmatrix} K_{13} \\ K_{23} \end{bmatrix} \quad (43)$$

Where i=1, 2, 3. Thus:

$$U_{in} = W_i \frac{K_{13}\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{23}}{\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right)\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}}$$

(44)

$$V_{in} = W_i \frac{K_{23}\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right) - K_{13}K_{21}}{\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right)\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}} \quad (45)$$

*Win:* can by assume unity.

Substituting eq. (44) and (45) into mode eq. (5) gives the natural modes eqs.(46, 47, 48), (response of free vibration)

$$U_1(x,\theta) = W_i \frac{K_{13}\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{23}}{\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right)\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}}\; cos\left(\frac{n\pi x}{L}\right) cos\left(\frac{n\theta}{2}\right)$$

(46)

$$U_2(x,\theta) = W_i \frac{K_{23}\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right) - K_{13}K_{21}}{\left(\frac{\rho(v+1)\omega_{ni}^2}{2e} + K_{11}\right)\left(\frac{\rho\,\omega_{ni}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}}\; sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\theta}{2}\right)$$

(47)

$$U_3(x,\theta) = W_n\, sin\left(\frac{n\pi x}{L}\right) cos\left(\frac{n\theta}{2}\right) \quad (48)$$

To obtain the total response of the system under variable turbulent velocity can be the particularly solutions are:

$$Uf = U1\sin\omega t$$

$$Vf = V1\sin\omega t$$

(49)

$$Wf = W1\sin\omega t$$

Now Sub eq. (49), into eqs. (33, 34, 35), and using the adjoin method (William, 2000), to obtain the response under variable velocity gives:

$$\begin{Bmatrix} U1 \\ V1 \\ W1 \end{Bmatrix} = [Z(\omega)]^{-1}\begin{Bmatrix} F1 \\ 0 \\ F2 \end{Bmatrix} = \frac{adj[Z(\omega)]\begin{Bmatrix} F1 \\ 0 \\ F2 \end{Bmatrix}}{|Z(\omega)|} \quad (50)$$

Where

$$adj[Z(\omega)] = \begin{bmatrix} Z11 & Z12 & Z13 \\ Z21 & Z22 & Z23 \\ Z31 & Z32 & Z33 \end{bmatrix} \quad (51)$$

Where

$$Z11 = (\rho \quad h \quad \omega^2 - K_{33})\left(\frac{\rho\omega^2(-v^2+1)}{2e} + K_{22}\right) - K_{23}K_{32}$$

$$Z21 = -K_{21}(\rho \quad h \quad \omega^2 - K_{33}) + K_{23}K_{31}$$

$$Z31 = -K_{31}\left(\frac{\rho\omega^2(-v^2+1)}{2e} + K_{22}\right) + K_{21}K_{32}$$

$$Z12 = -K_{12}(\rho \quad h \quad \omega^2 - K_{33}) + K_{13}K_{32}$$

$$Z22 = (\rho \quad h \quad \omega^2 - K_{33})\left(\frac{\rho(v+1)\omega^2}{2e} + K_{11}\right) - K_{13}K_{31}$$

$$Z32 = -K_{32}\left(\frac{\rho(v+1)\omega^2}{2e} + K_{11}\right) + K_{12}K_{31}$$

$$Z13 = -K_{13}\left(\frac{\rho\omega^2(-v^2+1)}{2e} + K_{22}\right) + K_{12}K_{23}$$

$$Z23 = -K_{23}\left(\frac{\rho(v+1)\omega^2}{2e} + K_{11}\right) + K_{13}K_{21}$$

$$Z33 = \left(\frac{\rho(v+1)\omega^2}{2e} + K_{11}\right)\left(\frac{\rho\omega^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}$$

Substituting eq. (37, 38, and 51) into mode eq. (50) and (49) gives the force modes eqs. (52, 53, 54) (Response of force vibration):

$$Uf = \frac{1}{|Z(\omega)|}\left(Z11 * \underline{F1} + Z13 * \underline{F2}\right)\sin\omega t \qquad (52)$$

$$Vf = \frac{1}{|Z(\omega)|}\left(Z21 * \underline{F1} + Z23 * \underline{F2}\right)\sin\omega t \qquad (53)$$

$$Wf = \frac{1}{|Z(\omega)|}\left(Z11 * \underline{F1} + Z13 * \underline{F2}\right)\sin\omega t \qquad (54)$$

The final equations of the response of thin pipe are:

$$(55)\,U_1(x,\theta) = \sum_{n=1}^{\infty}\left[W_n \frac{K_{13}\left(\frac{\rho\omega_{n1}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{23}}{\left(\frac{\rho(v+1)\omega_{n1}^2}{2e} + K_{11}\right)\left(\frac{\rho\omega_{n1}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}} \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\theta}{2}\right) + \frac{1}{|Z(\omega)|}\left(Z11 * \underline{F1} + Z13 * \underline{F2}\right)\sin\omega t\right]$$

$$(56)\,U_2(x,\theta) = \sum_{n=1}^{\infty}\left[W_n \frac{K_{23}\left(\frac{\rho(v+1)\omega_{n1}^2}{2e} + K_{11}\right) - K_{13}K_{21}}{\left(\frac{\rho(v+1)\omega_{n1}^2}{2e} + K_{11}\right)\left(\frac{\rho\omega_{n1}^2(-v^2+1)}{2e} + K_{22}\right) - K_{12}K_{21}} \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\theta}{2}\right) + \frac{1}{|Z(\omega)|}\left(Z21 * \underline{F1} + Z23 * \underline{F2}\right)\sin\omega t\right]$$

$$(57)\,U_3(x,\theta) = \sum_{n=1}^{\infty}\left[W_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\theta}{2}\right) + \frac{1}{|Z(\omega)|}\left(Z11 * \underline{F1} + Z13 * \underline{F2}\right)\sin\omega t\right]$$

## 3- Numerical Analysis

*ANSYS Workbench-12* (CFX-ANSYS), commercial code have been developed that model the interaction between the fluid and the structure.

In this research, the numerical solution packages, **ANSYS Workbench-12** has been used to simulate the behaviors of thin cylinder model under variable velocity of turbulent. The model of structure and fluid simulation can be built by Multi step, see **Fig. 4**, the procedures of numerical analysis are passing through number of phase analyses are:

- Generate shell model, see **Fig. 5**
- Generate fluid model, see **Fig. 6**
- CFX mesh fluid model, see **Fig. 7**
- Construction CFX-ANSYS model, see **Fig. 8**
- Solution a fluid model under CFX package, **Fig. 9**, and show the simple result. i.e. velocity result, **Fig. 10**
- Mesh of pipe structure ,see **Fig. 11**
- Construction boundary condition in quarter model (symmetry), simply supported see **Fig.12**
- Imported the important parameters from CFX solution of fluid modal, see **Fig. 13**
- Construction static model, see **Fig. 4**
- Construction modal analysis(free response), see **Fig.4**
- Construction Response Spectrum (force response) , see **Fig. 4**
- Construction Interface through all above models, see **Fig. 4**

## 4- Result s and Discussions

In this paper, an investigation has been made into the effect of variable loads represented by fluid flow on the vibration characteristics of thin cylinder edges elastically restrained simple support using FEM and Hamilton method (energy principle). General new frequency equation with and without including the effect of variable loads has been obtained. This equation is used to investigate the effect of variable loads on natural frequency for different classical parameter of flow. The approach is simple and allows the solution of a rather difficult elastodynamics problem. An advantage of using FEM simulation for complete picture of the deflection fields is obtained. The verification of variable loads and natural frequency are obtained by using Eq. (33) and Eqs. (42, 43, and 44), with FEM, see **Figs. 16 and 17**. **Figs. (18-27)** shows the results obtained analytically for thin cylinder in different states (that has been calculated).

**Figs. 18, 19, and 20**, Show the displacement in longitudinal, tangential, and normal to surface (i.e. x,$\theta$, and U3) respectively. at the selected line **Fig. 21**. For the first three mode shape study at Free State (without flow), we notice the displacement at U3 direction is clearly, but contrarily at x, and $\theta$ direction. Also we notice the failure state firstly can be start under longitudinal direction.

**Figs. 22, 23, and 24**, Show the displacement in longitudinal, tangential, and normal to surface (i.e. x,$\theta$, and U3) respectively. At the same selected line **Fig. 21**. For the first three mode shape study at force state (with flow), we notice the displacement at tangential direction $\theta$ is clearly, but contrarily at x, and normal direction. Also we notice the failure state firstly can be start under torsional load.

**Figs. 25, 26, and 27**, Show the displacement in direction of normal to surface (U3), for mode 1, 2, and 3 under different velocity of flow at the selected line fig. (21). we notice the displacement at U3 direction is clearly and increase when the velocity increasing. Also we notice the failure state firstly can be start under mode three when the velocity is over 10 m/s.

**Figs. 29, 30, and 31**, views the relation between transmissibility factors with frequency ratio, at the same selected line fig (21), which shows three phases appears respectively. The transmissibility factor which calculated for modes 1, 2, and 3, and $\omega_{in}$ $where\ i = n = 1,2,3.$ we notice at same selected line a three phases and motion are appears oscillatory motion, non-oscillatory motion, and oscillatory motion , which on the longitudinal, rotational, and transverse directions.

## 5- Conclusions

The general energy method (Hamilton principle) was employed for analyzing the vibration characteristics of thin circular pipe with and without including the effect of variable loading because high velocity flows. Comparisons were made with finite element results. The following words tried to explore the general conclusions extended from the entire present work

1. The new equations (eqs. 42, 46-48, 55-57) of natural frequency, displacement with and without loading are obtain, and can be applied on any thin circular pipe under turbulent flow for any liquid, which gives good agreement result.
2. The vibration in perpendicular direction U3 of the pipe shell gives clear and regular shape of mode shape put other longitudinal and circulation direction are reverse.
3. According to the result showing in the previous section. Detected the maximum displacement appears in longitudinal vibration state, and because the model have limited movement in longitudinal direction, a maximum residual stress in solid structure are generate.
4. The failure criterion in rotational vibration is increase when the velocity of flow is increasing.
5. When the velocity of fluid is increased the displacement of all mode shape are increasing, also the failure criterion is increasing and appear in third mode shape when the velocity is over 10 m/s.

6. The effect of frequency ratio appears on longitudinal and transverse vibrations and the critical value at $\left(\frac{\omega}{\omega_{ni}} = 2\right)$ , put at rotation vibration the critical value is $\left(\frac{\omega}{\omega_{ni}} = 2.5\right)$ , it follows the failures may occurred in longitudinal or transvers displacement large than in rotational displacement.

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Table (1): Re. and Mach number

| $v_{water}$ (m/s) | *diameter of pipe m* | *Re. number* | *Mach number M* | $(\epsilon/D)$ | *Friction factor f* |
|---|---|---|---|---|---|
| 5 | 0.3 | 1497494.7 | 0.003401361 | 0.00015 | 0.014 |
| 5 | 0.4 | 1996659.6 | 0.003401361 | 0.0001125 | 0.0135 |
| 5 | 0.5 | 2495824.5 | 0.003401361 | 0.00009 | 0.0133 |
| 10 | 0.3 | 2994989.4 | 0.006802721 | 0.00015 | 0.0136 |
| 10 | 0.4 | 3993319.2 | 0.006802721 | 0.0001125 | 0.0134 |
| 10 | 0.5 | 4991649 | 0.006802721 | 0.00009 | 0.0122 |
| 20 | 0.3 | 5989978.8 | 0.013605442 | 0.00015 | 0.013 |
| 20 | 0.4 | 7986638.4 | 0.013605442 | 0.0001125 | 0.0121 |
| 20 | 0.5 | 9983298 | 0.013605442 | 0.00009 | 0.0119 |
| 100 | 0.3 | 29949894 | 0.068027211 | 0.00015 | 0.0128 |
| 100 | 0.4 | 39933192 | 0.068027211 | 0.0001125 | 0.0121 |
| 100 | 0.5 | 49916490 | 0.068027211 | 0.00009 | 0.0118 |

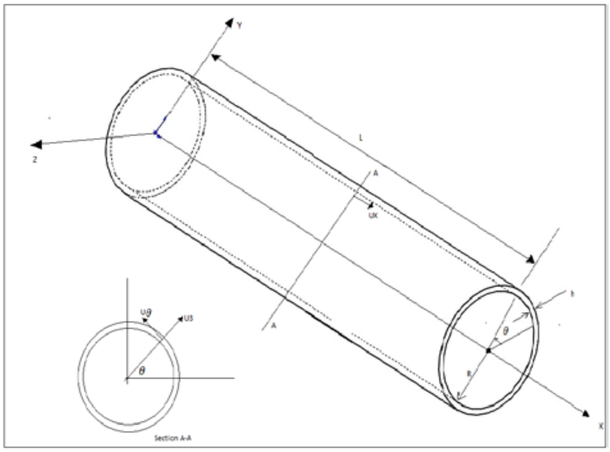

Fig. 1: Thin Pipe Model (U$\theta$=U2; UX=U1; U3=U3)

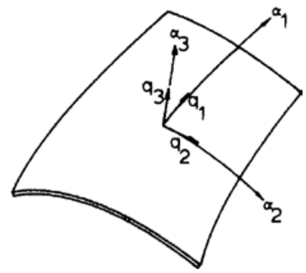

Fig. 3: distributed load

Table 2: Properties of Pipe Model

| Length *mm* | Diameter *mm* | Thickness *mm* | roughness *ϵ mm* |
|---|---|---|---|
| 10000 | $400^{+100}_{-100}$ | 10 | 0.045 |
| | | | |
| Weight density *(kg/m³)* | Poisson's ratio | Modulus of elasticity *E Pa* | Modulus of rigidity *G Pa* |
| 7750.372 | 0.28 | 189.6 | 74.1 |

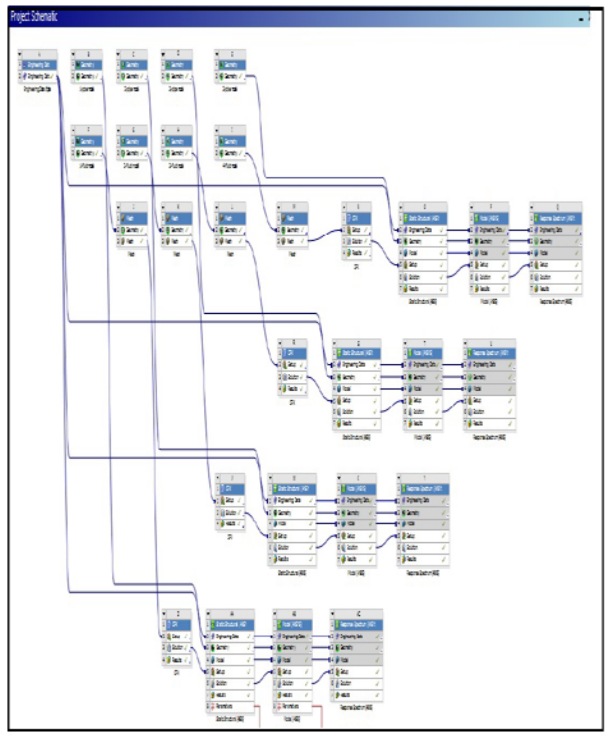

Fig. 4: Flow Chart of Numerical Solutions

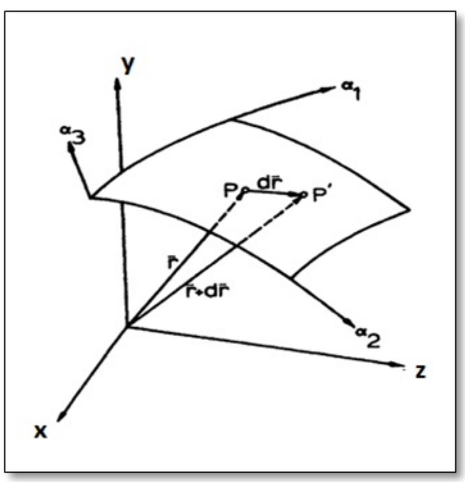

Fig. 2: General neutral shell surface

(Reference surface)

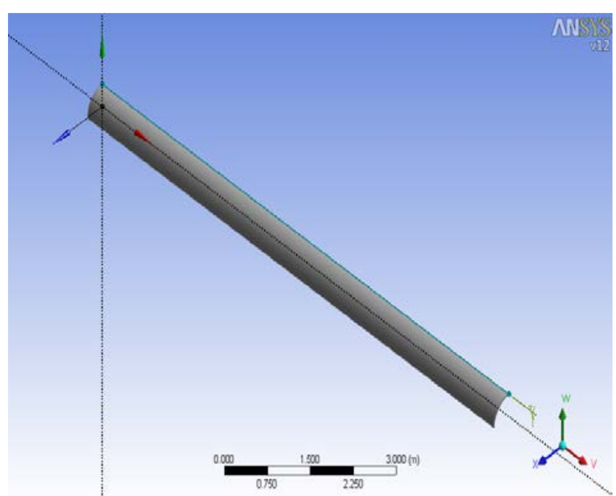

Fig. 5: Shell Model

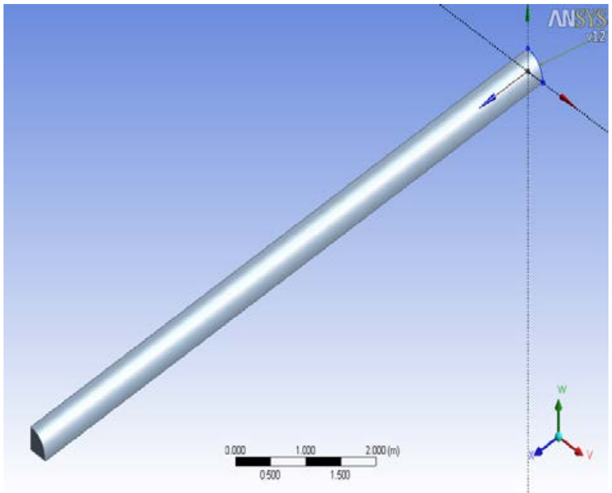


**Fig. 6: Fluid Model**

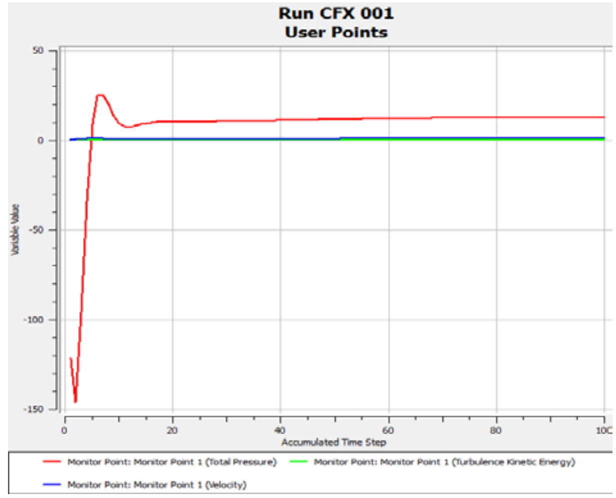


**Fig. 9: Number of Iteration**

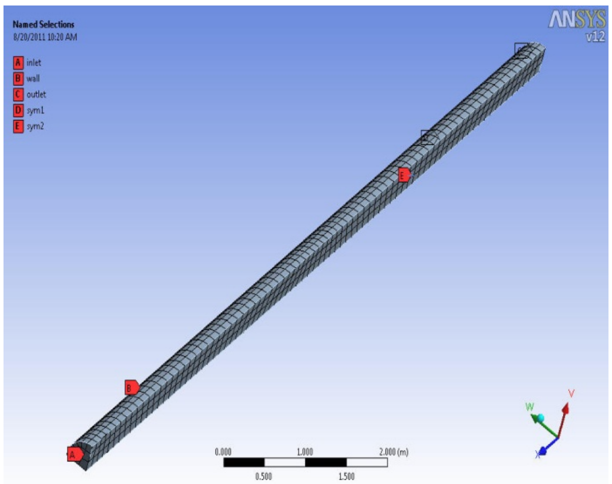


**Fig. 7: Construction of Fluid Model**

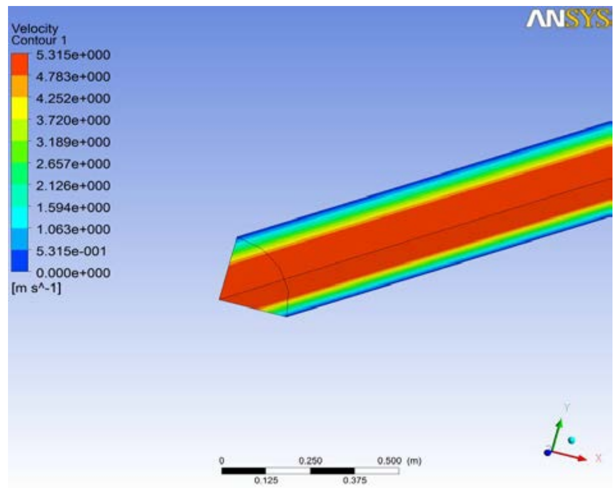


**Fig. 10: CFX Result-Velocity Profiles**

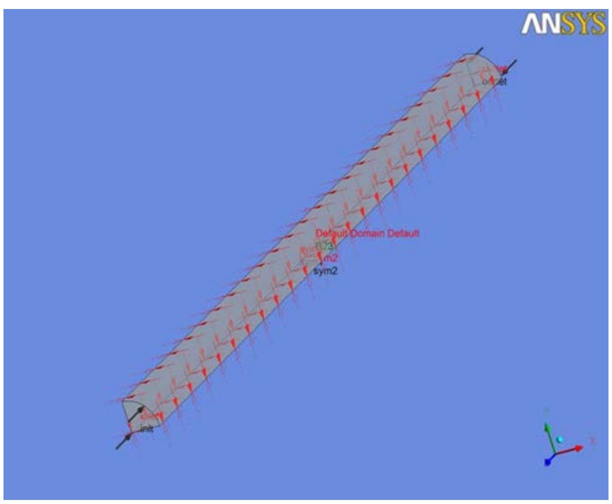


**Fig. 8: B.C of CFX Solution**

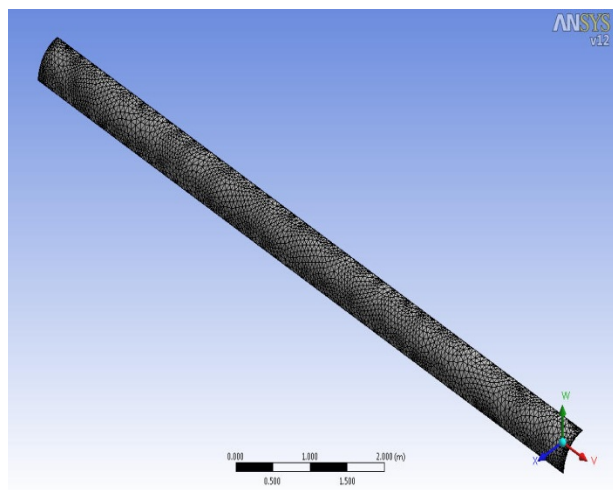


**Fig. 11: Mesh of Pipe**

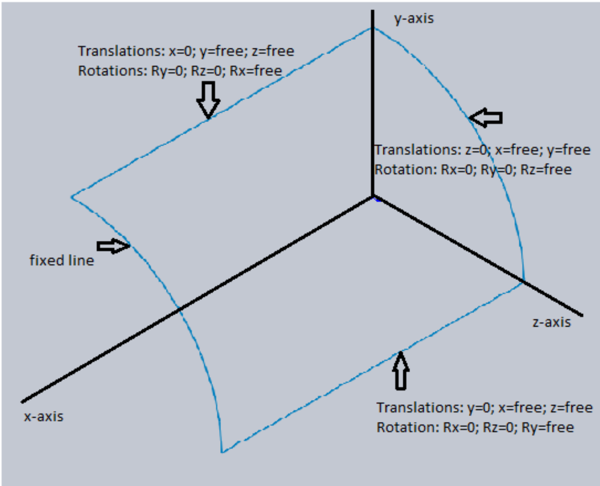


Fig. 12: B.C. Numerical Model

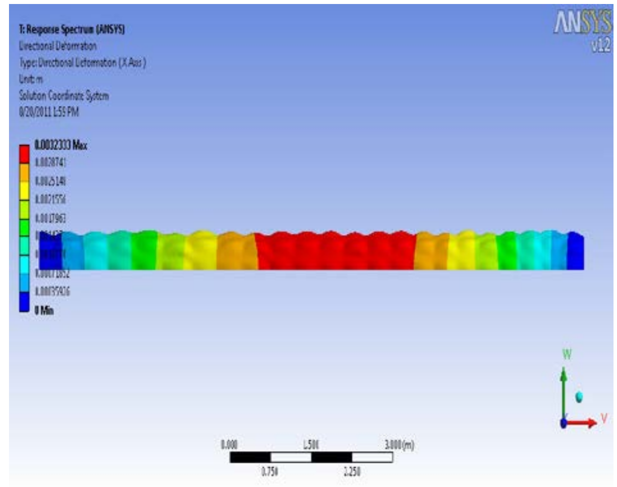


Fig. 15: Response

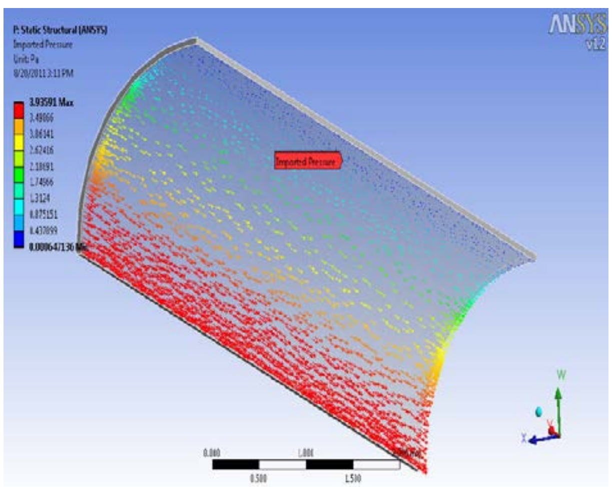


Fig. 13: Importing Presser Load

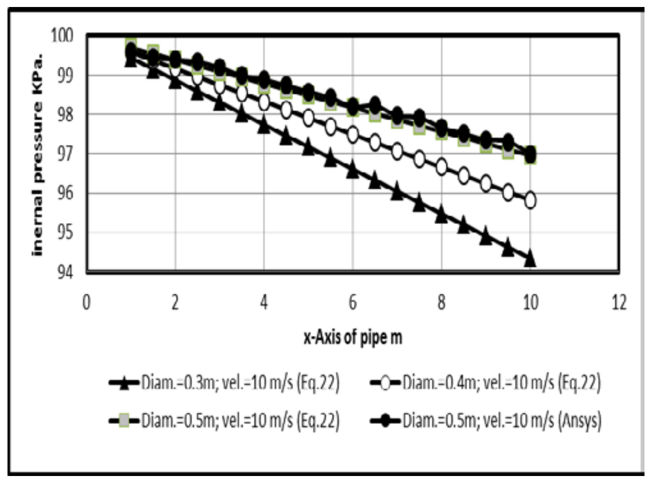


Fig. 16: Verification of Distributed the Internal Pressure

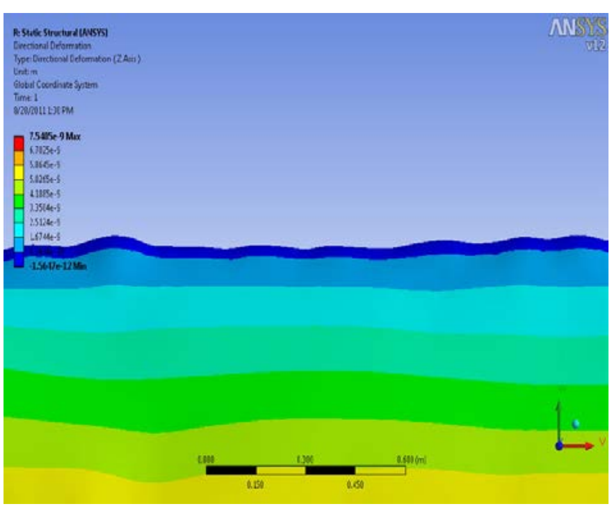


Fig. 14: Response

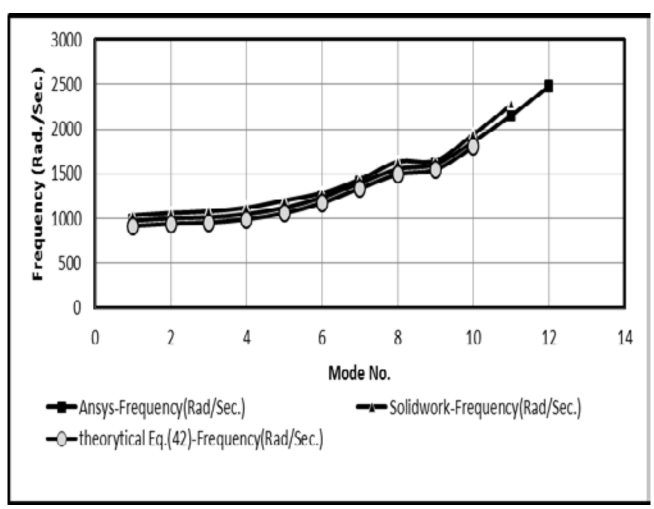


Fig. 17: Verification of Natural Frequency

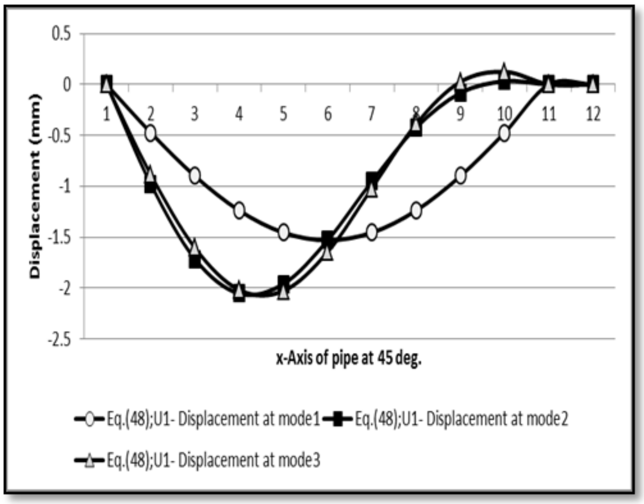


Fig. 18: Longitudinal Displacements (free)

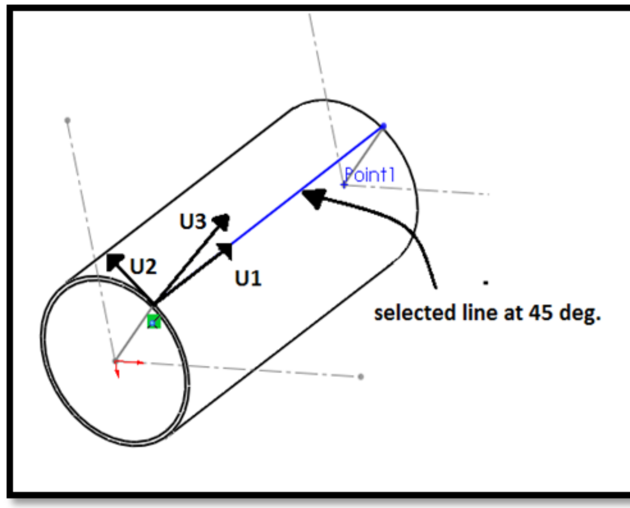


Fig. 21: Selected Line for Study

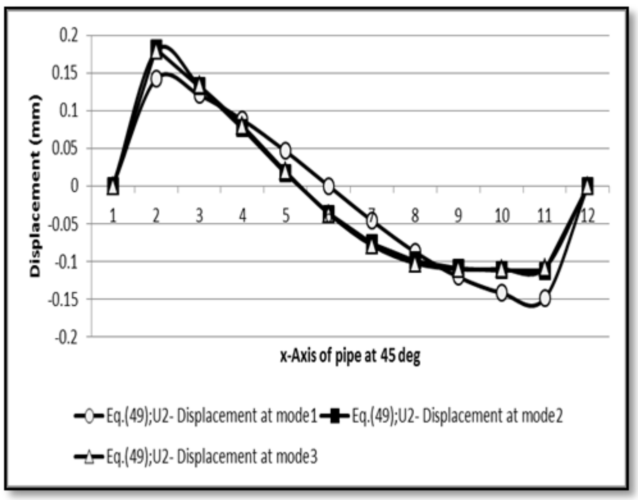


Fig. 19: Angular Displacements (free)

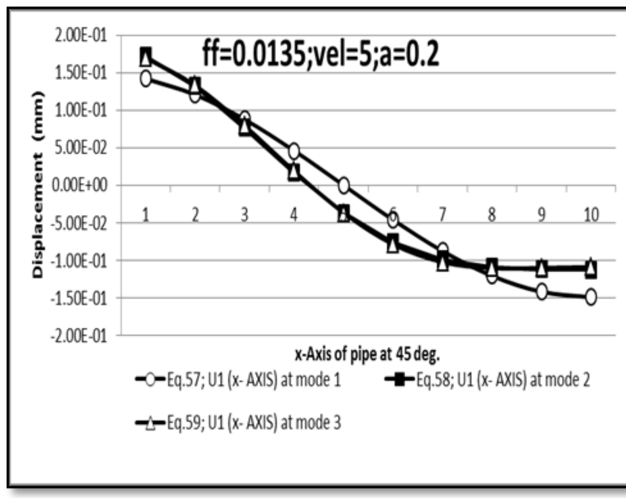


Fig. 22: Longitudinal Displacements (force)

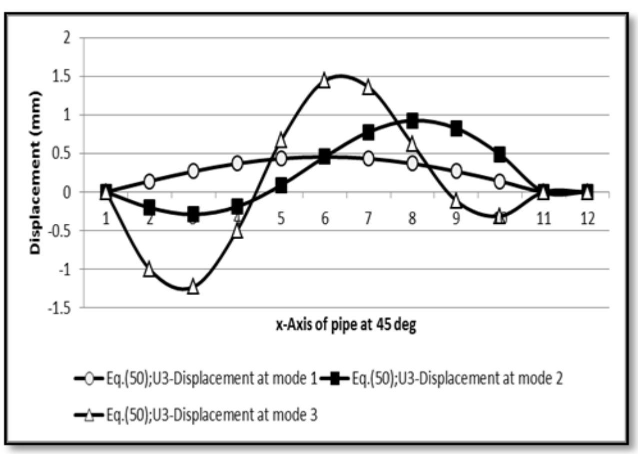


Fig. 20: Transversal Displacements (free)

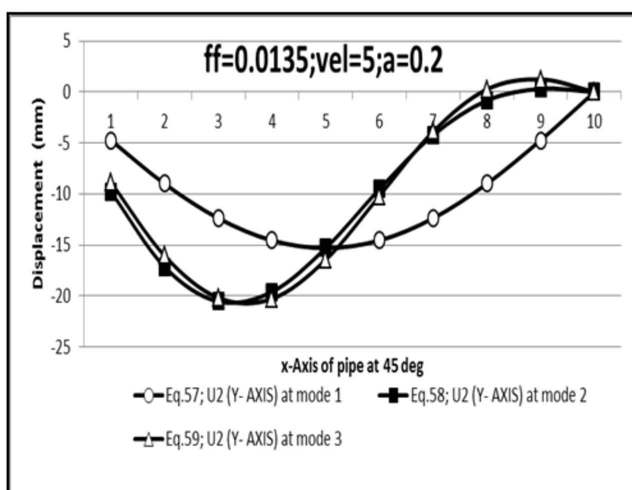


Fig. 23: Angular Displacements (force)

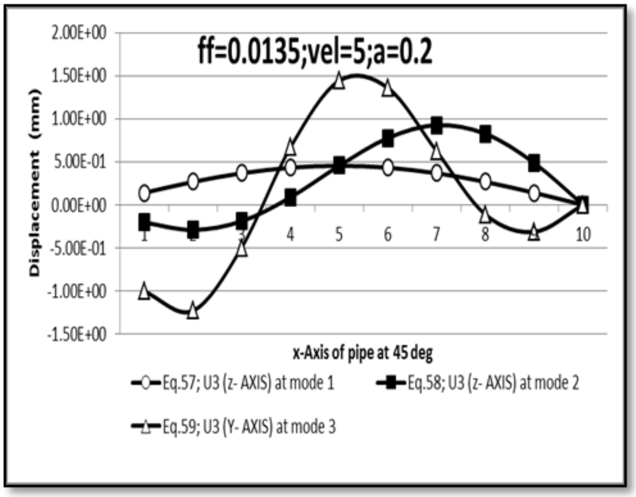


**Fig. 24: Transversal Displacements (force)**

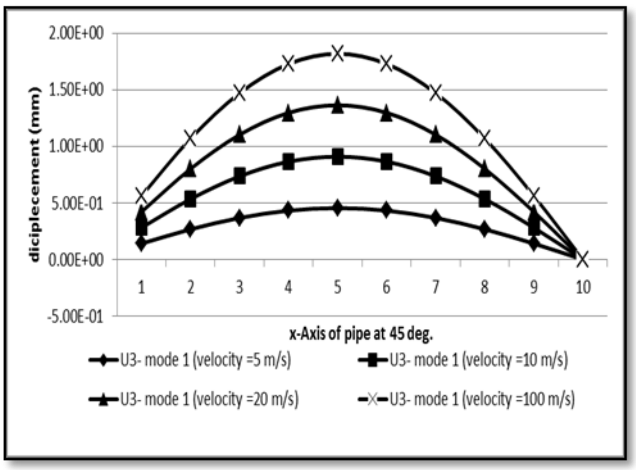


**Fig. 25: Transversal Displacements of Variable Load (Variable Velocity)-mode1**

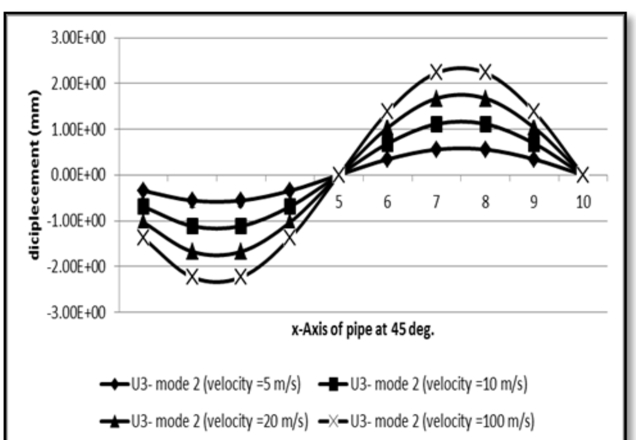


**Fig. 26: Transversal Displacements of Variable Load (Variable Velocity)-mode2**

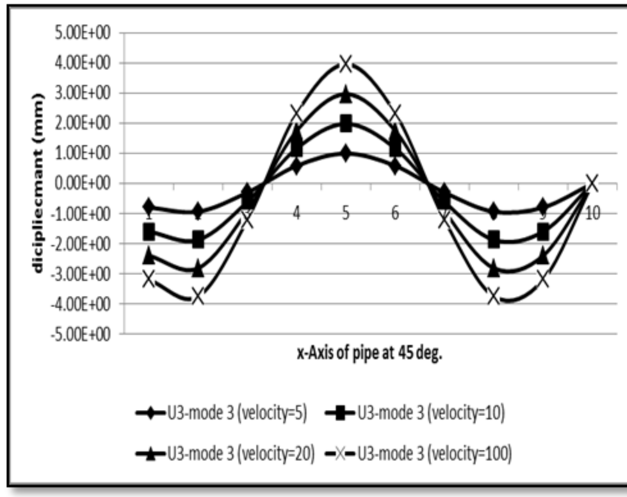


**Fig. 27: Transversal Displacements of Variable Load (Variable Velocity)-mode3**

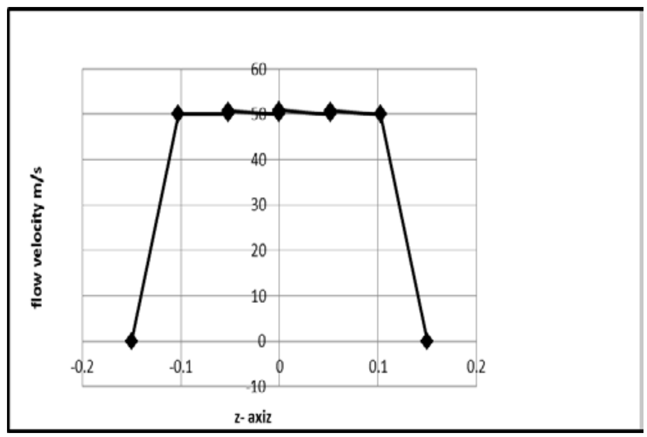


**Fig. 28: Velocity Profile in Internal Pipe-CFX Ansys**

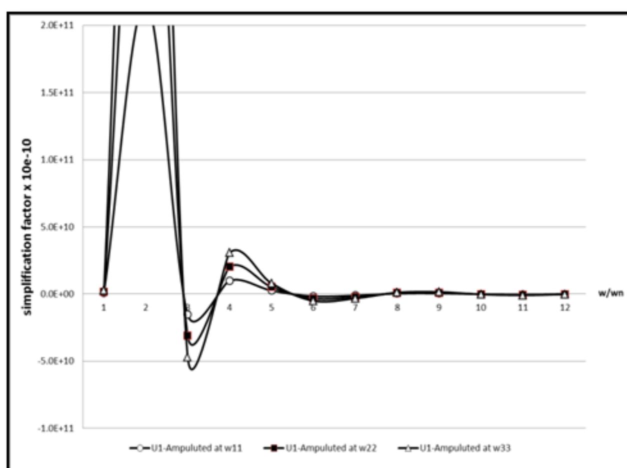


**Fig. 29: Longitudinal Displacement Transmissibility with Frequency Ratio**

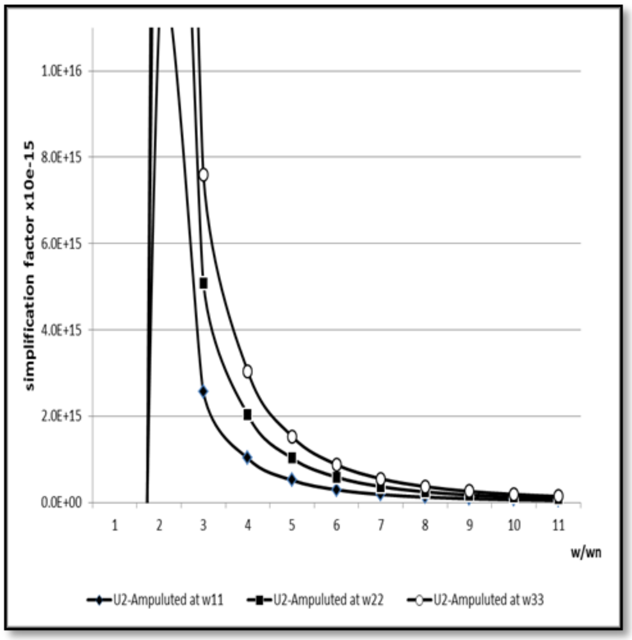


**Fig. 30: Rotational Displacement Transmissibility with Frequency Ratio**

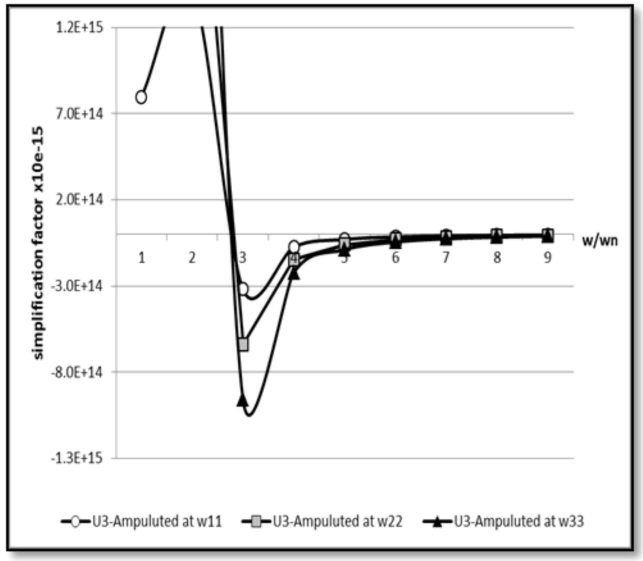


**Fig. 31: Transverse Displacement Transmissibility with Frequency Ratio**