



Three-Dimensional Finite Element Simulation of the Buried Pipe Problem in Geogrid Reinforced Soil

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ABSTRACT

Buried pipeline systems are commonly used to transport water, sewage, natural oil/gas and other materials. The beneficial of using geogrid reinforcement is to increase the bearing capacity of the soil and decrease the load transfer to the underground structures.

This paper deals with simulation of the buried pipe problem numerically by finite elements method using the newest version of PLAXIS-3D software. **Rajkumar and Ilamaruthi's study, 2008** has been selected to be reanalyzed as 3D problem because it is containing all the properties needed by the program such as the modulus of elasticity, Poisson's ratio, angle of internal friction. It was found that the results of vertical crown deflection for the model without geogrid obtained from PLAXIS-3D are higher than those obtained by two-dimensional plane strain by about 21.4% while this percent becomes 12.1 for the model with geogrid, but in general, both have the same trend. The two dimensional finite elements analysis predictions of pipe-soil system behavior indicate an almost linear displacement of pipe deflection with applied pressure while 3-D analysis exhibited non-linear behavior especially at higher loads.

Keywords: buried flexible pipe, finite elements, static loads, soil reinforcement.

تمثيل ثلاثي الابعاد لمسألة الانابيب المدفونة في تربة مسلحة بشبكة بطريقة العناصر المحددة

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الخلاصة

تستخدم شبكات الأنابيب المدفونة عادة لنقل المياه والصرف الصحي والنفط الطبيعي / الغاز وغيرها من المواد. من فوائد استخدام شبكة تسليح في التربة زيادة قدرة تحملها وتقليل نقل الحمل الواصل إلى المنشآت المشيدة تحت الأرض. يتناول هذا البحث محاكاة مسألة الأنابيب المدفونة عددياً باستخدام طريقة العناصر المحددة باستخدام أحدث نسخة من برنامج (2013) PLAXIS-3D وتم اخذ دراسة سابقة ل (راجكومار والمبورثي، 2008) أعيد تحليل المسألة كمسألة ثلاثية الابعاد وتم اختيار هذه الدراسة لأنها تحتوي على جميع الخصائص التي يحتاجها البرنامج مثل معامل المرونة، ونسبة بواسون، زاوية الاحتكاك الداخلي.



وقد وجد أن نتائج الانفعال الرأسي لقمة الانبوب لنموذج من دون شبكة جيولوجية تم الحصول عليها من PLAXIS-3D هي أعلى من تلك التي حصل عليها رجوكمار عندما حلل المسألة كثنائية الأبعاد بنحو 21.4، في حين تصبح هذه النسبة 12.1 للنموذج مع شبكة تسليح، ولكن بصورة عامه لديهما نفس الاتجاه. بصورة عامة النتائج في تحليل المسألة كثنائية الأبعاد يظهر تصرف خطي بينما في المسألة الثلاثية يظهر تصرف لاخطي خصوصا بالأحمال الكبيرة.
الكلمات الرئيسية : الانابيب المرنة المدفونه، العناصر المحددة، الاحمال الساكنة، تسليح التربة

1. INTRODUCTION

A number of engineering problems that not always could have been resolved through analytical calculations can now be solved with numerical analyses using the finite element method (FEM). The finite element method, known for nearly 50 years, has been effectively implemented in various computer software packages aiding the designing and analysis of engineering structures, such as ABAQUS, ANSYS, PLAXIS, **Kliszczewicz, 2013**.

A numerical model of the soil-pipe system is necessary to analyze or predict the detailed behavior of buried pipelines. Analytical theories of soil-structure interaction such as Burns-Richard method provide mathematical model that are used to design or analyze buried pipelines. These theories do not account for the actual interaction between the pipe and the surrounding soil during construction, service, and ultimate conditions, due to the nonlinearities, no homogeneity and other complexities. Numerical modeling is the best approach to adequately model the pipe-soil system. The finite element method is the most common numerical technique that can be used to analyze and design buried pipelines, **Bashir, 2000**.

A contemporary engineer is thus endowed with tools enabling to perform multi-variant analyses of a structure, its optimization or expert evaluation of failure modes. The use of extended, geotechnical oriented computer packages (e.g. Plaxis) for solving the aspects of buried structures, especially for analyzing buried piping systems, seems to be particularly attractive. Such structures are interworking with the surrounding soil in a specific manner, **Gerscovich et al., 2008, Goltabar and Shekarachi, 2010**. When such interaction is taken into account in classical calculations, soil is usually replaced with a largely simplified analogue, and the considered calculation scheme is far from the reality. Unlike in analytical calculations where a pipe ring (2-dimensional) is usually considered, numerical analyses allow to take into consideration the spatial character of a pipeline with the surrounding soil (3-dimensional) ,**Janson, 1996, Madryas et al., 2002, Kuliczowski, 2004**.

The major difference between the analysis of a continuum and a framed structure is that even though a continuum is only connected to adjacent elements at its nodal points, it is necessary to maintain displacement compatibility between adjacent elements. Special shape functions are used to relate displacements along the element boundaries to the nodal displacements and to specify the displacement compatibility between adjacent elements. Once the continuum has been idealized, an exact structural analysis of the system is performed using the stiffness method of analysis, **Zienkiewicz, 1977**.

The output of a finite element analysis includes the stresses and strains at any point in the system and, more importantly, the displacement, moment, shear, and thrust at any point in the buried pipe.

The power of using the finite element method is that once the model is set up, many cases can be analyzed and the sensitivity of assumptions can be tested. Furthermore, a finite element analysis can



be performed for many buried pipe applications that are difficult to analyze using conventional analysis procedures. Multiple pipes in a trench, the effect of excavating a trench adjacent to an existing pipe, and other applications can conveniently be accommodated by a finite element analysis. There are limitations in using a finite element analysis. The problem is usually analyzed as a two-dimensional problem, even though the system is clearly three-dimensional. For long culverts, treating the problem as a plane-strain two-dimensional problem is generally not a serious limitation, but in some cases the three-dimensional effects cannot be neglected. There are three-dimensional finite element analysis programs available, but generally they do not have the proper constitutive relationship for modeling soil and do not provide interface elements that allow slip between the soil and the pipe. In these cases, it may be necessary to compare two- and three-dimensional solutions for conditions that can be modeled and then to extrapolate to the real case.

Moghddas Tafreshi and Khalaj, 2008 assessed the behavior of small- diameter high-density polyethylene (HDPE) pipes (110 mm diameter and 4.03 mm wall thickness) buried in reinforced sand that was then subjected to repeated loading (of amplitude 550 kPa). They examined the influence of between 1 and 5 layers of reinforcement in soil having relative densities of 42%, 57%, and 72%. The pipes were embedded at depths 1.5- 3 times their diameter. Testing was performed in a trench of 550 mm width. It was reported, that the proportion of vertical pipe diameter change and soil surface settlement can be reduced by up to 40% and 51%, respectively, when using the most reinforcement in backfill of the highest density when the pipe is at its deepest embedment.

Bildik et al., 2012 studied the behavior of buried pipe numerically by finite elements method. Their results showed, the pipe behavior influenced by the intensity of the surcharge load. The pipe displacements increase linearly with increase in surcharge load. Also their results showed that the pipe displacement decrease with increase of embedment ratio. This behavior can be explained using stress- displacement behavior. The vertical stresses decrease with increase on embedment ratio. The variation of vertical stress with embedment ratio from the PLAXIS analyses showed generally similar behavior with Boussinesq theory. The pipe behavior is strongly influenced by the relative density of sand. The displacement of the pipe decrease with the increase in relative density of sand. The results show that pipe displacement decrease with increase on rigidity of pipe and the concrete pipe displacements are less than PE pipes.

Kluszczewicz, 2013 presented 3D numerical analysis of interaction of a pipeline structure with stratified subsoil loaded across a certain area. The analysis enabled to evaluate the effort state of the pipe and the changes taking place in the soil mass. The impact of the load was particularly evident in the sub-surface soil layers immediately within the load working area. A distribution zone of the stresses excited by a load working within the entire soil mass, especially in the direct surrounding of the pipelines, can also be identified. Considering the stratification of the subsoil with a layer of low-bearing ground with varied thickness and the fact of varied material parameters in the zones of virgin soil, bedding and backfill in the excavation, one can observe clear disturbances in the distribution of stresses in the direct surrounding of the pipe (excavation) and in the further zones of the soil. As the load is situated specifically as shifted in relation to the pipe axis, the deformation and effort state of the pipe side surface is non-uniform. This signifies irregular distribution of generalized internal forces in such structure. Such results of the activity of surface loads onto the pipe structure situated in stratified subsoil are identifiable only by building numerical pipe- soil system models and by analyzing their behavior when simulating the activity of loads. The reliability



of the outcomes obtained is linked to the correct construction of the model including correct model dimensions, discretization density, selection of appropriate material parameters and an adequate constitutive model of soil and of the modeled structure. Numerical analyses can be regarded as an attractive tool for examining limit states of the bearing capacity and serviceability of buried piping.

The objective of the present study is to investigate the dimensional effects of numerical simulation of the buried pipe problem. The problem is conventionally analyzed as two-dimensional plane strain. Geogrid reinforcement layer is placed above the pipe to reduce load transfer.

2. PLAXIS-3D NONLINEAR SOLUTION STRATEGY

PLAXIS-3D employs a solution strategy known as the direct iterative method, or more simply called trial and error. This method has proven to be robust and readily accommodates the wide variety of nonlinear models such as tensile cracking and elastic-plastic behavior of pipe models, hyperbolic constitutive laws for soil models, frictional sliding and separation for interface models, and geometric nonlinearity for large deformation analysis, **Plaxis Manual, 2013**.

When two consecutive iterations produce the same stiffness matrices for all elements within small error limits, then the solution has converged and we proceed to the next load step. Once a converged solution increment has been found, all the mechanical responses are updated based on the last iteration solution, **Plaxis Manual, 2013**.

3. MATERIAL PROPERTIES

Rajkumar and Ilamparuthi (2008) described numerically and experimentally the interaction between the soil and flexible PVC pipes buried in sand bed and subjected to surface pressures. The tests were conducted with and without Netlon Geogrid reinforcement. They studied the behavior of the soil-pipe interaction by using of the 2D Finite element analysis software PLAXIS. Moreover, in the presence and absence of geogrid reinforcement, they measured the variation of the vertical crown deflection due to the applied surface pressure, with a noticeable difference between the numerical and experimental results for both cases.

Table 1 shows the values of material properties that were used by, **Rajkumar and Ilamparuthi, 2008**. PLAXIS-2D software was used by, **Rajkumar and Ilamparuthi, 2008** to model the behavior of PVC pipes buried in dense sand under surface loads. The results were compared with those measured experimentally and expected numerically by, **Rajkumar and Ilamparuthi, 2008**.

4. BOUNDARY CONDITIONS AND MODELING

The finite element analysis was performed to model the response of the buried flexible plastic pipes in different backfills, embedded at different levels and compare the behavior with the experimentally obtained results.

The dimensions of the soil model adopted by, **Rajkumar and Ilamparuthi, 2008** are (1200 mm × 600 mm). The pipe model system is considered a plane strain condition by, **Rajkumar and Ilamparuthi, 2008** with 15-node elements. Mohr-Columbe plasticity model was specified to solid element which symbolizes soil around the pipe. The pipe used in the analysis has a diameter of 200 mm and wall thickness of 0.5 mm. To exactly simulate the experimental model the right hand boundary was selected at 1.5D away from the trench center with a restricted horizontal displacement

and free vertical displacement. The bottom boundary was located at 1.2 m below the surface with a restricted vertical displacement and free horizontal displacement. Six circular segment elements were used to represent the pipe. Fifteen noded plane strain triangular elements were used by, **Rajkumar and Ilamparuthi, 2008** to model the backfill. The numerically simulated model is as shown in **Fig. 1**.

The study consists of two stages, the first one deals with the vertical crown deflection on the pipe under 50, 100, 150 kPa surface loads with 400 mm backfill cover without geogrid reinforcement as shown in **Fig. 2**. To reduce the effect of surface load on the pipe and increase the performance of it, the geogrid are used in the second stage as shown in **Fig. 2**. **Table 1** summarizes the material properties of sand, pipe and geogrid. **Fig. 3** shows the problem in Plaxis-3D.

5. Results and Discussion

In the presentation of results, standard terms have been used throughout presentation of the results. These terms are defined below:

The definition of the Vertical Diametric Strain (VDS) of a pipe is shown in **Fig. 4** and relates to the change of the internal vertical diameter of a pipe compared to its external diameter. The Horizontal Diametric Strain (HOS) conversely relates to the change of the internal horizontal diameter of the pipe compared to its external diameter. A positive VDS or HOS denotes a decrease in pipe diameter and both are expressed as a percentage.

In order to inquest the Plaxis-3D software, the model has been prepared and run by using the same data and boundary conditions under the same load stages. Then the results of crown strain and diametrical strain with applied surface load for both loose and dense sand are compared with those obtained by, **Rajkumar and Ilamparuthi, 2008** as shown in **Figs 5 to 8**.

It can be seen from **Fig. 5** that the results of vertical crown deflection for the model without geogrid obtained from PLAXIS-3D are higher than those obtained by two-dimensional plane strain, **Rajkumar and Ilamparuthi, 2008** by about 21.4% while this percent becomes 12.1 for the model with geogrid, but in general, both have the same trend. The difference increases as the applied surface load increases.

The match with the experimental data was reasonably good owing to inadequacies of the 2D predictions involving the assumption of a rigid side boundary. The walls of the laboratory test box may not have been perfectly rigid and this factor could have influenced the finite element analysis predictions. The correspondence of the finite elements analysis output with the horizontal pipe strain was less satisfactory. It is also apparent that the two dimensional finite elements analysis predictions of pipe-soil system behavior indicate an almost linear displacement of pipe deflection with applied pressure while 3-D analysis exhibited non-linear behavior especially at higher loads.

In **Figs 5 and 6**, the applied pressure and the corresponding crown deflection of the pipe are compared for 400 mm of backfill cover with and without geogrid reinforcement in dense and loose conditions of sand, respectively.

Geogrid functions in two ways: reinforcement and separation which are the techniques of improving poor soil with geo-grid, to increase the stiffness and load carrying capacity of the soil through frictional interaction between the soil and geo-grid material.

A geogrid reinforced soil is stronger and stiffer and gives more strength than the equivalent soil without geo-grid reinforcement. Geo-grids provide improved aggregate interlock in stabilizing road infrastructure through soil restraint reinforcement applications. Geogrid reinforcement provided between the soil layers carries the shear stress induced by vehicular loads.

Geogrid mesh provides better interlocking with the soil particles thus ensuring adequate anchorage during loading. The improvement in the load carrying capacity could be attributed to improved load dispersion through reinforced soil. This in turn, results in lesser intensity of stresses getting transfer to underlying soil, thus leading to lesser distress in the pipe.

Figs 7 and 8 show the results of diametric and crown strain of the pipe both vertically and horizontally plotted against the surface pressure under backfill cover of 400 mm versus the applied surface load which was obtained from ,**Plaxis 3D, 2013** program by entering the data of **Table 1**. The results of, **Rajkumar and Ilamparuthi, 2008** are drawn on the same Fig. to facilitate the comparison process. The upper half of the diagram consists of negative diametric strains or compression of the pipe at the crown. The lower half consists of positive or extensions of the pipe at the level of the spring line. The Fig. shows that 3-D analysis reveals higher strains than 2-D.

The pipe response to the applied pressure is almost linear both in 2-D and 3-D analysis which results in an elliptical deformed shape of the pipe. The pipe deformations were quite localized. The greatest reduction in diameter vertically occurred under the centre of the loading plate. The pipe crown deflected most directly beneath the centre of the loading plate. The invert of the pipe suffered little movement but tended to rise slightly well away from the loaded area. It can be seen in **Figs 7 and 8** which represent the pipe strain with applied pressure that the vertical diametric strain is usually significantly higher than the horizontal strain. This is true in 2-D and 3-D analyses.

The deformation response of the pipe soil system to the external loading was nonlinear. The pipes were usually observed to regain their shapes after they were recovered from the buried pipe installation.

It is noticed from **Fig. 8** that there is clear convergence in results. The small difference between the two results is because the problem is solved by ,**Rajkumar and Ilamparuthi, 2008** as plane strain while the current analysis considers the problem as three dimensional problems. It is acceptable difference.

These results are expected and compatible with those obtained by ,**Hosseini and Moghddas Tafreshi, 2002** who studied the laboratory tests of small diameter pipes buried in reinforced sand and found that the deflection behavior and failure mechanism of the system highly depend on soil density. The results are also compatible with the findings of, **Arockiasamy et al., 2006** who studied the soil pipe interaction, and interpreted that whatever the soil is well compacted it absorbs the bulk of the load transferred to the pipe and thus reduce the strain of the pipe's wall. However, in relatively loose soils, due to weak contacts and poor interlocking of the grains and special arrangement of the soil fabric, regardless of the embedment depth, even under low loads, the failure of the system

usually occurred in low applied load due to local buckling or large deflection of the pipe together with excessive settlement of the loading plate.

Figs 9 to 17 present visualization of the output map results generated in PLAXIS 3D software.

A uniformly distributed load of the surcharge causes soil mass deformation shown with a deformed net of the pipe soil system model **Fig. 9** presents the deformed mesh. **Fig. 10** presents the total volumetric strain ϵ_v , while **Fig. 11** shows the total displacement $|u|$ (which represents absolute total displacement). **Fig. 12** presents the total horizontal displacement u_x , the resultant displacements are displayed in characteristic sections of the model, i.e. in planes perpendicular and parallel to the pipe axis. It is pointed out by analyzing the maps that the impact of the surcharge load is most important directly in the place of its application, covering a significant area of the soil mass and reaching the pipeline placement zone.

Fig. 14 presents total mean stress p , **Fig. 15** shows the deviator stress q which indicates that the most stressed zone extends to a depth of about (1.0-1.5) the footing width (loaded area).

Fig. 16 presents Cartesian total stress σ_{zz} , it can be shown that the influence zone by the surface load extent to reach the pipe while **Fig. 17** shows the plastic points around the pipe. The impact of the load is particularly evident in the sub-surface soil layers immediately within the load working area. A distribution zone of the stresses excited by a load working within the entire soil mass, especially in the direct surrounding of the pipelines, can also be identified.

The maximum vertical displacement of nodes on the soil mass model reaches 0.02 m. The maximum vertical displacement of the pipe model nodes are 0.003 m and occur in the central part of the surface model.

6. CONCLUSIONS

1. The results of vertical crown deflection for the model without geogrid obtained from PLAXIS-3D are higher than those obtained by two-dimensional plane strain by about 21.4% while this percent becomes 12.1 for the model with geogrid, but in general, both have the same trend.
2. The two dimensional finite elements analysis predictions of pipe-soil system behavior indicate an almost linear displacement of pipe deflection with applied pressure while 3-D analysis exhibited non-linear behavior especially at higher loads.
3. The pipe response to the applied pressure (strain) is almost linear both in 2-D and 3-D analysis which results in an elliptical deformed shape of the pipe. The pipe deformations were quite localized. The greatest reduction in diameter vertically occurred under the centre of the loading plate.
4. The impact of the surcharge load is most important directly in the place of its application, covering a significant area of the soil mass and reaching the pipeline placement zone.

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NOMENCLATURE

D_0 = original external diameter (mm).

V.D.S= vertical diametric strain.

δ = change in internal diameter (mm).

σ_{zz} = vertical stress.

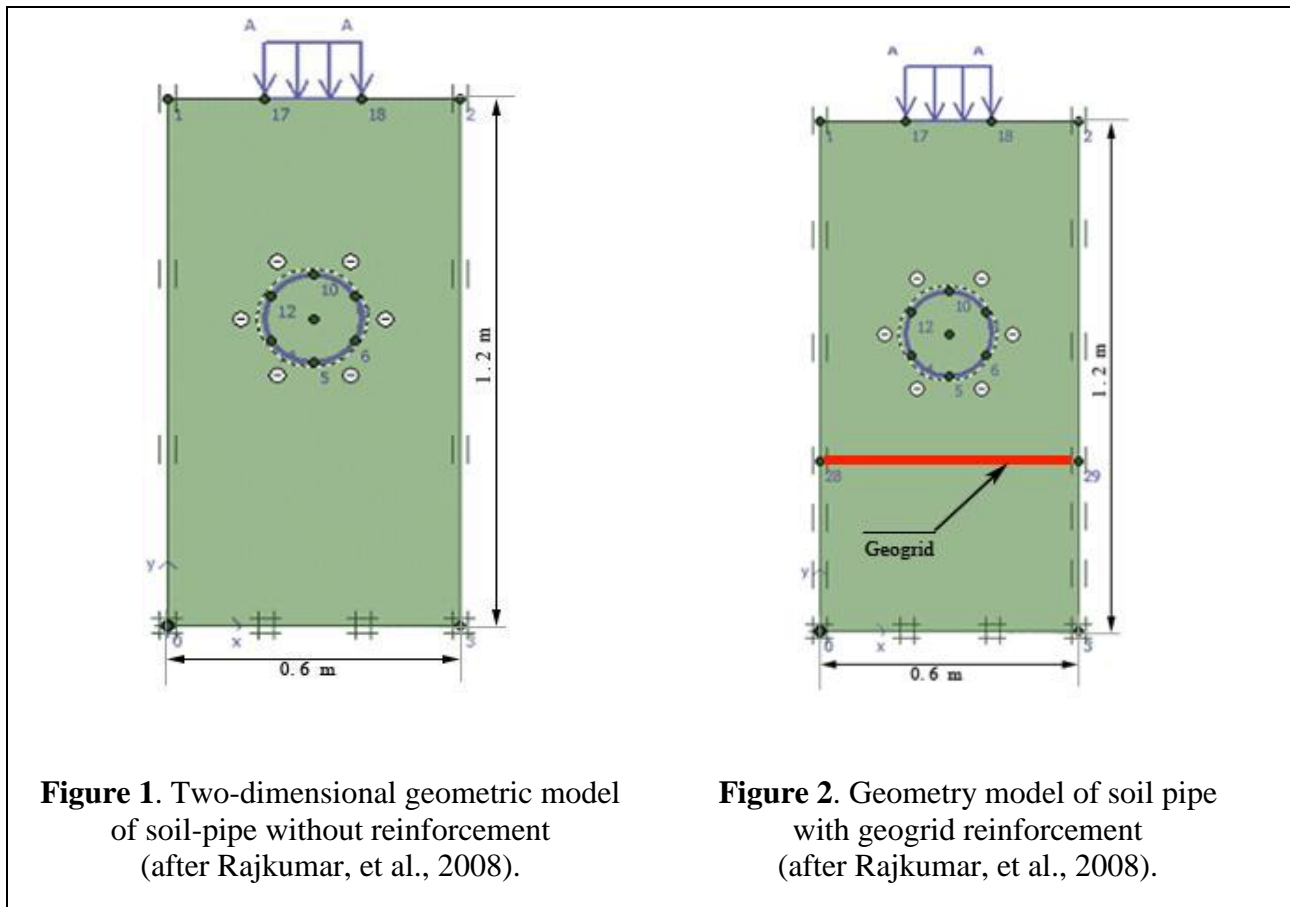
ϵ_v = volumetric strain.

$|u|$ = absolute total displacement.

u_x = the total horizontal displacement.

Table 1. Material properties of the soil-pipe system **Rajkumar and Ilamparuthi, 2008.**

Properties	Loose sand	Dense sand	PVC pipe	Netlon geogrid
Dry unit weight (kN/m ³)	15	17	–	–
Modulus of elasticity E (kN/m ²)	9000	19000	0.933*10 ⁶	–
Poisson ratio ν	0.3	0.3	0.31	–
Friction angle ϕ	32	42	–	–
Dilation angle ψ	2	12	–	–
Axial stiffness (kN/m)	–	–	–	60



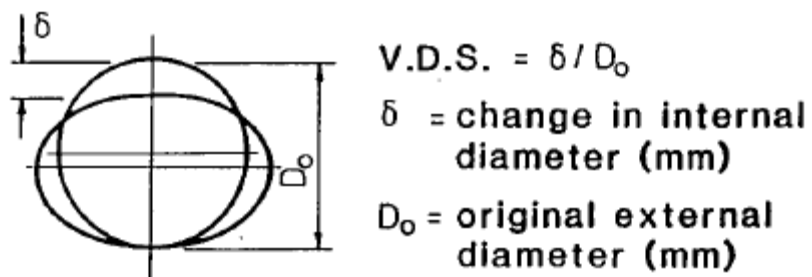
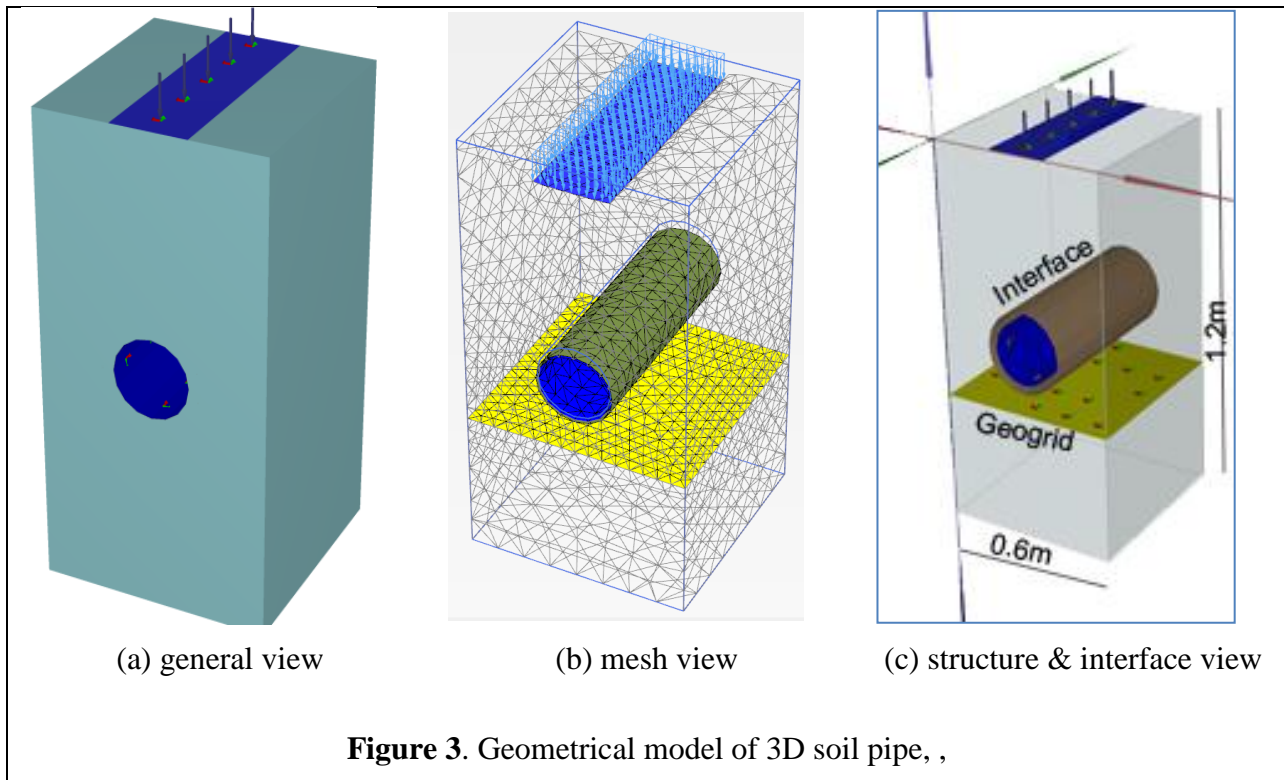


Figure 4. Definition of vertical diametric strain.

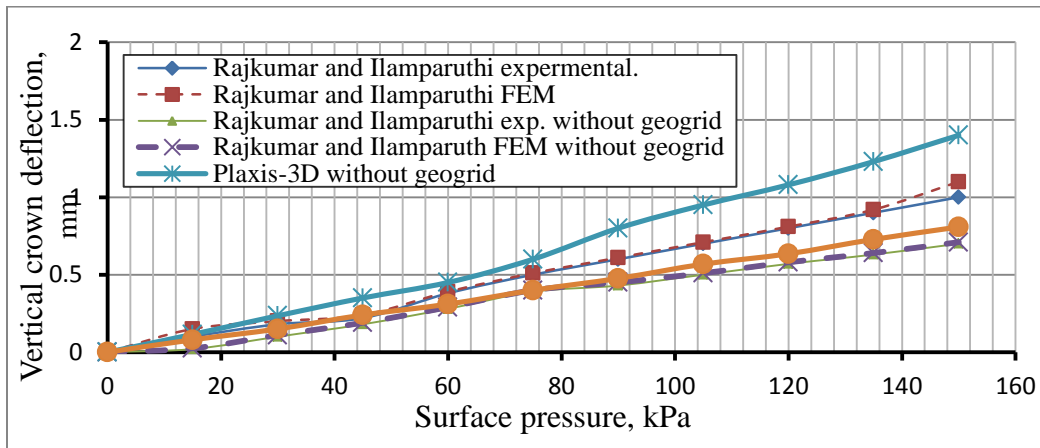


Figure 5. Comparison of the vertical crown deflection of the pipe obtained from PLAXIS-3D with experimental and numerical results of Rajkumar and Ilamparuthi, 2008 with and without geogrid reinforcement in dense sand.

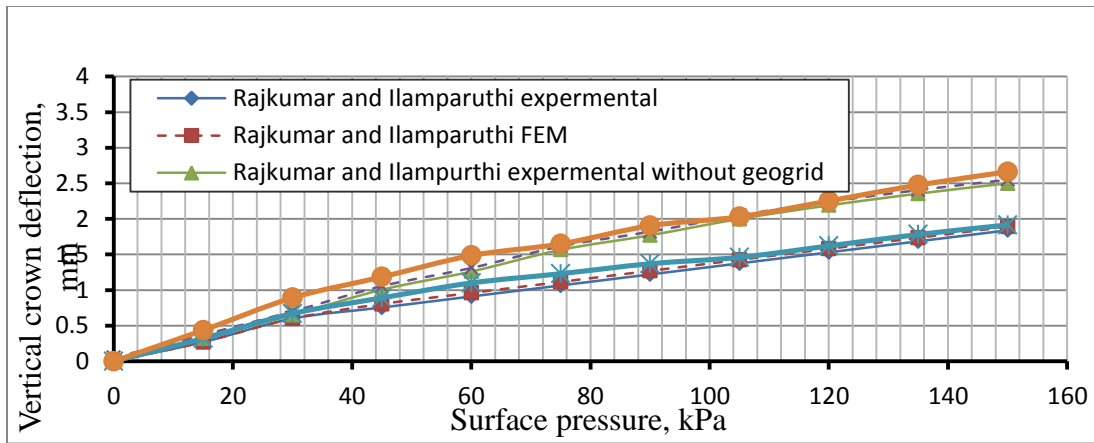


Figure 6. Comparison of vertical crown deflection of the pipe obtained from PLAXIS =3D with experimental and numerical results of Rajkumar and Ilamparuthi, 2008 with and without geogrid reinforcement in loose sand.

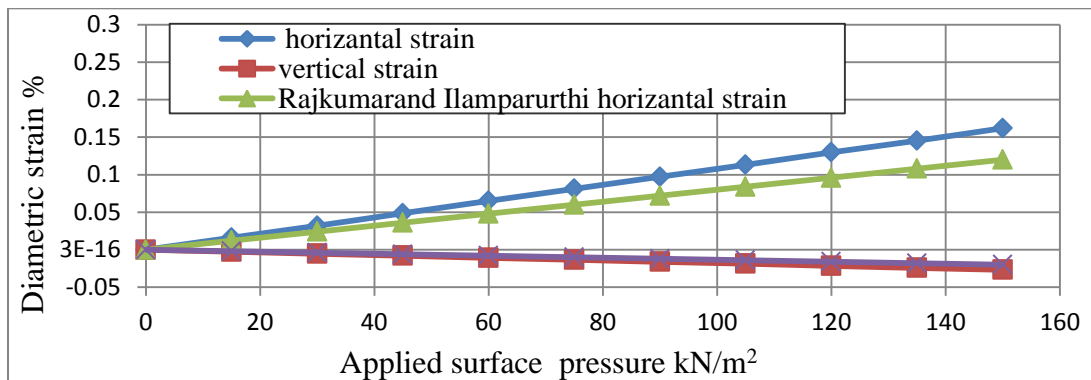


Figure 7. Comparison of diametric strain of the pipe obtained from PLAXIS-3D with the results of Rajkumar and Ilamparuthi, 2008 in loose sand.

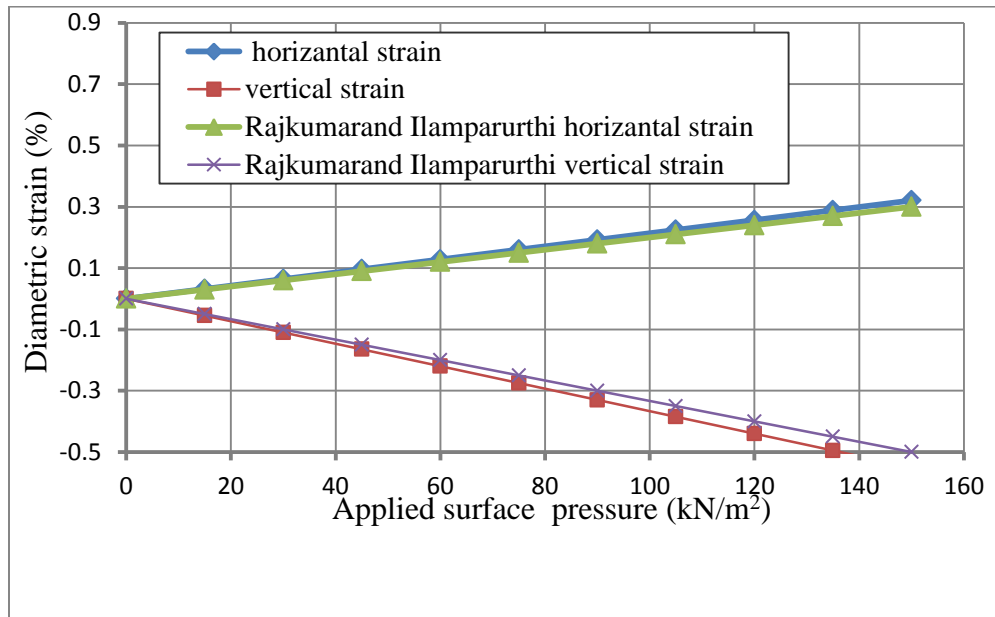


Figure 8. Comparison of crown strain of the pipe obtained from PLAXIS-3D with the results of Rajkumar and Ilamparurthi, 2008 in dense sand.

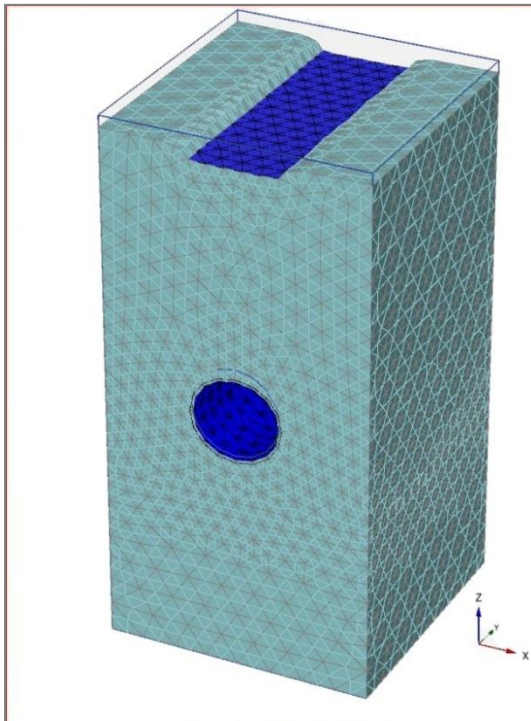


Figure 9. Results of Plaxis 3D representing the deformed mesh.

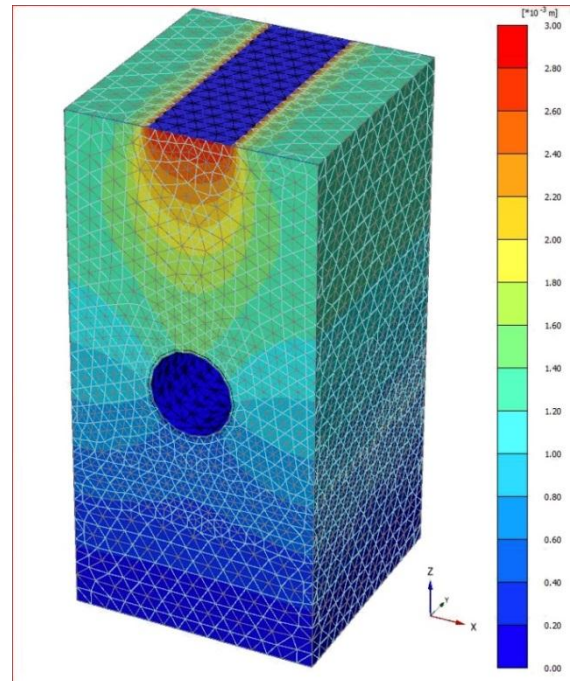


Figure 10. Results of Plaxis 3D representing total volumetric strain ϵ_v .

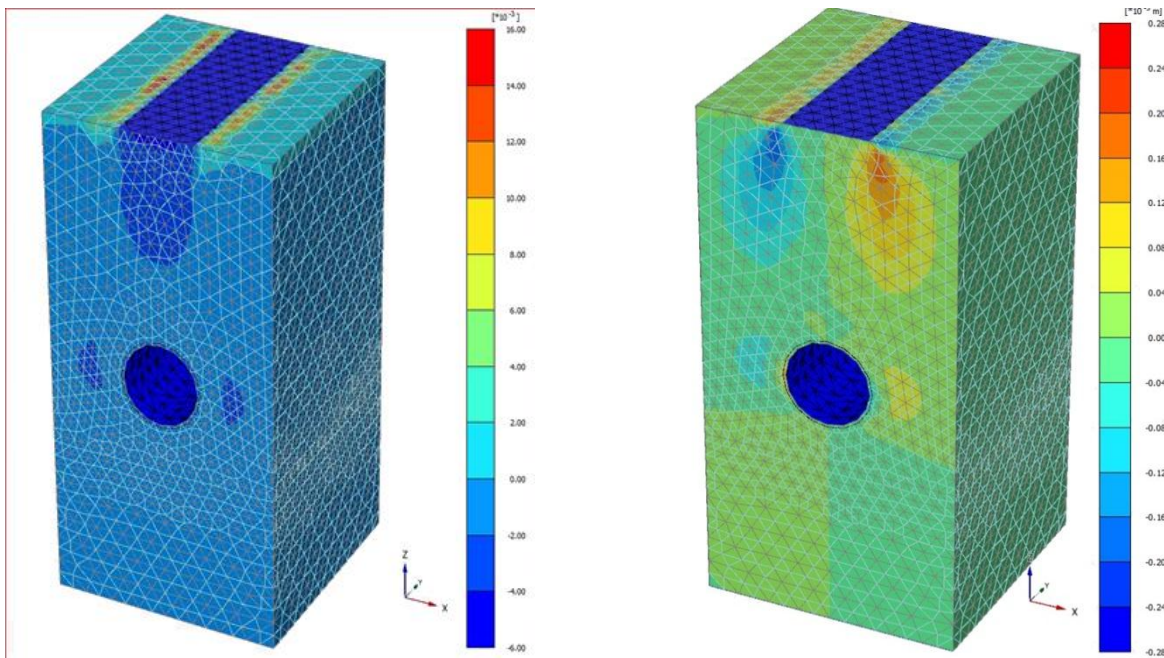


Figure 11. Results of Plaxis 3D representing the total absolute displacement (m).

Figure 12. Results of Plaxis 3D representing the total horizontal displacement u_x

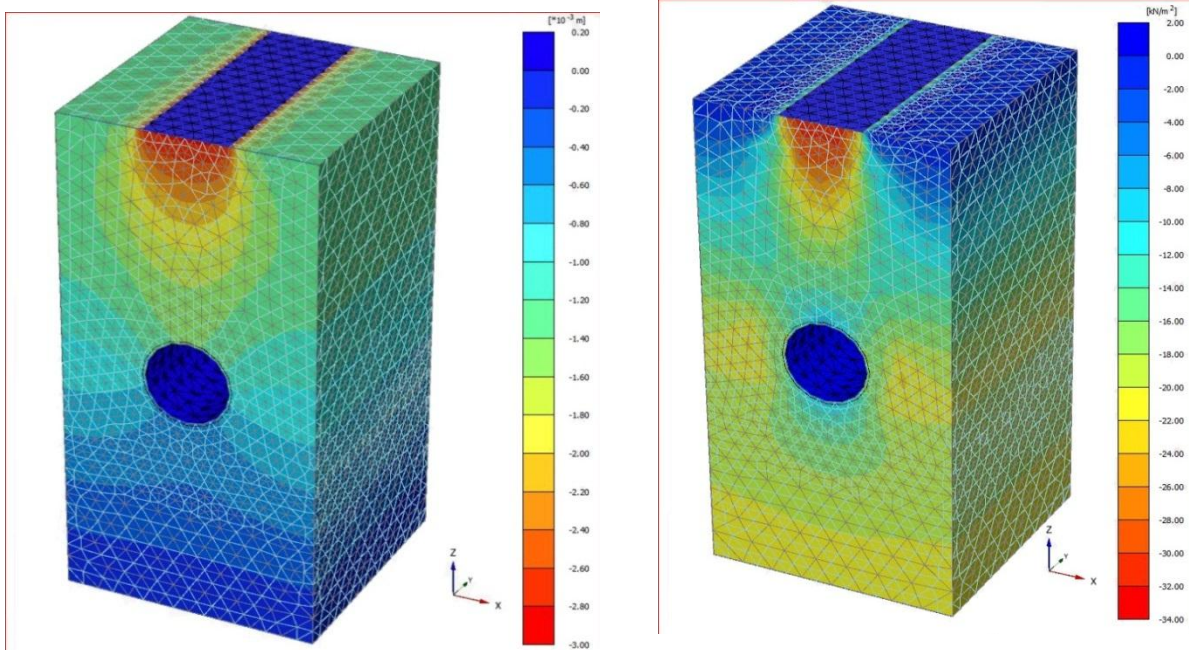


Figure 13. Results of Plaxis 3D representing the total vertical displacement, u_z .

Figure 14. Results of Plaxis 3D representing the total mean stress p (kN/m^2).

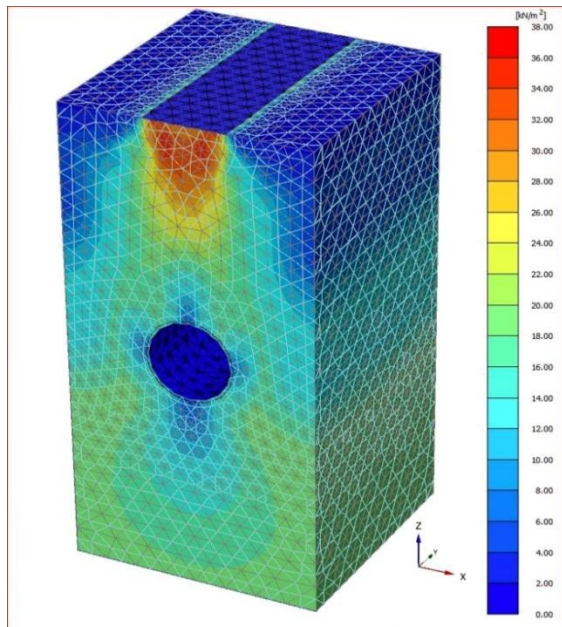


Figure 15. Results of Plaxis 3D representing the deviatoric stress q (kN/m^2).

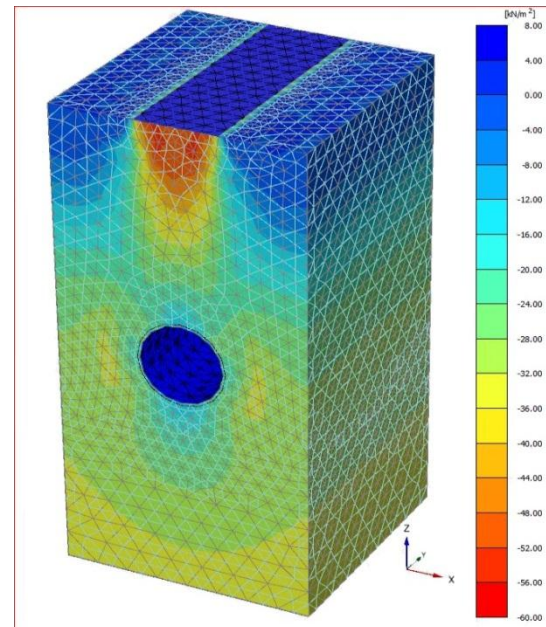


Figure 16. Results of Plaxis 3D representing the total vertical stress σ_{zz} (kN/m^2).

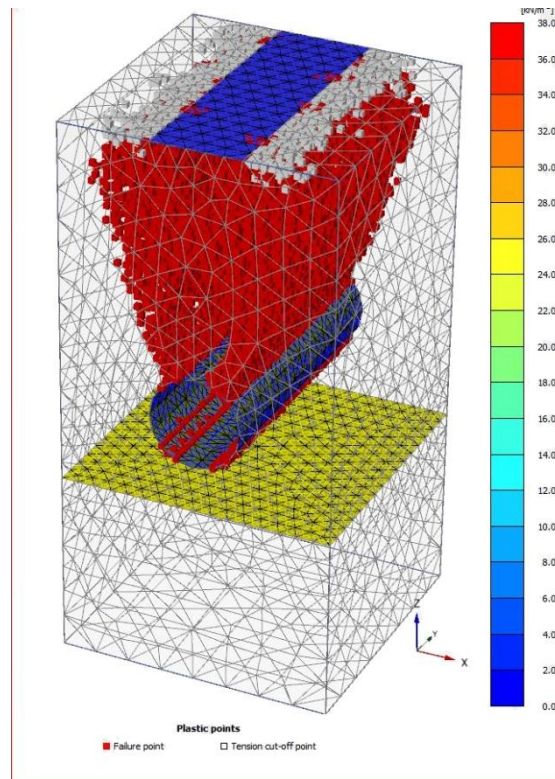


Figure 17. Result of Plaxis 3D represent plastic point.