Monitoring of the Vertical Settlement In Heavy Structures By Precise Levelling

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ABSTRACT

Monitoring and analysing of the vertical deformations or the settlements of the structures is one of the main research fields in geodetic applications, which is considered a precise periodic measurement, made at different epochs to investigate these deformations on heavy structures.

In this research, the deformation measurements were carried out on one of Baghdad University buildings," Building of Computers Department" of dimensions (70.0 * 81.3 m.). Due to some cracks observed in their walls, it was necessary to monitor the vertical displacement of this building at some particular monitoring points by constructing a vertical network and measured in different epochs. The first epoch (zero epoch) was carried out in April 2006, the second in July 2006, the third in October 2006 and the last one in October 2012.

These four epochs include precise levelling measurements were adjusted by Least Squares Adjustment with the aim of investigating the settlement of this building. The two approaches "the Global Congruency test" and "the simple test" are carried out to detect if there any deformation. These two approaches were employed in the analysis and found the difference in elevations between two epochs most be ensured and found that if the monitoring points (P_1 to P_4) stayed really stable, when compared with the time interval or not?

Then according to the analysis procedure to determine the localization of settlement at specific points in the case may change in elevation must be applied. The results showed in two different statistical techniques a significant settlement in four selected corner points on building (P1, P2, P3 and P4). The statistics are based on the probability 95% test and the congruency test with Fisher distribution table.

Keywords: Geodetic applications –Monitoring - Vertical Deformations – Settlements – precise Levelling Global Congruency test- Least Squares Adjustment – Heavy Structures.

مراقبة الهطول العمودي في الأبنية الضخمة باستخدام التسوية الدقيقة

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الخلاصة

تعتبر مراقبة وتحليل الإزاحة العمودية أو الهطول في المنشآت أحد حقول البحث الرئيسية في التطبيقات الجيوديسية، والمتضمنة قياسات دقيقة دورية تجرى في فترات زمنية مختلفة لفحص الإزاحة في الأبنية الضخمة.

أخذت قياسات الإزاحة في هذا البحث على أحد بنايات جامعة بغداد ألا وهي "بناية قسم الحاسبات" ذات الأبعاد (70.0 * 81.3 m.) بسبب التشققات الظاهرة في جدرانها، لذا أصبح من الضروري مراقبة الإزاحة الرأسية للبناية عند نقاط المراقبة المعينة من خلال إنشاء شبكة رأسية ورصدها في فترات زمنية مختلفة. حيث أخذت الرصدة الأولى (والتي تدعى بالصفرية) في نيسان 2006، ثم الثانية في تموز 2006، والثالثة في تشرين الأول 2006، و الرصدة الأخيرة كانت في تشرين الأول 2012.

تتضمن هذه الرصدات الأربع قياسات باستخدام التسوية الدقيقة، ثم تصحح القياسات باستخدام تصحيح المربعات الصغرى لغرض فحص الهطول في البناية. تم استخدام طريقتين للكشف عن أي تشويه وهما "اختبار التطابق الشامل Global Congruency test" والاختبار البسيط "simple test" حيث استخدمت هاتين الطريقتين في تحليل اختلاف المناسيب بين رصدتين لنفس النقاط من أجل التأكد هل إن النقاط (P_1 to P_4) بقيت مستقرة عند المقارنة ضمن فترة زمنية أم لا؟

وفي حالة وجود اختلاف في المنسوب يطبق التحليل الإحصائي لتعيين مقدار الإزاحة في النقاط المحددة. حيث أشارت النتائج باستخدام الطريقتين الإحصائيتين أن هناك هطول ملحوظ في النقاط الأربع (P1, P2, P3 and P4) الموجودة على أركان البناية. إن الإحصائيات مستندة على اختبار الاحتمالية (95%) واختبار التطابق باستخدام القيم الجدولية لتوزيع (F- Fisher).

1. INTRODUCTION

Al-jadriya lake was constructed in 2002 for touring purposes, then cracks were observed in Baghdad University buildings nearest the boundary of this lake (especially the building of computers department that was built in 1993), so a settlement or vertical deformations study is needed, in order to analyse the effect of the water level in the lake on the nearby buildings, **Fig. 1**,

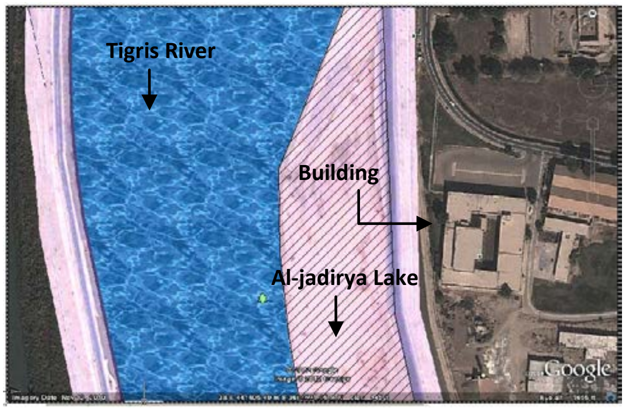

Figure 1. Al-jadriya Lake and Building of Computers Department

It is obvious, the movements and deformation effects on building objects and structures due to own weight, water pressure (changes of ground water level), inner temperature and other factors. [**Vladimir and Miloš, 2004]**

There are a lot of deformation monitoring studies for determining and analysing different kinds of engineering structures such as high-rise buildings, dams, bridges, etc., are implemented. During these studies, the used measurement techniques and systems, this could be geodetic or non-geodetic. [**Erol et al., 2004]**

The deformation monitoring may be divided into two parts: planimetry or horizontally (Δx, Δy) and altimetry or vertically (Δz) [**Baselga et al., 2011**], the combination between them is a three dimensional monitoring. This study will be discussed the vertical deformation analysis using precise levelling measurements at some particular monitoring points on the building.

In general, the deformation analysis is evaluated in four fundamental steps in a geodetic network:

1. The first step, *measurement collection*,which were carried out in t1 and t2 measurement epochs.
2. Adjusted every epoch separately according to the *Least Squares adjustment* method.
3. *Test procedure,* which is carried out to ensure that if the network points stayed really stable in the time interval ($\Delta t = t2 - t1$) or not?
4. *Deformation detection* by analysing to determine the localization of height changes. [**Erol et al., 2004], [Erol et al., 2004]**

2. SETTLEMENT MONITORING

When some cracks appeared in the walls of the *building of computers department*, of dimensions (70.0 * 81.3 m.), with a height of about (8 m.), as shown in **Fig. 2**.

A precise vertical deformation monitoring was proposed to be studying the building stability by determination possible settlement at some main particular monitoring points. It was established in one monitoring point over each corner placed on the columns as it is illustrated in **Fig. 2**,. So there are (four) monitoring points for frequent measuring to be of interest.

There are several methodologies are currently followed when a precise determination of settlement is required. The *precise levelling* is the most accurate method for detecting the smallest change in elevation associated with construction activity, with an accuracy of about (0.001 m.m.) in elevation, since the conclusion about movement must be made with statistical confidence.

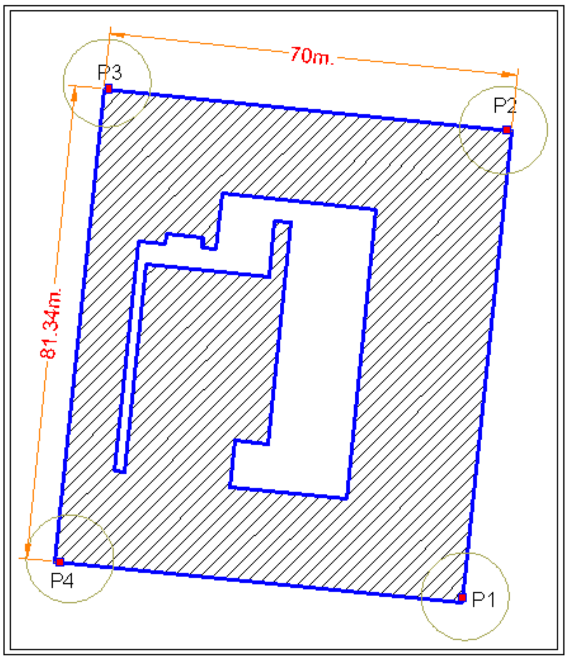

Figure 2. Monitoring points installed in the core wall of the Building of Computers Dept., then monitor its elevation by precise Levelling.

3. PREPARING THE PROCEDURE OF LEVELLING

To start the levelling procedure, a permanent access point of known height above the datum has been needed, which is a main benchmark that constructed far from the lake and the building in order to be free from possible deformation, it defines the height origin that determined by precise levelling , **Fig. 3**, shows the benchmark which is a monument of reinforced concrete has a metal rod in the middle with spherical head makes only one part at the top that can be used in measurements.

Figure 3. The levelling staff over the main benchmark ready to measure, erected in 2006.

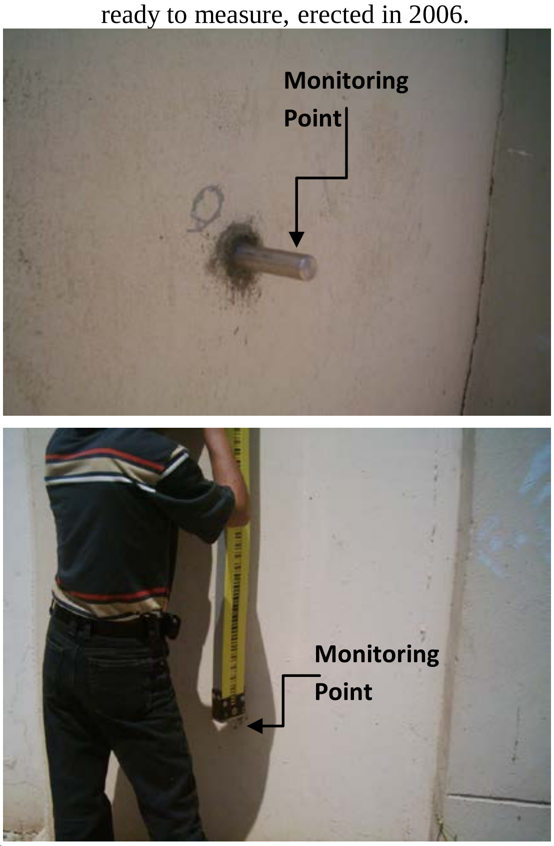

Figure 4. The levelling staff over the monitoring point ready to measure, erected in 2006.

The monitoring points were located in the walls designed from stainless steel rods driven to a point and set in concrete post or bedrock outcrops, with spherical head.

As a result, the site plan of points related to a vertical geodetic network illustrated in **Fig. 5,** which contains a singular benchmark, four (4) monitoring points and other turning points.

The *turning points* should be taken on the change plate in **Fig. 6,** which is made from a solid piece of steel and its weight is heavy.

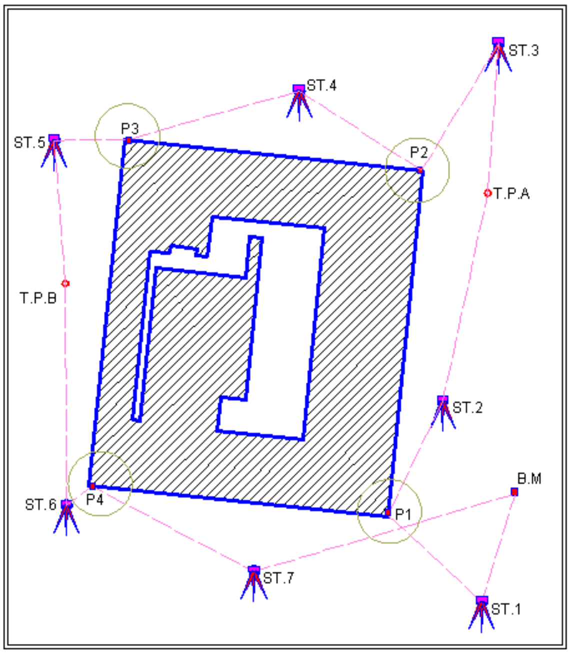

Figure 5. The vertical geodetic network referred to (4) monitoring points , benchmark and other Turning Points.

Figure 6. Precise levelling using change plate, the top is smooth, round and polished.

Then precise levelling was conducted with a *Topcon (DL-102*) Digital Level of the highest accuracy **Fig. 7,** which provides a reading by estimation to (0.0001 m.m.), and observing the codded invar staff shown in **Fig. 8,**

The digital level is an instrument that uses electronic image processing to evaluate the staff reading.For the most precise work, two invar codded staves are used beside the digital level.
All the data of the vertical staff readings and the horizontal distances of the instrument from the staff are automatically stored by the instrument. [**Schofield and Breach, 2007]**
Indeed the two components of precise levelling are precise equipment and precise procedures that need Least Squares Adjustment for a levelling net.

Figure 7. *Topcon (DL-102*) Digital Level

Figure 8. Shows Codded invar staff

4. MEASUREMENT PROCEDURE

After constructing a vertical control, it is necessary to perform a loop circuit for the observation of the points established in the body of the building and examine their elevations periodically for different epochs.

1. The first epoch (zero epoch) was carried out in 20/4/2006, as listed in **Table 1,**
2. The second epoch in 29/6/2006, **Table 2,**
3. The third epoch in 12/10/2006 , **Table 3,**
4. The last one on 23/10/2012, **Table 4 .**

These four epochs include precise levelling measurements adjusted by Least Squares Adjustment with the aim of investigating the settlement of this building

5. LEAST SQUARES ADJUSTMENT

Deformation controlling performs are able to create adjustment models forms and perform analysis when precise levelling is used. The solution follows a systematic procedure, any system of observation may be represented in matrix form as: [**Gilani and Wolf, 2012]**

$$\underset{m*n}{A} * \underset{n*1}{X} = \underset{m*1}{L} + \underset{m*1}{V} \quad (1)$$

where A: matrix of coefficients of the unknowns.
X: matrix of unknowns, adjusted quantities.
L: matrix of observations.
V: matrix of residuals.
m: number of unknowns.
n: number of observations.

$$X = (A^T A)^{-1} A^T L \quad (2)$$

For a system of weighted observation:

$$X = (A^T W A)^{-1} A^T W L \quad (3)$$

where W: is a diagonal matrix of weights.

To calculate the residual:

$$V = AX - L \quad (4)$$

The standard deviation of unit weight for a weighted adjustment is:

$$\sigma_o = \sqrt{\frac{VWV^T}{r}} \qquad r = n - m \quad (5)$$

The standard deviation of the individual adjusted quantities is:

$$\sigma_{x_i} = \sigma_o \sqrt{q_{x_i x_i}} \quad (6)$$

$(q_{x_i x_i})$ the diagonal element in $(A^T W A)^{-1}$ matrix, in the i[th] row and in the i[th] column, this matrix is called "*covariance matrix*" and symbolized by Q_{xx}.

All observations within this levelling network can be simultaneously adjusted using the method of Least Squares to obtain most probable adjusted elevations of points.

The covariance can be used to determine the error ellipsoids of an (n-dimensional random variables). In the practical application of adjustment the *variance and covariance* are often replaced by what should be called "*relative variance and covariance*" for these the terms "*weight coefficient* or *cofactors*" are in common use.
The term cofactor is selected and the letter "*q*" for one element and "*Q*" for a matrix are used as a symbol for it. [**Mikhail, 1976]**
A cofactor is related to a covariance by:

$$q_{ij} = \frac{\sigma_{ij}}{\sigma_o^2} \quad \text{or} \quad \sigma_{ij} = q_{ij} * \sigma_o^2 \qquad (7)$$

Eq.(7) related to the relation between a cofactor and the variance:

$$q_i = \frac{\sigma_i^2}{\sigma_o^2} \quad \text{or} \quad \sigma_i^2 = q_i * \sigma_o^2 \qquad (8)$$

6. STATISTICAL TESTS

Statistical tests are increasingly applied in engineering and in combination with the least Squares method. They are often used to compare results with previous ones or with given standards. In testing, one seeks adjustment as to whether some estimator function. [**Mikhail, E. M., 1976].**

In the case study of vertical network, when the differences in elevations occurred for the same point at different periods (will be presented) it is very important to distinguish between the "*error*" and the "*movement*" this is done by statistical tests.
The adjusted results, according to the Least Squares method, are based on several assumptions which give anchor to the reliability of the statistical test. [**Sansó, F. and Gil, 2006].**

Statistical detection in levelling measurement can be achieved by two statistical tests:

1. Simple deformation test.
2. Global congruency test.

6-1. Simple deformation Test

From the results of two epochs adjustment (i , f) with (n) points, it is possible to calculate the displacement (deformation) vector and its associated variance covariance matrix (Qxx). When the problem deals with a settlement that means one dimensional deformation required. So the simple deformation test depends on comparing the absolute displacement │dn│ in elevation for each point with the probable error at a (95%) confidence limit (e_n). [**Engineering Manual , 2002]**

$$|d_n| = h_f - h_i \qquad (9)$$

$$(e_n) = (1.96)\sqrt{\sigma_f^2 + \sigma_i^2} \qquad (10)$$

where:

$|d_n|$: for point n, is the magnitude of the displacement.
(e_n) : max dimension of combined 95% confidence ellipse for point *n.*
(σ_f) : is the standard error in elevation for the (final) epoch.
(σ_i) : is the standard error in elevation for the (initial) epoch.

If *computed* (d_n) < *theoretical* (e_n) : Accepted, that means no settlement. Otherwise rejected when *computed* (d_n) > *theoretical* (e_n) means there is deformation or settlement.

6-2. Global Congruency Test

The Global Congruency test is the most commonly methodology adopted for the detection of general deformations in a given area i.e. an overall change in shape. [**Fagir et al., 2007]**

After adjusting each epoch separately, then the procedure of deformation analysis is done step by step, with the Global Congruency test. If the elevations of repeated measurements with its variance covariance matrices of the elevations and its datum are available, the question, congruence between different epochs exist or not? [**Denli and Deniz, 2003]**

The problem of investigation of the stability of network points is solved by a test of the null hypothesis (H_o) "the common points of both epochs *(i, ,j)* are stable" and thus have:

The displacement vector (d) for two different epochs and its associated weight matrix (W_{dd}) from error propagation can be computed as:

$$d = h_i - hj \qquad (11)$$
$$W_{dd} = inv(Q_i + Qj) \qquad (12)$$

$H_o = E(d) = 0$: The null hypothesis (12-b)
$H_A = E(d) \neq 0$: The alternative hypothesis
E: indicates expectation,

When (H_o) is accepted (d = 0): the points are assumed to be stable, (the network is stable). Otherwise (H_o) is rejected ($d \neq 0$): the network has undergone a change (settlement).

The test begins with computed the pooled variance (σ_{ij}^2) for two epochs as follows: **[Grunding et al., 1985]**

$$\sigma_{ij}^2 = \frac{V_i^t P_i V_i + V_j^t P_j V_j}{r_i + r_j} \qquad (13)$$

$$\Omega^2 = \frac{d^{-1} * (W_{dd}) * d}{h} \qquad (14)$$

$$h = m - r_d \qquad (15)$$

where:

(V_i , V_j): The residual error vectors for epochs i, j
(r_i , r_j): The redundant observations for epochs i, j
(p_i , p_j): The weight matrices of observations for epochs i, j respectively.
(m): Number of observations
(r_d): Rank deficiency of variance covariance matrix (W_{dd}).
(σ_{ij}^2): Pooled variance.
(Ω^2): Estimated variance of displacements.

By comparing (σ_{ij}^2) with (Ω^2) then:

If <u>(H_o) is accepted</u>, (Ω^2) only mat be exceeded the quantity (σ_{ij}^2) by the effect of random errors.
Otherwise, <u>(H_o) is rejected</u>, accept (H_A) the alternative hypothesis. This is tested by Fisher test quantity.

$$F^* = \frac{\Omega^2}{\sigma_{ij}^2} \qquad (16)$$

By comparing (f- table) with (f- computed) to decide the case of settlement:

1- If (F^*: f- computed) < (f- table): accepted it means “no settlement” when ($F^* < F_{1-\alpha,\ f1,\ f2}$) taken by Fisher distribution. Where:

(α) = 0.05 Levels of significance.
(f1) = h: Degree of freedom.
(f2) = ($r_i + r_j$) : Total degrees of freedom.

2- If (F^*: f- computed) fits (f- table), there is no reason to reject the null hypothesis.

3- If there is a significant deviation from the theoretical distribution, the existence of the settlement must be accepted.
When (F^*: f- computed) > (f- table): reject the null hypothesis (H_o), “there is settlement”

Fisher distribution values obtained from prepared tables or interpolated from graph for each level of significance.

6-3. Localization of Elevation Changes

After determining a group of stable points as a result of global test, the following step of the analysis is the localization of elevation changes. For doing this (Ω^2) are calculated for the every network point, except the stable points, and they were compared with (F) critical value that is given in the fisher distribution table, **[Erol et al., 2003]**

$$d = h_i - hj$$
$$W_{dd} = inv(Q_i + Qj)$$
$$\sigma_{ij}^2 = \frac{V_i^t P_i V_i + V_j^t P_j V_j}{r_i + r_j}$$
$$\Omega^2 = \frac{d^{-1} * (W_{dd}) * d}{h}$$
$$F^* = \frac{\Omega^2}{\sigma_{ij}^2}$$

If (F^*) > ($F_{1-\alpha,\ f1,\ f2}$) , it is said that the elevation of the point changed significantly. Otherwise it is resulted that (d: elevation difference) is not a displacement but it is caused by the random measurement error.

7. CONCLUSIONS AND ECOMMENDATIONS

By application of the Least Squares method, the adjusted elevations of every epoch are determined. It was possible to compute the possible differences and the corresponding deformation after applying it to the statistical test.

The first third epochs did not show significant displacement, may be for the short period, but then when taking observations over six years the settlement will be appeared as shown in **Table (5).**

Additional measurement companies are suggested for the next years, in order to obtain a more reliable monitoring modelization.

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Zero Epoch:

Date: 20-4-2006

1-1) Table (1): Observations of Zero Epoch:

From	To	B.S		F.S		Elevation Difference	Total Dist (S_i)	Elevation (H)	Comul. Dist.
		Dist.	Read.	Dist.	Read.				
B.M	P1	27.00	1.63235	29.84	0.49498	Δh 1.13737	56.840	36.77467 37.91204	56.840
P1	T.P.A	29.87	0.49480	54.829	1.25624	-0.76144	84.699	37.91204 37.15060	
T.P.A	P2	34.986	1.27929	34.610	0.56569	0.71360	69.596	37.15060 37.86420	211.135
P2	P3	34.606	0.56580	41.857	0.53406	0.031745	76.463	37.8642 37.89595	287.598
P3	T.P.B	17.120	0.51505	33.631	1.22961	-0.71456	50.751	37.89595 37.18139	
T.PB	P4	52.0515	1.52836	7.432	0.83581	0.69255	59.484	37.18139 37.87394	397.832
P4	B.M	43.042	0.340415	62.820	1.43912	-1.09871	105.862	37.87394 36.77523	503.694
						Closure Error =	0.00056		

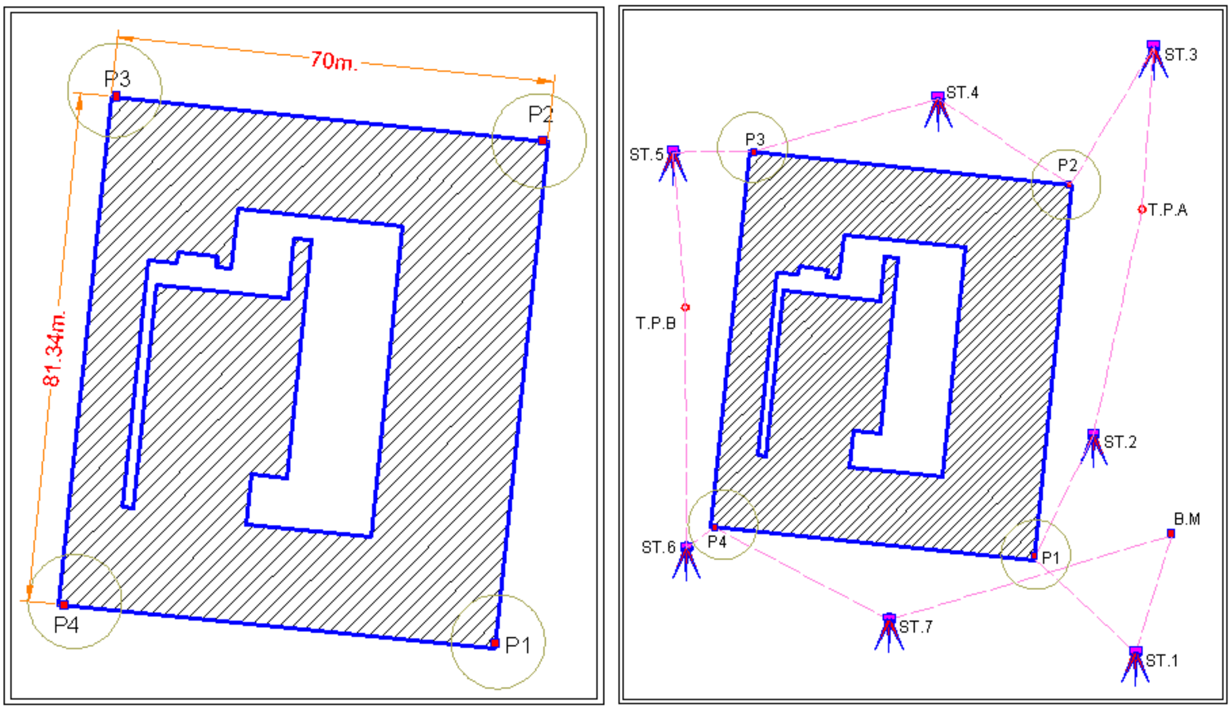

Fig (2), (5) : Illustrate the dimensions of Computers Department Building and vertical network.

1-2) Adjustment By Least Squares:

a) Formatting matrices: Observation Equations are written relating each line's measured elevation difference to its residual error, and the most probable value for adjusting unknown elevations of points as follows: $[A^* X = \underset{7*1}{L} + \underset{7*1}{V}]$

1) $P_1 = B.M + \Delta h_1 + V_1$
2) $TPA = P_1 + \Delta h_2 + V_2$
3) $P_2 = TPA + \Delta h_3 + V_3$
4) $P_3 = (P_2) + \Delta h_4 + V_4$
5) $TPB = P_3 + \Delta h_5 + V_5$
6) $P_4 = TPB + \Delta h_6 + V_6$
7) $B.M = P_4 + \Delta h_7 + V_7$

$$\underset{7*6}{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \underset{6*1}{X} = \begin{bmatrix} P_1 \\ TPA \\ P_2 \\ P_3 \\ TPB \\ P_4 \end{bmatrix} \quad \underset{7*1}{L} = \begin{bmatrix} 37.91204 \\ -0.76144 \\ 0.71360 \\ 0.031745 \\ -0.71456 \\ 0.69255 \\ 37.87394 \end{bmatrix} \quad \underset{7*1}{V} = \begin{bmatrix} V_1 \\ V2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$$

b) Computing the weight matrix (w_o), which is inversely proportional to course lengths,

$$\underset{7*7}{W_O} = \begin{bmatrix} (1/s_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1/s_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1/s_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1/s_4) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1/s_5) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (1/s_6) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (1/s_7) \end{bmatrix}$$

S_1 = 56.840 m.
S_2 = 84.699 m.
S_3 = 69.596 m.
S_4 = 76.463 m.
S_5 = 50.751 m.
S_6 = 59.484 m.
S_7 = 105.862 m.

$$\underset{7*7}{W_O} = \begin{bmatrix} 0.01759 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.01181 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.014369 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.013078 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.019704 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.016811 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.009446 \end{bmatrix}$$

c) Solution to find the most probable values of adjusted points.

- Compute: $[A^T W_o A]$ then $[\,Q_{xx} = (A^T WA)^{-1}]$
- Compute: $[A^T W_o L]$ then $[\,X = (A^T WA)^{-1}(A^T WL)]$
- Compute: $[V = AX - L]$ then $[\,V^T W_O V]$
- Compute: the standard deviation of unit weight $[\sigma_o]$ and
- Compute: the standard deviation of adjusted point $[\sigma_{xi}]$

Computation of Adjusted Elv.

Point	Adjusted Elev.(H^*)	Stand. dev.(σ)
B.M	36.77467	-----------
P1	37.91198	0.000176
T.P.A	37.15044	0.000249
P2	37.86397	0.000274
P3	37.89563	0.000275
T.P.B	37.18101	0.000261
P4	37.87350	0.000226

Computing the residuals (V)

From	To	Adjusted Elev.Diff.(Δh)	V
B.M	P1	1.137307	-0.0000626
P1	T.P.A	-0.761533	-0.0000933
T.P.A	P2	0.713523	-0.0000767
P2	P3	0.031661	-0.0000843
P3	T.P.B	-0.714616	-0.0000559
T.P.B	P4	0.692484	-0.0000655
P4	BM	-1.098827	-0.0001166

The Monitoring Points

P.	Adjusted Elev.(H^*)	Variance (σ^2)
P1	37.91198	3.10E-08
P2	37.86397	7.51E-08
P3	37.89563	7.56E-08
P4	37.87350	5.11E-08
By listing the Monitoring Points only		

(σ): is the Standard deviation of the (adjusted elevations :unknowns)

1-3) **Analysis of the Adjustment:**

Reference σ_o = ±0. 000025:

Degrees of freedom = n - m = 7 – 6 = 1

$$Q_{XX}=\begin{bmatrix}(\sigma_0^2)_{P1} & 0 & 0 & 0\\ 0 & (\sigma_0^2)_{P2} & 0 & 0\\ 0 & 0 & (\sigma_0^2)_{P3} & 0\\ 0 & 0 & 0 & (\sigma_0^2)_{P4}\end{bmatrix}=\begin{bmatrix}3.10E-08 & 0 & 0 & 0\\ 0 & 7.51E-08 & 0 & 0\\ 0 & 0 & 7.56E-08 & 0\\ 0 & 0 & 0 & 5.11E-08\end{bmatrix}$$

Q_{XX}=variance covariance, it is a diagonal matrix, assuming no correlation between observations

$$Q_o=\frac{1}{\sigma_0^2}*Q_{XX}=\begin{bmatrix}49.5616 & 0 & 0 & 0\\ 0 & 120.1216 & 0 & 0\\ 0 & 0 & 121.00 & 0\\ 0 & 0 & 0 & 81.7216\end{bmatrix}$$

1-4) **Statistical Tests:** No Statistical Test for this Epoch, but it exists for the next epochs.

Fig (9): Illustrate the Computers Department Building.

First Epoch:

Date: 29-6-2006

2-1) Table (2): Observations of First Epoch:

From	To	B.S		F.S		Elevation Difference	Total Dist (S_i)	Elevation (H)	Comul. Dist.
		Dist.	Read.	Dist.	Read.				
B.M		32.06	1.547265					36.77467	
	P1			39.479	0.410575	1.13669	71.538	37.91136	71.538
P1		39.468	0.41053					37.91136	
	T.P.A			39.545	1.07838	-0.66785	79.014	37.24351	
T.P.A		31.607	1.15373					37.24351	
	P2			27.778	0.53457	0.61916	59.385	37.86267	209.936
P2		27.786	0.53474					37.86267	
	T.P.B			39.857	1.29938	-0.764635	67.643	37.09804	
T.P.B		44.187	1.38968					37.09804	
	P3			16.034	0.59357	0.79611	60.221	37.89415	337.800
P3		16.038	0.59348					37.89415	
	T.P.C			48.841	1.23126	-0.63779	64.879	37.25636	
T.P.C		40.579	1.32863					37.25636	
	P4			7.716	0.71220	0.61643	48.295	37.87279	450.973
P4		7.714	0.71245					37.87279	
	T.P.D			37.307	1.75023	-1.03778	45.021	36.83501	
T.P.D		26.741	1.36267					36.83501	
	B.M			42.451	1.42389	-0.06121	69.192	36.77380	565.186
						Closure Error = 0.00087			

2-2) Adjustment By Least Squares:

a) Formatting matrices: $[\underset{9*8}{A} * \underset{8*1}{X} = \underset{9*1}{L} + \underset{9*1}{V}]$

1) $P_1 = B.M + \Delta h_1 + V_1$
2) $TPA = P_1 + \Delta h_2 + V_2$
3) $P_2 = TPA + \Delta h_3 + V_3$
4) $TPB = P_2 + \Delta h_4 + V_4$
5) $P_3 = TPB + \Delta h_5 + V_5$
6) $TPC = P_3 + \Delta h_6 + V_6$
7) $P_4 = TPC + \Delta h_7 + V_7$
8) $TPD = P_4 + \Delta h_8 + V_8$
9) $B.M = TPD + \Delta h_9 + V_9$

$$\underset{9*8}{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \underset{8*1}{X} = \begin{bmatrix} P_1 \\ TPA \\ P_2 \\ TPB \\ P_3 \\ TPC \\ P_4 \\ TPD \end{bmatrix} \quad \underset{9*1}{L} = \begin{bmatrix} 37.91136 \\ -0.66785 \\ 0.61916 \\ -0.76463 \\ 0.79611 \\ -0.63779 \\ 0.61643 \\ -1.03778 \\ 36.83501 \end{bmatrix}$$

b) Computing the weight matrix (w_1), which is inversely proportional to course lengths,

$$\underset{9*9}{W_1} = \begin{bmatrix} 0.013979 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.012656 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.016839 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.014783 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.016606 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.015413 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.020706 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.022212 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.014453 \end{bmatrix}$$

S_1 = 71.538 m.
S_2 = 79.014 m.
S_3 = 59.385 m.
S_4 = 67.643 m.
S_5 = 60.221 m.
S_6 = 64.879 m.
S_7 = 48.295 m.
S_8 = 45.021 m.
S_9 = 69.192 m.

C) Solution to find the most probable values of adjusted points.

Computation of Adjusted Elv.

Point	Adjusted Elev.(H^*)	Stand. dev.(σ)
B.M	36.77467	-----------
P1	37.91147	0.000291
T.P.A	37.24374	0.000387
P2	37.86300	0.000423
T.P.B	37.09846	0.000437
P3	37.89467	0.000429
T.P.C	37.25698	0.000396
P4	37.87348	0.000351
T.P.D	36.83577	0.000287

Computing the residuals (V)

From	To	Adjusted Elev.Diff.(Δh)	V
B.M	P1	1.136801	0.0001108
P1	T.P.A	-0.667728	0.0001223
T.P.A	P2	0.619252	0.0000919
P2	T.P.B	-0.764530	0.0001047
T.P.B	P3	0.796203	0.0000932
P3	T.P.C	-0.637690	0.0001004
T.P.C	P4	0.616505	0.0000748
P4	T.P.D	-1.037710	0.0000697
T.P.D	B.M	-0.061103	0.0001071

The Monitoring Points

P.	Adjusted Elev.(H^*)	Variance (σ^2)
P1	37.91147	8.47E-08
P2	37.86300	1.79E-07
P3	37.89467	1.84E-07
P4	37.87348	1.23E-07
By listing the Monitoring Points only		

(σ): is the Standard deviation of the (adjusted elevations: unknowns)

2-3) <u>Analysis of the Adjustment:</u>

Reference σ_1 = ±0. 000037:

Degrees of freedom = n - m = 9 – 8 = 1

=variance covariance, it is a diagonal matrix, Q_{XX} assuming no correlation between observations

$$Q_{XX} = \begin{bmatrix} (\sigma_1^2)_{P1} & 0 & 0 & 0 \\ 0 & (\sigma_1^2)_{P2} & 0 & 0 \\ 0 & 0 & (\sigma_1^2)_{P3} & 0 \\ 0 & 0 & 0 & (\sigma_1^2)_{P4} \end{bmatrix} = \begin{bmatrix} 3.10E-08 & 0 & 0 & 0 \\ 0 & 7.51E-08 & 0 & 0 \\ 0 & 0 & 7.56E-08 & 0 \\ 0 & 0 & 0 & 5.11E-08 \end{bmatrix}$$

$$Q_1 = \frac{1}{\sigma_1^2} * Q_{XX} = \begin{bmatrix} 49.5616 & 0 & 0 & 0 \\ 0 & 120.1216 & 0 & 0 \\ 0 & 0 & 121.00 & 0 \\ 0 & 0 & 0 & 87.7216 \end{bmatrix}$$

2-4) Statistical Tests of the First Epoch:

- **<u>The first statistical method</u>** : (Simple Test)

Point	Theoretical	Computed		
P1	0.000666564	0.00051	Accepted	If computed < theoretical =accepted
P2	0.000987819	0.00097	Accepted	
P3	0.000998766	0.00096	Accepted	
P4	0.000818231	2.00E-05	Accepted	

- **<u>The second statistical test :</u>** (Cogruency Test)

σ_{ij}^2	Pooled variance=	9.83E-10	$W_{dd} = (Q_o + Q_1)^{-1}$ $\Omega^2 = \frac{d_{01}^{-1} * W_{01} * d_{01}}{h}$ $h = m - r_d$ = 4 $\sigma_{01}^2 = \frac{V_o^t * W_o * V_o + V_1^t * W_1 * V_1}{r_o + r_1}$	$F^* = \frac{\Omega^2}{\sigma_{ij}^2}$ $d = Z_o^{\hat{}} - Z_1^{\hat{}}$
Ω^2	Estimated variance=	2.42E-09		
	f-table =	19.247		
F^*	f-computed =	2.4664		
If (f- computed) < (f-table) =accepted Accept: No Settlement				

Second Epoch:

Date: 12-10-2006

3-1) Table (3): Observations of Second Epoch:

From	To	B.S		F.S		Elevation Difference	Total Dist (S_i)	Elevation (H)	Comul. Dist.
		Dist.	Read.	Dist.	Read.				
B.M		26.61	1.51806					36.77467	
	P1			34.404	0.381325	1.136735	61.016	37.91141	61.016
P1		34.405	0.38138					37.91141	
	T.P.A			48.846	1.30456	-0.923185	83.251	36.98822	
T.P.A		38.441	1.43255					36.98822	
	P2			28.483	0.55821	0.87434	66.924	37.86256	211.191
P2		28.4825	0.55825					37.86256	
	P3			39.329	0.52668	0.031565	67.811	37.89413	279.002
P3		18.065	0.481485					37.89413	
	T.P.B			48.183	1.105725	-0.62424	66.248	37.26989	
T.PB		43.577	1.21796					37.26989	
	P4			8.485	0.61486	0.60310	52.062	37.87298	397.311
P4		8.500	0.61477					37.87298	
	T.P.C			54.246	1.76244	-1.14767	62.746	36.72531	
T.P.C		25.0535	1.52207					36.72531	
	B.M			25.106	1.47306	0.04901	50.159	36.77467	510.216
						Closure Error =	0.00035		

3-2) Adjustment By Least Squares:

a) Formatting matrices: $[\underset{8*7}{A} * \underset{7*1}{X} = \underset{8*1}{L} + \underset{8*1}{V}]$

1) $P_1 = B.M + \Delta h_1 + V_1$
2) $TPA = P_1 + \Delta h_2 + V_2$
3) $P_2 = TPA + \Delta h_3 + V_3$
4) $P_3 = P2 + \Delta h_4 + V_4$
5) $TPB = P_3 + \Delta h_5 + V_5$
6) $P_4 = TPB + \Delta h_6 + V_6$
7) $TPC = P_4 + \Delta h_7 + V_7$
8) $B.M = TPC + \Delta h_8 + V_8$

$$\underset{8*7}{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \underset{7*1}{X} = \begin{bmatrix} P1 \\ TPA \\ P2 \\ P3 \\ TPB \\ P4 \\ TPC \end{bmatrix} \quad \underset{8*1}{L} = \begin{bmatrix} 37.91141 \\ -0.923185 \\ 0.87434 \\ 0.031565 \\ -0.62424 \\ 0.60310 \\ -1.14767 \\ 36.72531 \end{bmatrix}$$

$S_1 = 61.016$ m.
$S_2 = 83.251$ m.
$S_3 = 66.924$ m.
$S_4 = 67.811$ m.
$S_5 = 66.248$ m.
$S_6 = 52.062$ m.
$S_7 = 62.746$ m.
$S_8 = 50.159$ m.

b) Computing the weight matrix (w_2), which is inversely proportional to course lengths,

$$\underset{8*8}{W_2} = \begin{bmatrix} 0.016389 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.012011 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.014942 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.014747 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.015095 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.019208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0159370 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.019937 \end{bmatrix}$$

c) Solution to find the most probable values of adjusted points.

Computation of Adjusted Elv.

Point	Adjusted Elev.(H^*)	Stand. dev.(σ)
B.M	36.77467	-----------
P1	37.91145	0.000112
T.P.A	36.98832	0.000155
P2	37.86270	0.000170
P3	37.89431	0.000172
T.P.B	37.27012	0.000161
P4	37.87325	0.000143
T.P.C	36.72563	0.000103

Computing the residuals (V)

From	To	Adjusted Elev.Diff.(Δh)	V
B.M	P1	1.136776	0.0000413
P1	T.P.A	-0.923130	0.0000563
T.P.A	P2	0.874385	0.0000453
P2	P3	0.031611	0.0000459
P3	T.P.B	-0.624200	0.0000448
T.P.B	P4	0.603135	0.0000352
P4	T.P.C	-1.14763	0.0000424
T.P.C	B.M	0.049044	0.0000339

The Monitoring Points

P.	Adjusted Elev.(H^*)	Variance (σ^2)
P1	37.91145	1.25E-08
P2	37.86270	2.89E-08
P3	37.89431	2.96E-08
P4	37.87325	2.04E-08
By listing the Monitoring Points only		

(σ): is the Standard deviation of the (adjusted elevations: unknowns)

3-3) Analysis of the Adjustment:

Reference σ_2 = ±0. 000015:

Degrees of freedom = n - m = 8 – 7 = 1

=variance covariance, it is a diagonal matrix, Q_{XX} assuming no correlation between observations

$$Q_{XX} = \begin{bmatrix} (\sigma_2^2)_{P1} & 0 & 0 & 0 \\ 0 & (\sigma_2^2)_{P2} & 0 & 0 \\ 0 & 0 & (\sigma_2^2)_{P3} & 0 \\ 0 & 0 & 0 & (\sigma_2^2)_{P4} \end{bmatrix} = \begin{bmatrix} 1.25E-08 & 0 & 0 & 0 \\ 0 & 2.89E-08 & 0 & 0 \\ 0 & 0 & 2.96E-08 & 0 \\ 0 & 0 & 0 & 2.04E-08 \end{bmatrix}$$

$$Q_2 = \frac{1}{\sigma_2^2} * Q_{XX} = \begin{bmatrix} 55.75111 & 0 & 0 & 0 \\ 0 & 128.4444 & 0 & 0 \\ 0 & 0 & 131.4844 & 0 \\ 0 & 0 & 0 & 90.8844 \end{bmatrix}$$

3-4) Statistical Tests

- **The first statistical method** : (Simple Test)

Point	Theoretical	Computed		
P1	0.000409	0.00053	Accepted	if computed < theoretical =accepted
P2	0.000632	0.00127	Accepted	
P3	0.000636	0.00132	Accepted	
P4	0.000524	0.00025	Rejected	

- **The second statistical test :** (Cogruency Test)

σ_{ij}^2	Pooled variance=	4.22E-10	$W_{dd} = (Q_o + Q_2)^{-1}$ $\Omega^2 = \frac{d_{02}^{-1} * W_{02} * d_{02}}{h}$ $\sigma_{02}^2 = \frac{V_o^t * W_o * V_o + V_2^t * W_2 * V_2}{r_o + r_2}$ $h = m - r_d$ = 4	$F^* = \frac{\Omega^2}{\sigma_{ij}^2}$ $d = Z_o^{\vdots} - Z_2^{\vdots}$
Ω^2	Estimated variance=	2.81E-07		
	f-table =	19.247		
F^*	f-computed =	9.7186		
If (f- computed) < (f-table) =accepted Accept: No Settlement				

Third Epoch:

Date: 23-10-2012

From	To	B.S		F.S		Elevation Difference	Total Dist (S_i)	Elevation (H)	Comul. Dist.
		Dist.	Read.	Dist.	Read.				
B.M		24.23	1.57610					36.77467	
	P1			38.104	0.44440	1.1317	62.329	37.90637	62.329
P1		26.312	0.90457					37.90637	
	T.P.A			43.414	1.56290	-0.65833	69.726	37.24804	
T.P.A		35.675	1.18970					37.24804	
	P2			32.643	0.58160	0.60810	68.318	37.85614	200.373
P2		30.431	0.75837					37.85614	
	P3			36.508	0.72580	0.03257	66.939	37.88871	267.312
P3		20.170	0.63870					37.88871	
	T.P.B			43.122	1.34417	-0.70547	63.292	37.18324	
T.PB		38.29	1.56877					37.18324	
	P4			15.776	0.88580	0.68297	54.066	37.86621	384.670
P4		18.324	0.89320					37.86621	
	T.P.C			49.564	1.91003	-1.01683	67.888	36.84938	
T.P.C		30.389	1.59320					36.84938	
	B.M			37.873	1.66730	-0.07410	68.262	36.77528	520.820
						Closure Error = 0.00061			

4-1) <u>Table (4): Observations of Third Epoch:</u>

4-2) <u>Adjustment By Least Squares:</u>

a) Formatting matrices: $[\underset{8*7}{A} * \underset{7*1}{X} = \underset{8*1}{L} + \underset{8*1}{V}]$

1) $P_1 = B.M + \Delta h_1 + V_1$
2) $TPA = P_1 + \Delta h_2 + V_2$
3) $P_2 = TPA + \Delta h_3 + V_3$
4) $P_3 = P2 + \Delta h_4 + V_4$
5) $TPB = P_3 + \Delta h_5 + V_5$
6) $P_4 = TPB + \Delta h_6 + V_6$
7) $TPC = P_4 + \Delta h_7 + V_7$
8) $B.M = TPC + \Delta h_8 + V_8$

$S_1 = 62.329$ m.
$S_2 = 69.726$ m.
$S_3 = 68.318$ m.
$S_4 = 66.939$ m.
$S_5 = 63.292$ m.
$S_6 = 54.066$ m.
$S_7 = 67.888$ m.
$S_8 = 68.262$ m.

$$\underset{8*7}{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad \underset{7*1}{X} = \begin{bmatrix} P1 \\ TPA \\ P2 \\ P3 \\ TPB \\ P4 \\ TPC \end{bmatrix} \quad \underset{8*1}{L} = \begin{bmatrix} 37.90637 \\ -0.65833 \\ 0.60810 \\ 0.03257 \\ -0.70547 \\ 0.68297 \\ -1.01683 \\ 36.84938 \end{bmatrix}$$

c) Computing the weight matrix (w_2), which is inversely proportional to course lengths,

$$\underset{8*8}{W_3} = \begin{bmatrix} 0.016044 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.014341 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.014637 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.014939 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.015800 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.018496 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.014730 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.014649 \end{bmatrix}$$

d) Solution to find the most probable values of adjusted points.

Computation of Adjusted Elv.

Point	Adjusted Elev.(H^*)	Stand. dev.(σ)
B.M	36.77467	-----------
P1	37.91145	0.000112
T.P.A	36.98832	0.000155
P2	37.86270	0.000170
P3	37.89431	0.000172
T.P.B	37.27012	0.000161
P4	37.87325	0.000143
T.P.C	36.72563	0.000103

Computing the residuals (V)

From	To	Adjusted Elev.Diff.(Δh)	V
B.M	P1	1.131627	-0.000073
P1	T.P.A	-0.65841	-0.000082
T.P.A	P2	0.608092	-0.000008
P2	P3	0.032492	-0.000078
P3	T.P.B	-0.70554	-0.000074
T.P.B	P4	0.682907	-0.000063
P4	T.P.C	-1.01684	-0.000008
T.P.C	B.M	-0.07411	-0.000008

The Monitoring Points

P.	Adjusted Elev.(H^*)	Variance (σ^2)
P1	37.91145	1.25E-08
P2	37.86270	2.89E-08
P3	37.89431	2.96E-08
P4	37.87325	2.04E-08
By listing the Monitoring Points only		

(σ): is the Standard deviation of the (adjusted elevations: unknowns)

4-3) Analysis of the Adjustment:

Reference σ_3 = ±0. 000027:

Degrees of freedom = n - m = 8 – 7 = 1

=variance covariance, it is a diagonal matrix, Q_{XX} assuming no correlation between observations

$$Q_{XX} = \begin{bmatrix} (\sigma_3^2)_{P1} & 0 & 0 & 0 \\ 0 & (\sigma_3^2)_{P2} & 0 & 0 \\ 0 & 0 & (\sigma_3^2)_{P3} & 0 \\ 0 & 0 & 0 & (\sigma_3^2)_{P4} \end{bmatrix} = \begin{bmatrix} 3.92E-08 & 0 & 0 & 0 \\ 0 & 8.82E-08 & 0 & 0 \\ 0 & 0 & 9.30E-08 & 0 \\ 0 & 0 & 0 & 7.18E-08 \end{bmatrix}$$

$$Q_3 = \frac{1}{\sigma_3^2} * Q_{XX} = \begin{bmatrix} 53.7777 & 0 & 0 & 0 \\ 0 & 121.000 & 0 & 0 \\ 0 & 0 & 127.6063 & 0 \\ 0 & 0 & 0 & 98.5240 \end{bmatrix}$$

4-4) Statistical Tests

- **The first statistical method** : (Simple Test)

Point	Theoretical	Computed		
P1	0.000519	0.00568	Rejected	if computed < theoretical =accepted
P2	0.000792	0.00806	Rejected	
P3	0.000805	0.00723	Rejected	
P4	0.000687	0.00774	Rejected	

- **The second statistical test :** (Cogruency Test)

σ_{ij}^2	Pooled variance=	6.63E-10	$W_{dd} = (Q_o + Q_2)^{-1}$ $\Omega^2 = \frac{d_{03}^{-1} * W_{03} * d_{03}}{h}$ $\sigma_{03}^2 = \frac{V_o^t * W_o * V_o + V_3^t * W_3 * V_3}{r_o + r_3}$ $h = m - r_d = 4$	$F^* = \frac{\Omega^2}{\sigma_{ij}^2}$ $d = Z_o^{\ddagger} - Z_3^{\ddagger}$
Ω^2	Estimated variance=	2.81E-07		
	f-table =	19.247		
F^*	f-computed =	424		
If (f- computed) < (f-table) =accepted Accept: No Settlement				

4-5) Localization of Displacement

Table 5. The localization of settlement at monitoring points (P1-P4).

σ_{ij}^2	Pooled variance=	6.63E-10	$W_{dd} = (Q_o + Q_2)^{-1}$ $\Omega^2 = \frac{d_{03}^{-1} * W_{03} * d_{03}}{h}$ $\sigma_{03}^2 = \frac{V_o^t * W_o * V_o + V_3^t * W_3 * V_3}{r_o + r_3}$	$F^* = \frac{\Omega^2}{\sigma_{ij}^2}$ $d = Z_o^{\ddagger} - Z_3^{\ddagger}$ $h = m - r_d = 4$
Ω^2	Estimated variance=	2.81E-07		
F	f-table =	19.247		
F^*	f-computed =	424		

Localization of displacement			if (f-computed) < (f-table) =accepted
			 if (f-computed) > (f-table) =rejected
			 Reject the null hypothesis Ho
			 There is a settlement!!!!!
			 Localization of displacement must be computed	

	F-table	F-computed		
Shifted-P1	470.97	18.513		
Shifted-P2	406.44	18.513		
Shifted-P3	317.19	18.513		
Shifted-P4	501.39	18.513		