Strengthening and Closing Cracks for Existing Reinforced Concrete Girders Using Externally Post-Tensioned Tendons

Nazar K. Ali Oukaili [(1)] and Ihab Nabeel Issa Al-Shawi [(2)]

[1] Prof. Dr., College of Engineering, University of Baghdad, Iraq. email: dr_nazar12000@yahoo.com

[2] M.Sc. Student, College of Engineering, University of Baghdad, Iraq. email: ehabshawi@yahoo.com

ABSTRACT

This research is devoted to study the strengthening technique for the existing reinforced concrete beams using external post-tensioning. An analytical methodology is proposed to predict the value of the effective prestress force for the external tendons required to close cracks in existing beams. The external prestressing force required to close cracks in existing members is only a part from the total strengthening force.

A computer program created by **Oukaili (1997)** and developed by **Alhawwassi (2008)** to evaluate curvature and deflection for reinforced concrete beams or internally prestressed concrete beams is modified to evaluate the deflection and the stress of the external tendons for the externally strengthened beams using Matlab 7.0.

The analytical investigation is implemented on three ideal reinforced concrete beam models, each model is considered to be strengthened using three types of external tendon profile (straight, draped and double draped), where each type of tendon profile is analyzed separately. No comparisons were made with analytical or experimental investigations, because no publications for this kind of studies were found.

KEYWORDS: Strengthening, Post-Tensioning, Draped Tendons, Deflection, Curvature and Reinforced Concrete.

تقوية و غلق الشقوق في الروافد الخرسانية المسلحة المتواجدة بأستعمال الاجهاد المسبق الخارجي

نزار كامل علي العقيلي (1) و ايهاب نبيل عيسى الشاوي (2)

(1) أستاذ دكتور، كلية الهندسة، جامعة بغداد، العراق. (2) طالب ماجستير، كلية الهندسة، جامعة بغداد، العراق.

الخلاصة

ان الغرض من هذا البحث هودراسة تقنية تقوية الروافد الخرسانية المسلحة المتواجدة باستعمال الاجهاد المسبق الخارجي. حيث تم اقتراح طريقة تحليلية لحساب قوة الاجهاد المسبق للحديد الخارجي الكافية لغلق الشقوق في الروافد الخرسانية المنشأة. ان قوة الاجهاد المسبق للحديد الخارجي المطلوبة لغلق الشقوق هي جزء من القوة الكلية المطلوبة للتقوية.

تم تعديل برنامج حسابي اقترحه (العقيلي) وطورته (الهواسي) والذي يقوم بحساب الهطول و التقوس في الروافد الخرسانية المسلحة الاعتيادية او الروافد الخرسانية المسبقة الاجهاد داخليا ليقوم بحساب هطول الروافد الخرسانية المقواة بالأوتاد الفولاذية الخارجية المسبقة الاجهاد و حساب اجهاد الحديد الخارجي.

تم تطبيق الطريقة الرياضية المقترحة على ثلاثة نماذج تحليلية لثلاثة روافد خرسانية مسلحة ليتم تقويتها باستعمال ثلاثة اشكال من الحديد المسبق الاجهاد خارجيا (مستقيم ، منحرف بموقع واحد، منحرف بموقعين). حيث تم تحليل كل شكل على حدى. حيث لم يتم اجراء مقارنة نتائج الطريقة المقترحة مع البيانات المختبرية و ذلك لعدم توفر ابحاث منشوره بهذا الخصوص.

الكلمات الرئيسية: التقوية، الاجهاد المسبق، الاوتار ذات الشكل المنحرف، الهطول، التقوس، الخرسانة المسلحة

INTRODUCTION

During the service life, concrete members may severe different types of deteriorated conditions in addition to the progressive structural aging lead to extreme cracking and deflection that may affect their performance for the rest of their lifetime. Serviceability requirements might be changed and required higher loading capacity than designed due to many possible reasons, like: increasing traffic volume in bridges and culverts or changing the type of floor occupancy in a building.

Cracking is a usual behavior in concrete structures in service, due to low tensile strength of concrete, therefore the internal steel reinforcement have the full responsibility to resist the tensile forces. Since, the cracking behavior is unavoidable; the steel reinforcement will be exposed to the exterior environment within the serviceability lifetime.

Accordingly, two alternatives can be considered: either to demolish and replace the existing members or to rehabilitate and restore the strength of the structures. The latter alternative, which is preferred economically, involves either strengthening or repairing the member.

The external post-tensioning is an attractive technique for strengthening existing structures. In which it introduce many advantages, like: increasing the load carrying capacity, improving serviceability performance and ease of installation and maintenance.

CRACKING BEHAVIOUR OF REINFORCED CONCRETE BEAMS

Cracking is a disadvantageous phenomenon in concrete structures. Cracks formed in the concrete tension zone when the tensile stress exceeds the low tensile concrete strength. Cracking causes reduction in stiffness of the member that leads to larger curvature value at crack locations (with respect to uncracked location within the constant moment region), and exposing the steel reinforcement to the exterior environment. Cracks in concrete beams can be classified into three types:

1- *Normal-Flexural cracks*: formed due to the effect of the flexural tensile stresses, and mostly located within the middle third of the span. Flexural cracks are formed in the tensile zone and have a wedge shape, with a maximum width at the extreme bottom fiber and zero width at the tip of the crack.

2- *Inclined shear-flexural cracks*: occurs when critical combination of flexural and shear stresses develops near the top of a flexural crack.

3- *Web-Shear cracks*: formed when no flexural cracks are formed and occurred due to shear stresses being higher than flexural stresses in the web portion of the member at region near support. Thinner web encourage this type of cracking.

The formation of each type of cracking depends on the relative stiffness of the member and type of loading. The type of the crack that under consideration in this study is the flexural crack and the other types are not discussed.

Crack Width

Several formulas for prediction of crack width were developed by various investigators. These formulas contains miscellaneous set of variables, where no general agreement among these investigations on the significant variables affecting the crack width. The collected expressions are:

1. **Clark (1956)** :

$$\omega_{\max} = 1.29\left(\frac{\phi(h-d_s)}{10^6 * d_s\rho_s}\right)\left(1.56 f_s - \left(\frac{1}{\rho_s}+8\right)\right) \quad (1)$$

Where h: Total depth of the member; d_s: Depth of the tensile non-prestressed steel reinforcement measured from extreme top fiber; ρ_s: Reinforcement ratio for the tension steel; f_s: Steel stress for the tensile reinforcement; ϕ:Bar diameter

2. **Chi and Kirestein (1958)** :

$$\omega_{\max} = \frac{5\tau\phi}{E_s}\left(f_s - \frac{438}{\tau\phi}\right) \quad (2)$$

Where τ : Coefficient related to the assumed effective concrete area in tension to the area of a single bar; E_s: Modulus of elasticity of steel

3. **Kaar and Mattock (1963)** :

$$\omega_{\max} = 1.57 * 10^{-5} f_s \sqrt[4]{A_t} \quad (3)$$

Where A_t : Effective concrete area in tension

4. **Gergely and Lutz (1968)** :

$$\omega_{\max} = 11.02*10^{-6} \beta . f_s \sqrt[3]{d_c A_e} \quad (4)$$

Where β: Ratio of distances from the extreme bottom fiber and from the steel centroid to the center gravity of the section; d_c: Thickness of concrete cover measured from the extreme bottom fiber to the center of bar closest to that fiber; A_e: Concrete area surrounding one bar, equal to total effective tension area of concrete surrounding reinforcement and having same centriod divided by number of bars

5. **Venkateswarlu and Gesund (1972)** :

$$\omega_{\max} = \frac{2.4*10^{-5} \phi (1462 - f_s)}{(1 + n\rho_s)(662 - f_s)} \quad (5)$$

Where n : Modular ratio

6. **Frotch (1999)** :

$$\omega_{\max} = 2 \frac{f_s}{E_s} \beta \sqrt[2]{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (6)$$

Where s : Maximum bar spacing

7. **Beeby (1979)** :

$$\omega_{\max} = \frac{3 a_{cr} \varepsilon_{my}}{\left(1 + 2 \frac{(a_{cr} - d_{cl})}{(h - c)}\right)} \quad (7)$$

Where ε_{my} : Mean strain of concrete at the selected level; c : Depth of the neutral axis; d_{cl} : Clear concrete cover; a_{cr} : Distance from the bar surface to the point where the crack width is calculated.

A simple comparison is made for the collected expressions of crack width and plotted with respect to the experimental data taken form (**Hong el a. (2008)**) for beam (F30-2D19-10), as shown in **Fig. (1)**. the expression which was proposed by **Gergely and Lutz (1968)** shows good agreement with the experimental data.

STRESS-STRAIN RELATIONSHIP

The model of **Korpenko (1986)** for concrete and steel is adopted in this study; the model takes the following form:

$$\sigma_m = \varepsilon_m E_m \nu_m \quad (8)$$

In which ($E_m \nu_m$) represent the secant modulus of elasticity at the nonlinear portion of the stress-strain curve, while (ν_m) equals to (1) in the linear portion and less than (1) in the nonlinear portion of the stress-strain curve. **Korpenko** derived the following expression for (ν_m):

$$\nu_m^2 \left[1 + \frac{e_{2m}(\nu_o - \hat{\nu}_m)^2 \cdot \tilde{\varepsilon}_m^2}{\hat{\nu}_m}\right] + \left[\hat{\nu}_m^2 - (\nu_o - \hat{\nu}_m)^2\right] - \nu_m \left[2\hat{\nu}_m - e_{1m}\left(\frac{\tilde{\varepsilon}_m (\nu_o - \hat{\nu}_m)^2}{\hat{\nu}_m}\right)\right] = 0 \quad (9)$$

In which

$$\hat{\nu}_m = \frac{\sigma_{peak}}{|\varepsilon_o| E_m} \quad ; \quad \tilde{\varepsilon}_m = \left|\frac{\varepsilon_m}{\varepsilon_o}\right| \quad ; \; e_{2m} = 1 - e_{1m}$$

Where subscript (m) = refers to material.

σ_{peak} : Ultimate strength of material.

ε_o : Material strain corresponding to (σ_{peak}).

E_m : Initial modulus of elasticity.

$\tilde{\varepsilon}_m$: Material strain level.

e_{1m}, e_{2m} : Factors depend on material type.

ν_o : Factor depends on the stress level.

$\hat{\nu}_m$: It is the value of (ν_m) which correspond to the stress (σ_{peak}).

Fig. (2) shows the stress strain diagram for concrete as suggested by **Korpenko**.

MOMENT-CURVATURE MODEL

An iteration method for analysis which adopted by **Oukaili (1997)** is programmed using Matlab 7.0. This method requires section meshing. The Cartesian coordinates for a cross-section is shown in **Fig. (3),** in which positive signs convention are shown for each force. The method is based on the following assumptions:

1. Strain of the concrete and reinforcement is proportional to the distance from the neutral axis in accordance to Bernoulli's hypothesis "Cross-section shall remain plane after bending".

2. Shear and torsion stresses are ignored.

3. The behavior of steel and concrete is considered to follow **Korpenko**'s model, where all stresses in concrete and steel are related to secant modulus of elasticity.

4. Perfect bond exists between concrete and the internal reinforcement, the strain of the ordinary reinforcement due to external load is compatible with the strain of the concrete fiber exists at the center gravity of that reinforcement. Also, the strain increment of the bonded prestressed steel is equal to the concrete fiber strain which exists at its center gravity.

5. Concrete is divided into a group of small cells having sizes related to the required accuracy conditions. The individual steel reinforcement will not be meshed. Thus the reinforcement element acts as a system of linear elements exposed to axial compression or tension.

6. External tendons are not incorporated in this analysis.

the general relation between forces vector $|F|$ $|F|$ strain vector $|\bar{\varepsilon}|$ and stiffness matrix $[C]$ can be expressed as follows:

$$|F| = [C] * |\bar{\varepsilon}| \tag{10}$$

This expression can be detailed as follows:

$$\begin{vmatrix} N \\ M_x \\ M_y \end{vmatrix} = \begin{bmatrix} C_{(1,1)} & C_{(1,2)} & C_{(1,3)} \\ C_{(2,1)} & C_{(2,2)} & C_{(2,3)} \\ C_{(3,1)} & C_{(3,2)} & C_{(3,3)} \end{bmatrix} \begin{vmatrix} \varepsilon_o \\ K_x \\ K_y \end{vmatrix} \tag{11}$$

The elements of the stiffness matrix can be expressed as follows:

$$C_{(1,1)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} + \sum_{i=1}^{p} E_{si} A_{si} \tag{12}$$

$$C_{(1,2)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} y_{ci} + \sum_{i=1}^{p} E_{si} A_{si} y_{si} \tag{13}$$

$$C_{(1,3)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} x_{ci} + \sum_{i=1}^{p} E_{si} A_{si} x_{si} \tag{14}$$

$$C_{(2,2)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} y_{ci}^2 + \sum_{i=1}^{p} E_{si} A_{si} y_{si}^2 \tag{15}$$

$$C_{(3,3)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} x_{ci}^2 + \sum_{i=1}^{p} E_{si} A_{si} x_{si}^2 \tag{16}$$

$$C_{(2,3)} = \sum_{i=1}^{r} \overline{E_{ci}} A_{ci} x_{ci} y_{ci} + \sum_{i=1}^{p} E_{si} A_{si} x_{si} y_{si} \tag{17}$$

The direct iteration method proposed by **Cooke (1981)** is adopted to solve a non-linear problem for determination of the strain vector for a cross-section subjected to known forces.

The evaluation of each element of the stiffness matrix $[C]$ is dependent on secant modulus of elasticity in which it depends on the unknown value of strain in each material. The strain value depends on the strain vector as shown in the following equation:

$$\varepsilon_m = \varepsilon_a + K_x y_m + K_y x_m \tag{18}$$

Where ε_a: Axial strain; K_x: Curvature of the member longitudinal axis in OYZ plane; K_y: Curvature of the member longitudinal axis in OYX plane. In other word, the matrix $[C]$ is function of strain vector. Accordingly eq. (10) can be rewritten in the following form:

$$|\bar{\varepsilon}| = C\left(|\bar{\varepsilon}|\right)^{-1} * |F| \tag{19}$$

In the first iteration all materials in section shall be assumed to be linear and the value of strain vector equal to zero. So the stiffness matrix can be calculated easily. Eq. (18) is used then to evaluate the strain vector resulted from the first iteration. For further iterations, the stiffness matrix will be updated according to the strain vector calculated from the previous iteration as shown below:

$$|\bar{\varepsilon}|_i = C\left(|\bar{\varepsilon}|_{i-1}\right)^{-1} * |F| \tag{20}$$

Where subscript (i): iteration number

The procedure is repeated until the convergence of the load vector satisfies the following condition:

$$\left(|F|_i - |F|_{(i-1)}\right) = \lambda F \tag{21}$$

Where λF : Convergence limit for the force vector which is considered a very small value

LOAD-DEFLECTION MODEL

The method of **Newmark (1943)** is adopted to determine deflection at each node from curvature values at these nodes. This method is based on the conjugate beam method, in which the beam is subjected to a fictitious load equal to (M/EI) which represents the curvature distribution along the beam. Hence the moment and shear values at a location in the conjugate beam represent the slope and deflection; respectively; values at that location in the actual beam. The model based on the following assumptions:

1. The beam is considered prismatic in which the cross-section geometry is same along the beam length. But the change of eccentricity of the internal prestressed tendon will be permitted, because the change in coordinate for the tendons between different locations has a marginal effect on member stiffness.

2. The applied load on the beam is considered symmetric.

3. The end supports of the beam are assumed to be simply supports only.

4. Sign convention for deflection is positive for downward deflection and negative for upward camber.

5. The beam shall be divided into segments of equal length, as shown on **Fig. (3)**.

The (M/EI) curve between two nodes can be represented by second order polynomials (parabola). Accordingly, the following equations can be used for evaluating deflection depending on curvature values determined at each node.

$$\Delta_{(i)} = \sum_{j=2}^{i} \left(\theta_{(j)}\right) * \Delta x \tag{22}$$

$$\theta_{(j)} = \left(\sum_{j=i+1}^{c-1} \overline{K}_{(l)} \right) + \overline{K}_{(l)} / 2 \tag{23}$$

$$\overline{K}_{(l)} = \frac{\Delta x}{12} \left(K_{(l-1)} + 10 K_{(l)} + K_{(l+1)} \right) \tag{24}$$

Where subscript (i) referes to node location

$\Delta_{(i)}$: deflection; $\theta_{(j)}$: rotation; K: curvature

PROPOSED ANALYTICAL METHODOLOGY FOR CLOSING CRACKS IN EXISTING REINFORCED CONCRETE BEAMS

The main goal is to predict the value of the effective prestress force for the external tendons required to close all cracks in an existing beam. The major given data are the maximum crack width, crack spacing and number of cracks, while the corresponding output is the external prestressing force.

Assumptions

1. The beam cross section and the external tendons are considered to be symmetric about a principle axis of the member that is parallel to member's depth, creating no transverse curvature about that axis.

2. The external tendons are not incorporated in the strain compatibility conditions, and they are not compatible with surrounding concrete.

3. The stress increase in the external tendons beyond the effective prestress is member dependent rather than section dependence.

4. The stress and strain in the external tendons are uniform along tendons' length.

5. An idealized beam model is adopted in which. the deflection equals to $\left(\Delta cr_j\right)$ when the beam is subjected to an externally uniform distributed load $\left(Wcr_j\right)$.

6. At the moment when the deflection of the idealized beam attains $\left(\Delta cr_j\right)$, the tensile strain at the extreme bottom fiber attains $\left(\varepsilon_r\right)$, where $\left(\varepsilon_r\right)$the strain is corresponds to the modulus of rupture of concrete $\left(f_r\right)$.

Method of Analysis

Based on the assumptions mentioned above, the analysis will be performed for four separate models:

1- *Existing Beam Model*: Based on the measured crack width $\left(\omega_{\max}\right)$ and eq. (4), the stress of the steel at the extreme bottom layer $\left(f_s\right)$ can be determined. This model deals with the calculated value of $\left(f_s\right)$ and by using the analytical moment-curvature model, the analytical uniformly distributed load $\left(W_{ex}\right)$ can be evaluated. Accordingly, the load $\left(W_{ex}\right)$ is that load which produces a maximum crack width at mid span equal to $\left(\omega_{\max}\right)$.

Also, the depth of the cracks $\left(Ycr_j\right)$ can be estimated from the analysis. In which subscript $\left(j\right)$ refers to crack number as shown in **Fig. (7)**.

Fig. (9) shows the flow chart for the programming procedure to determine $\left(W_{ex}\right)$.

2- *Idealized Beam Model*: Based the analytical moment-curvature model, the beam is subjected to external incremental load to reach $\left(Wcr_j\right)$ for each crack. The external uniformly distributed load $\left(Wcr_j\right)$ produces curvature $\left(K_i\right)$ and deflection $\left(\Delta cr_j\right)$ at location $\left(j\right)$ at this moment, the tensile strain at the extreme bottom fiber attains $\left(\varepsilon_r\right)$.

This model deals with determining $\left(\Delta cr_j\right)$ at location $\left(Xcr_j\right)$ from the support as shown in **Fig. (8)**. **Fig. (10)** shows the flow chart for the programming procedure to determine $\left(\Delta cr_j\right)$.

3- *Strengthening to close cracks:* the beam is subjected to combination of uniformly distributed load $\left(W_{ex}\right)$ and the incremental external prestressing force $\left(\Delta Fcr_j\right)$. The external prestressing force will be increased to attain $\left(Fcr_j\right)$. The value of $\left(Fcr_j\right)$ is the one that close crack number $\left(j\right)$ when the following condition is achieved:

$$\left(\Delta s_j - \Delta cr_j\right) \le \lambda cr$$

Where Δs_j : Deflection value resulted from a combination effect of the load $\left(W_{ex}\right)$ and the prestressing force $\left(Fcr_j\right)$ for crack $\left(j\right)$.

λcr : Convergence limit for the deflection value (taken as 0.005)

It is worth to mention that, the depth of the cracks $\left(Ycr_j\right)$ calculated from the existing beam model were used in this model to express the real stiffness of the beam by excluding concrete cells along the depth of cracks. **Fig. (11)** shows the flow chart for the programming procedure to determine $\left(Fcr_j\right)$.

4- *Optimum Strengthening:* This model is similar to model (3), in which the beam is subjected to combination of uniformly distributed load $\left(W_{ex}\right)$ and the incremental external prestressing force $\left(\Delta F_{st}\right)$. The external prestressing force will be increased to attain $\left(F_{st}\right)$. The value of $\left(F_{st}\right)$ is reach when the following condition is achieved:

$$\left(\varepsilon_{ct,\,j=1} - \varepsilon_r\right) \le \lambda r$$

Where $\varepsilon_{ct,\,j=1}$: Strain of the extreme top fiber at mid span.

λr : Convergence limit for the strain value (taken as 0.005).

It is worth to mention that, the external prestressing force $\left(F_{st}\right)$ used for optimum strengthening is larger than the calculated external prestressing forces $\left(Fcr_j\right)$ to close cracks.

Numerical Applications on the Proposed Methodology

The proposed methodology is implemented on three ideal beam models reinforced with nonprestressed reinforcement under service load, to be strengthened by external prestressed strands to close existing cracks.Three types of external profile for strengthening (straight, draped (one devaitor at mid span) and draped (two deviators at one-third span distance from each support)). The analytical study includes determining the increment percentage in load carrying capacity for the models after strengthening for each chosen profile. Models are:

1. *Model-01*: the geometric and material properties for this beam are shown in **Fig. (4)** and **Table (1)**, respectively. The cracking is described in **Table (3)**.The properties of the external prestressing reinforcement used for strengthening is shown in **Table (2)**.The output results for the beam analysis before and after strengthening is shown in **Fig. (12)**.the increment in load carrying capacity after strengthening is shown in **Table (4)**.

2. *Model-02*: the geometric and material properties for this beam are shown in **Fig. (5)** and **Table (5)**, respectively. The cracking is described in **Table (7)**.The properties of the external prestressing reinforcement used for strengthening is shown in **Table (6)**.The output results for the beam analysis before and

after strengthening is shown in **Fig. (13)**.the increment in load carrying capacity after strengthening is shown in **Table (8)**.

3. *Model-03*: the geometric and material properties for this beam are shown in **Fig. (6)** and **Table (9)**, respectively. The cracking is described in **Table (11)**.The properties of the external prestressing reinforcement used for strengthening is shown in **Table (10)**.The output results for the beam analysis before and after strengthening is shown in **Fig. (14)**.the increment in load carrying capacity after strengthening is shown in **Table (12)**.

CONCLUSION

1. External prestressing is a very effective technique for strengthening existing concrete members, in which it allows to increase the load carrying capacity of the member to (111%) for straight tendon profile, (104%) for draped tendon profile (one deviator at mid span) and (103%) for draped tendon profile (two deviators at one third distance from the support).

2. The calculated external prestressing force that is required to close cracks for the existing concrete members is found to be ((0.41) for straight tendon profile, (0.48) for draped tendon profile (one deviator at mid span) and (0.46) for draped tendon profile (two deviators at one third distance from the support)) of the calculated external prestressing force required for optimum strengthening.

3. Strengthening using the straight tendon profile requires higher prestressing force than the draped tendon profile by about (62%).

4. The empirical formula of Gergely **and lutz (1968)** for calculating the maximum crack width shows a very good agreement with the experimental data than others.

REFERENCE

- ***ACI Committee 318***, “Building Code requirements for Reinforced Concrete (ACI 318-2008),” American Concrete Institute.

- ***Alhawwassi I. F.,*** “Short term deflection of Ordinary, partially prestressed and GFRB Bars reinforced Concrete Beams” M.Sc thesis, Baghdad university, Iraq, 2008.

- ***Beeby, A. W.,*** “The Prediction of Crack width in Hardened Concrete Cracking” Journal of Structural Engineering, Vol. 57, No. 1, pp 9-17, 1979.

- ***Chi, M. and Kirestein, A. F.*** , “Flexural Cracks in reinforced Concrete Beams” ACI Structural journal, Proceeding, Vol. 54, No. 10, pp 865-878, 1958.

- ***Clark, A. P.***, “Cracking in reinforced Concrete Flexural members” ACI Structural journal, Proceeding, Vol. 52, No. 8, pp 851-862, 1956.

- ***Frotch R.***, “Another Look at Cracking Crack Control in Reinforced Conrete” ACI Structural journal, Vol.96, No. 3, pp437-442, 1999.

- ***Gergely, P. and lutz, L. A.***, “Maximum crack width in Reinforced Concrete, Causes, Mechanism, and Control of Cracking Concrete”, SP-20, ACI Structural journal, pp87-177, 1968.

- ***Hong Sung Nam, Han Kyoung Bong, Kim Tae Wan, Beak Kyeong Seok , Park Sun Kye, and Ko Won Jun***, “ Estimation of Flexural Crack Width in Reinforced Concrete Members” ,The 3rd ACF international conference – ACF/VCA 2008.

- ***Kaar , P. H., and Mattock, A. H.*** ” High Strength Bar as Concrete Reinforcement ’’, Part 4, Control of Cracking, Journal of Portland Cement Association Research and Development Laboratories , Vol. 7, No.1, pp 42-5.

- ***Korpenko, N. I.***, ***Mukhamediev, T.A.*** and ***Petrov, A. N.***, “The Initial and Transformed Stress - Strain Diagrams of Steel and Concrete. “ Special Publication, Stress-Strain Condition for Reinforced Concrete Construction, Reinforced Concrete Research Center, Moscow, 7 – 25, 1986.

- ***Newmark, N. M.,*** "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads", Transactions, ASCE, 1-8, 1943.

- ***Oukaili, Nazar K. Ali***, “Strength of Partially Prestressed Concrete Elements with Mixed Reinforcement by Highly Strength Strands and Steel Bars", PH.D. Thesis, Moscow Civil Engineering University, Moscow, 1991.

- ***Oukaili, Nazar K. Ali***, "Moment Capacity and Strength of Reinforced Concrete Members Using Stress-Strain Diagrams of Concrete and Steel", Journal of King Saud University, Vol. 10, pp. 23-44, 1997.
- ***Venkateswarlu, B. and Gesund, H.***, "Cracking and Bond Slip in Concrete Beams" Journal of Structural Engineering, ASCE, Vol. 98, No. ST11, pp 2663-2885, 1972.
- ***Cook R.D.***, "Concept and Application of Finite Element Analysis", Second Edition, John Wiley and Sons, New York, 1981.

NOTATION

A_{ci}: Distance from the center of gravity to the concrete cell (i).

A_e: Concrete area surrounding one bar, equal to total effective tension area of concrete surrounding reinforcement and having same centriod divided by number of bars.

A_{si}: Area of the internal reinforcement for bar (i).

A_t: Effective concrete area in tension.

a_{cr}: Distance from the bar surface to the point where the crack width is calculated.

c: Depth of the neutral axis.

d_s: Depth of the tensile non-prestressed steel reinforcement measured from extreme top fiber.

d_c: Thickness of concrete cover measured from the extreme bottom fiber to the center of bar closest to that fiber.

d_{cl}: Clear concrete cover.

E_m: Modulus of elasticity of material (m).

E_s: Modulus of elasticity of steel.

$\overline{E}$: Secant modulus of elasticity.

e_{1m}, e_{2m} : Factors depend on material type.

Fcr_j: External prestressing force for closing crack (j).

F_{st}: External prestressing force for optimum strengthening.

f_s: Steel stress for the tensile reinforcement.

f_r : Modulus of rupture of the concrete.

h: Total depth of the member.

K_x : Curvature of the member longitudinal axis in OYZ plane.

K_y: Curvature of the member longitudinal axis in OYX plane.

M_x : Bending moment about x-axis.

M_y : Bending moment about y-axis.

N : Normal force.

n : Modular ratio.

s : Maximum bar spacing.

x_{ci}, y_{ci}: Distance to the center of gravity of concrete cell (i).

x_{si}, y_{si}: Distance to the center of gravity of non-prestrssed steel for bar (i).

W_{ex}: Applied uniform load for beam in service.

Wcr_j: Idealized cracking load.

Ycr_j : Crack depth for crack (i).

β: Ratio of distances from the extreme bottom fiber and from the steel centroid to the center gravity of the section.

Δ: Deflection.

θ: Rotation.

Δcr_j: Deflection at cracking for crack (i).

ε_a: Axial strain.

ε_m: Strain in material.

ε_s: Strain of the tensile reinforcement.

ε_{my}: mean strain of concrete at the selected level.

ε_r: Strain correspond to (f_r).

ε_{ct}: Strain in the concrete top fiber at ultimate

ε_o: Material strain corresponding to (σ_{peak}).

$\tilde{\varepsilon}_m$: Material strain level.

ν_m: Material elastic modulus factor that expresses the ratio of elastic strains to the total strains.

ν_o: Factor depends on the stress level.

$\hat{\nu}_m$: It is the value of (ν_m) which correspond to the stress (σ_{peak}).

ρ_s : Reinforcement ratio for the tension steel.

σ_m : Stress in material.

σ_{peak} : Ultimate strength of material.

ϕ : Bar diameter

$\omega_{\max}$: Maximum crack width at extreme bottom fiber.

τ : Coefficient related to the assumed effective concrete area in tension to the area of a single bar.

$|F|$: Force vector.

$[C]$: Stiffness matrix.

$|\bar{\varepsilon}|$: Strain vector.

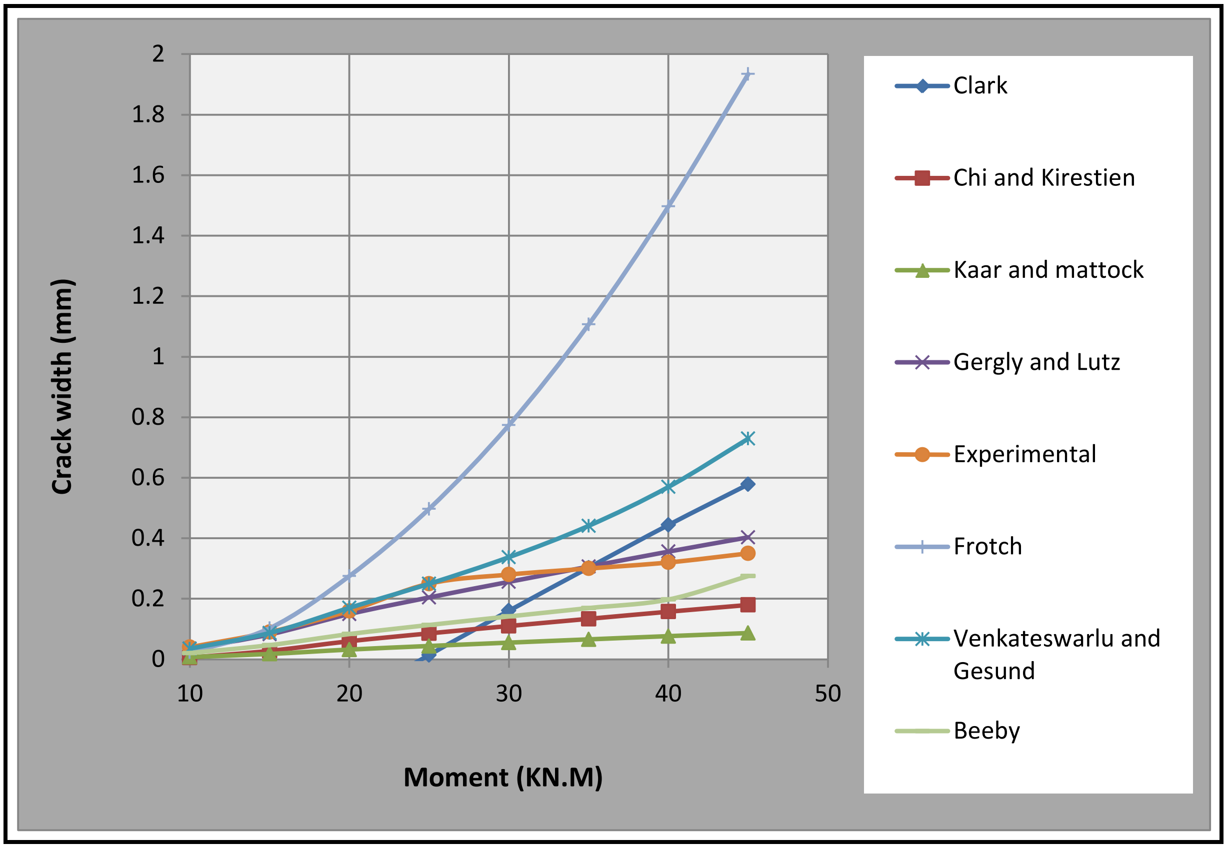


Figure (1) Mid span moment - Maximum crack width

(Hong el a. (2008) - beam (F30-2D19-10))

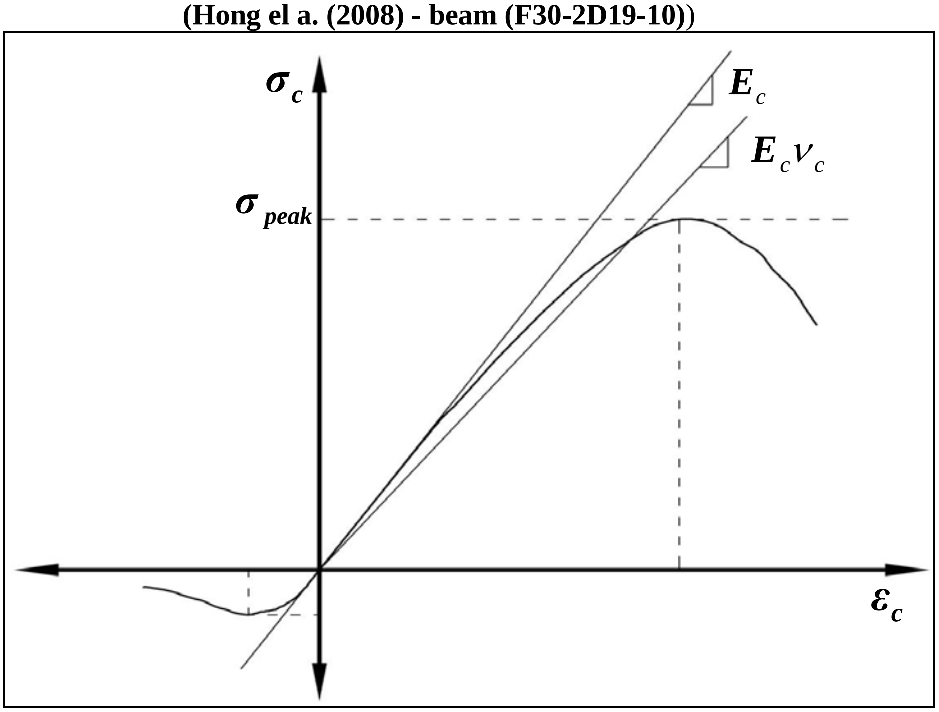


Figure (2) Stress-Strain diagram for concrete

(**Korpeko (1986)**)

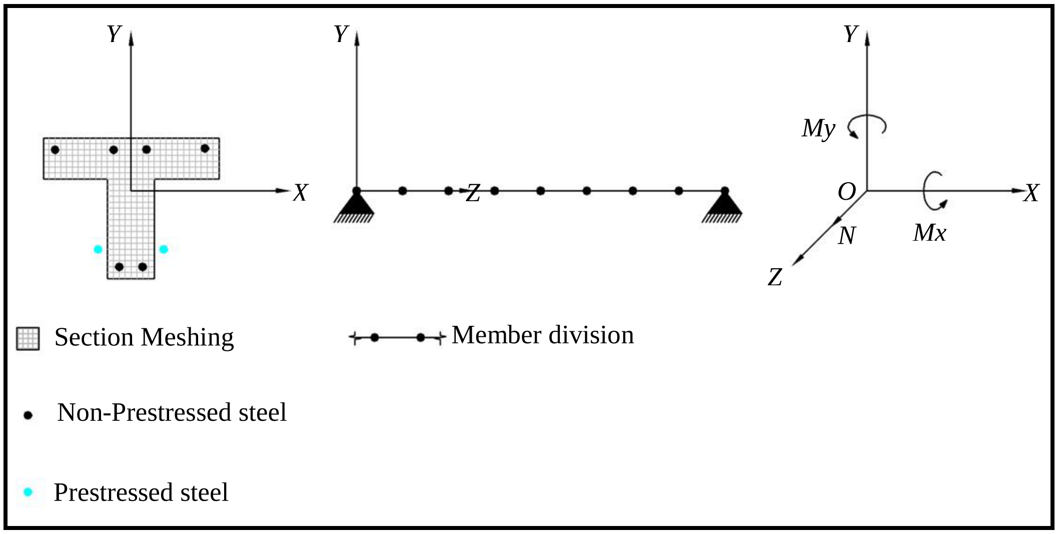


Figure (3) Sign Convention for internal forces

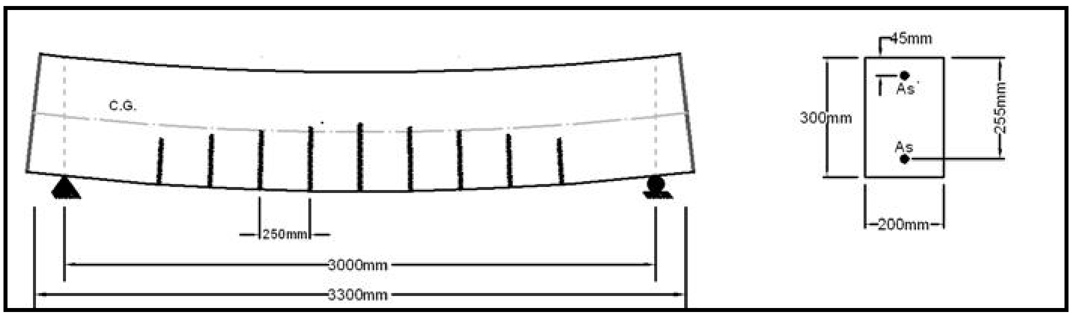


Figure (4) Geometric properties [Model-01]

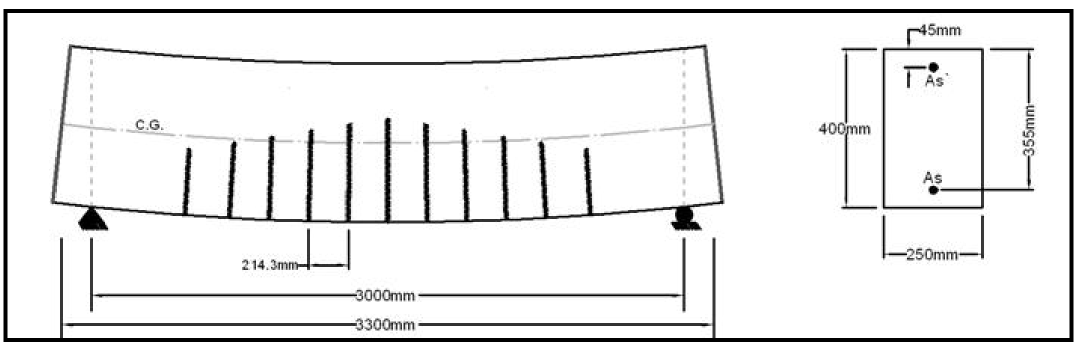


Figure (5) Geometric properties [Model-02]

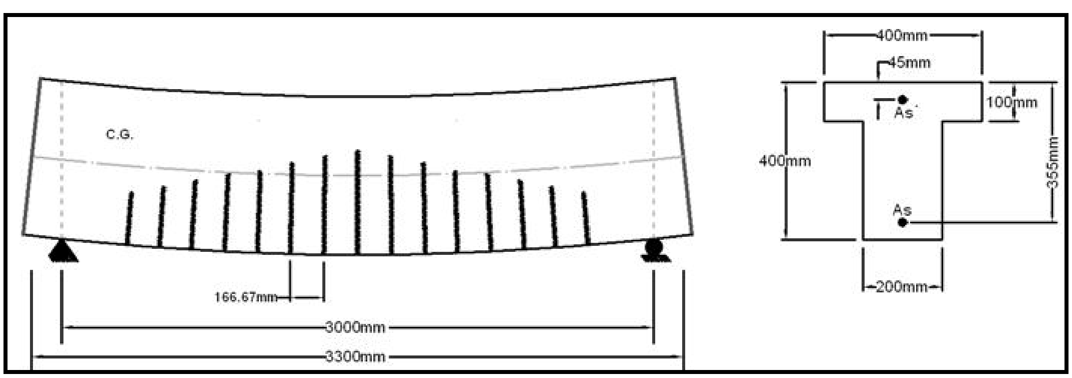


Figure (6) Geometric properties [Model-03]

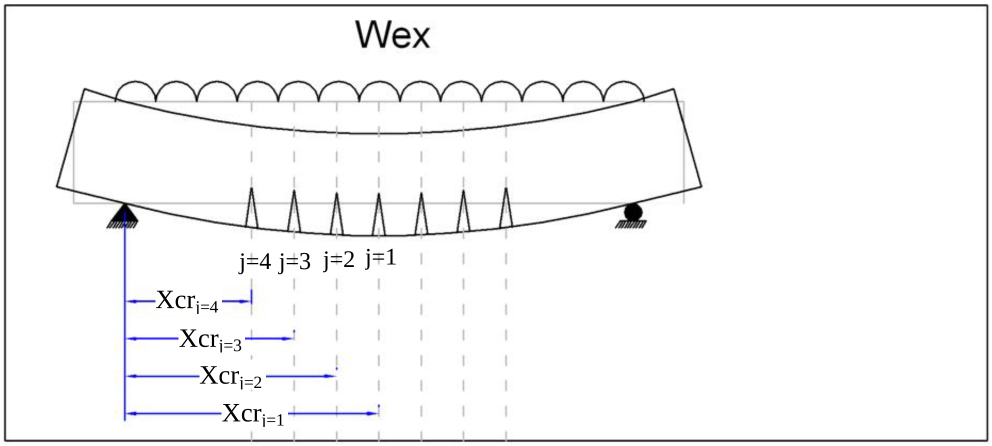


Figure (7) Existing beam model

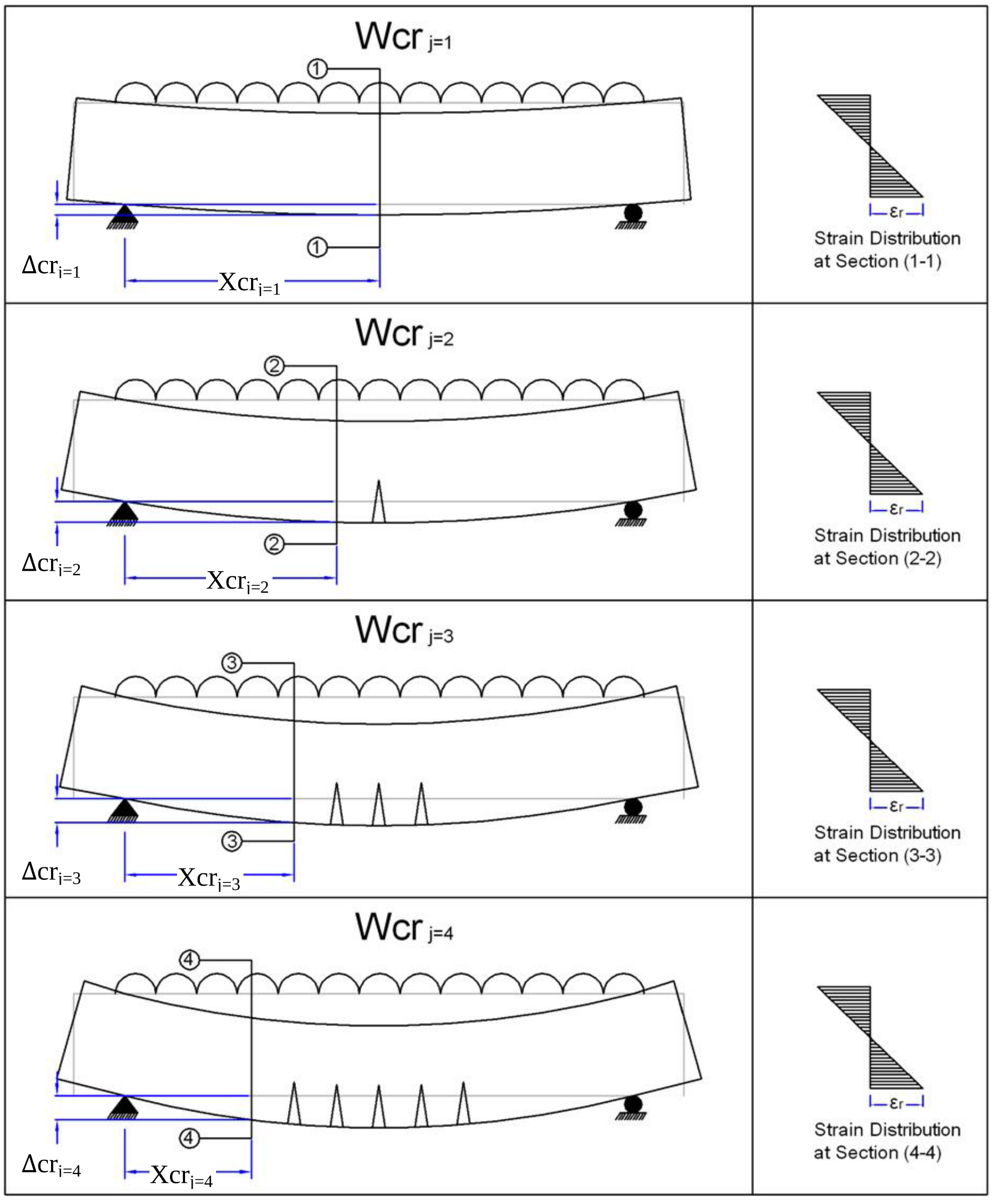


Figure (8) Idealized beam model

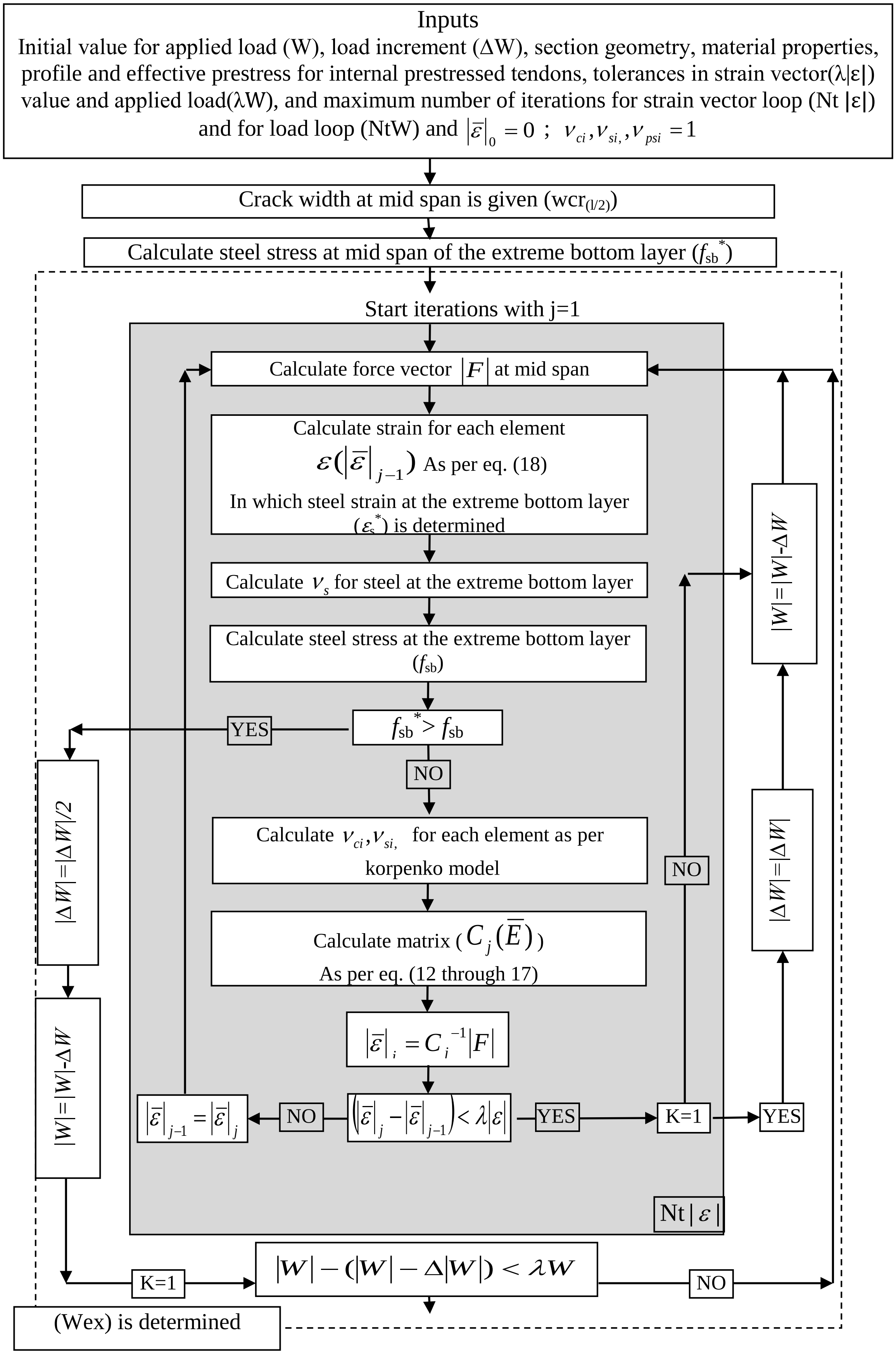


Figure (9) Flow chart for the proposed existing beam model

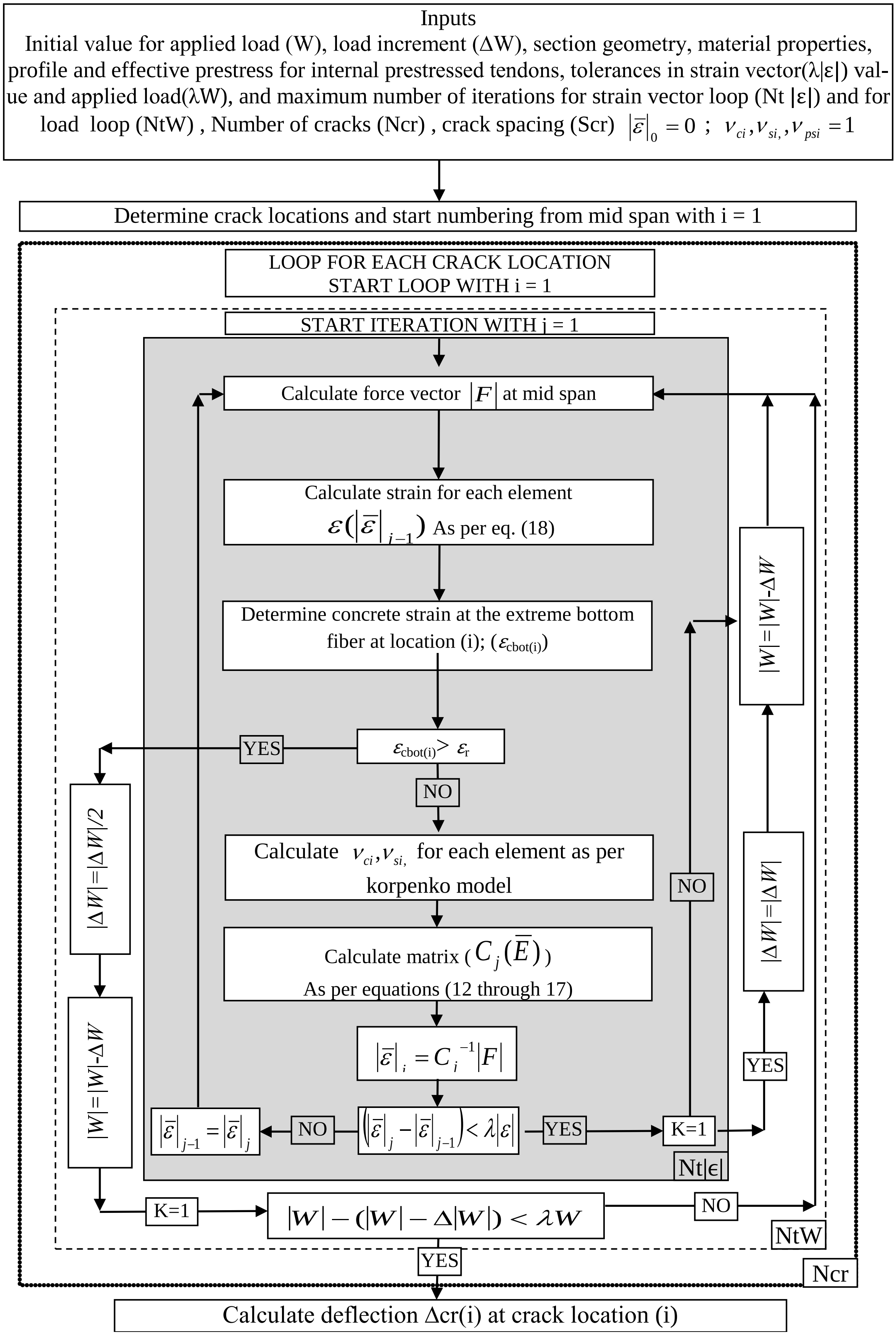


Figure (10) Flow chart for the proposed idealized beam model

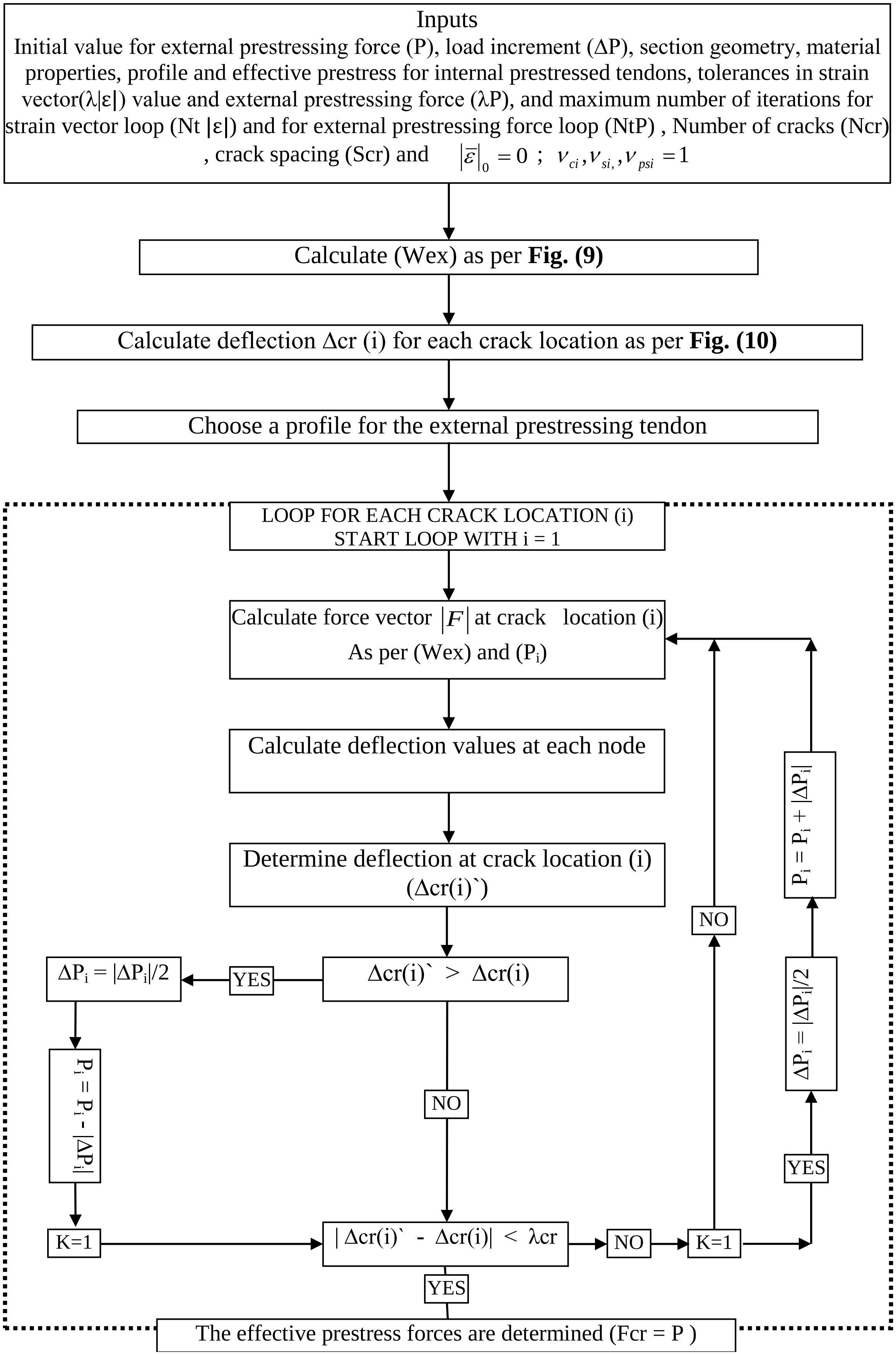


Figure (11) Flow chart for the strengthened beam model

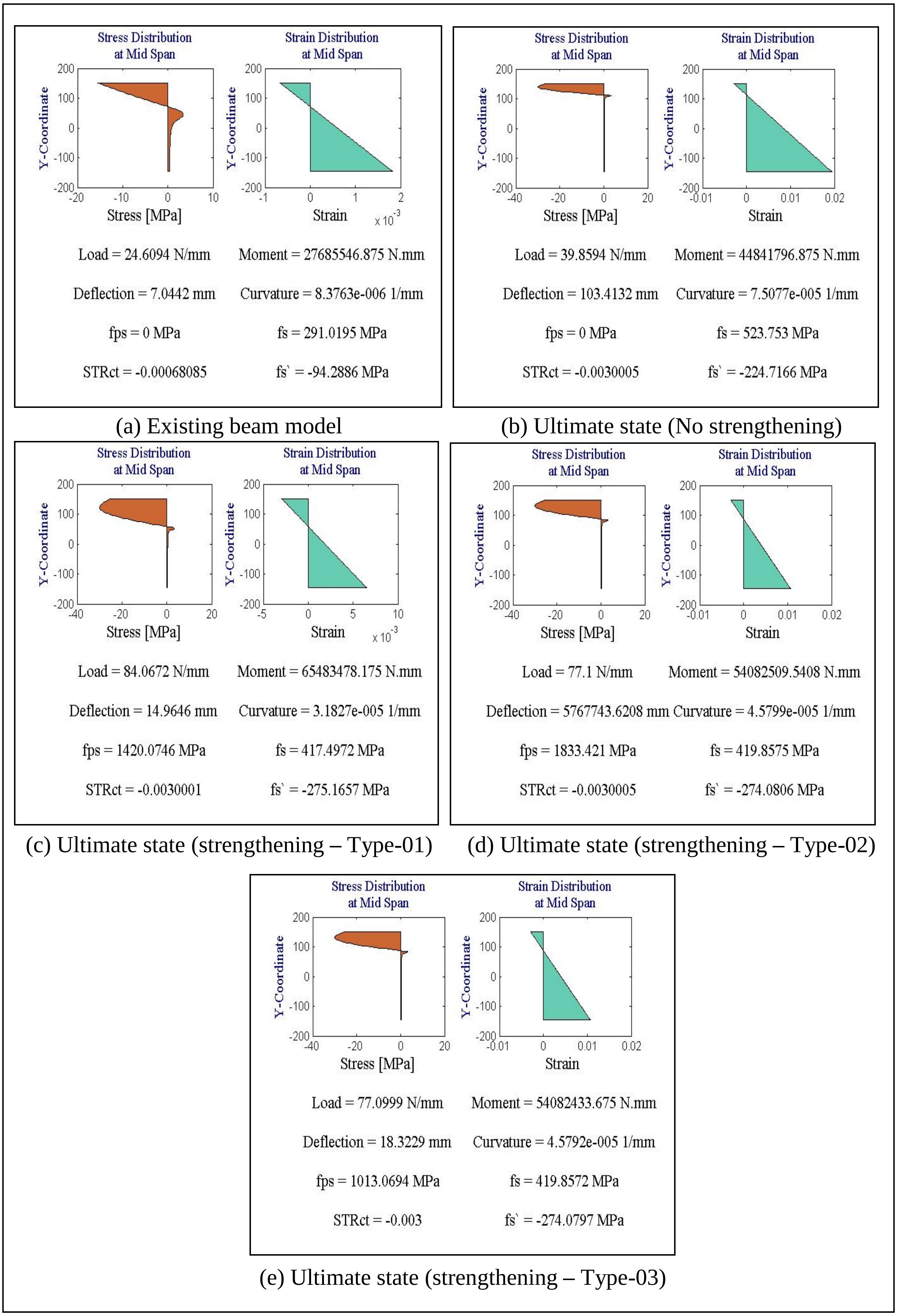
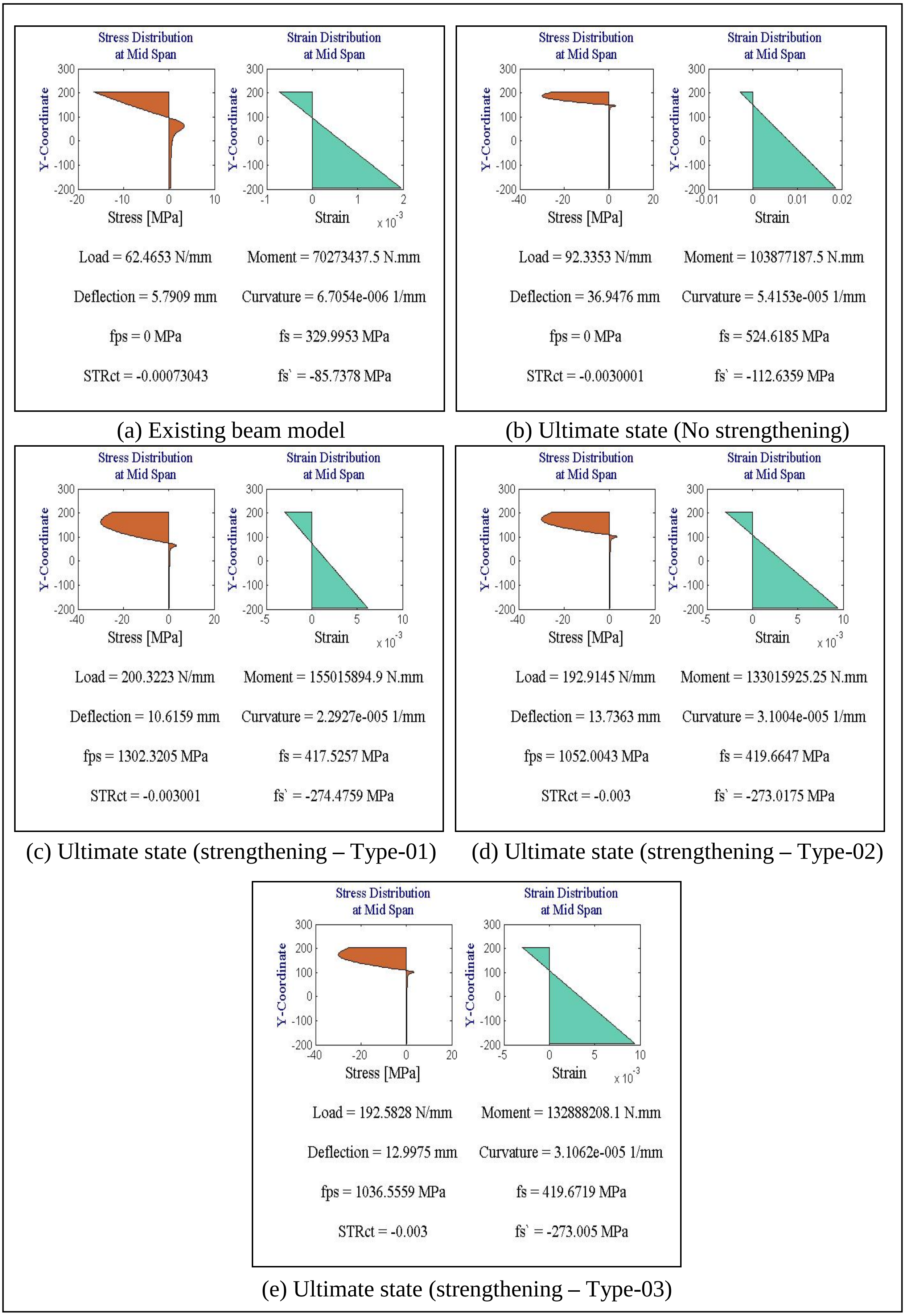


Figure (12) Output results for beam model-

Note : STRct stand for the strain of concrete at extreme top fiber



(a) Existing beam model

(b) Ultimate state (No strengthening)

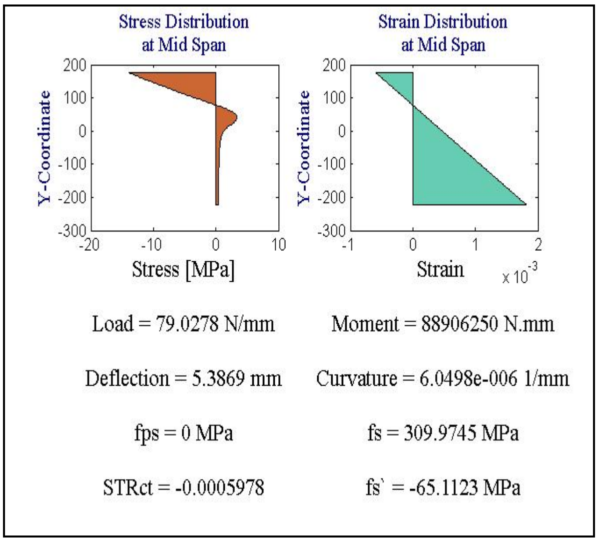
(c) Ultimate state (strengthening – Type-01)

(d) Ultimate state (strengthening – Type-02)

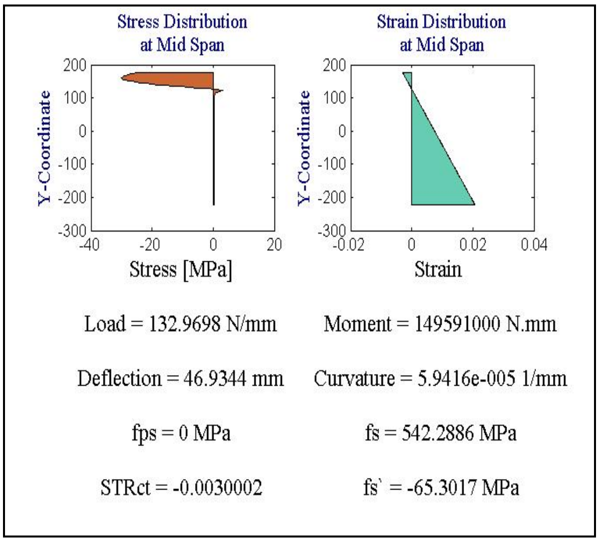
(e) Ultimate state (strengthening – Type-03)

Figure (13) Output results for beam model-

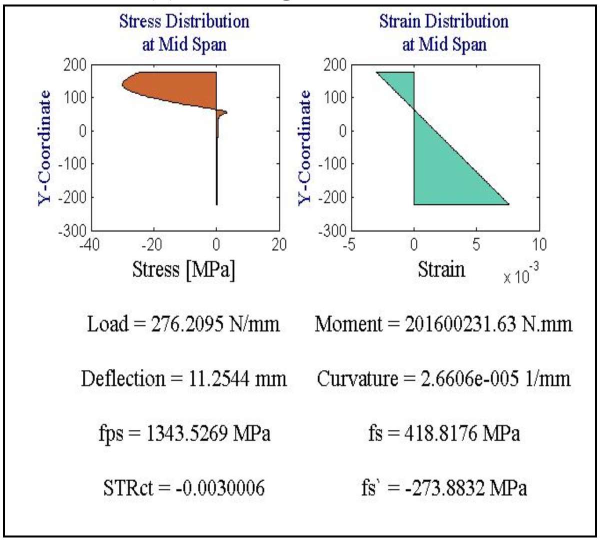
Note : STRct stand for the strain of concrete at extreme top fiber



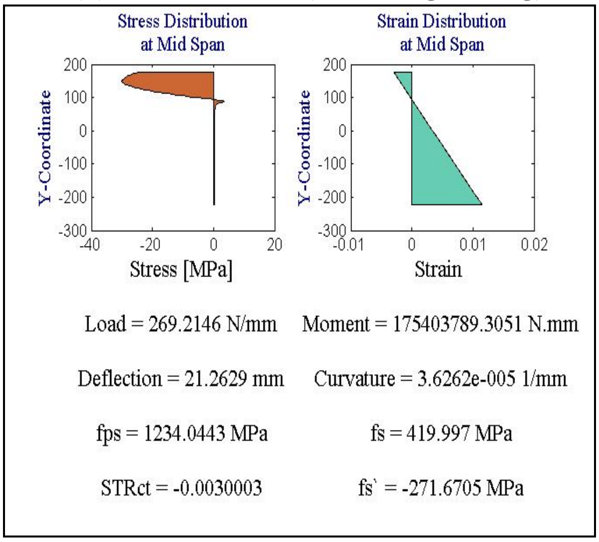
(a) Existing beam model



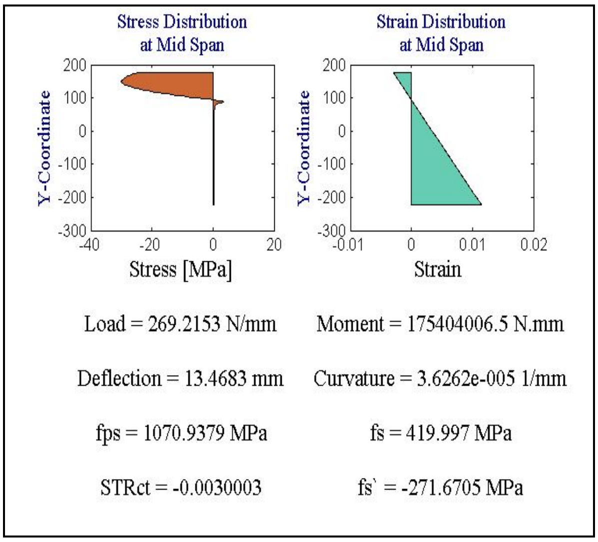
(b) Ultimate state (No strengthening)



(c) Ultimate state (strengthening – Type-01)



(d) Ultimate state (strengthening – Type-02)



(e) Ultimate state (strengthening – Type-03)

Figure (14) Output results for beam model-

Note : STRct stand for the strain of concrete at extreme top fiber

Table (1) Material properties [Model-01]

Concrete Properties				
Ultimate Compressive Strength (f'_c) MPa	Modulus of Elasticity (E_c) MPa	Strain corresponds to (f'_c) (ε_o)	Ultimate Compressive Strain (ε_{cu})	Modulus of Rupture (f_r) MPa
30.00	25923.7	0.0020	0.003	3.41

Internal Steel Properties (Nonprstressed)					
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Depth [mm]
420	280	200000	0.20	100	45
620	420	200000	0.125	339	255

Table (2) External tendon properties used for strengthening [Model-01]

External Prestressed Strand Properties						
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Prestress MPa	Effective Depth [mm]
1860	1625	180000	0.05	231	As per prestressing force	As per profile

Table (3) Cracking description in existing beam [Model-01]

Crack width at mid span (thousandth mm)	Crack Spacing (mm)	No of cracks
431	250	9

Table (4) Prestressing forces for different external prestressing profiles [Model-01]

Tendon profile					Prestressing force to close all cracks (kN)	Prestressing force for optimum strengthening (kN)	Ultimate load for the strengthened beam (kN/m)	Increment percentage in load carrying Capacity (%)
Type	e_s (mm)	e_d (mm)	No. of deviators	x_d (mm)				
1	98.2	-	-	-	122.656	296.25	84.07	110.9
2	98.2	197.3	1	1500	85.937	165.87	77.1	93.43
3	98.2	197.3	2	1000 - 2000	81.250	165.51	77.09	93.4

Table (5) Material properties [Model-02]

Concrete Properties				
Ultimate Compressive Strength (f'_c) MPa	Modulus of Elasticity (E_c) MPa	Strain corresponds to (f'_c) (ε_o)	Ultimate Compressive Strain (ε_{cu})	Modulus of Rupture (f_r) MPa
30.00	25923.7	0.0020	0.003	3.41

Internal Steel Properties (Nonprstressed)					
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Depth [mm]
420	280	200000	0.20	226	45
620	420	200000	0.125	565	355

Table (6) External tendon properties used for strengthening [Model-02]

External Prestressed Strand Properties						
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Prestress MPa	Effective Depth [mm]
1860	1625	180000	0.05	462	As per prestressing force	As per profile

Table (7) Cracking description in existing beam [Model-02]

Crack width at mid span (thousandth mm)	Crack Spacing (mm)	No of cracks
475	214.3	11

Table (8) Prestressing forces for different external prestressing profiles [Model-02]

Tendon profile					Prestressing force to close all cracks (kN)	Prestressing force for optimum strengthen-ing (kN)	Ultimate load for the strengthened beam (kN/m)	Increment percentage in load carrying Capacity (%)
Type	e_s (mm)	e_d (mm)	No. of deviators	x_d (mm)				
1	129.8	-	-	-	232.031	541.96	200.3	116.92
2	129.8	244.7	1	1500	170.703	343.33	192.91	108.93
3	129.8	244.7	2	1000 - 2000	164.453	342.3	192.58	108.56

Table (9) Material properties [Model-03]

Concrete Properties				
Ultimate Compressive Strength $(f_c^{'})$ MPa	Modulus of Elasticity (E_c) MPa	Strain corresponds to $(f_c^{'})$ (ε_o)	Ultimate Compressive Strain (ε_{cu})	Modulus of Rupture (f_r) MPa
30.00	25923.7	0.0020	0.003	3.41

Internal Steel Properties (Nonprstressed)					
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Depth [mm]
420	280	200000	0.20	339	45
620	420	200000	0.125	800	355

Table (10) External tendon properties used for strengthening [Model-03]

External Prestressed Strand Properties						
Ultimate Tensile Strength (f_u) MPa	Yield Strength (f_y) MPa	Modulus of Elasticity (E_s) MPa	Ultimate Tensile Strain (ε_{ult})	Area [mm^2]	Effective Prestress [Mpa]	Effective Depth [mm]
1860	1625	180000	0.05	616	As per prestressing force	As per profile

Table (11) Cracking description in existing beam [Model-03]

Crack width at mid span (thousandth mm)	Crack Spacing (mm)	No of cracks
400	166.67	15

Table (12) Prestressing forces for different external prestressing profiles [Model-03]

Tendon profile					Prestressing force to close all cracks (KN)	Prestressing force for op-timum strengthening (KN)	Ultimate load for the strengthened beam (KN/m)	Increment percentage in load carrying Capacity (%)
Type	e_s (mm)	e_d (mm)	No. of deviators	x_d (mm)				
1	148.94	-	-	-	293.750	732.75	276.21	107.72
2	148.94	273.4	1	1500	211.718	467.82	269.21	102.46
3	148.94	273.4	2	1000 - 2000	201.562	466.22	269.22	102.47