THE AXISYMMETRIC DYNAMICS OF ISOTROPIC CIRCULAR PLATES WITH VARIABLE THICKNESS UNDER THE EFFECT OF LARGE AMPLITUDES

Dr. Ahmed A. Al-Rajihy
College of Eng., University of Kerbalaa

ABSTRACT

This paper presents a study of the geometrically non-linear vibrations of clamped circular plates with variable thickness by taking the effect of large amplitude motion. The maximum thickness is considered to be at the plate center and it is taken to be twice the value of thickness at the edge. The problem is solved by the numerical iteration procedure to obtain the results of vibration amplitudes up to twice the maximum plate thickness. The results are presented for the first two modes of vibration. The obtained results indicate that increasing the ratio of thickness has the effect of increasing the nonlinear frequency and modify the corresponding mode shape.

الخلاصة

في هذ البحث تمت دراسة الإ هتزازات اللا خطية لصفيحة دائرية محكمة الإسناد على المحيط ، ذات سمك متغير وذلك بأخذ تأثير السعة العالية على الأهتزاز . لقد تم إعتبار أقصى سمك عند مركز الصفيحة ومساويا لضعف السمك عند الحافة . لقد تم إستخدام طريقة التكرار العددي للحصول على النتائج وذلك بأعتبار أن الأزاحة مسوية لضعف السمك الأقصى ، وأخذت النتائج للنسقين الأوليين فقط .لقد بينت النتائج أن زيادة نسبة السمك (أقصى سمك الى السمك عند الحافة) يزيد من قيمة الذبذبة اللاخطية ويؤثر على شكل الموجة التابعة للنسق .

KEYWORDS: Non-Linear Vibration, Circular Plate, Variable Thickness, Large Amplitudes

INTRODUCTION

Thin plates are used in various modern engineering problems and they are often subjected to severe dynamic loading. In some cases this may result in large amplitudes vibration which leads to a behavior different from that predicted by the classical linear theory. Thus it is necessary to include the geometrical non-linearity. In the literature, the Von Ka'rma'n relations is the most widely used. The governing equations are coupled non-linear partial differential equations of motion. Also no general and symmetric approach to nonlinear problems is available which allows all or most of the various non-linear effects to be described in a unified manner **(Benamar 1990)**.

In the study of geometrically non-linear axi-symmetric vibrations of clamped circular plates, the common approach has been to use an assumed space or time mode. The different methods of solution used in the literature related to the subject of interest have been presented in **(Benamar 1991)**. In the very recent works, the finite element method has been applied to study the nonlinear vibrations of hinged orthotropic circular plates with a concentric rigid mass using Von Ka'rma'n equations **(Huang 1998)** and geometrically nonlinear free vibrations of polar orthotropic circular plates with various boundary conditions, using the three-dimensional elasticity theory with all of the non-linear terms retained in the strain expressions **(liu 1996)**. If the single mode approach is used, this approach is not completely useful for studying the geometrically non-linear vibration of thin structures, therefore multimode analyses are used.

In the present paper the nonlinear vibration of a clamped circular plate with linearly varied thickness is studied taking both the in-plane and the transverse motions into account. The method of solution depends on the explicit approach. This approach is based on the linearization of the set of algebraic equations in the neighbourhood of each resonance.

MATHEMATICAL ANALYSIS

The plate is considered to have a radius R and variable thickness h clamped along its edge. The variation of the plate thickness is assumed as linear. The origin of the coordinate system is taken at the center of the plate, as shown in **Fig. 1**.

The plate is assumed to be elastic with homogeneous isotropic mechanical properties. For circular plate having large amplitude vibrations, the strains are given by the following equation **(Hung, 1971)** :

$$\varepsilon_r = \frac{\partial U}{\partial r} + \frac{1}{2}\left(\frac{\partial W}{\partial r}\right)^2 - z\frac{\partial 2W}{\partial r^2}$$
$$\varepsilon_\theta = \frac{U}{r} - \frac{z}{r}\frac{\partial W}{\partial r} \qquad (1)$$

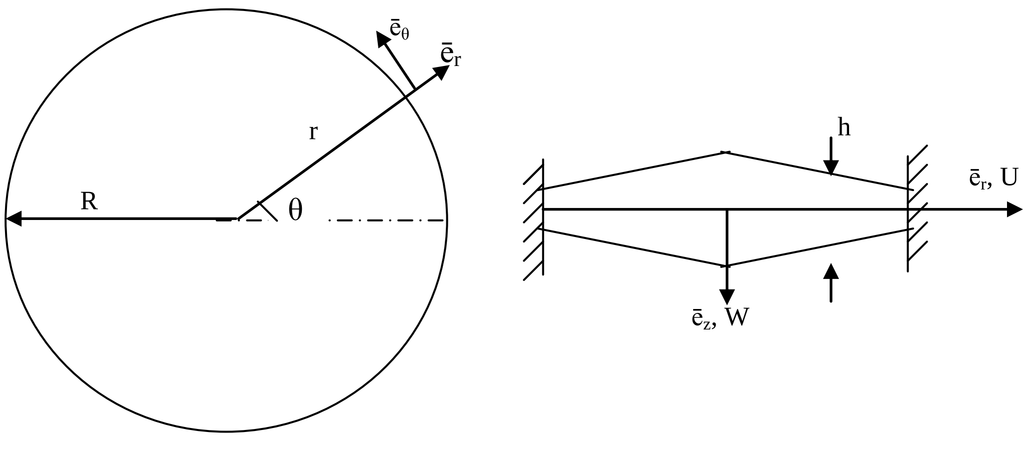

Fig. 1: Plate and Coordinate Notation

In large amplitude vibration, the strain energy is the sum of strain energy due to bending and the strain energy due to membrane, that is:

$$V = V_b + V_m \qquad (2)$$

the bending strain energy of the clamped circular plate with axi-symmetric vibrations is **(Haterbouch 2003)** :

$$V_b = \pi D \int_0^a \left\{ \frac{\partial 2W}{\partial r^2} + \frac{1}{r^2}\left(\frac{\partial W}{\partial r}\right)^2 \right\} r dr \tag{3}$$

where, D=Eh^3/ 12(1-ν^2). The membrane strain energy of the circular plate is given by **(Timoshinko 1959**) :

$$V_m = \frac{12\pi D}{h^2} \int_0^a \left\{ \left(\frac{\partial U}{\partial r}\right)^2 + \frac{U^2}{r^2} + 2\nu \frac{U}{r}\frac{\partial U}{\partial r} + \left(\frac{\partial W}{\partial r}\right)^2 \frac{\partial U}{\partial r} + \frac{1}{4}\left(\frac{\partial W}{\partial r}\right)^4 + \nu \frac{U}{r}\left(\frac{\partial W}{\partial r}\right)^2 \right\} r dr \tag{4}$$

Now the total strain energy is:

$$V = \pi D \int_0^a \left\{ \frac{\partial 2W}{\partial r^2} + \frac{1}{r^2}\left(\frac{\partial W}{\partial r}\right)^2 \right\} r dr +$$
$$\frac{12\pi D}{h^2} \int_0^a \left\{ \left(\frac{\partial U}{\partial r}\right)^2 + \frac{U^2}{r^2} + 2\nu \frac{U}{r}\frac{\partial U}{\partial r} + \left(\frac{\partial W}{\partial r}\right)^2 \frac{\partial U}{\partial r} + \frac{1}{4}\left(\frac{\partial W}{\partial r}\right)^4 + \nu \frac{U}{r}\left(\frac{\partial W}{\partial r}\right)^2 \right\} r dr ....(5)$$

The kinetic energy of the circular plate with neglecting the rotary inertia is:

$$T = \pi \rho h \int_0^a \left\{ \left(\frac{\partial W}{\partial t}\right)^2 + \left(\frac{\partial U}{\partial t}\right)^2 \right\} r dr \tag{6}$$

The most common approach in seeking an approximate solution of geometrically non-linear vibration is by separation of space and time functions. The transverse displacement function is:

$$W(r,t) = w(r)\cos(\omega t)$$

(7)

and the in-plane radial displacement is given by **(Haterbouch 2004)** :

$$U(r,t) = u(r)\cos^2(\omega t) \tag{8}$$

The spatial functions u(r) and w(r) are expanded in the form of finite series of p_i and p_o in-plane u_i(r) and transverse motion w_i(r) basic functions, respectively as:

$$w(r) = a_i w_i(r),$$
$$u(r) = b_i u_i(r) \tag{9}$$

Now the discretized forms for the total strain and kinetic energies are:

$$V = \frac{1}{2} a_i a_j k_{ij}^{\;1} \cos^2(\omega t) + \frac{1}{2}\left[a_i a_j a_k a_l b^1{}_{ijkl} + a_i b_j k^2{}_{ij}\right]\cos^4(\omega t) \tag{10}$$

$$T = \frac{1}{2}\omega^2[a_i a_j m_{ij}^1 \sin^2(\omega t) + b_j b_j m^2{}_{ij} \sin^2(2\omega t)]$$
(11)

where, $m^1{}_{ij}$, $m^2{}_{ij}$, $k^1{}_{ij}$, $k^2{}_{ij}$ are the mass and stiffness tensors associated with W and U respectively, and b_{ijkl} & c_{ijk} are fourth order and third order non-linearity tensors respectively. These tensors are:

$$m^1{}_{ij} = 2\pi\rho h\int_0^a w_i w_j r dr \quad (12a)$$

$$m^2{}_{ij} = 2\pi\rho h\int_0^a u_i u_j r dr \quad (12b)$$

$$k^1{}_{ij} = 2\pi\rho h\int_0^a \left(\frac{d2w_i}{dr^2}\frac{d2w_j}{dr^2} + \frac{1}{r^2}\frac{dw_i}{dr}\frac{dw_j}{dr} \right) r dr \quad (12c)$$

$$k^2{}_{ij} = \frac{24\pi D}{h^2}\int_0^a \left(\frac{du_i}{dr}\frac{du_j}{dr} + \frac{1}{r^2}u_i u_j + \frac{\nu}{r}\frac{d_i}{dr}u_j + \frac{\nu}{r}u_i\frac{du_j}{dr} \right) r dr \quad (12d)$$

$$c_{ijk} = \frac{24\pi D}{h^2}\int_0^a \left(\frac{dw_i}{dr}\frac{dw_j}{dr}\frac{du_k}{dr} + \frac{\nu}{r}\frac{dw_i}{dr}\frac{dw_j}{dr}u_k \right) r dr \quad (12e)$$

$$b^1{}_{ijkl} = \frac{6\pi D}{h^2}\int_0^a \left(\frac{dw_i}{dr}\frac{dw_j}{dr}\frac{dw_k}{dr}\frac{dw_l}{dr} \right) r dr \quad (12f)$$

Hamilton's principle is powerful to govern the dynamics of structures, which is written in its general symbolic form as:

$$\delta\int_0^{2\pi}(V - T)dt = 0 \quad (13)$$

Substituting **Eq. (5)** and **(6)** into **Eq. (13)** and after integrating the time functions and differentiating with respect to a_i's & b_i's results the following set of non-linear algebraic equations:

$$2a_i k^1{}_{ir} + 3a_i a_j a\ a_k b^1{}_{ijkr} + \frac{3}{2}a_i b_k c_{irk} - 2\omega^2 a_i m^1{}_{ir} = 0, \ldots\ldots. r = 1, \ldots. p_o \quad (14a)$$

$$\frac{3}{4}(a_i a_j c_{ijs} + 2b_i k^2{}_{is}) - 2\omega^2 b_i m^2{}_{is} = 0, \ldots\ldots\ldots s = 1, \ldots\ldots\ldots p_i \quad (14b)$$

In order to generalize the analysis, the following non-dimensional displacements may be used;

$$r^* = \frac{r}{R}, ..w^*{}_i(r^*) = \frac{w_i(r)}{h_0}, ...u^*{}_i(r^*) = \frac{w_i(r)}{\lambda h_0} \quad (15)$$

where, $\lambda = \frac{h_o}{R}$

Now **Eq. (14)** may be written to take the form:

$$2a_i k^{1*}{}_{ij} + 3a_i a_j a_k b^{1*}{}_{ijkr} + \frac{3}{2} a_i b_k c^*{}_{irk} - 2\omega^{*2} a_i m^{1*}{}_{ir} = 0, ...r = 1,....p_o \qquad (16)$$

$$\frac{3}{4}(a_i a_j c^*{}_{ijs} + 2b_i k^{2*}{}_{is}) - 2\lambda^2 \omega^{*2} b_i m^{2*}{}_{is} = 0, ...s = 1, ...p_i$$

where ω^* is the non-dimensional non-linear frequency, which is defined by:

$$\omega^{*2} = \frac{\rho h_o R^4 \omega^2}{D} \qquad (17)$$

The dimensional terms in (12) may be written in non-dimensional forms as:

$$m^{1*}{}_{ij}, m^{2*}{}_{ij}) = \frac{1}{2\pi\rho R^2 h_o{}^3}(m^1{}_{ij}, m^2{}_{ij} l\lambda^2),$$

(18)

$$(k^{1*}{}_{ij}, k^{2*}{}_{ij}, c^*{}_{ijk}, b^{1*}{}_{ijkl}) = \frac{R^2}{2\pi D h_o{}^2}(k^1{}_{ij}, k^2{}_{ij}, c_{ijk}, b^1{}_{ijkl})$$

These non-dimensional terms are given by:

$$m^{1*}{}_{ij} = \int_0^1 w_i^* w_j^* r^* dr^* ....,...(19a)$$

$$m^{2*}{}_{ij} = \int_0^1 u_i^* u_j^* r^* dr^* ....(19b)$$

$$k^{1*}{}_{ij} = \int_0^1 \left(\frac{d2w_i^*}{dr^{*2}} \frac{d2w_j^*}{dr^{*2}} + \frac{1}{r^{*2}} \frac{dw_i^*}{dr^*} \frac{dw_j^*}{dr^*}\right) dr^* ....(19c)$$

$$k^{2*}{}_{ij} = 12\int_0^1 \left(\frac{du_i^*}{dr^*} \frac{du_j^*}{dr^*} + \frac{1}{r^{*2}} u^*{}_i u^*{}_j + \frac{\nu}{r^*} \frac{du_j^*}{dr^*} u_j^* + \frac{\nu}{r^*} u_i^* \frac{du_j^*}{dr^*}\right) r^* dr^* ....(19d)$$

$$c^*{}_{ijk} = 12\int_0^1 \left(\frac{dw_i^*}{dr^*} \frac{dw_j^*}{dr^*} \frac{du_k^*}{dr^*} u_j^* + \frac{\nu}{r^*} \frac{dw_i^*}{dr^*} \frac{dw_j^*}{dr^*} u_k\right) r^* dr^* ....(19e)$$

$$b^{1*}{}_{ijkl} = 3\int_0^1 \left(\frac{dw_i^*}{dr^*} \frac{dw_j^*}{dr^*} \frac{dw_k^*}{dr^*} \frac{dw_l^*}{dr^*}\right) r^* dr^* ....(19f)$$

The transverse functions $w^*{}_i(r^*)$ for the clamped axisymmetric circular plate are written as **(Hatrbouch 2003)** :

$$w^*{}_i(r^*) = A_i \left[J_o(\beta_i r^*) - \frac{J_o(\beta_i)}{I_o(\beta_i)} I_o(\beta_i r^*) \right]$$

(20)

where, β_i 's are the real positive roots of:

$$J_1(\beta_i)I_o(\beta_i) + J_1(\beta_i)I_o(\beta_i) = 0 \qquad (21)$$

In this equation J_n , I_n are the Bessel and the modified Bessel functions of the first kind of order n. The parameter β_I related to $(\omega^*_l)_I$ by;

$$\beta_i^2=(\omega^*_l)_i \qquad (22)$$

The values of β can be found from **Eq. (21).**
The in-plane basic functions $u_I^*(r^*)$ for the immovable axisymmetric circular plate are **(Lee 1971)** :

$$u^*{}_i(r^*) = \beta_i J_1(\alpha_i r^*) \qquad (23)$$

where, α_I is the ith real root of ;

$$J_1(\alpha)=0 \qquad (24)$$

The functions $w_I^*(r)$ and $u_I^*(r)$ should be normalized as:

$$m_{ij}{}^{1*} = \int_0^1 w^*{}_i w^*{}_j r^* dr^* = \delta_{ij}$$
$$m_{ij}{}^{2*} = \int_0^1 u^*{}_i u^*{}_j r^* dr^* = \delta_{ij} \qquad (25)$$

The values of k^{1*}_{ij} , k^{2*}_{ij} , c^*_{ijk} and b^{1*}_{ijkl} given by **Eq. (19)** were computed by Simpson's rule. The set of nonlinear algebraic equations (16), which called the amplitude equation, can be written in matrix form as:

$$([K^{1*}]+[K_{nl}^*])\{A\}-\omega^{*2}[M^{1*}]\{A\}=\{0\} \qquad (26)$$

where, $([K^{1*}]$, $[M^{1*}]$ and $[K_{nl}^*]$ are respectively the non-dimensional linear stiffness, mass and non-linear geometrical stiffness matrices. The terms of the matrix $[K^*_{nl}]$ are;
$(K^*_{nl})_{ij} =(3/2)a_k a_l b^*_{ijkl}$. Neglecting the term $[K^*_{nl}]$ from **Eq. (26)** gives the classical eigenvalue problem;

$$[K^{1*}]\{A\}=\omega^{*2}[M^{1*}]\{A\} \qquad (27)$$

In this equation each eigenvalue have a corresponding eigenvector while the nonlinear **Eq. (26)** lead to a set of amplitude-dependent eigenvectors with their amplitude-dependent associated eigenvalues.

The single mode assumption, which neglects all of the coordinates except the single resonant coordinate, has been used widely in the geometrical non-linearities due to the great simplifications it introduces (**Azrar,1999)**. Also this approach does not give any information about the amplitude dependence between the deflection shape and distribution of stresses **(El Kadiri 2002)**. Therefore the explicit method of solution is used because it remedies this insufficiency of the single mode approach.

If the effect of λ in **Eq. (16)**is neglected due to its very small values, it can be rewritten according to this approach as:

$$a_i k^{1*}{}_{ir} + \frac{3}{2} a^3{}_i b^3{}_{111r} - \omega^{*2} a_i m^{1*}{}_{ir} - 0, ... r = 1, ... p_o \quad (28)$$

For r=1, we have,

$$\omega^{*2} = \frac{k_{11}{}^{1*}}{m_{11}{}^{1*}} + \frac{3}{2} \frac{b^*{}_{1111}}{m_{11}{}^{1*}} a_1{}^2 \quad (29)$$

The (p_o-1) remaining equations are:

$$\left(k_{rr}{}^{1*} - \omega^{*2} m_{rr}{}^{1*}\right) \varepsilon_r = \frac{-3}{2} a^3{}_1 b^*{}_{111r}, ... r = 2, ... p_o \quad (30)$$

where, ε_r is the contribution coefficient of the non-resonant modes which is given by:

$$\varepsilon_r = -\frac{3 a_1{}^3 b^*{}_{111r}}{2\left(k^{1*}{}_{rr} + \omega^{*2} m^{1*}{}_{rr}\right)}, ... r = 2, ..., p_o \quad (31)$$

substituting **Eq. (29)** into **(31)** gives:

$$\varepsilon_r = -\frac{3 a_1{}^3 b^*{}_{111r}}{2\left(k^{1*}{}_{11} + \frac{3}{2} a_1{}^2 b^*{}_{1111} - k^{1*}{}_{rr}\right)}, ... r = 2, ..., p_o \quad (32)$$

Eq.(32) is an explicit formula, allowing direct calculation of higher order contribution corresponding to the first mode shape. Thus the first non-linear amplitude dependent clamped circular plate mode shape, $w^*{}_{nl1}(r^*, a)$ can be defined in a series form as:

$$w^*{}_{nl1}(r^*, a_1) = a_1 w^*{}_1(r^*) + \sum_{r=2}^{p_o} \frac{3 a_1{}^3 b^*{}_{111r}}{2(k^{1*}{}_{11} + \frac{3}{2} a_1{}^2 b^*{}_{1111} - k^*{}_{rr})} w^*{}_r(r^*) \quad (33)$$

In this equation the predominant term in which proportional to the first linear mode shape is $a_1 w^*{}_1(r^*)$ and the others which corresponding to the higher linear mode shapes $w^*{}_2(r^*)$, ……… $w^*{}_{po}(r^*)$ are corrections due to the non-linearity.

In order to determine the distribution of membrane stresses, the in-plane displacement coefficients b_I should be determined. As mentioned above, because of the very small values of λ, **Eq. (16)** gives:

$$b_i = a_j a_l d^*{}_{jli} ...., i = 1, ... p_i \quad (34)$$

where, $d^*{}_{ijk} = -\frac{1}{2} k^{2*}{}_{kl} c^{1*}{}_{ijl}$, is a third order terms expressing the coupling between the transverse and in-plane motions, the tensor $k^{2*}{}_{ij}$.

If the first and second order terms in the expression $a_i a_j d^*{}_{ijk}$ are neglected, the in-plane contribution coefficients are simply given by:

$$b_i = a_1{}^2 d^*{}_{11I}, \; i = 1, ...p_I \quad (35)$$

Thus the in-plane shape function is given by:

$$u^*(r^*) = a_1{}^2 d^*{}_{11i} u^*{}_i(r^*) \tag{36}$$

If the first order term $a_1\varepsilon_1 d^*{}_{1li}$ is added, the in-plane basic function contribution coefficients, **Eq. (35)**, are given by:

$$b_i = a_1{}^2 d^*{}_{11i} + \sum_{l=2}^{p_o} a_1\varepsilon_l d^*{}_{1li} \quad , \quad i=1,\ldots\ldots p_I \tag{37}$$

now the in-plane function is:

$$u^*(r^*) = a_1{}^2[d^*{}_{11i} + \sum_{r=2}^{p_o} \frac{3a_1{}^2 b^*{}_{111r} d^*{}_{1ri}}{(2k^{1*}{}_{11} + 3d_1{}^2 b^*{}_{1111} - 2k^{1^r}{}_{rr})}] w^*{}_i(r^*) \tag{38}$$

This equation improves significantly the membrane stress estimates for amplitudes higher than those permitted by expression (35).

RESULTS AND DISCUSSIONS

The dependence of the non-linear frequency on the amplitude of vibration is shown in **Fig. (2)** for thickness ratios of 1, 1.5 and 2. This figure is plotted for the first two axisymmetric mode shapes. The ratio of thickness (h_i/h_o) has the effect of magnifying the frequency ratio ($\omega^*{}_{nl}/\omega^*{}_l$). Also it is seen that a spring hardening effect is present and this effect increased with increasing the amplitude ratio. The plot also shows that the first mode shape exhibits less change in frequency with the vibration amplitude than does the second non-linear mode shape. This is because that the deflection shape associated with the first mode shape produces less induced tensile forces than does that associated with the second mode shape for the same maximum displacement amplitudes. This figure shows that the nonlinear frequency increased with increasing the ratio of thickness. This is because of the bending effect arise due to the geometry of the plate. This effect increased with increasing the ratio of thickness.

Figs. (3a) and (3b) show the effect of thickness ratio on the non-linear mode shape. The non-linear mode shapes are plotted for the first two axisymmetric modes. The effect of amplitude ratio is presented in **(Haterbouch 2003 & Haterbouch 2004)**, therefore it is not presented here. The values of thickness ratio has the effect of keeping away the mode shape in the direction of the plate edge. Also it can be seen that the mode shapes become more flatening near to the centre of the circular plate with the increase of vibration amplitude. But here it is shown that the effect of thickness ratio is more pronounced than the amplitude ratio.

The effect of amplitude of vibration and thickness ratio on the normalized in-plane displacement shape functions is shown in **Figs. (4a) and (4b)** respectively for the first two modes. Increasing the ratio of thickness has the effect of pulling the in-plane mode shape in the direction of plate centre. This effect because that the inertia force near the centre of plate is higher than that near the edge.

Fig. (5) shows that the normalized amplitude is affected by the ratio of thickness in which increasing this ratio cause a shift to higher values at dimensionless radius values between 0.2 and 0.8. This trend is due to the high inertial values which cause higher values of deflection.

Conclusions

From the presented results the following two conclusions can be drawn;

1- It is shown that both of the amplitude of vibration and thickness nonuniformity have a clear effect on the nonlinear frequency and the corresponding mode shape.

2- Increasing these two parameters cause an increase in the nonlinear frequency and change the mode shape.

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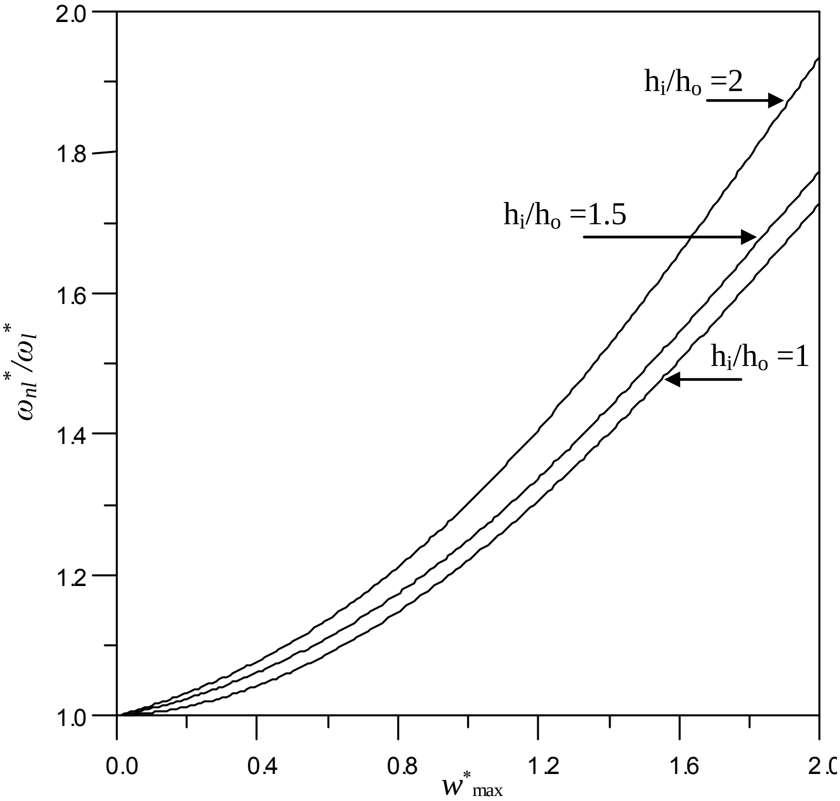

Fig. 2: Effect of Thickness Ratio and Maximum Vibration Amplitude on the nonlinear Frequency.

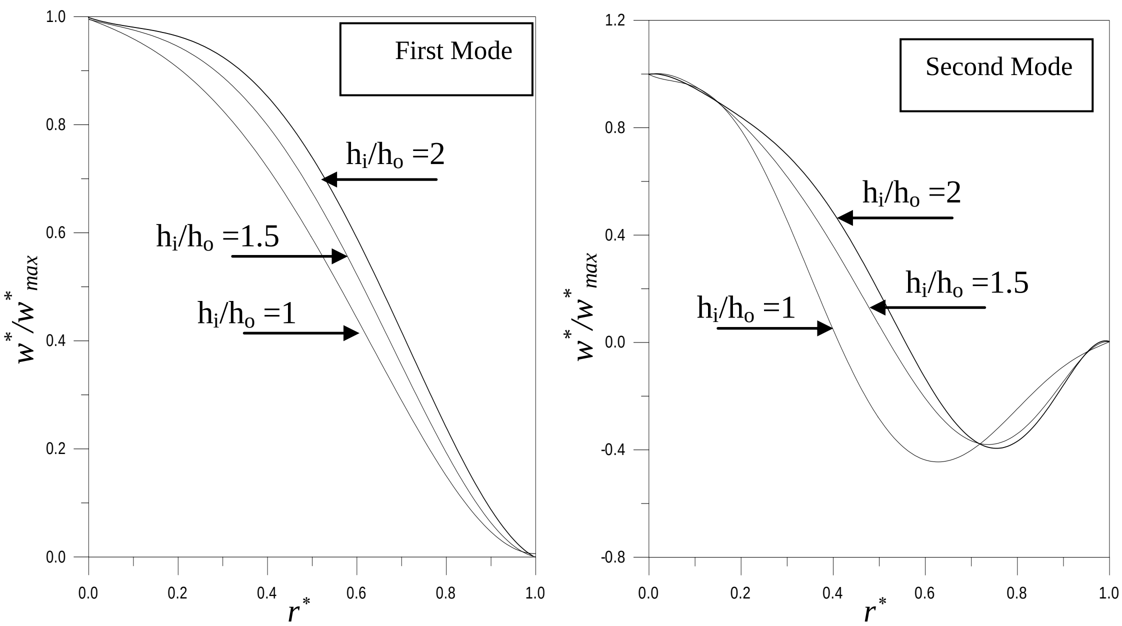

Fig. 3: Effect of Thickness Ratio on the normalized mode shape of the first two nonlinear Axisymmetric modes of the clamped circular plate, $w^*_{max} = 2$.

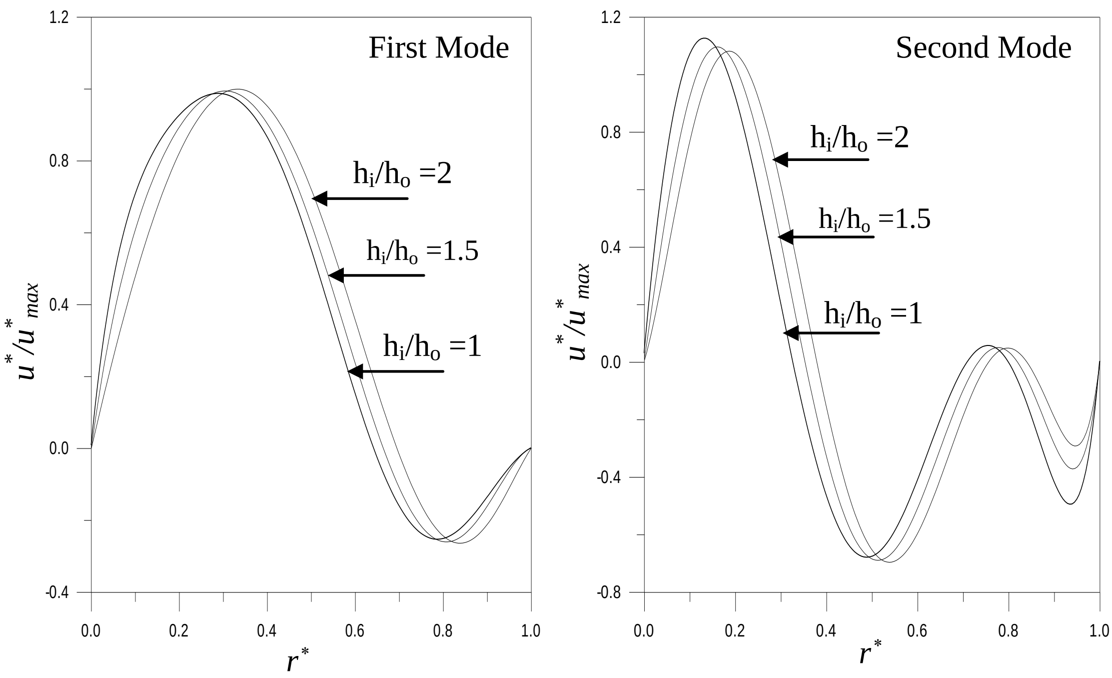

Fig.4: Effect of Thickness Ratio the Normalized in-Plane Shape Functions of the nonlinear Axisymmetric Modes of a Clamped Circular Plate, $w^*_{\max} = 2$

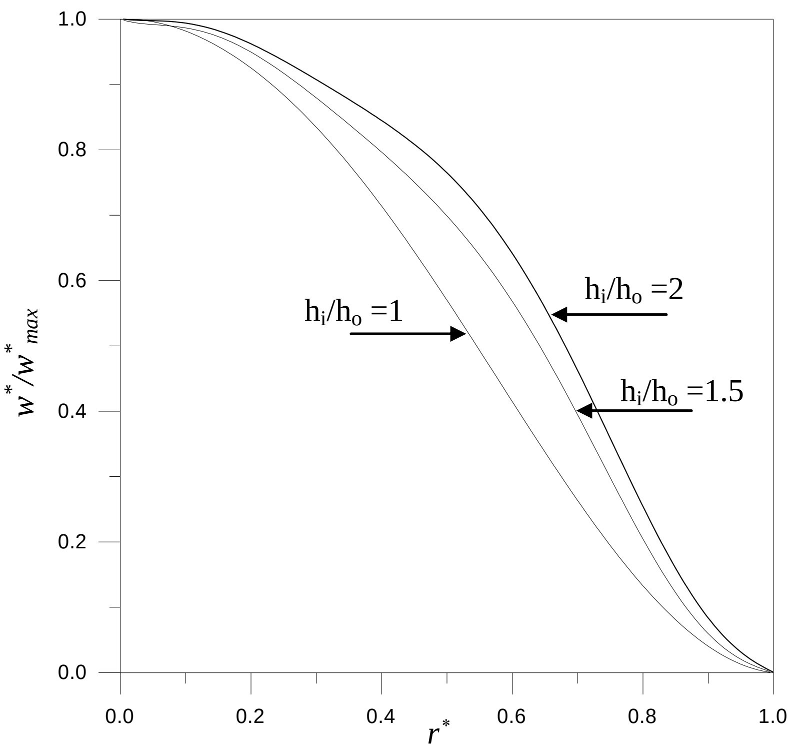

Fig. 5: Effect of Thickness Ratio on the First Mode Shape from Model with $w \; and \, u, w^*_{\max} = 2$.