SOLVING PROBLEMS OF UNSTEADY – CONFINED FLOW TO PUMPED WELLS BY COMPUTER

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ABSTRACT

In this research, a procedure to solve problems of unsteady flow to wells in confined aquifers by the computer is developed. The solution is based on the **[THEIS:1935]** procedure to solve such problems. The developed procedure is applied to (14) different sets of well – test data, including a predicted ideal one. The respective problems are solved completely by the computer without the need to construct or to refer to tables or nomographs; this, accordingly, deletes the role of personal judgment and the need to a high skill.

The applications indicate that the developed solution procedure is simple, easy to use, elaborate, superiorly fast in giving the required results, and comparatively accurate.

Despite that the developed solution procedure has been set for the case of a pumped well in an ideal confined aquifer, it is basically general; the computer program can be easily modified to fit the solution of problems of the other cases of groundwater flow to wells after introducing the additions that consider the respective boundary conditions.

KEYWORDS:
Wells hydraulics; Unsteady confined flow; Theis equation; Solution by computer.

الخــلاصــة

تمّ بهذا البحث تطوير طريقة لحل مسائل الجريان غير الثابت الى الآبار في الحشارج المحصورة وذلك باستعمال الحاسوب, واستند الحل على طريقة **[ثايس:1935]** في حل مثل هذه المسائل. جرى تطبيق الطريقة المطوّرة على (14) مجموعة مختلفة لرصودات فحص الآبار، منها مجموعة مثالية مستنبطة. لقد تمّ حل المسائل ذات العلاقة كلياً بواسطة الحاسوب دون الحاجة الى إعداد أو الرجوع الى جداول أو رسوم بيانية, وهذا بالنتيجة ألغى دور الاجتهاد الشخصي والحاجة الى الخبرة العالية.

لقد بيّنت التطبيقات ان طريقة الحل المطوّرة هي بسيطة وسهلة الاستعمال ومُتقنة وفائقة السرعة في إعطاء النتائج المطلوبة ودقيقة نسبيّاً.

ورغم ان طريقة الحل المطوّرة وُضعت لحالة بئر ضخّ في حشرج محصور مثالي, فإنها بشكل أساس عامة إذ بالإمكان تعديل برنامج الحاسوب بسهولة ليوائم حل مسائل الحالات الأخرى لجريان الماء الجوفي الى الآبار وذلك بعد تضمين برنامج الحاسوب الإضافات التي تأخذ بعين الاعتبار الأوضاع التُخميّة ذات العلاقة.

INTRODUCTION

Groundwater represents a major water resource – if not the only one – in too many localities all over the world. Beside its relatively good quality, in general, the attainable quantity of the world's groundwater (at depths less than 760 m) has been estimated at (3, 853, 213 km^3) **[UNESCO:1978]**, which is more than (33) times of the estimated total quantity of fresh water in all rivers, reservoirs, and lakes.

Groundwater is extracted mainly by means of pumped wells. The flow to a well is in fact unsteady. However, a steady state may be practically assumed after a continuous pumping from a well at a constant rate for a considerably long time. The pumped aquifers may be unconfined, semi – confined (leaky), or confined; the basic hydraulic principles for the aforementioned flow cases are essentially the same.

Practical problems of most concern in the field of groundwater hydrology and hydraulics fall in one of the following categories :

(1). Estimating the average values of the aquifer characteristics, namely, the transmissivity (T) and storativity (S).
(2). Determining the safe yield of a well (or an aquifer, or even a basin) for known (T) and (S).
(3). Predicting the drawdown (z) at the pumped well or in an observation well at a distance (r) from the pumped well at any time (t) since the start of pumping, provided that aquifer's (T) and (S) are known.

Pumping tests are the major tool for establishing the field data which are necessary for the solution of the aforementioned problems. Results of pumping tests are usually recorded in the form of tables giving the depth of water (d) in the considered observation well (measured from the ground surface) versus the respective time elapsed since the start of pumping, (t). The water surface in a well represents the instantaneous level of the piezometric surface in the confined aquifer (or that of the water table in an unconfined aquifer).

As it is discussed later, solution of any problem related to the discussed subject necessates the reference to some pre – established tables. Besides, the available procedures for solving problems of category (1) are graphical (in which human judgment plays a major role).

Realizing that the basic principles behind the Theis equation and Theis procedure of solution (mentioned hereafter) are unquestionable, this research aims at developing a simple and accurate methodology to solve problems of any of the three aforementioned categories. The solution is to be performed completely by computer and, consequently, there will be no further reference to tables, nomographs, or graphical solution for which high personal skill and experience are prerequisites. An ideal confined aquifer will be considered in establishing the methodology. The validation of the methodology is checked through application to some selected case studies.

THE GOVERNING EQUATION

Actual groundwater flow is three – dimensional and unsteady, with the storage characteristics of the aquifer playing a major role. Thus, an actual soil – water system is so complex that the solution of any problem concerning it, no matter how simple is, cannot be performed straightforward and accurately; wells hydraulics is one such example. Consequently, accompanying simplifying assumptions are always indispensable.

For an assumed laminar groundwater – flow, the general governing differential equation is developed by combining Darcy's equation and the continuity (mass balance) equation. The resulting equation can be written as **[MCWHORTER** and **SUNADA:1977]** :

$$\frac{\partial}{\partial x}(Kx\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(Ky\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(Kz\frac{\partial h}{\partial z}) = S_s\frac{\partial h}{\partial t} \quad [1]$$

where, for a confined aquifer, the specific storage (S_s) is a measure of compressibility of the aquifer and water.

The solutions of Eq. (1) give the time and space distributions of the piezonetric head (h) in heterogeneous, anisotropic, confined aquifers, in the usual three Cartesian coordinates (*x*, *y*, *z*).

To overcome the big difficulty and the complexity involved in solving Eq. (1), some practical simplifying assumptions, beside the assumption of laminar flow, are usually introduced. Such assumption involve **[CHOW:1964] :**

1. The aquifer is homogeneous, isotropic, of infinite areal extent, bounded by impermeable (confining) strata above and below, and has constant coefficients of transmissivity and storage in all directions at all times.
2. The discharging well is of infinitesimal diameter and completely penetrates the aquifer.
3. Pumping is maintained at a constant rate (Q).
4. Water is released instantaneously with decline in head.

It is to be noted, however, that for an assumed horizontal confined aquifer of a relatively uniform thickness (b) and an extensive areal extent, when average values of the effective soil characteristics, namely, the hydraulic conductivity (K) and the specific storage (S_s), are considered then the assumptions of homogeneity and isotropy are implicitly justified.

Thus, under the aforementioned assumptions, Eq. (1) reduces to :

$$\frac{\partial^2 h}{\partial x^2} = \frac{S_s}{K}\frac{\partial h}{\partial t} \quad [2]$$

where : K = overall average hydraulic conductivity of the aquifer.

For a radial flow towards a well, Eq. (2), written in polar coordinates, becomes :

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S_s}{K}\frac{\partial h}{\partial t} \quad [3]$$

With the assumption that the aquifer is of a uniform thickness (b), and defining the aquifer's storativity (S) and transmissivity (T) as :

$$S = S_s\ .\ b \quad [4]$$

$$T = K\ .\ b \quad [5]$$

then Eq. (3) could be written as :

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \quad [6]$$

which is the governing equation for the groundwater flow described hereinbefore.

SOLVING THE GOVERNING EQUATION

The difficulty in solving Eq. (6) led C.V. Theis to present a formula based on heat – conduction analogy, accounting for the effects of the storage characteristics of the aquifer and the time. For the conditions of ($h = h_0$) at ($t = 0$) and ($h \rightarrow h_0$) as ($r \rightarrow \infty$) for ($t \geq 0$), the equation was **[THEIS:1935]** :

$$h_0 - h_r = z_r = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y}\, dy \qquad \textbf{[7]}$$

which is the nonequilibrium or Theis equation. The parameters (h_0), (h_r), and (z_r) are as shown in **Fig. (1)**. The dimensionless parameter (u) is given by :

$$u = r^2 S / 4\, T\, t \qquad \textbf{[8]}$$

In groundwater literature, the integral in Eq. (7), known in mathematics as the exponential integral, is commonly denoted by [$W(u)$] and called "the well function of (u)"; the log – log plot of [$W(u)$] versus [u] or [$1/u$] is called the type curve. Thus, Eq. (7) is commonly written as :

$$z_r = \frac{Q}{4\pi T}\, W(u) \qquad \textbf{[9]}$$

No analytic solution is available for [$W(u)$] but a numerical representation by the infinite series :

$$W(u) = -0.5772 - \ln u - \sum_{n=1}^{\infty} \frac{(-)^n u^n}{n.n!} \qquad \textbf{[10]}$$

Based on that, **[WENZEL:1942]** prepared a table for the values of [$W(u)$] versus values of [u], available for the range ($1 \times 10^{-15} \leq u \leq 10$). The solution of any problem in this respect necessates the reference to Wenzel's table.

There is only one equation, the governing equation, which is available for solving the faced problems. Problems of categories (2) and (3) involve one unknown only and, therefore, can be solved directly, of course with the aid of Wenzel's table. However, the case is not so for a problem of category (1) because it involves two unknowns, namely, (T) and (S). For this **[THEIS:1935]** proposed his well – known graphical solution. Later, **[COOPER** and **JACOB:1946]** proposed a modified graphical procedure to solve problems of this category, provided that (u) has a small value. **[CHOW:1952]** developed a method that avoids the curve fitting of the Theis method and not being restricted to small values of (u). Nevertheless, the method is also graphical and necessates the use of a second table for a new function (Fu), beside Wenzel's table.

In an extensive study to solve some subsurface flow problems by the use of a computer, **[AL-ASSAF:1976]**, supported by some hypothetical and real examples, established the respective computer programs. One such a program is to estimate the aquifer characteristics directly from well test data through an iterative approach and in two stages. The first stage involves approximating the Theis equation and solving the approximated version for the considered data. The second stage uses the approximate results obtained in the first stage to solve the exact Theis equation and thus, obtaining the exact results aimed at.

PRACTICAL CONSIDERATIONS

To keep the computer work within some reasonable limits, the following practical considerations would be helpful. However, the developed solution procedure, in general, and the computer work, in particular, are by no means limited to these considerations as they are only indicative.

[A] Values of (*u*)

Theoretically, the dimensionless parameter (u), as defined in Eq. (8), may take any value in the range ($0 < u < \infty$). However, in real practice, the following may be considered.

(1): Values of (*r*)

With a specified constant pumping rate (Q), the value of the radial distance (r) from the pumped well depends mainly on the characteristics of the aquifer. Values of (r) of (5 – 100 m) are common in practical use. Reasonable values of (10 – 50 m) will be considered in the following analysis.

(2): Values of (*S*)

Values of the aquifer storativity (S) in the range ($1 \times 10^{-6} - 5 \times 10^{-3}$) have been recorded. Values of (1×10^{-4}) and (1×10^{-3}) will be used in the following analysis.

(3): Values of (*T*)

The hydraulic conductivity (K) of a natural soil is the most variable soil characteristic; it ranges from as low as (1×10^{-7} m/day) for clay to as high as (10^{3} m/day) or even more for gravel **[CHOW:1964]**. However, establishing a pumped – wells scheme in an aquifer of low permeability is not a feasible practice. Consequently, practical values of (2 – 20 m/day) seem reasonable for the purposes of this research.

The average thickness (b) of the source confined aquifer could be from few meters to several hundreds of meters. For the purposes of this research, values in the range (20 – 100 m) shall be considered.

Based on the considered values of (K) and (b) hereinbefore, the extreme values of (T), as defined by Eq. (5) would be (40 – 2000 m^2/day).

(4): Values of (*t*)

Duration of pumping, (t), in a pumping test, i.e., the time of continuous pumping from its start until stoppage, depends on the characteristics of the aquifer and the degree of accuracy of results aimed at. Values of (t) in the range (0.5 – 2 day) are practically reasonable.

On considering the extreme values of (r), (S), (T), and (t) mentioned hereinbefore, the respective values of (u) would be in the range ($6.25 \times 10^{-7} - 3.125 \times 10^{-2}$). Consequently, the practical range of (u) for the purposes of this research will be considered as ($1 \times 10^{-7} - 0.1$).

[B] Values of (*Q*)

The pumping rate (Q) should be high enough to produce measurable drawdowns in the respective observation wells. Values of ($5\ l/s \leq Q \leq 40\ l/s$) shall be considered when needed in this research.

THE DEVELOPED PROCEDURE OF SOLUTION

The type curve, {the *log – log* plot of [$W(u)$] vs. [u]}, resembles the theoretically – expected trend of the observed data of a pumping test when drawn as [(z) vs. (r^2 / t)] on a log – log paper of the same scale as that of the type curve. Two basic assumptions in the derivation of Eq. (9) are that (Q) is constant throughout the test, and water is released instantaneously with decline of head. Practically, these two assumptions are commonly not well met during the first few observations. Consequently, less weight is usually given to the data of such observations.

With the progress of the pumping, the plot of the pumping – test data, [(z) on an arithmetic scale versus (t) on a logarithmic scale], will be close to a straight line, the longer (t) is the more the closeness will be. For such a straight line, the data could be represented by :

$$z = C + m \; log(t) \quad \textbf{[11]}$$

where : (C) is the logarithmic intercept; (m) is the slope of the line, calculated between any two points, [(t_a, z_a) and (t_b, z_b); ($t_b > t_a$)] as :

$$m = \frac{z_b - z_a}{log(t_b) - log(t_a)} = \frac{\Delta z}{log(t_b / t_a)} \quad \textbf{[12]}$$

If the two points were chosen such that they cover a complete logarithmic time cycle, i. e., ($t_b = 10\ t_a$), then [$log\ (t_b / t_a) = 1$] and Eq. (12) reduces to ($m = \Delta z$).

Each observation shall be denoted by (R) with a subscript to denote its serial number. Thus, an observation will be [(R_i); i = 0, 1, …, N; N = the serial number of the last observation]. Consequently, and if the available data permit, the analysis will consider (J) observations [from (R_a) or (R_{N-J+1}) to (R_N)] that cover the last complete time logarithmic – cycle [that is from (t_a) to (t_N), where ($t_a = t_N / 10$)]. If (t_a) is not the time of an existing observation then an existing observation (R_a') is to be considered such that the time (t_a') is just preceding (t_a).

The developed solution procedure is summarized in the following steps :

[A] Verification of the observed data :

(1): Imagine that all data points have been located on a semi – log plot, (z) on the vertical arithmetic scale versus (t) on the horizontal logarithmic scale. After deciding on the (J) data points to be considered in the analysis (as mentioned hereinbefore), imagine that each two consecutive data points of the chosen ones are connected by a chord.

(2): Use Eq. (12) to calculate the slope of the established chords, (m_j; j = 1, 2, …, J–1). Of course, (m_j) as calculated would never be (–ve); otherwise, a point creating a (–ve) (m_j) should be considered as an outlier and, consequently, discarded.

It is to be noted that whenever an observation is discarded from the analysis, the number of involved observations (J) is decreased by one for the following calculations.

(3): For the considered data points to form a single, reasonably – acceptable straight line [outlined by the established (J–1) chords], the computed values of (m_j) should be insignificantly different.

[B] For the (J) observations still under consideration, the best – fit straight line is found by the least squares method.

Denoting [$log(t)$] as (X) and (z) as (Y), then Eq. (11) becomes :

$$Y = C + m\,X \quad \textbf{[13]}$$

According to the least – squares fitting **[MONTGOMERY et al.:1998]** :

$$C = \frac{(\sum Y)(\sum X^2) - (\sum X)(\sum XY)}{J(\sum X^2) - (\sum X)^2} \quad \textbf{[14]}$$

$$m = \frac{J(\sum XY) - (\sum X)(\sum Y)}{J(\sum X^2) - (\sum X)^2} \quad \textbf{[15]}$$

and the respective regression coefficient (R) is computed as :

$$R = \sqrt{\frac{Explained\ variation}{Total\ variation}} \quad \textbf{[16]}$$

which, for linear regression, could be set in a form simpler for calculation as :

$$R = \frac{J(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[J(\sum X^2) - (\sum X)^2][J(\sum Y^2) - (\sum Y)^2]}} \quad \textbf{[17]}$$

[C] The function (*Fu*) of [CHOW:1952]

To connect the observed data with the type curve, **[CHOW:1952]** defined a function (Fu) such that :
In relation to the considered observed data :

$$Fu = z_i / m \quad \textbf{[18]}$$

where (z_i) is the drawdown of an observation (R_i) chosen arbitrarily from the considered set of observations.
In relation to the type curve :

$$Fu = \frac{1}{ln\ 10} W(u)\ .\ e^u \quad \textbf{[19]}$$

where (Fu), [$W(u)$], and (u) are all evaluated at (R_i).

The value of (Fu) is calculated by Eq. (18). Then, the solution of Eq. (19) would give unique values of (u) and [$W(u)$] evaluated at (R_i). This would enable solving Eq. (9) for (T) and then solving Eq. (8) for (S).

[D] Solving for [*W*(*u*)] and (*u*)

What is mentioned in item [C] hereinbefore sounds attractive. However, it involves an obstacle in that, despite of that [$W(u)$] being a function of (u) (and vice versa), Eq. (19) could not be solved explicitly, neither for (u) nor for [$W(u)$].

To overcome the involved dilemma, **[CHOW:1952]** prepared a table (and also a *log – log* nomograph) for values of (Fu) corresponding to a range of values of [$W(u)$] and their respective values of (u).

For a solution to be performed completely by a computer, the reference to tables or nomographs should be avoided. Accordingly, the case under consideration could be tackled by a computer through a trial – and – error solution. Such a solution would involve assuming a reasonable value for (u), computing the respective value of [$W(u)$] by Eq. (10), computing the respective value of (Fu) by Eq. (19), and comparing this value with that computed by Eq. (18); the two values should be insignificantly different (within a pre-specified tolerance limit). If the two values were not so, the calculations are repeated for modified values of (u) until the aimed goal is achieved. However, this would be a cumbrous and tedious task. An elaborate alternative is to provide the necessary explicit mathematical relationships between each two of the three involved parameters, namely, (Fu), [$W(u)$], and (u). This implies the establishment of the following functional relationships : {$W(u) = f_1[u]$}, {$u = f_2[W(u)]$}, and {$u = f_3[Fu]$} or {$W(u) = f_4[Fu]$}.

Taking into consideration the practical range of values of (u) of (1×10^{-7} – 0.1) mentioned before, the following has been deduced in this respect.

(1): {$W(u) = f_1[u]$}

Such a relationship is already given by Eq. (10). However, it is logical and more practical that for small values of (u), the series may be truncated after the second term. Thus, Eq. (10) becomes :

$$W(u) = -0.5772 - \ln u = \ln (1 / 1.781\, u) \qquad \textbf{[20]}$$

To check the validity of such an approximation, consider values of (u) of (0.001), (0.01), and (0.1). Moreover, and for illustrative purposes and based on the practical considerations outlined before, consider average practical values of the involved parameters, say [Q = 10 l/s; T = 400 m^2/d]. The drawdowns calculated for the aforementioned cases are summarized in **Table (1)**. For drawdowns calculated to the nearest millimeter, the results show that the approximation of [$W(u)$] for the considered values of (u) will yield a relative percent decrease (D_1) in the computed drawdowns of (0.0), (0.29), and (5.35), respectively. The relative percent decrease (D_2) would be (0.0), (0.0), and (3.23), respectively, if the drawdowns were calculated to the nearest centimeter. This indicates that the use of Eq. (20) is well justified, at least for the purposes of this research, for ($u < 0.01$).

(2): {$u = f_2[W(u)]$}

(i): Since Eq. (20) was found satisfactorily accurate for ($u < 0.01$) [for which $W(u) > 4.0379$], then the inverse functional relationship will be valid too. That is :
For [$W(u) > 4.0379$] :

$$u = 1 / \{1.781\, exp.\, [W(u)]\} \qquad \textbf{[21]}$$

(ii): For [$4.0379 \geq W(u) \geq 1.8229$], [that is $0.01 \leq u \leq 0.1$] :

For purposes other than this research, the researcher solved Eq. (10) up to (n = 34) (which was the limit of the capacity of the used computer) and prepared a table similar to Wenzel's table and for the same values of (u) involved therein. The respective type curve is then divided into several overlapping hypothetical sectors. The data of each sector were regressed, [$W(u)$ on (u) and vice versa]; the sectors were shortened or elongated until the best regression, i. e., the highest regression coefficient (R), was obtained.

For the aforementioned relationship, the obtained one was :

$$u = Exp.\, \{-[1.04\, W(u) + 0.41]\};\ \{R = 1.0\} \qquad \textbf{[22]}$$

(3): {$W(u) = f_4[Fu]$}

(43) values of (u), covering the range ($1\times10^{-7} \leq u \leq 0.1$) were selected. The respective values of [$W(u)$] were calculated by Eq. (10) or Eq. (20), as the case indicated. Then, Eq. (19) is used to calculate the corresponding values of (Fu).

The software GRAPHER was used to perform the regression of [$W(u)$] on (Fu). Of the various styles the software offers, the best functional relationship obtained in this respect was a third – degree polynomial which, after modification, came as :

$$a_0 = -0.34;\ a_1 = 2.55;\ a_2 = -0.0567;\ a_3 = 0.004124;\ \{R = 1.0\} \qquad \textbf{[23]}$$

[E] Solving for (T) and (S)

With known (Q), (r), and [$W(u)$] in respect to the chosen observation (R_i) with known (t) and (z), the solution for (T) and (S) would be systematic and straightforward and as follows :

(1): Solve Eq. (5) for (T).

(2): Solve Eq. (22) [or Eq. (21) as the case may be] for (u); then, solve Eq. (4) for (S).

It is known in statistics that when a set of data points {X, Y} is fitted linearly, a point ($\overline{X}$, $\overline{Y}$), whether real or predicted, would locate on the fitted straight line. Consequently, it was thought that such a point would be an appropriate choice for (R_i). Thus, the established computer program has been set accordingly; it computes ($\bar{t}$, $\bar{z}$) of the considered (J) data points and uses the respective values to represent the chosen observation (R_i).

APPLICATION

The researcher was hoping to apply the developed solution procedure to some recent, dependable well – test data, Iraqian in particular. However, he was almost unfortunate in this respect. Nevertheless, (14) different data sets were used to form the application cases. The information regarding these sets are outlined hereinafter.

[1] For a rigid check of the applicability of the developed procedure, an ideal hypothetical data set has been fabricated as follows :

(a) (24) values of (u) representing the practically – common range of ($2.5E–1 \geq u \geq 4E–5$) have been selected. The respective values of [$W(u)$] were calculated by Eq. (10) [for ($u \geq 0.01$)] or by Eq. (20) [for ($u < 0.01$)].

(b) For assumed reasonable values of (Q = 20 l/s = 1728 m^3/d), (r = 25 m), (T = 400 m^2/d), and (S = 2.5E–4), the values of (z) [by Eq. (9)] corresponding to the calculated values of [$W(u)$] and then the values of (t) [by Eq. (8)] corresponding to the chosen values of (u), were calculated.

(c) The resulting (24) (t, z) data points formed the application set [1] for the present research.

[2] To illustrate his developed graphical approach, **[CHOW:1952]** used the well - test data shown in his Fig. (3). The values of (22) data points (t, z) abstracted from the aforementioned figure formed the application set [2].

[3] Test data for well (1) at Gridley, Illinois is used as an illustrating example in **[WALTON:1970]**, (P 229). The data of the example formed the application set [3].

[4] Data constituting (29) observations are given in Problem (4-1), P 283 in **[WALTON:1970]**. He indicated the solution to be (T = 358000 gpd/ft ; S = 4.7E–4) without mentioning the procedure of solution. This data formed the application set [4].

[5]; **[6]**; **[7]**: Data from the pumping test Qude Korendjik are given in **[KRUSEMAN** and **DE RIDDER:1970]**, (P 53). Observations were made in three piezometers, (P1), (P2), and (P3), located at radial distances from the pumped well of (30 m), (90 m), and (215 m), respectively. The basic assumptions mentioned before were closely satisfied in the test.

[KRUSEMAN and **DE RIDDER:1970]** used the aforementioned data to estimate (T) and (S). The procedures of solution used were as follows :

(a)[THEIS:1935] procedure applied to the data of the three piezometers collectively.

(b)[COOPER and **JACOB:1946]** procedure. This was used in three approaches :

(i): The traditional approach, that is : [(z) vs. (t)] for constant (r), for each piezometer separately.

(ii): [(z) vs. (r)] for constant (t); solved for (t = 140 min.).

(iii): [(z) vs. (t/r^2)] for the data of the three piezometers collectively.

(c) **[CHOW:1952]** procedure applied to the data of (P1).

The data of (P1), (P2), and (P3) have been considered as the application sets [5], [6], and [7], respectively.

[8]; **[9]**; **[10]**: **[LOHMAN:1972]** used data observed at three observation wells (*N*–1), (*N*–2), and (*N*–3) in a solution by the Theis procedure. These data have been considered as the application sets [8], [9], and [10], respectively.

[11]: **[LINSLEY et al.:1988]** present an illustrative example on (P 179). They solved for (T) and (S) by the Theis and the Jacob procedures. The data of the example have been considered as the application set [11].

[12]; **[13]**; **[14]**: **[ABDULLA:2001]** used data sets observed at seven observation wells in the Jolak Basin (Al-Ta'miem Governate, Iraq), each corresponds to a certain pumped well. Those corresponding to the wells Yarimja, Kurzi, and Nabi Awah have been considered as the application sets [12], [13], and [14], respectively, for the purposes of the present research.

RESULTS AND ANALYSIS

The basic results of applying the developed solution procedure to the considered (14) data sets are summarized in **Table (2)**. In this respect, the following is worth mentioning :

[A] No data point has been found to be as an outlier.

[B] The respective values of the regression coefficient (R) were high enough to indicate excellent linear fittings.

[C] Values of (T) and (S) estimated by the solution procedure developed in this research are designated as (CT) and (CS), respectively. The corresponding reference values are designated (RT) and (RS), respectively; those concerning data set [1] are the assumed values; those concerning the other data sets are the values estimated by the respective researchers; the abbreviations (TH), (JA), and (CH) denote that the respective values have been estimated by the **[THEIS:1935]**, **[COOPER** and **JACOB:1946]**, and **[CHOW:1952]** procedures, respectively.

[D] Values of (T) and (S) could be described as exact only when they are obtained by a purely – theoretical solution (without any added assumptions other than those accompany the derivation of the governing equation) and for ideal well – test data. This is virtually the case of (RT) and (RS) of application case [1].

The values (CT) and (CS) obtained by the solution procedure developed in this research and for the same data set [1] were identical to the reference values. This clearly proves the elaborateness and accuracy of the developed procedure.

[E] The developed procedure solves the involved problem completely by the computer without the need neither to construct nor to refer to any table or graph. In fact, when the computer program is ready in the computer, the time to obtain the results is just that time for inputting the data and pressing the "run" button.

[F] Recalling that the keystone in the solution of the considered problems is the Theis equation. However, actual field conditions are never ideal; the percent deviation between the two could be anywhere between the two extreme limits, (0) and (100). The error in the results due to ignoring such a deviation may be exaggerated when using different data sets for the same aquifer, using different approximate procedures of solution, or when the solutions are performed by different persons and by procedures in which the personal skill and judgment play a significant role. Consequently, the comparison of different mere results for the same data would be indicative only.

All the known procedures of solution in this respect are issued from the same theoretical basis. The differences between them are basically the additional assumptions, simplifications, and approximations to facilitate and accelerate the solution process while keeping the results closely comparable to some references ones, usually taken as those of the Theis procedure of solution. Accordingly, one can pronounce that no other solution procedure may be more accurate than the one developed in this research since it adds no further assumptions or conditions, except that the well test should last long enough to make the plot [(z) vs. (log t)] closely approximates the expected straight line (which is, in fact, a general prerequisite to any well test). Moreover, the procedure involves no approximations other than those required to give the highest reasonable computation accuracy the computer provides. Yet, the fully – computerized solution by the developed procedure leaves no role to personal judgment or the necessity to highly – skilled personnel.

Keeping in mind the aforementioned merits of the developed solution procedure, the obtained values (*CT*) and (*CS*) for the application cases [2] through [14], given in **Table (2)**, indicate that they were :

- Close to (*RT*) and (*RS*) for [2] by (*CH*).
- Identical to (*RT*) and (*RS*) for [3] by (*TH*).
- Somehow comparable to (*RT*) and (*RS*) for [4] (for which the reference do not mention his procedure of solution).
- Somehow different from (*RT*) and (*RS*) for [5] as obtained by all the five applied procedures.
- Somehow comparable to (*RT*) and (*RS*) for [6] as obtained by all the five applied procedures.
- Identical to (*RT*) and (*RS*) for [7] by (*JA*–1).
- Different from the average (*RT*) and (*RS*) obtained for [8], [9], and [10] collectively, by (*TH*).
- Very closely comparable to those obtained for [11] by both (*TH*) and (*JA*).
- Ditto, for [12].
- (*CT*) was identical to (*RT*) for [13] obtained by both (*TH*) and (*JA*). However, (*CS*) was identical to (*RS*) by (*TH*) but different from that by (*JA*) {unless there is a misprint}.
- Almost identical to (*RT*) and (*RS*) for [14] by (*TH*) and closely comparable to those by (*JA*).

CONCLUSIONS

In summary, the following conclusions could be stated :

1. The fully – computerized procedure developed in this research for solving problems of unsteady flow to wells in confined aquifers is simple, elaborate, superiorly fast in giving the required results, and without the need of graphics or the reference to tables or nomographs. Moreover, on considering the results obtained by the available solution procedures for the same data, the accuracy of the results of the developed procedure are comparatively unquestionable.

2. The procedure is basically general; it is equally applicable to the different problems encountered in wells hydraulics (such as those for data of water – level recovery, for unconfined or semi – confined aquifers, or when accretion is present) after introducing the additions that count for the relevant boundary conditions.

Table (2) : Summary of results [The cases and symbols are explained in the text]

Item	Parameter	Units	Case Number						
			(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	**Q**	**M^3/d**	1728	4088	1200	8176	788	788	788
2	**r**	**m**	25	245	251.2	91.44	30	90	215
3	**N**	**---**	24	22	22	29	34	35	9
4	**J**	**---**	9	16	13	11	11	15	9
5	**C**	**m**	0.4763	-0.8848	-1.2823	0.2346	0.4169	0.0256	-0.1679
6	**m**	**---**	0.7914	0.6519	1.6993	0.2890	0.2297	0.2390	0.1450
7	**R**	**---**	1.0000	0.9984	0.9988	0.9982	0.9977	0.9991	0.9948
8	**$\bar{t}$**	**min.**	1524.41	166.79	142.45	503.09	281.53	274.88	274.44
9	**$\bar{Z}$**	**m**	2.995	0.564	2.377	1.015	0.982	0.608	0.186
10	**ΔZ**	**m**	0.792	0.652	1.699	0.289	0.230	0.239	0.145
11	**F(u)**	**---**	3.782	0.865	1.399	3.512	4.270	2.544	1.283
12	**W(u)**	**---**	8.7084	1.8260	3.1278	8.0867	9.8320	5.8578	2.8470
13	**u**	**---**	9.28E-5	9.94E-2	2.57E-2	1.73E-4	3.02E-5	1.60E-3	3.44E-2
14	**CT**	**M^2/d**	399.8	1053.1	125.6	5183.2	627.8	604.1	959.7
15	**CS**	**---**	2.51E-4	8.08E-4	2.00E-5	1.50E-4	1.60E-5	9.10E-5	5.44E-4
16a	**RT**	**M^2/d**	**(TH)** 400	**(CH)** 1008.1	**(TH)** 125.24	**(?)** 4439.2	**(TH)** 418		
17a	**RS**	**---**	2.50E-4	8.83E-4	2.00E-5	4.70E-4	1.7E-4		
16b	**RT**	**M^2/d**					**(JA-1)** 401	480	960
17b	**RS**	**---**					1.7E-4	1.8E-4	5.8E-4
16c	**RT**	**M^2/d**					**(JA-2)** 355		
17c	**RS**	**---**					4.5E-4		
16d	**RT**	**M^2/d**					**(JA-3)** 438		
17d	**RS**	**---**					1.7E-4		
16e	**RT**	**M^2/d**					**(CH)** 375		
17e	**RS**	**---**					2.2E-4		

Table (2) : [Continued]

Item	Parameter	Units	Case Number						
			(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	Q	M^3/d	2718.4	2718.4	2718.4	3672	3120	691.2	907.2
2	r	m	61	122	244	30	50	30	30
3	N	---	25	25	25	11	17	20	25
4	J	---	12	12	12	10	15	17	19
5	C	m	0.5382	-0.2137	-0.8924	-5.3440	-0.5443	-0.1741	-0.1632
6	m	---	1.3184	1.3003	1.2545	2.6451	0.6516	1.1292	0.4103
7	R	---	0.9999	0.9998	0.9994	0.9829	0.9961	0.9991	0.9987
8	$\bar{t}$	min.	83.38	83.38	83.38	422.34	69.53	249.59	453.64
9	$\bar{Z}$	m	3.071	2.284	1.518	1.601	0.656	2.533	0.927
10	Δ Z	m	1.318	1.300	1.255	2.645	0.652	1.130	0.411
11	F(u)	---	2.330	1.757	1.210	0.605	1.006	2.242	2.255
12	W(u)	---	5.3650	4.0456	2.6698	1.1829	2.1721	5.1624	5.1923
13	u	---	2.63E-3	9.83E-3	4.13E-2	1.94E-1	6.93E-2	3.22E-3	3.12E-3
14	CT	M^2/d	377.9	383.1	380.4	215.9	822.0	112.1	404.3
15	CS	---	6.2E-5	5.9E-5	6.1E-5	5.46E-2	4.40E-3	2.78E-4	1.78E-3
16a	RT	M^2/d	(TH) 1267.0			(TH) 196.7	(TH) 856.2	(TH) 114.6	(TH) 424.7
17a	RS	---	1.98E-4			6.4E-2	4.8E-3	2.8E-4	1.7E-3
16b	RT	M^2/d				(JA) 200.8	(JA) 878.9	(JA) 110.0	(JA) 437.2
17b	RS	---				5.6E-2	4.9E-3	1.53E-3	4.3E-3

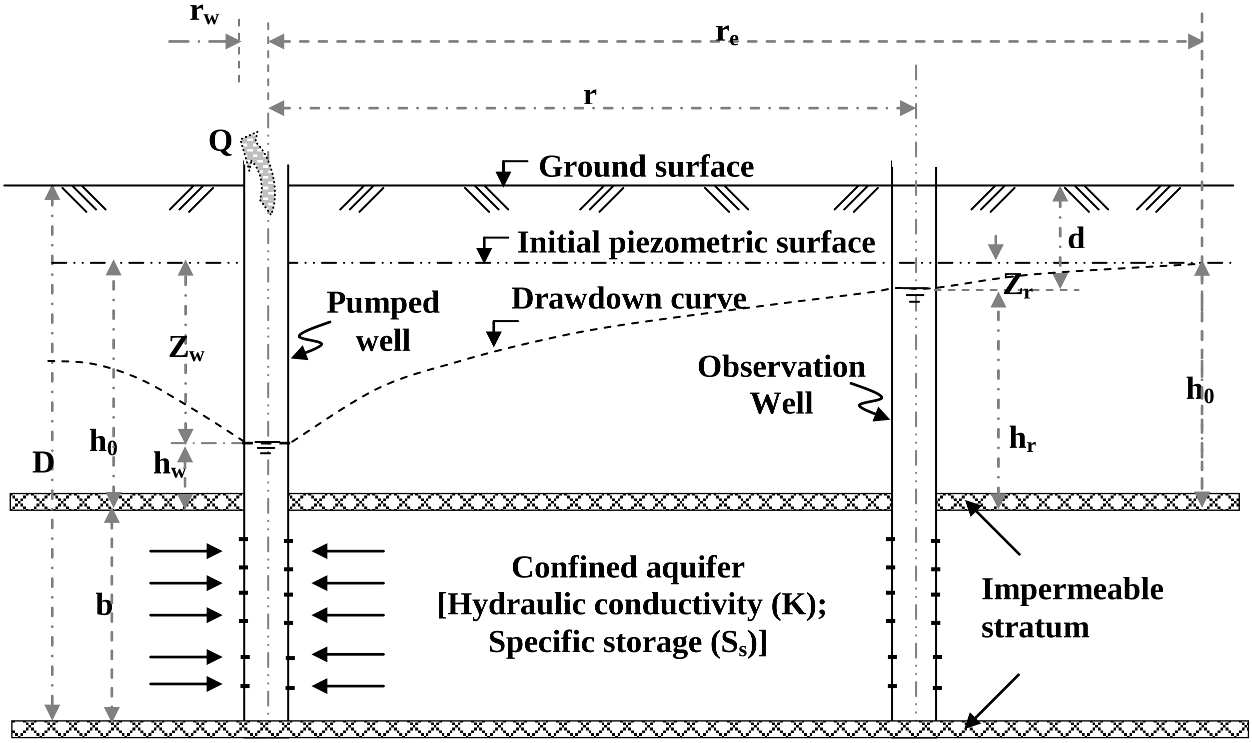

Fig. (1) : Definition sketch for a confined flow to a pumped well.

Table (1) : Results by approximating $W(u)$. [Q = 10 l/s ; T = 400 m^2/d]

Parameter	Units	Values of (u)		
		0.001	**0.01**	**0.1**
$W(u)$ { Eq. (10) }	----	6.3315	4.0379	1.8229
Z_1 { Eq. (9) }	m	1.088	0.694	0.313
$W(u)$ { Eq. (15) }	----	6.3306	4.0280	1.7255
Z_2 { Eq. (9) }	m	1.088	0.692	0.297
$D_1^{(*)}$	%	0	0.29	5.35
$D_2^{(*)}$	%	0	0	3.23

(*) D = $\frac{Z_1 - Z_2}{Z_1}$ × 100 ; (D_1) and (D_2) are calculated for values of (Z) computed to the nearest millimeter and centimeter, respectively.

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