



EVALUATION OF MATHEMATICAL TECHNIQUES USED FOR PRODUCING DIGITAL ELEVATION MODEL (DEM)

Dr.Hussain Z. Ali
Ministry of Science
and Technology (M.O.S.T)
Image Processing/Expert

Mr. Bashar S. Abbas
University of Baghdad
College of Engineering
Department of Surveying

Miss. Afrah M. Daham
University of Baghdad
College of Engineering
Department of Surveying

ABSTRACT

The Digital Elevation Model (DEM) considered a common tool in producing topographic maps, orthophotos and civil engineering projects besides other different engineering applications. As a result many software packages were developed and used to produce DEM from different sources like field surveys, scanned topographic maps and stereo photos exposed from air or space.

This paper is devoted to evaluate the most suitable and accurate interpolation method in producing digital elevation model for the data gathered from existed topographic maps which are also compared with data gathered from field survey. Map scale (1:25000) with contour interval (50m) were chosen in the tests. The accuracy tests based on the National Mapping Accuracy Standards (NMAS) by comparing the result of Root Mean Square Error (RMSE) in elevations with the typical standard deviation (σ_z) proposed by (NMAS) which depends on the scale of maps and contour intervals.

From testing four interpolation techniques ((Kriging, Triangulated Irregular Network (TIN), Inverse Distance Weighting (IDW) and Polynomial)) it was found that kriging is the best method followed by TIN method while IDW method failed in some tests, and the polynomial model failed in all tests.

الخلاصة:

اصبح نموذج الارتفاع الرقمي (DEM) في الوقت الحاضر شائع الأستعمال في مجال انتاج الخرائط الطبوغرافية ومشاريع الهندسة المدنية هذا وبالإضافة إلى استخدامه بشكل واسع في مجالات اخرى مثل ابراز الأرتفاعات والتخطيط والأستكشافات الجيولوجية والجيوفيزيائية وكنتيجه لذلك فان العديد من البرامج الجاهزة قد تم تطويرها واستخدامها لأنتاج نموذج الارتفاع الرقمي ومن مختلف المصادر مثل المسوحات الأرضية، النماذج المجسمة للصور الجوية والفضائية والخرائط الطبوغرافية المشطية. وتماشيا مع هذا التوجه فان هذا البحث يتناول استخدام اكثر من طريقة رياضية لأنتاج النموذج وباستخدام برنامج (Autodesk Field Survey) ومن مصدرين مختلفين المصدر الأول هو المسوحات الأرضية للمنطقة سد طق طق لأغراض المقارنة والمصدر الثاني وهو طريقة الخرائط الكنتورية المشطية (Scanned Contour maps) لمنطقة الدراسة وذات مقياس (1:25000) ويفتره كنتورية مقدارها (50m) وبالأعتماد على نفس البرنامج.

وبعد انتاج النموذج فقد تم فحص الدقة للنماذج والطرق الرياضية و بالأعتماد على معيار دقة الخارطة الوطنية (National Mapping Accuracy Standards (NMAS)) الذي يعتمد على مقياس الخارطة ومقدار الفترة الكنتورية وتم استخراج جذر معدل مربع الخطأ ((Root Mean Square Error (RMSE)) لقيم الارتفاع ولكل النماذج وبكل الطرق الرياضية المستخدمة (طريقة الشبكة المثلثية الغير منتظمة (TIN), طريقة ال Kriging, طريقة الوزن بمعكوس المسافات (IDW) و طريقة ال (Polynomial) وتم مقارنة الأتحراف المعياري المحسوب (Computed Standard Deviation) مع الأتحراف المعياري النموذجي ((Typical Standard Deviation (σ_z)). وبعد اختبار الدقة فكانت افضل النتائج هي للطريقة ال (Kriging) وتليها طريقة ال (TIN) , اما طريقة ال (IDW) نتائجها كانت مقبولة في بعض الفحوصات وطريقة ال (Polynomial) فنتائجها كانت فاشلة في جميع الفحوصات.

KEY WORD

Digital Elevation Model (DEM), Applications, Interpolation Techniques, Evaluation of Interpolation Techniques.

INTRODUCTION

Digital elevation modeling is one of the modern methods for representing the topographic surface of the terrain, (i.e., how the elevation of the ground surface is changing with position). Traditionally this has been done by contour lines on topographic maps. DEMs have been developed and studied for more than 40 years [Kennie, 1993]. The development has revealed the significance of several main aspects such as data acquisition, interpolation methods, accuracy, computer programs and application, all that will be discussed in this chapter.

For the future, the use of terrain modeling methods will undoubtedly continue to develop and expand, particularly with continued improvements in the price performance ratio of computer systems. National and regional terrain data based on existing topographic maps are now being developed in many parts of the world, and these will play an increasingly important role in terrain visualization during the preliminary planning stage of engineering projects [Anderson, 1998].

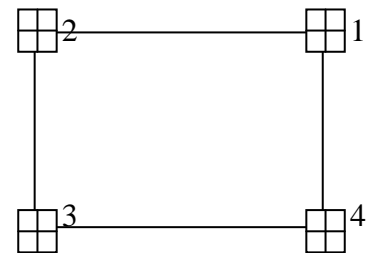
The term DEM originally referred to the use of cross sectional height data to describe the terrain. Nowadays, however, the definition is including both grid and non grid data sets. Several other terms are also used to describe the terrain surface. Among the more common are Digital Elevation Model (DEM), Digital Height Model (DHM), Digital Terrain Model (DTM), Digital Ground Model (DGM) and Digital Terrain Elevation Data (DTED) [Kennie, 1993].

THE STUDY AREA:

The study area is described through the following conditions:

- The area is called (**TaqTaq**), which covers about (12.974km²), the survey for this area were accomplished by AL-Aali for engineering consultancies by Total Station Topcon, its borders coordinates are shown :

Pt	E (m)	N (m)
1	468460.657	3973469.559
2	464468.958	3973469.559
3	464468.958	3970219.393
4	468460.657	3970219.393



These points help to obtain real-world coordinates, the same tic numbers and locations will be recorded and used for each separate layer of contour lines. The coordinate system of this data set is defined by using the following values which has the following projection data:

<u>Projection</u>	<u>Units</u>	<u>Zone</u>
UTM	Meter	38

This area were covered by a topographic map of a scale 1:25000 with contour interval (50m), It extends between latitude ($35^{\circ} 52' 30'' - 36^{\circ} 00' 00''$, North) and longitude ($44^{\circ} 30' 00'' - 44^{\circ} 00' 00''$, East). (See fig.1).

- Valleys and mountains is the most active features of land cover in the region due to high slopes of this region.
- The **Small Zap River** passes through this region.

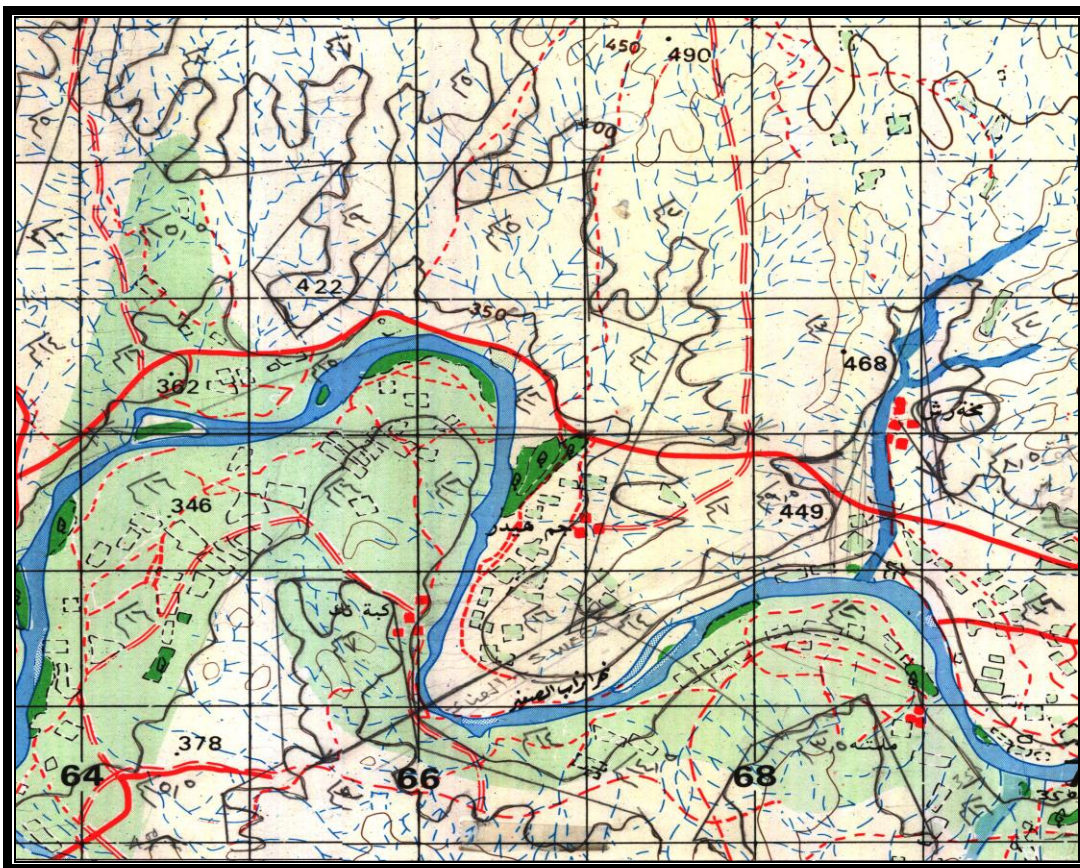


Fig (1): The study area (TaqTaq), (1/25000) scale.

GRIDDING INTERPOLATION ALGORITHM

Gridding produces a regularly spaced array of Z values from irregularly spaced XYZ data. The term "irregularly spaced" implies that the points are randomly distributed over the extent of the map area meaning that the distance between data points is not consistent over the map [Lebrer, 1973]. When the XYZ data is randomly spaced over the map area, there are many "holes" in the distribution of data points. Gridding fills in the holes by extrapolating or interpolating Z values in those locations where no data exists [Surfer, 1995].

- **Interpolation:** the process of estimating the values of an attribute (e.g., elevation) at internal unsampled sites using measurements made at reference points. The interpolation point lies within the range defined by the reference points [Habib, 2003].
- **Extrapolation:** the process of predicting the values of an attribute (e.g., elevation) at external unsampled sites using measurements made at reference points. The extrapolation point lies outside the range defined by the reference points [Habib, 2003].

There are several gridding methods. Each method calculates grid node values using a different algorithm, and can result in a some what different interpretation of input data [Surfer, 1995].

Inverse Distance to a Power Interpolation Method

The Inverse Distance to a Power gridding method is a weighted average interpolator, and can be either an exact or a smoothing interpolator. The Power parameter controls how the weighting factors drop off as distance from a grid node increases. For a larger power, closer data points are given a higher fraction of the overall weight; for a smaller power, the weights are more evenly distributed among the data points.

The weight given to a particular data point when calculating a grid node is proportional to the inverse of the distance to the specified power of the observation from the grid node. When calculating a grid node, the assigned weights are fractions, and the sum of all the weights is equal to 1. When an observation is coincident with a grid node, the observation is given a weight of essentially 1.0, and all other observations are given a weight of almost 0. In other words, the grid node is assigned the value of the coincident observation. This is an exact interpolator [Surfer, 1995].

One of the characteristics of inverse distance is the generation of "bull's-eyes" surrounding the position of observations within the gridded area.

The smoothing parameter can be assigned during inverse distance gridding. If it was greater than zero assures that no one observation is given all the weight at a particular grid node, even if the observation is coincident with the grid node. The smoothing parameter reduces the "bull's-eye" effect by smoothing the interpolated grid.

This method basically depends on estimating the height of the unknown point by computing the distances from this point to the other known points. Weights are proportionally by inverse to distances. Whenever the point being far away, its effect reduce the following equation explain that:

$$Z(X, Y) = \frac{\sum_{i=1}^n \left[\frac{Z_i}{d_i^p} \right]}{\sum_{i=1}^n \left[\frac{1}{d_i^p} \right]} \quad (1)$$

$$Z(X, Y) = \sum \lambda_i \cdot Z_i \xrightarrow{\text{With}} \sum \lambda_i = 1$$

(2)

d_i Is the planimetric distance between the reference point and the i^{th} interpolation point [21].

$$d_i = \sqrt{(X_i - X)^2 + (Y_i - Y)^2} \quad (3)$$

$Z(x,y)$: is the predicted value at the unsampled location x,y .

n : is the number of measured sample points within the neighborhood defined for x,y .

Z_i : is the observed value at location i .

λ : are the distance-dependent weights associated with each sample point.

d_i : is the distance between the prediction location x,y and the measured location i .

p : is the power parameter that defines the rate of reduction of the weight as distance increases.

We have mention that the weight might be inverse to a distance which rise to the power n , thus $n = 1,2,3,4,5,.....$

Kriging Interpolation Method

Kriging is a geostatistical gridding method, the geostatistical method is defined as that use spatial coordinates to help in the formulation of models, which are used in the estimation and prediction. In the Geostatistical Analyst: Exploratory and interpolation methods that used information of the spatial coordinates [ESRI, 2001].

Kriging method has proven useful and popular in many fields. This method produces visually appealing contour and surface plots from irregularly spaced data [Ibrahem, 1993]. Kriging attempts to express trends that are suggested in our data, so that, for example, high points might be connected along a ridge, rather than isolated by bull's-eye type contours.

Kriging is a very flexible gridding method. It can be custom fit to a data set by specifying the appropriate variogram model. It incorporates anisotropy and underlying trends in an efficient and natural manner [Habib, 2003].

Kriging is based on the assumption that the parameter being interpolated can be treated as a regionalized variable. A regionalized variable is intermediate between a truly random variable and a completely deterministic variable in that it varies in a continuous manner from one location to the next and therefore points that are near each other have a certain degree of spatial correlation, but points that are widely separated are statistically independent [Habib, 2003]. Kriging is a set of linear regression routines which minimize estimation variance from a predefined covariance model.

This method uses to calculate the auto correlation between the data points and produce a minimum variance estimate. Kriging method produces much more accurate interpolation results than the other methods.

Kriging can be achieved by the following steps:-

- Using the observed height in the Ground Control Points (GCP) to compute experimental covariance function $C\{d\}$.
- Choose covariance sample similar to experimental covariance function $C\{d\}$.
- Using least square for computing unknowns, which related to function covariance sample.
- Using product equation to interpolate heights.

The kriging interpolation techniques include:-

- a. Ordinary Kriging.
- b. Simple Kriging.
- c. Universal Kriging.

a. Ordinary Kriging: it is presented statistically what can be named Best Linear Unbiased estimation (**BLUE**) and in this way the summation of weights are one, λ is Lagrange multipliers is a powerful method for finding the maximum and minimum of constrained functions and k is the value of elements equal one, but that mean complex in mathematics

from other side, also the using of this method require solving a huge number of equations [Surfer, 1995]

$$\hat{Z}_p = w^T Z_r, \text{ and } \sum_{i=1}^n w_i = 1$$

$$\begin{bmatrix} w_{n \times 1} \\ \lambda \end{bmatrix} = \begin{bmatrix} C\{z_r, z_r\} & K \\ K^T & O \end{bmatrix}_{(n+1) \times (n+1)}^{-1} \begin{bmatrix} C\{z_r, z_p\} \\ 1 \end{bmatrix}_{(n+1) \times 1} \quad (4)$$

b. Simple Kriging: Simple kriging is similar to ordinary except that the following equation is not added to the set of equations:

$$w_1 + w_2 + w_3 = 1 \quad (5)$$

And the weights do not sum to unity. Simple kriging uses the average of the entire data set while ordinary kriging uses a local average (the average of the scatter points in the kriging subset for a particular interpolation point). As a result, simple kriging can be less accurate than ordinary kriging [Habib, 2003].

Simple kriging is presented statistically what can be named Best Linear Estimation (**BLE**) and next the method interpolation achieve as shown:

Interpolated Height

Quality of the Interpolated Height

$$\hat{Z}_p = w^T Z_r$$

$$w_{n \times 1} = [C\{z_r, z_r\}]_{n \times n}^{-1} [C\{z_r, z_p\}]_{n \times 1}$$

$$D\{\hat{Z}_p = w^T C\{z_r, z_r\}\} w = w^T C\{z_r, z_p\}$$

- Where:

$$C\{z_r, z_r\}_{n \times n} = \begin{bmatrix} C\{0\} & C\{d12\} & C\{d13\} & \dots & C\{d1n\} \\ C\{d21\} & C\{0\} & C\{d23\} & \dots & C\{d2n\} \\ C\{d31\} & C\{d32\} & C\{0\} & \dots & C\{d3n\} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ C\{dn1\} & C\{dn2\} & C\{dn3\} & \dots & C\{0\} \end{bmatrix} \quad (6)$$

$$C\{z_r, z_p\}_{n \times 1} = \begin{bmatrix} C\{d_{1p}\} \\ C\{d_{2p}\} \\ C\{d_{3p}\} \\ \vdots \\ C\{d_{np}\} \end{bmatrix} \quad (7)$$

- Note: $C\{d_{ij}\} = C\{d_{ji}\} \Rightarrow C\{z_r, z_r\}$ is symmetric

z_r : vector, its height value of Ground Control Point (GCP).

$C\{z_r, z_r\}$: Covariance between ground control points with each other.

$C\{z_r, z_p\}$: Covariance between ground control points with unknown point.

And it is named $C\{z_r, z_r\}$ and $C\{z_r, z_p\}$ semi-variance.

c. Universal Kriging: One of the assumptions made in kriging is that the data being estimated are stationary. That is, as you move from one region to the next in the scatter point set, the average value of the scatter points is relatively constant. Whenever there is a significant spatial trend in the data values such as a sloping surface or a localized flat region, this assumption is violated. In such cases, the stationary condition can be temporarily imposed on the data by using of a drift term [Surfer, 1995]. The drift is a simple polynomial function that models the average value of the scattered points. The residual is the difference between the drift and the actual values of the scattered points. Since the residuals should be stationary, kriging is performed on the residuals and the interpolated residuals are added to the drift to compute the estimated values. Using a drift this function is often called "universal kriging." the mathematical model of universal kriging is as follow [Habib, 2003]:

$$\begin{bmatrix} \gamma\{d_{11}\} & \gamma\{d_{12}\} & \gamma\{d_{13}\} & \gamma\{d_{14}\} & \gamma\{d_{15}\} & 1 & X_1 & Y_1 \\ \gamma\{d_{21}\} & \gamma\{d_{22}\} & \gamma\{d_{23}\} & \gamma\{d_{24}\} & \gamma\{d_{25}\} & 1 & X_2 & Y_2 \\ \gamma\{d_{32}\} & \gamma\{d_{32}\} & \gamma\{d_{33}\} & \gamma\{d_{34}\} & \gamma\{d_{35}\} & 1 & X_3 & Y_3 \\ \gamma\{d_{41}\} & \gamma\{d_{42}\} & \gamma\{d_{43}\} & \gamma\{d_{44}\} & \gamma\{d_{45}\} & 1 & X_4 & Y_4 \\ \gamma\{d_{51}\} & \gamma\{d_{52}\} & \gamma\{d_{53}\} & \gamma\{d_{54}\} & \gamma\{d_{55}\} & 1 & X_5 & Y_5 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ X_1 & X_2 & X_3 & X_4 & X_5 & 0 & 0 & 0 \\ Y_1 & Y_2 & Y_3 & Y_4 & Y_5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ \lambda \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \gamma\{d_{1p}\} \\ \gamma\{d_{2p}\} \\ \gamma\{d_{3p}\} \\ \gamma\{d_{4p}\} \\ \gamma\{d_{5p}\} \\ 1 \\ X_p \\ Y_p \end{bmatrix} \quad (8)$$

Where:-

λ : is Lagrange multipliers.

a_1 and a_2 : the coefficients expressing the trend (two coefficients).

3. Polynomial Regression Interpolation Method

Polynomial Regression is used to define large scale trends and patterns in our data. There are several options can be used to define the type of trend surface. Polynomial Regression is not really an interpolator because it does not attempt to predict unknown Z values [Lebrer, 1973].The form of the 2nd degree polynomial is as follow:

$$Z(X, Y) = a_0 + a_1X + a_2Y + a_3X^2 + a_4XY + a_5Y^2$$

(9)

TIN INTERPOLATION ALGORITHM

TINs represent surfaces as contiguous non overlapping triangular faces. A surface value can be estimated for any location by simple or polynomial interpolation of elevations in a triangle. Because elevations are irregularly sampled in a TIN, a variable point density to areas where the terrain changes sharply can be applied, yielding an efficient and accurate surface model [ESRI, 2001].

A TIN preserves the precise location and shape of surface features. Area features such as lakes and islands are represented by a closed set of triangle edges. Linear features such as ridges are represented by a connected set of triangle edges. Mountain peaks are represented by a triangle node.

TINs support a variety of surface analyses such as calculating elevation, slope, and aspect performing volume calculations, and creating profiles on alignments. The disadvantage of TINs is that they are often not readily available and require data collection. TINs are well suited for large-scale mapping applications where positional accuracy and shapes of surface features is important [Surfer, 1995]. The TIN data structure can be used in representing any type of surface accurately. Not only can elevations be interpolated for any location within a TIN, but natural features that form breaks in a surface's slope, such as ridges and streams, can also be stored. After constructing TIN we can interpolated the height of unknown point that its location inside the triangulation network as shown in figure (2) one of the next mathematical techniques could be used [Habib, 2003]:

- 1.Linear Interpolation.
- 2.Second Exact Fitted Surface Interpolation.
- 3.Quintic Interpolation.

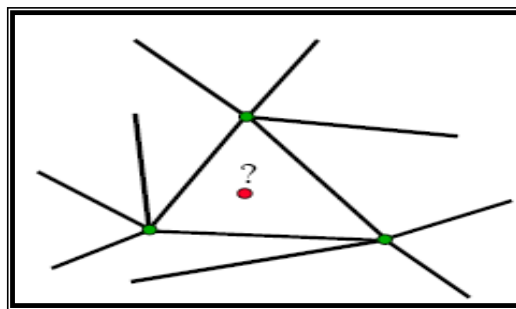


Figure (2): Represent the point unknown elevation [Habib, 2003].

1. Linear Interpolation: it is basically depending on the following equation:

$$Z(X, Y) = a_0 + a_1 X + a_2 Y^2 \tag{10}$$

By known vertices of triangle and by matrix form, so that:

$$I_z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \mapsto A = \begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{bmatrix} \mapsto x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \mapsto I_{z_{3 \times 1}} = A_{3 \times 3} x_{3 \times 1} \tag{11}$$

$$\text{or } \hat{x}_{3 \times 1} = (A^T A)^{-1} A^T I_z \quad \hat{x}_{3 \times 1} = A^{-1} I_z$$

Estimate the height at the interpolation point

$$Z(X_P, Y_P) = \begin{bmatrix} 1 & X_P & Y_P \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_3 \end{bmatrix}$$

(12)

2. Second Exact Surface: depend on a second degree equation, as shown in equation (13) [Habib, 2003].

$$Z(X, Y) = a_0 + a_1X + a_2Y + a_3X^2 + a_4XY + a_5Y^2$$

(13)

Or in matrix form:-

$$I_z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{bmatrix} \mapsto A = \begin{bmatrix} 1 & X_1 & Y_1 & X_1^2 & X_1Y_1 & Y_1^2 \\ 1 & X_2 & Y_2 & X_2^2 & X_2Y_2 & Y_2^2 \\ 1 & X_3 & Y_3 & X_3^2 & X_3Y_3 & Y_3^2 \\ 1 & X_4 & Y_4 & X_4^2 & X_4Y_4 & Y_4^2 \\ 1 & X_5 & Y_5 & X_5^2 & X_5Y_5 & Y_5^2 \\ 1 & X_6 & Y_6 & X_6^2 & X_6Y_6 & Y_6^2 \end{bmatrix} \mapsto x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \quad I_{z_{6 \times 1}} = A_{6 \times 6} x_{6 \times 1} \quad (14)$$

$$\hat{x}_{6 \times 1} = (A^T A)^{-1} A^T I_z \quad \text{or} \quad \hat{x}_{6 \times 1} = A^{-1} I_z \quad (15)$$

$$Z(X_P, Y_P) = \begin{bmatrix} 1 & X_P & Y_P & X_P^2 & X_P Y_P & Y_P^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \quad (16)$$

3. Quintic Interpolation: is produces a continuous surface. Considers the surface model to be smooth, that is, the normal to the surface varies continuously within each triangle and between triangles [Habib, 2003]. This smooth characteristic is accomplished by considering the geometry of the neighbouring triangles when interpolating the z value of appoint in a TIN triangle,quintic interpolation depend on a five degree polynomial in x and y as the follow:

$$Z(X, Y) = \sum_{i=0}^5 \sum_{j=0}^{5-i} a_{ij} X^i Y^j \quad (20)$$

$$Z(X, Y) = a_0 + a_1X + a_2Y^2 + a_3X^2 + a_4XY + a_5Y^2 + a_6X^3 + a_7x^2y + a_8XY^2 + a_9Y^3 + a_{10}X^4 + a_{11}X^3Y + a_{12}X^2Y^2 + a_{13}XY^3 + a_{14}Y^4 + a_{15}X^5 + a_{16}X^4Y + a_{17}X^3Y^2 + a_{18}X^2Y^3 + a_{19}XY^4 + a_{20}Y^5$$

Elevation Data Collection

In this work it was depended on collecting the data of elevation from field survey of TaqTaq area, also on the scanned topographic maps by separating the layer of contour lines where the value of the elevation for each line is known, elevation data will be assigned as shown in figure (3).

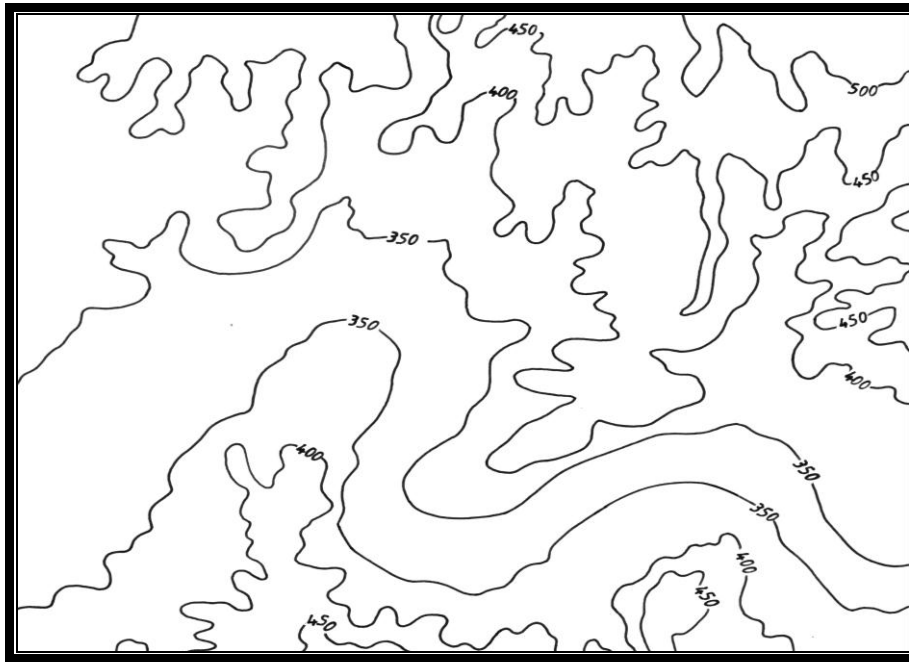


Fig (3): Contour lines of the study area (TaqTaq), from map (1/25000) scale.

Data Modeling

Data Modeling by Using Field Survey

Digital elevation model (DEM) can also be generated from field survey. The accuracy of all field survey method is very high but they are really practical and economical to be implemented over relatively small area of terrain. The field surveys were used here just for compartment purposes with those collected from topographic maps.

The used field survey are for TaqTaq area which is about 32000 surveyed points. The stage of producing the digital elevation model from field survey was the following as:-

The DEM building using the field survey. At first were drawn the points by using (Autodesk field survey) software as shown figure (4), then the points, which are considered in data base and information correlated with digital elevation model, are selected. The model building will be dependent on the triangulation method and gridding method. This means the information that is saved in surface file will be converted to (Triangulated Irregular Network (TIN), Inverse Distance Weighting (IDW), Universal Kriging and 2nd polynomial). After that the digital elevation model is displayed as a perspective figure for a three dimensional vision. (See fig. (5), which are plotted by ArcView ver.3.2, (6), (7), and (8)) are illustrate the digital elevation model of the study area (TaqTaq) by using mathematical techniques.

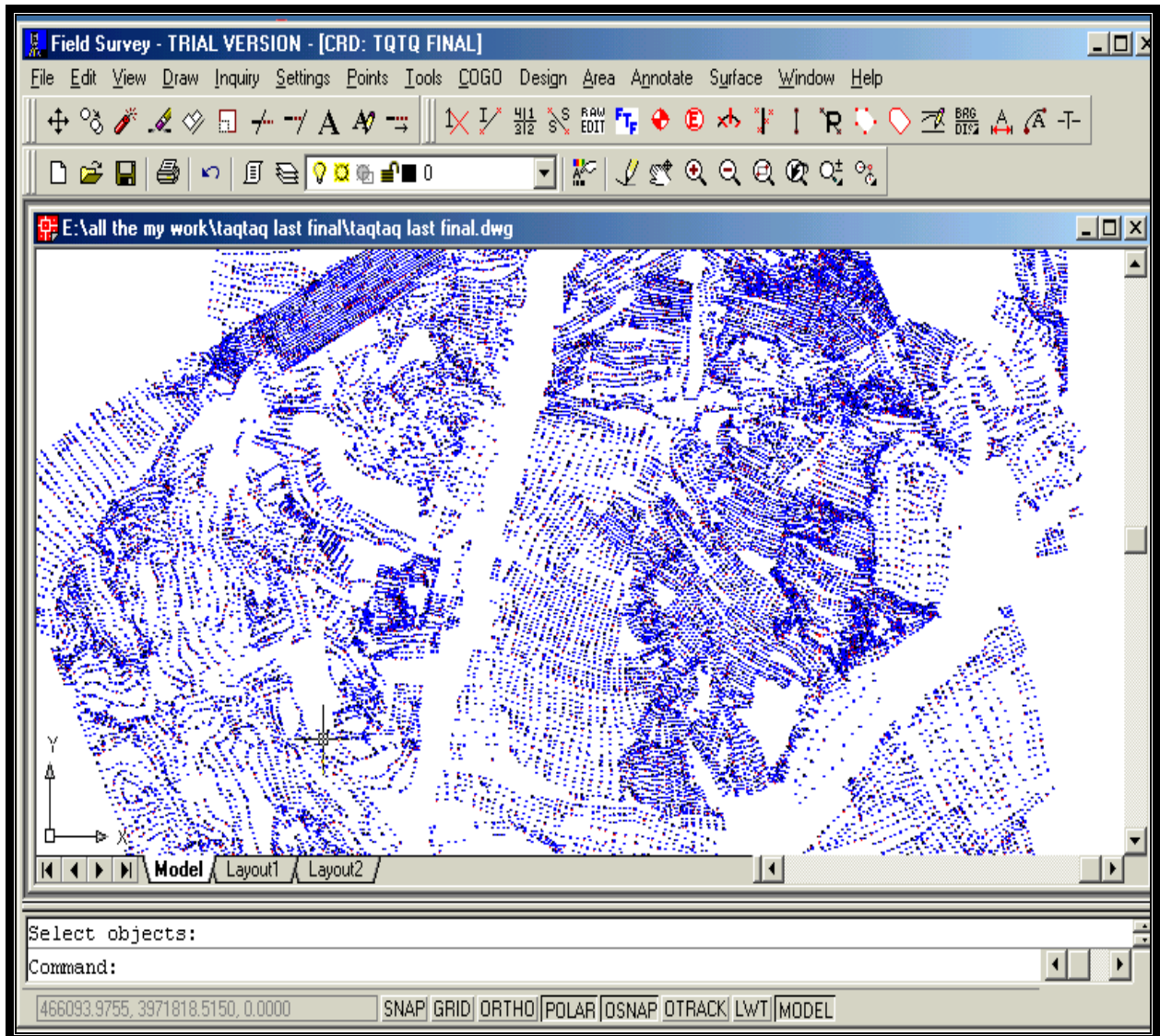


Fig (4): Represent the drawing of the field survey data by using (Autodesk Field Survey) software.

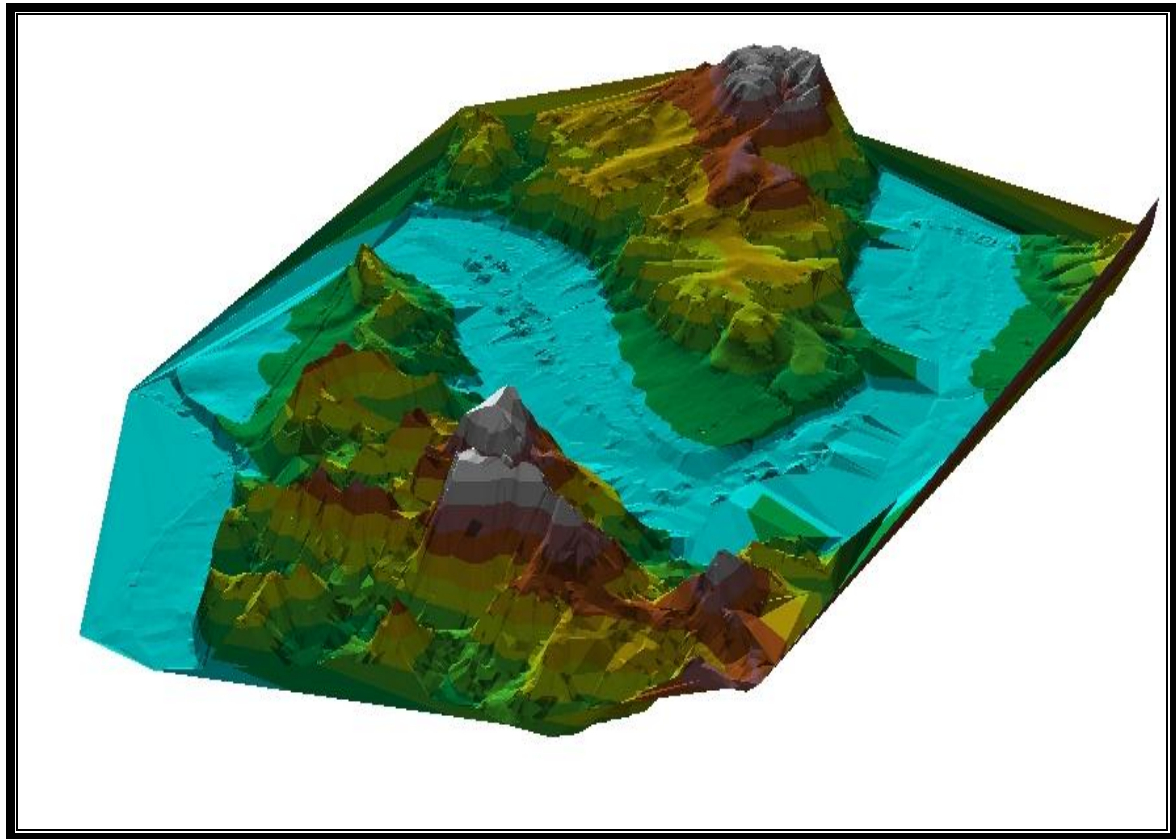


Fig (5): DEM for field survey data of TaqTaq area by (TIN) method.

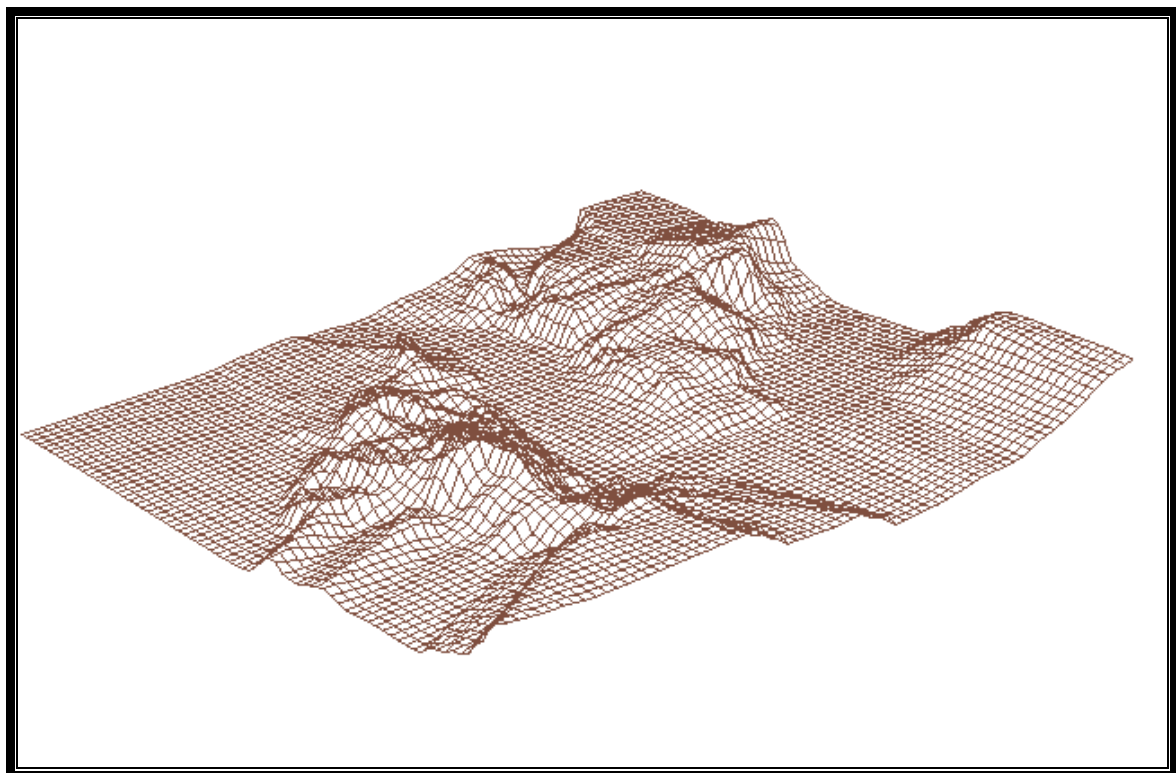


Fig (6): EM for field survey data of TaqTaq area by (Kriging) method.

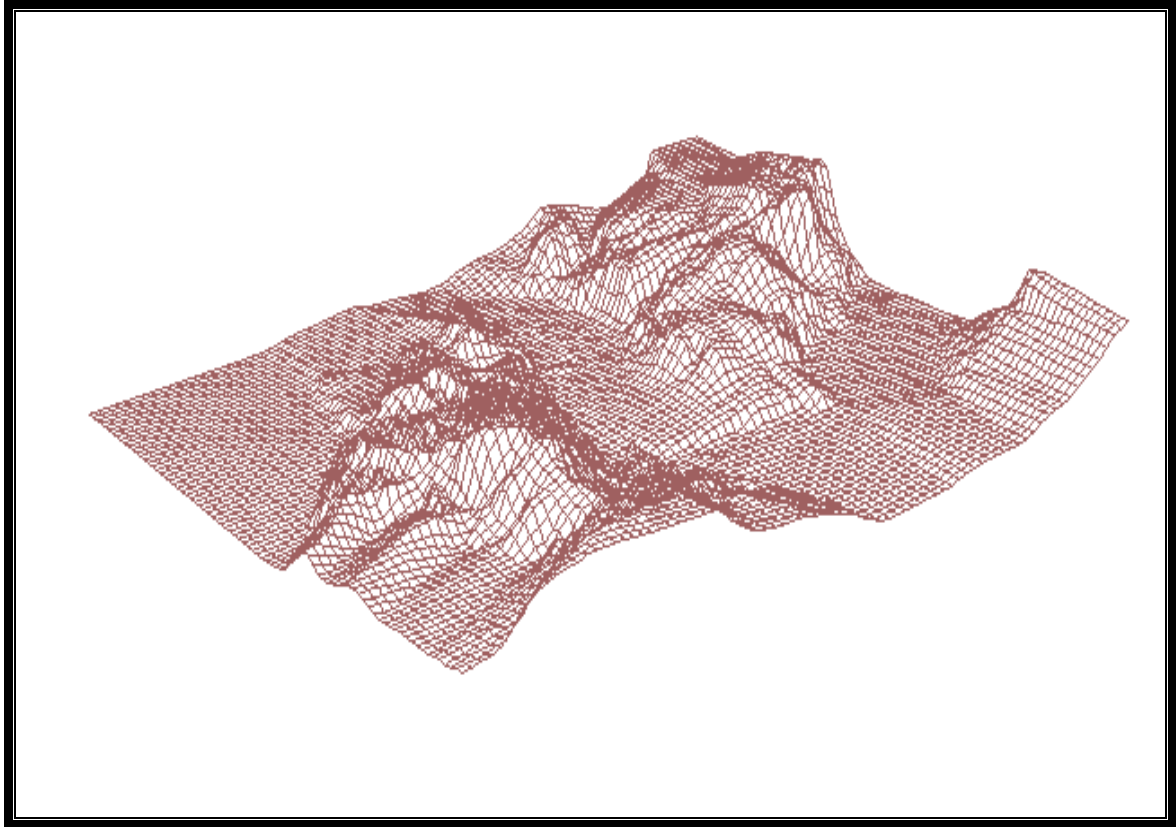


Fig (7): DEM for survey data of TaqTaq area by (IDW) method.

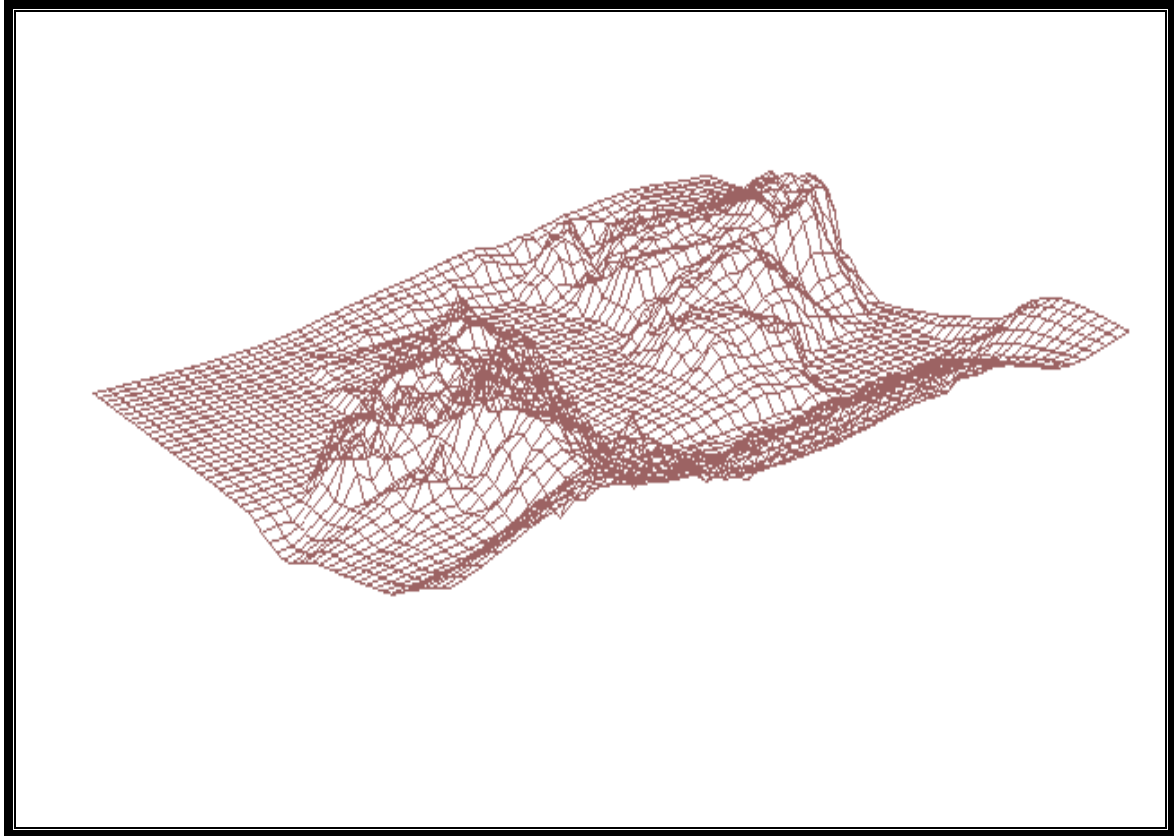


Fig (8): DEM for field survey data of TaqTaq area by (Polynomial) method.

Data Modeling by Using Topographic Map

Digital Elevation Model (DEM) can also be generated from the existing contour map. In this part topographic map with scale (1:25000) have been used to generate the digital elevation model from it. The procedures of DEM generation contain many phases as illustrated below:-

1- Separate contour lines layer from the study areas topographic maps, by projecting it on a trace paper used for this purpose, with indicating for the value of each line. Understanding this step, with more details can be made from the illustration contour line layer which will be given in figure (3) of the study area known as contour map.

2- Input the contour map to the computer memory. This means a converting a digital form (Raster) by using scanner instrument with a resolution accuracy of about 500dpi, and save it in a special work folder.

3- Convert contour maps from raster format to vector format. By what is called on screen digitizing this operation is fulfilled by using (**Autodesk Field Survey**) software, which shows the contour lines in the first case as separate picture elements (pixels) having a gray-scale value and identified by (sample, line) coordinate format. In the second case, the contour line is identified by a starting point (x_1, y_1) and ending point (x_n, y_n) coordinates. Between these two points there are many points known as (nodes), representing the changing in contour line direction. In this part of scanning and digitizing existing maps, prior to scanning the ground control points or grid points on maps have been fixed on the maps as register points, with known value (Easting, Northing). The values were inserted in the software (Autodesk Field Survey) with scanned maps images to control the coordinates system for DEM as the same as previous model. These control points obtained from the topographic maps of the study areas using a universal transverse meractor (UTM) projection. Each point was chosen in the

topographic maps and the scan images, which should be identified easily, the coordinates of these points for the study area are listed in table:

Table (1): GCP of the Study Area.

Point no.	E(m)	N(m)
1	468460.657	3973469.559
2	464468.958	3973469.559
3	464468.958	3970219.393
4	464468.958	3970219.393

4-Errors may result, when the contour lines are drawn. These errors, , are dependent on the region (study area) nature. When some of regions like (taqtaq and halbja) includes high slopes (i.e. very high contour line density), the probability of obtaining errors is increased.

The probable error defined as error resulting from the drawing of the dense contour lines. In addition, the processing includes also the errors in the map obtained as a result of the vectorization. These errors result from the cutting in the lines or the removing of the extra lines. Then reconstruction of contour lines is made in order to be ready for editing and identifying as contour lines having certain height values [Wolf, 1982].

The above processing was done by using (**AutoCAD map, ver. 5.0**). This process is called clean up.

5-DEM building, using the contour maps prepared in the previous steps. At the first step editing the contour lines and giving each contour line its real height value, then select the surface that considered in saving the data and information correlated with DEM building. This model will be depending on the triangulation and gridding method. This means the information that is saved in surface file will be converted to a ["Triangulated Irregular Network" (TIN), "Inverse Distance Weighting"(IDW), Kriging and Polynomial]. After that the digital elevation model is displayed as a perspective figure for a three-dimensional vision. As shown figures:-

Figure (9a) which illustrates the digital elevation model produced for the study areas by TIN method.

Figure (9b) which illustrates the digital elevation model produced for the study areas by Kriging method.

Figure (9c) which illustrates the digital elevation model produced for the study areas by IDW method.

Figure (9d) which illustrates the digital elevation model produced for the study areas by Polynomial method.

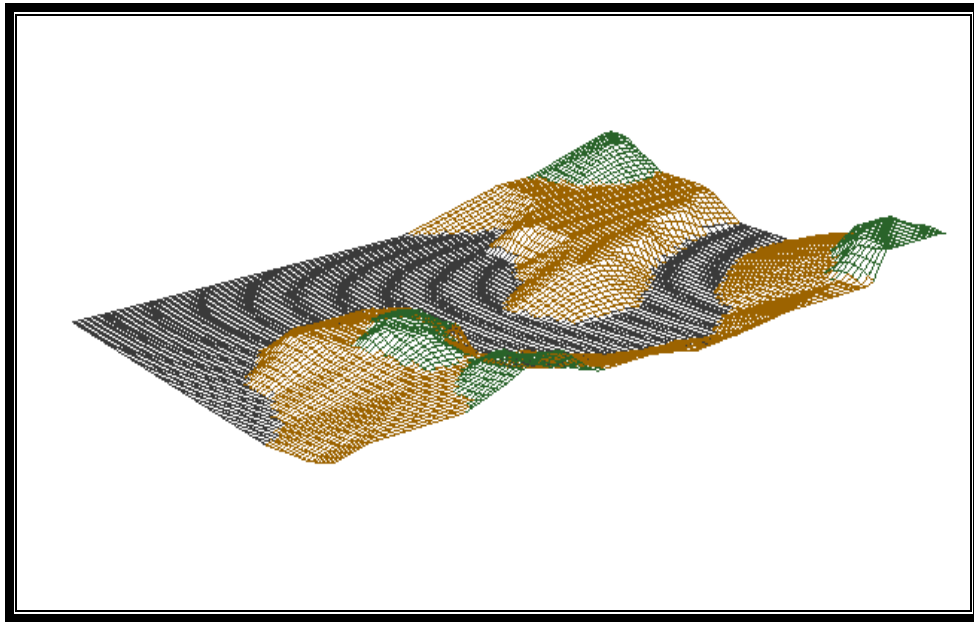


Fig (9a): DEM of the study area (Taq Taq) by TIN method.

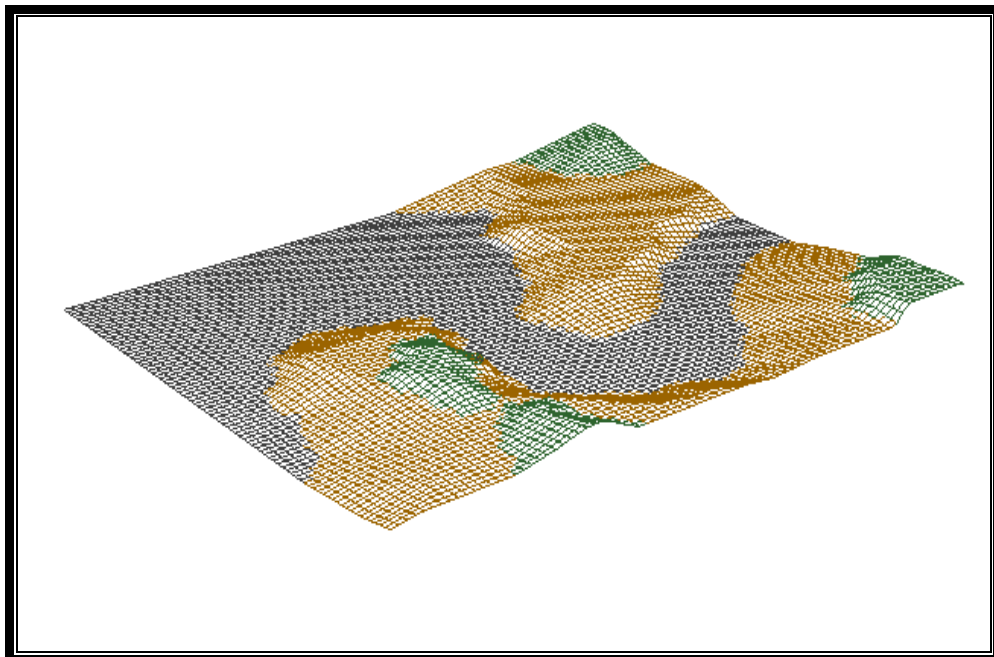


Fig (9b): DEM of the study area (Taq Taq) by Kriging method.

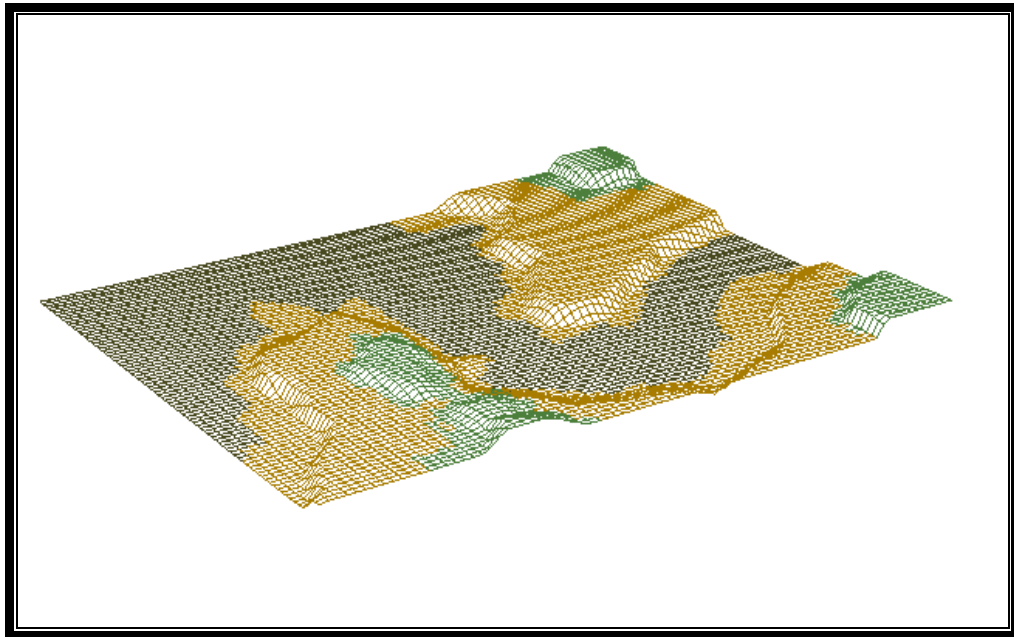


Fig (9c): DEM of the study area (Taq Taq) by IDW method.



Fig (9d): DEM of the study area (Taq Taq) by Polynomial method.

Evaluation of the Mathematical Techniques Accuracy

The Evaluation of Mathematical Techniques Accuracy for the DEM produced from Scanned Topographic Maps

To evaluate the accuracy of the mathematical techniques used for producing the DEM from scanned topographic maps, for study areas, the following procedure should be followed:

After producing the DEM from the scanned topographic maps with various mathematical methods, the models are transferred (Raster Digital Elevation Model) by using the (ERDAS Imagine ver.8.7) software. Figures (10) represent the raster digital elevation model for the study areas.

Thirty random points were chosen on each transferred models and its coordinates were recorded (E, N, Elev.), then simplifying or projecting them in to topographic maps which are related to the same model, using the value of elevation by traditional methods and taking in consideration the distance between the two lines, the point lies between them and the contour interval value, then compute the value of (RMSE) in elevations with the typical standard deviation proposed by National Mapping Accuracy Standard (NMAS) depending on the used map scales and contour intervals, for all models with the mathematical methods.

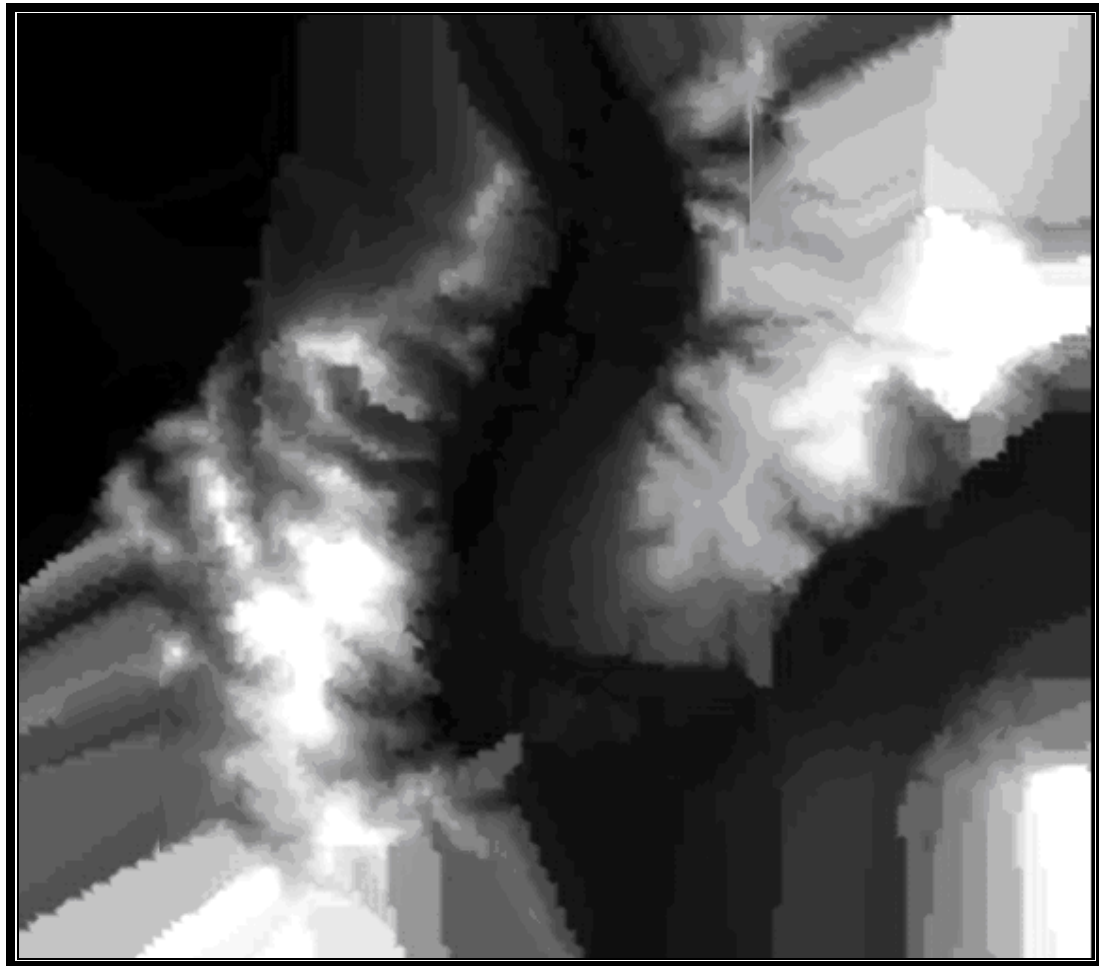


Fig (10): Raster DEM for field survey data of the study area.

The Root Mean Square Error (RMSE) were computed for each mathematical method by equation (21) and the Vertical Map Accuracy Standard (VMAS) also computed as described in section (2.4) for each method and comparative between the computed standard deviation and typical standard deviation as illustrated in table (2) and graphically by figures (11).

$$\sigma_z = \left[\frac{\sum_{i=1}^n (\delta_z - \bar{\delta}_z)^2}{n-1} \right]^{\frac{1}{2}} \quad (21)$$

Table (2): The computed Standard Deviation (σ_z) of DEM for the study areas by using different mathematical techniques.

Case Study. Methods.	1
TIN	9.694m
Kriging	7.465m
IDW	11.896m
Polynomial	15.146
Vertical Map Accuracy Standard (VMAS)	25m
Typical Standard Deviation	15.2m

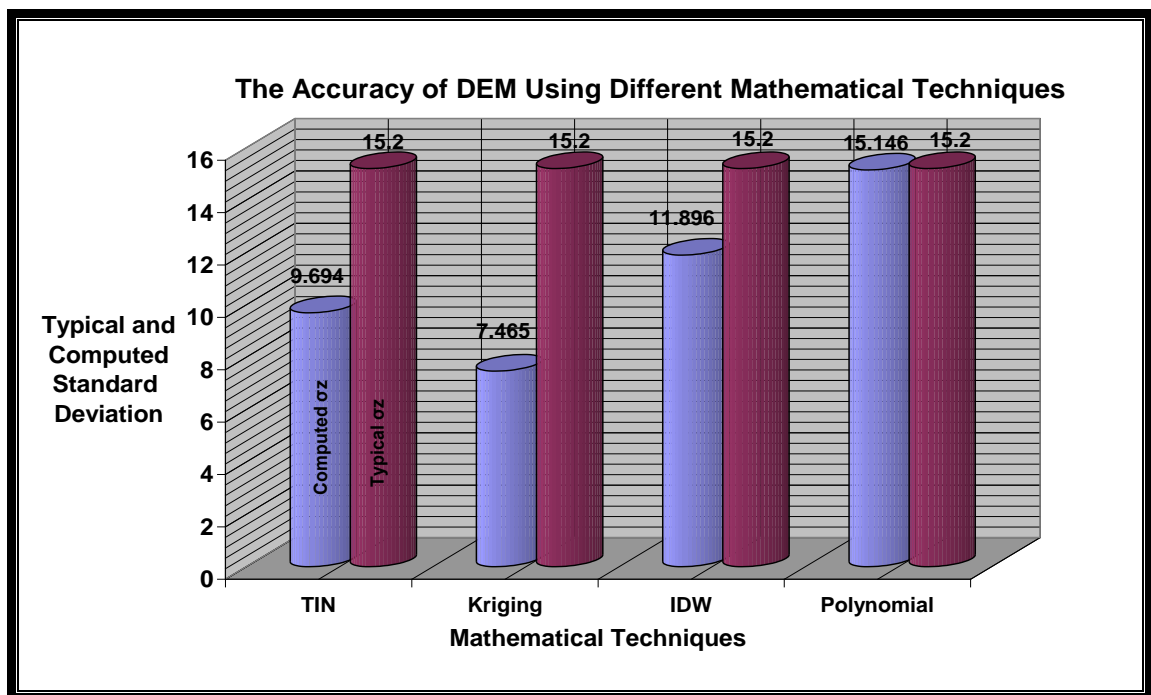


Fig (11): The Accuracy of DEM for the Study area by Using Different Mathematical Techniques Statistically.

The Evaluation of Mathematical Techniques Accuracy for the DEM produced from Field Survey and Topographic Map

To evaluate the mathematical techniques accuracy for the DEM produced from scanned topographic map for TaqTaq area, a compartment have been done by subtracting the produced DEMs from both data types first the evaluation accuracy was made from the model to the map, then an evaluation for the accuracy was made from model to the field survey with many mathematical methods, as follows:-

To examine the accuracy of the model, we consider the layer, of DEM produced from the field survey for TaqTaq area is the main layer which is produced by Universal Kriging method, but the layer of DEM produced from scanned topographic map is the sub-layer produced by the following methods (Universal Kriging, IDW, TIN and Second Degree Polynomial). By using the (Autodesk Field Survey) software, we subtract the main layer from each sub-layer by recalling the two layers and determining them from work file. This process was named (layer subtraction) were results are the differences of elevations (difference in size of cut and fill) see figure (12). Fifty points were selected for checking the accuracy of mathematical methods, in both Digital Elevation Models (DEMs) (main and sub-layer), the difference were computed in (z value), also the value of Root Mean Square Error (RMSE) computed by using a special equation (21).

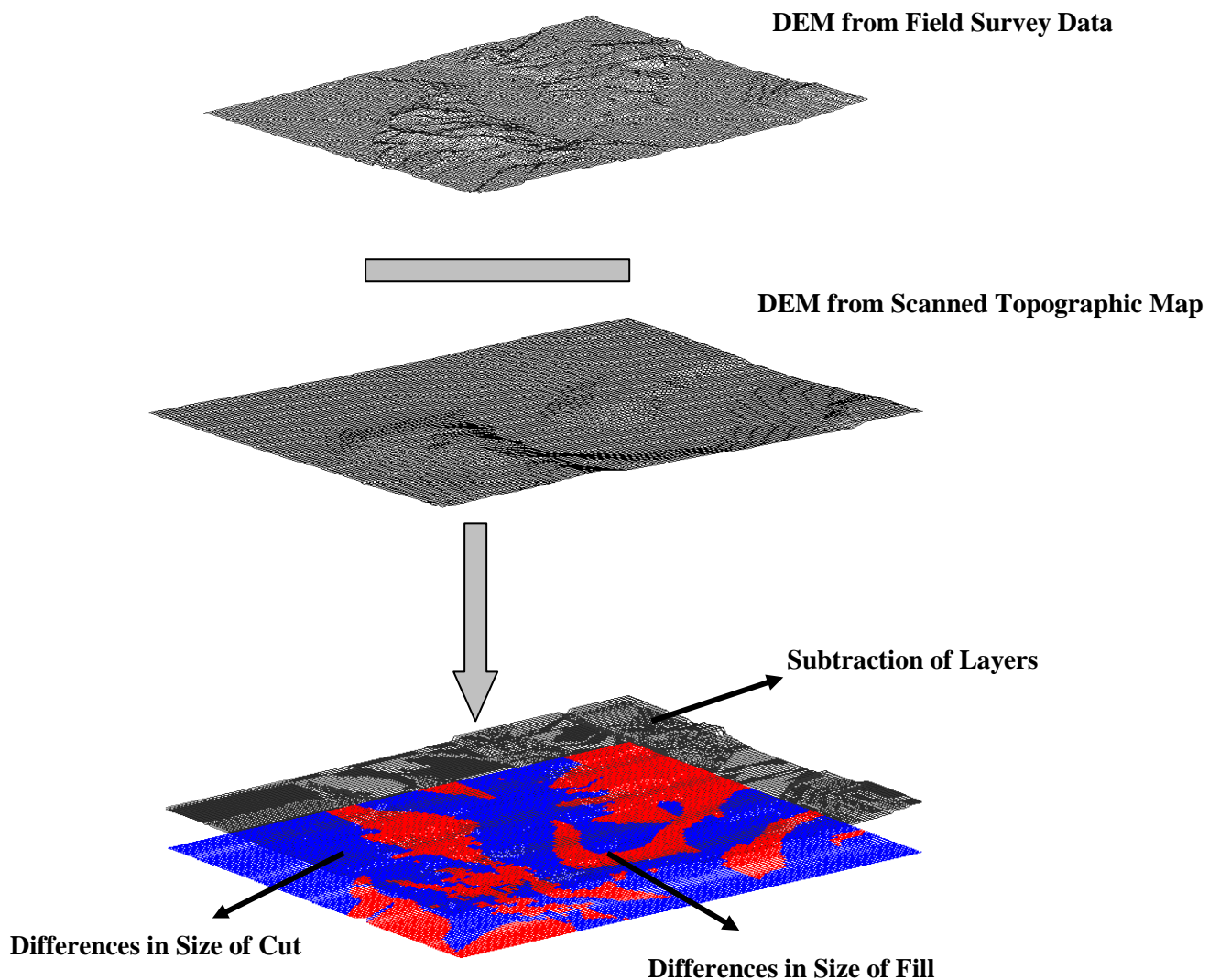


Figure (12): The process of Layers Subtraction by using Autodesk Field Survey software.

Results:

It can be noticed that the differences in elevations values between those obtained from field survey DEM and the obtained one scanned contour map, is within the contour interval limit which is (50m), i.e. this difference is within the acceptable range, but the differences in some

points has exceeded the (50m) contour interval, which caused a "gap" in some areas between the two models.

The reasons why the difference between the points which lies in two models (elevations) exceeds the acceptable range of the contour interval can be caused by one of the following:

1- The scale of the drawing. The larger the scale used the greater the accuracy obtained in line drawing, and hence the greater accuracy in elevation model (DEM). This can be obtained for scales larger than 1/25,000, for e.g. (1/10,000, 1/5,000), and

2- An error has occurred in the drawing of the contour line from aerial photographs in the "gaps" area between the two models.

The Root Mean Square Error (RMSE) were computed for each mathematical technique by equation (21) and also the Vertical Map Accuracy Standard (VMAS) computed. Table (3) summarize the predicted accuracy for all these mathematical methods.

Table (3): The computed standard deviation for the model by using mathematical techniques.

Mathematical Techniques.	σ_z
Triangulated Irregular Network (TIN).	14.439m
Kriging interpolation.	11.849m
Inverse Distance Weighting (IDW).	16.709m
Polynomial interpolation.	17.989m
Vertical Map Accuracy Standard (VMAS)	25m
Typical Standard Deviation	15.2m

A graphical representation of table (3) illustrates the difference in accuracy more as in fig.(13)

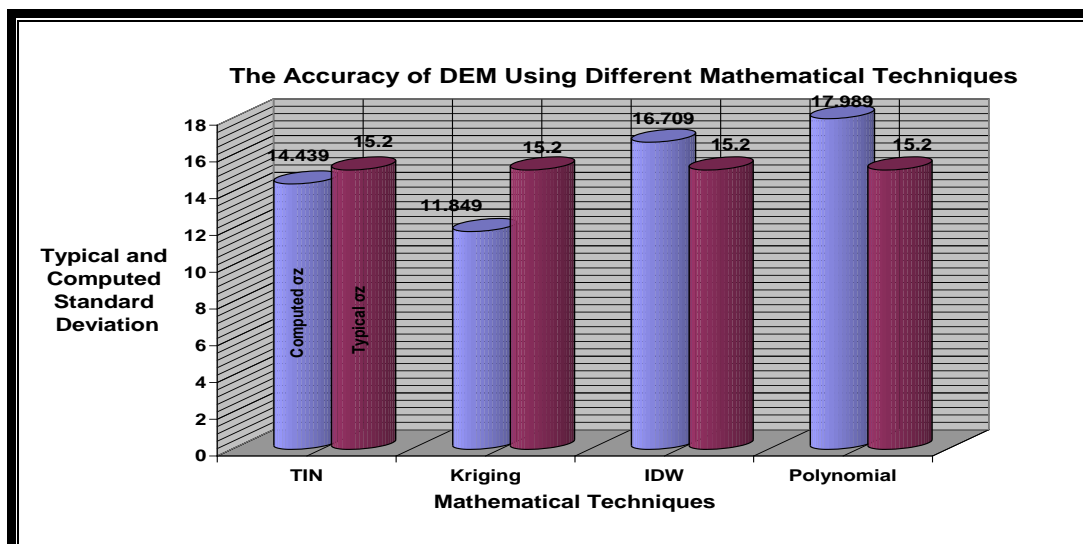


Fig (13): The Accuracy of DEM Using Different Mathematical Techniques Statistically.

Conclusions:

From the results obtained in table (3), the following conclusions extracted:-

1. The DEM's produced by four mathematical methods were compared on the basis of computing the Root Mean Square Error (RMSE). It was found that the best interpolation method is Kriging which gives the best results as illustrated in figure (14).

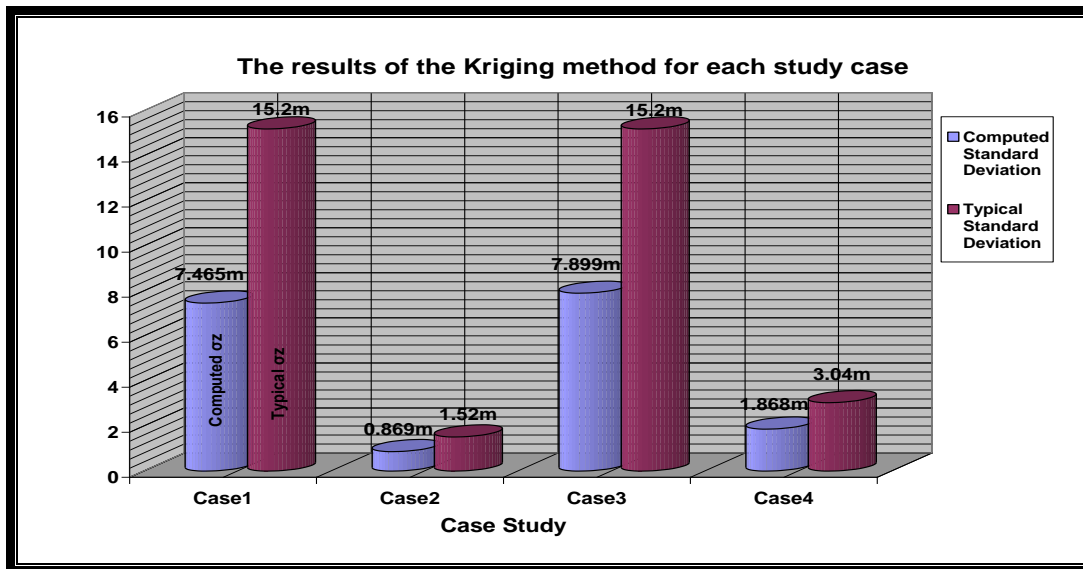


Fig (14): The results of the Kriging method for each study area.

2. The digital elevation model could be produced from scanned topographic maps with acceptable accuracy standards by using suitable interpolation method. It was found that the scale of maps and the contour intervals affect the accuracy besides the interpolation method.

3. When dealing with Raster format DEM it was found that the produced DEMs by mathematical methods is the one produced by Kriging technique which is a model distinguished for its high clarity, it was the reason to produce the photo textured DEM depends on the obtained kriging model.

4. Digital Elevation Models (DEMs) proved the effectiveness of solving many scientific and engineering problems like the intervisibility analysis, terrain analysis, volumetric computations and production of photo-realistic DEMs which is very useful in military applications.

**REFERENCES:**

Anderson J. M. and Mikhail E. M., “*Surveying Theory and Practice*”, 7th edition, University of California Berkeley, pp (792-793), (1998).

ESRI, “*What is Arc GIS*”, Computer Program, (2001).

Habib, A. “*Digital Terrain Model*”, 2003,
<http://www.geomatics.ucalgary.ca/~habib/courses/engo573/AKAM-CH-3-1.PDF>

Ibrahim, T. A. “*The Utilization of local resources to develop a geographical information system in survey and route design*”. M.SC. Thesis, college of engineering, Department of Surveying, Baghdad University, (1993).

Kennie, T.J.M. and Peterie, “*Engineering Surveying Technology*”. John wiley and Sons, New York, (1993).

Lebrer, F. “*Interpolation in Square grid DTM*”, ITC Journal, No.5, , pp (15-25), (1973).

Surfer(tm) help for Windows Version 6.xx Golden Software Incorporated, 1995 [Help of Surfer Package, (1995)].