NONLINEAR ANALYSIS OF ONE AND TWO DIMENSIONAL CONSOLIDATION PROBLEMS FOR UNBOUNDED SATURATED SOIL MEDIA

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ABSTRACT

Soil in general is not a linear material, in which the relation between stress and strain is more complicated than the simple, linearly elastic relation. To make accurate deformation analysis of clay deposits, the incremental Biot's theory of consolidation is used together with elasto-viscoplastic constitutive relations based on the critical state concept using the modified Cam-clay model. One and two dimensional consolidation problems were analyzed numerically by the infinite-finite elements. Results show that the proposed method can describe the effect of sample thickness on consolidation phenomena. Also the two dimensional behavior of a clay foundation during the construction of embankment was analyzed.

الخــلاصــــة

إن التربة وبصورة عامة لا تعتبر كمادة خطية، حيث إن العلاقة بين الإجهاد و الانفعال معقدة جداً مقارنة بالعلاقة الخطية المرنة البسيطة.

لذلك و لغرض الحصول على تحليل دقيق لتشوهات التربة، فقد استعملت نظرية(**Biot's**) للانضمام المتزايد مع علاقات النموذج المرن -اللزج اللدن اعتماداً على مفهوم الحالة الحرجة للتربة و باستخدام النموذج المعروف بـ ـ ـ ـ ـ (**Modified Cam clay**). تم تحليل مسائل الانضمام للتربة ذات البعد الواحد و البعدين

عددياً باستخدام العناصر الغير محددة و المحددة، حيث أظهرت النتائج أن الطريقة المقترحة قادرة على وصف تأثير سمك النموذج على ظاهرة الانضمام، وكذلك تم دراسة سلوكية الأساس الطيني أثناء إنشاء سدة ترابية.

INTRODUCTION

Most work on the analysis of consolidation problems using numerical technique has considered linear elastic behavior of soil. Unfortunately, Terzaghi's consolidation theory does not always describe the relation between excess pore pressure and settlement during consolidation of a clay deposits. Mesri and Choi (1979) reported that the settlement of a soft clay deposits take place with almost constant values of effective stress and pore water pressure. Similar phenomena have been observed for sensitive and aged clays, where this phenomena is due to both time-independent (plastic behavior), and time-dependent (creep) of clay.

In order to make accurate predictions of consolidation problem, it is necessary to use a constitutive model for clay that describes not only in viscid behavior, such as strain hardening, but rate sensitive behavior as well.

In this work, linear elastic Biot's consolidation is modified by extending the media to include infinite domain using an elasto-viscoplastic constitutive relations to model the non-linear soil stress-strain relations; Abdul-Hameed (1998).

CONSTITUTIVE EQUATIONS FOR SATURATED SOIL MEDIA

The general idea of elasto-viscoplastic behavior linking the strains and effective stresses in the soil skeleton are introduced for a general stress state. Perzyna (1966) proposed the following flow rule for viscoplastic deformation in the simple case of an infinitesimal strain field:

$$\dot{\varepsilon}^{vp} = \bar{\gamma} \rangle \phi(F) \langle \frac{\partial F}{\partial \sigma'} \qquad (1)$$

in which $\bar{\gamma}$ is a fluidity parameter controlling the viscoplastic flow rate, $\phi(F)$ represents the strain rate effect on the yielding of the material, and defined as:

$$\rangle\phi(F)\langle = \begin{cases} 0 & F < 0 \\ \phi(F) & F \geq 0 \end{cases} \qquad (2)$$

The functional form of $\phi(F)$ is given by Zienkiewicz et al. (1975) as follows:

$$\phi(F) = \left(\frac{F}{F_o}\right)^N \qquad (3)$$

where F_o the yield value is in uni-axial stress and N is an exponent, which is simply used equal to unity.

The function F is an appropriate yield function. The mechanical behavior of clays can be described by the modified Cam-clay model (Roscoe and Burland, 1968). The yield surface of this model is an ellipse in **p-q** plot as shown in Fig. (1), and mathematically is expressed as:

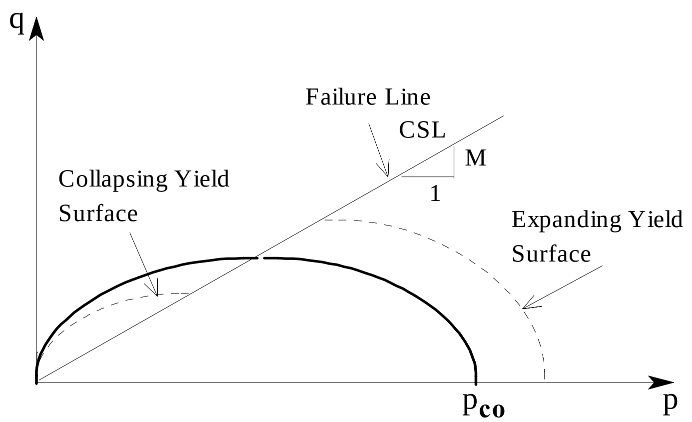

Fig. (1) Modified Cam-Clay Model in The Space of The Two Stress Invariants P and q.

$$F = \frac{q^2}{M_{c.s}^2} + p^2 - p.p_c\left(\varepsilon_{vp}^v\right) \qquad (4)$$

where **p, q, p_c,** and **$M_{c.s}$** are respectively the mean effective stress, the deviator stress and are related to the stress invariants as given in **[**Appendix **A],** the current diameter of the ellipse (yield surface) in p-direction and is analogous to a pre-consolidation pressure and the value of the stress ratio (**q/p**) at the critical state conditions; $\mathbf{M_{c.s}}$ is related to the angle of internal friction ϕ obtained from tri-axial compression test by:

$$M_{c.s} = \frac{6.\sin\phi}{3-\sin\phi}.g(\theta) \tag{5}$$

The shape of the failure surface in the deviatoric π- plane can be made to coincide with Mohr-Coulomb principles with a smooth variation (Yousif, 1984):

$$g(\phi) = \frac{2K}{(1+K)-(1-K).\sin 3\theta} \tag{6}$$

In which θ is the Lode angle defined in the deviatoric plane (Yousif, 1984):

$$K = \frac{3-\sin\phi}{3+\sin\phi} \tag{7}$$

When yielding occurs, the yield surface will expand or contract as the soil hardens or softens as shown in Fig. (1). The initial size of the ellipse is governed by the maximum pre-consolidation pressure ($\mathbf{p_{co}}$) to which the soil has previously been subjected during its past history. The change in the current value of the hardening parameter (**pc**) is related to the viscoplastic volumetric strain increment through the following equation:

$$\Delta p_c = \frac{\lambda-\kappa}{1+e_o}.p_{co}.\Delta\varepsilon_{vp}^{v} \tag{8}$$

where $\mathbf{e_o}$ is the initial void ratio, λ is the slope of the normal consolidation line in **e-ln p** space, and **κ** is the slope of swelling and recompression line in **e-ln p** plot as shown in Fig.(2).

CONSOLIDATION ANALYSIS BY THE FINITE ELEMENT METHOD

The consolidation problem may be discretized using finite elements. A pore pressure degree of freedom is added to the usual displacement degree of

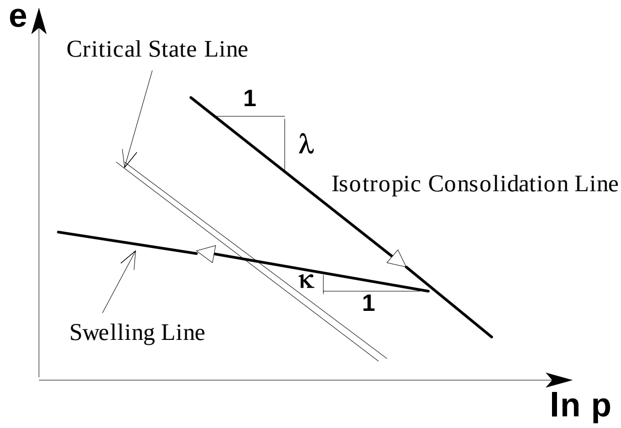

Fig. (2) Consolidation and Swelling Lines in e-ln p plot.

Freedom at the corner nodal points only. The spatial integrations may then be performed using Gaussian quadrature, and a time marching process is used to perform the time integration.

Shape functions {N} and $\{N_p\}$ are used to relate the continuous values of displacements u, and pore pressure p to the to the nodal values u^n , and p^n as follows (Zienkiewic , 1977) :

$$\left.\begin{aligned}\{u\} &= \{N\}^T.\{u^n\} \\ \{p\} &= \{N_p\}^T\{p^n\}\end{aligned}\right\} \tag{9}$$

The coupled equations for an element may be written in matrix form as [detailed of which are given by Yousif (1984) and Abdul-hameed (1998)]:

$$\begin{bmatrix} K_T & L \\ L^T & -0.5\Delta t.H \end{bmatrix}\begin{Bmatrix} du^n \\ dp^n \end{Bmatrix} = \begin{Bmatrix} df + C \\ 0 \end{Bmatrix} \tag{10}$$

In which the gravity load $\overline{F}$ is set equal to zero.

Where the element stiffness matrix $[K_T]$ is given in the usual way by:

$$[K_T] = \int_v B^T.D.B.dv \tag{11}$$

In which D is the elasticity matrix either for plane strain or axi-symmetric conditions. The matrix [L] couples the stiffness equations with those of fluid flow and is given as:

$$[L] = \int_v B^T.m.N_p^T.dv \tag{12}$$

Where

$$m^T = (1 \quad 1 \quad 0 \quad 1) \tag{13}$$

The fluid flow [H] matrix is given by:

$$[H] = \int_v (\nabla N_p^T)^T \frac{\{k\}}{\gamma_w} \nabla N_p^T.dv \tag{14}$$

Where {k} is the permeability matrix, ∇ is the del-operator defined as:

$$\nabla^T = \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right) \tag{15}$$

{df} is the incremental load vector, due to boundary or body force loading and is given as:

$$df = \int_v N^T.db.dv \quad + \quad \int_s N^T.dt.ds \tag{16}$$

Where b and t are body forces and boundary tractions respectively. And finally, C is the additional load vector due to viscoplastic flow or stress relaxation and is given as:

$$[C] = \int_v B^T.D.\dot{\varepsilon}^{vp}.\Delta t_n.dv \tag{17}$$

The symmetry of the final formulation is assumed providing K_T is symmetric as in the case of using associated plasticity theory. For a specified time step Δt, and prescribed incremental loadings, the unknown increments of displacements $u^n(\Delta t)$ and pore pressure $p^n(\Delta t)$, can be obtained.

COMPOSITE INFINITE ELEMENT FOR CONSOLIDATION PROBLEMS

This type of element can be formulated to show the different types of decay rates applied for the field variables. For consolidation problems the decay rate applied can be obtained from the available analytical solutions.

Solutions for the instantaneous loading condition at ($t=0^+$) and at the end of consolidation ($t=\infty$) responses indicate that the vertical displacement at the surface of the half-space decays approximately in inverse proportion to the radial distance from loaded area.

The element with a (1/r) order decay rate was found to yield the most accurate solutions for various consolidation problems, Selvadurai and Gopal (1986).

Hence, the same type of decay rate (1/r) is employed in this research work to develop the composite infinite element used for consolidation problems. Five-nodded infinite element was shown in Fig. (3), represent an infinite medium in the ξ-direction. This element is a combination of five-nodded infinite element which representing quadratic variation for the displacement field and a two-nodded super parametric infinite element which is representing a linear variation for the pore pressure field.

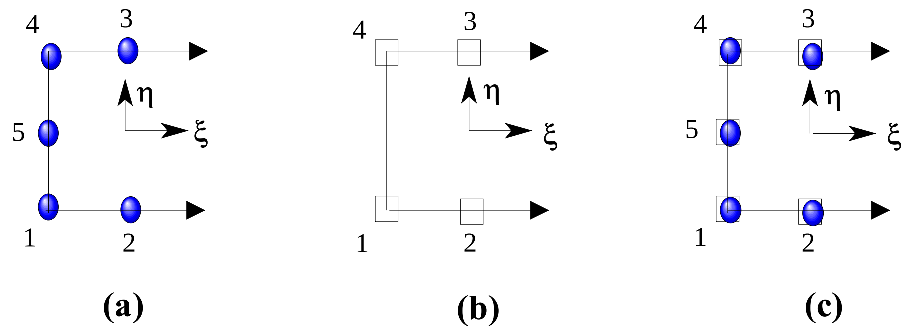

Fig.(3) Composite Infinite Element;
One)Five-Nodded Element for Displacement;
Two)Two-Nodded Super parametric Element for Pore Pressure; and
Three)Superposed Element [After Selvadurai and Rajagopal (1989)].

The pore pressure shape functions for the two-nodded super parametric element are obtained directly from those of the four-nodded linear quadrilateral element. Node 2 and 3 of this element are used only for mapping and are not used for interpolation.

Displacement shape functions for the five-nodded element are obtained from those of the eight-nodded quadratic element. Mapping and element shape functions can be found in Abdul-Hameed (1998).

The requirements for completeness and monotonic convergence are satisfied by these elements and therefore, the mapping is independent of the choice of the coordinate system.

The implementation of the infinite element required the following modifications:

* The jacobian is formulated on the basis of the singular mapping functions.
* The interpolation is carried on the basis of the variable at two or five nodes as indicated in Fig. (3).
* The mapping functions referring to the particular geometry of infinite elements is that the nodded coordinates of outer nodes (2 & 3) shown in Fig. (3 c) are taken twice as those of the corresponding inner nodes (1 & 4).

NUMERICAL APPLICATION

The coupled field equations presented by Eq. (11) have been incorporated into a finite element program called **"EVPCON" written in FORTRAN 77,** developed for this study to analyze one and two dimensional consolidation problems.

<u>One-Dimensional Consolidation Analysis</u>

An external uniform surface load (**T = 100 kN/m^2**) is applied to a clay layer and drainage is allowed to occur in the vertical direction only. The finite element mesh used in this analysis is shown in Fig. (4). The material properties needed for the nonlinear modified Cam-clay model are listed in Table (1). Euler's time integration scheme is used to calculate the viscoplastic strain increments.

The predicted settlements and pore pressure are presented for different values of over consolidation ratios in Figs. (5) And (6) for sample thickness of H=5cm. The pore pressure distribution are non-dimensionalized with respect to the initial pore pressure (p =T00), and settlements with respect to the final settlement ($S\infty$) predicted by Terzaghi's classical solution. The non-dimensional time factor Tv was used as the abscissa.

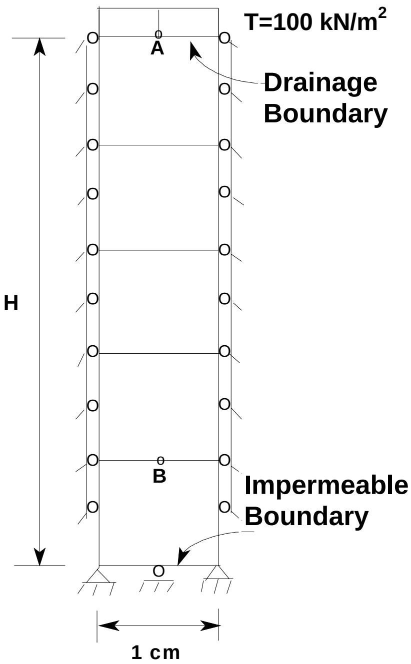

Fig. (4) Finite Element Mesh and Boundary Conditions for 1-D Nonlinear Consolidation.

Table (1) Material Properties Used for 1-D Consolidation Analysis

Material Properties	Soil
Young's Modulus, E′ (kN/m^2)	10000
Poisson's Ratio, ν′	0.333
Coefficient of Permeability, k (m/day)	1*10^{-4}
Fluidity Parameter, $\bar{\gamma}$ (day^{-1})	8.64*10^{-5}
Initial Void Ratio, e_o	1.5
Slope of Normal Consolidation Line, λ	0.231
Slope of Swelling Line, κ	0.05
Angle of Internal Friction, ϕ′	22
Initial Vertical Effective Stress, σ'_{yo} (kN/m^2)	200

The inclusion of the viscoplasticity effects decreases the magnitude of dissipation of pore pressure for Tv between 0.03 and 0.5. Therefore, the predicted pore pressure is higher than those predicted by the elastic analysis.

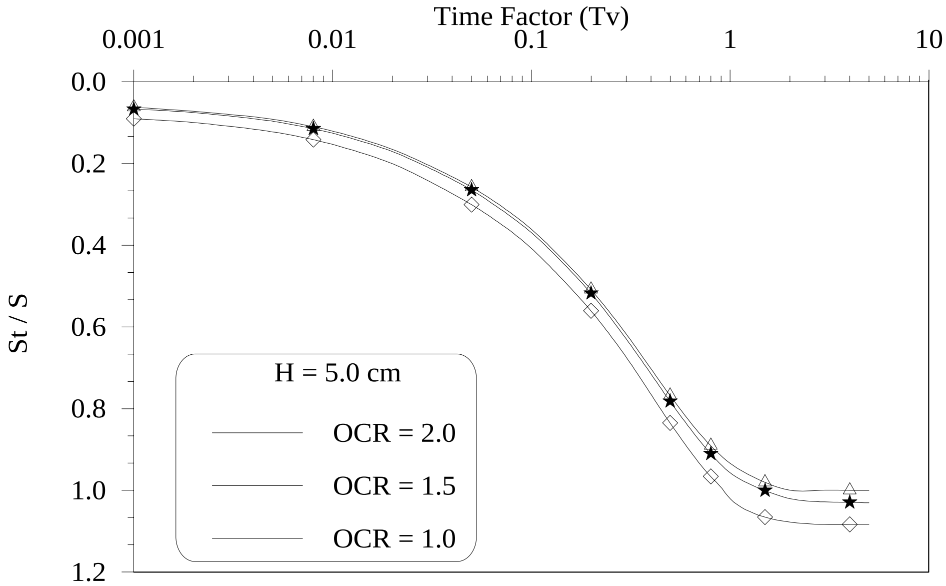

Fig. (5) One-Dimensional Consolidation; Surface Settlements versus Time Factor at (node A) for Different OCR.

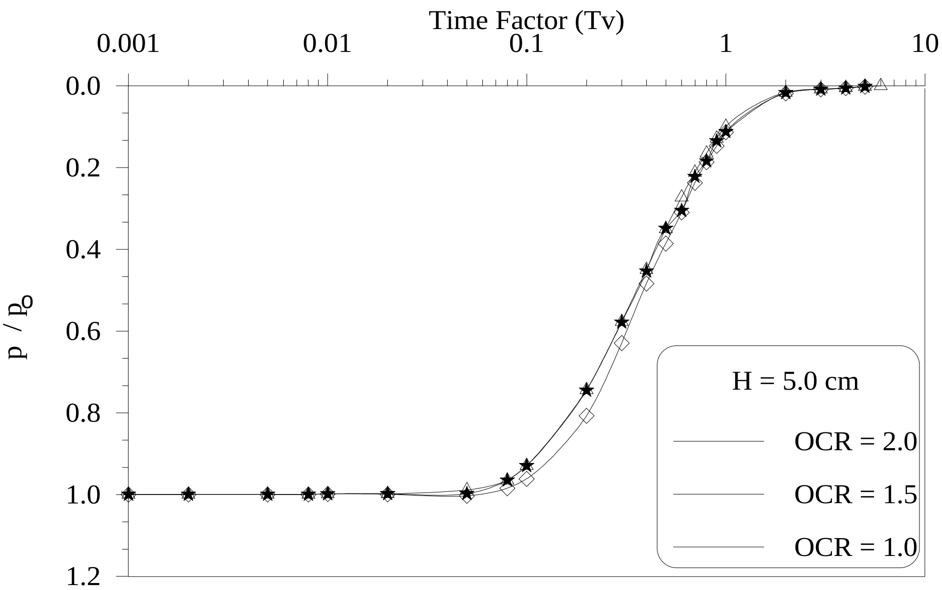

Fig. (6) One-Dimensional Consolidation; Pore Pressure versus Time Factor at (node B) for Different OCR.

When (OCR=2). Similarly the predicted settlements are higher than those predicted by the elastic theory. Figures (5) and (6) also show the effect of using different over consolidation ratio (OCR) on the predicted results. As (OCR) values increase the behavior is approaching that of the elastic solution obtained by Terzaghi's theory. For

(OCR) equal 2 elastic behaviors is predicted within the yield surface as the state of stress approaches the yield point viscoplastic is predicted till the end of consolidation. For (OCR) equal 1 non-linear analysis is predicted immediately upon loading.

Figure (7) shows the predicted vertical settlement-time curves for clay layer of (OCR=1) but of different heights, in which the effect of drainage path has been investigated.

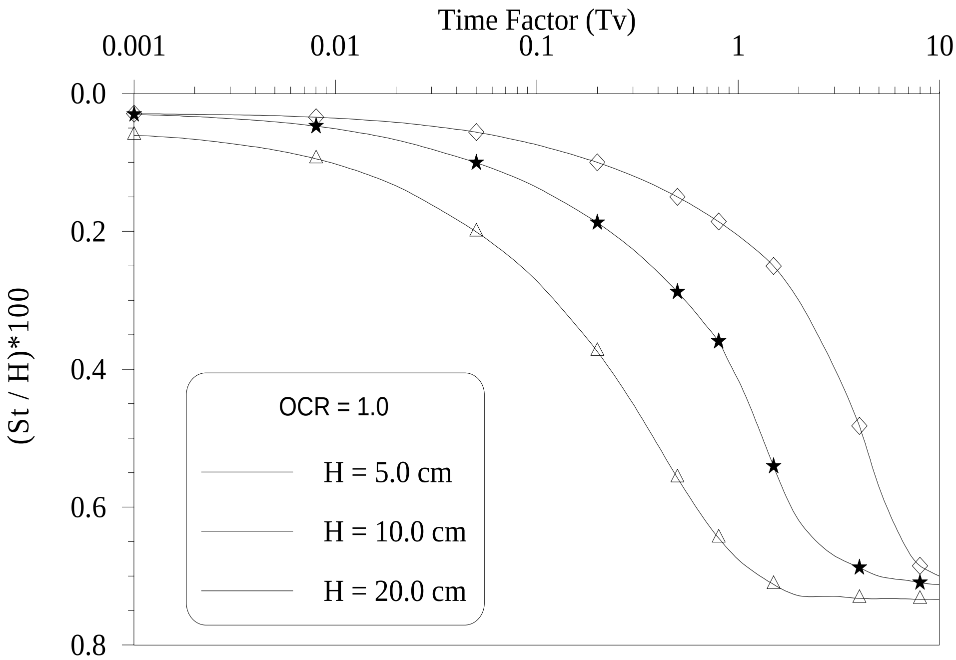

Fig.(7) One-Dimensional Consolidation; Surface Settlements Versus Time Factor at (node A) for Different OCR.

The predicted viscoplastic strain for the cases investigated was found to be proportional inversely to soil thickness. This behavior is experimentally supported by Aboshi (1973), who carried out several one-dimensional consolidation tests with different specimen heights on clay exhibiting creep settlement, it was found that the viscoplastic volumetric strain rate ($\dot{\varepsilon}^{vp}$) for the thick sample is lower than that of the thin sample.

Two-Dimensional Consolidation Analysis

The response of clay foundation during the construction of embankment has been investegated by Tavenas and Leroueil (1980). They reported that none of the existing analytical methods can describe field data. Oka et al. (1986) were the first who tried to account for this behavior using elasto-viscoplastic constitutive equations based on Cam-clay model and Biot's consolidation theory.

The construction of a similar embankment is analyzed numerically using the developed elasto-viscoplastic constitutive equations and Biot's consolidation theory under plane strain conditions. The infinite-finite elements idealization and boundary conditions are shown in Fig. (8). The material properties used for the nonlinear visco-plastic using the modified Cam clay model are listed in Table (2). The rate of loading required by the analysis is taken equal to 2 kN/m^2/day. The calculations were stopped at 100 days at the end of construction.

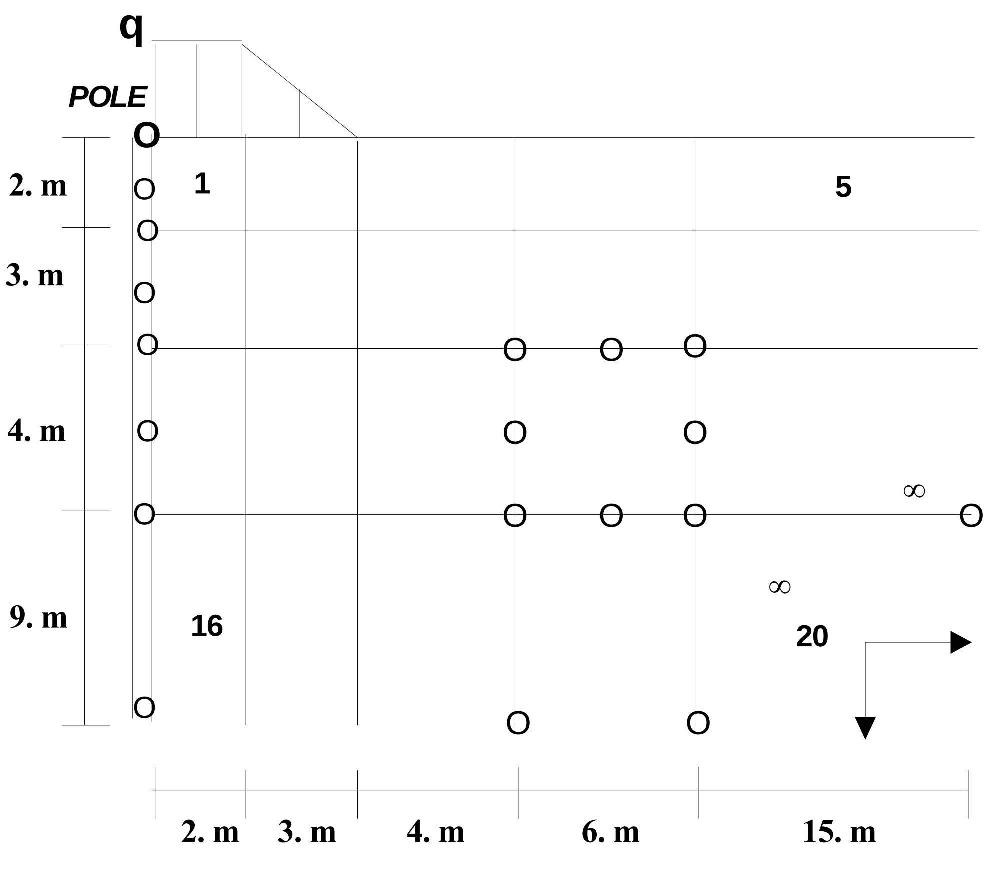

Fig. (8) Infinite-Finite Elements Mesh and Boundary Conditions.

Table (2) Material Properties Used for 2-D Consolidation Analysis

Material Properties	Soil
Young's Modulus, E' (kN/m^2)	5350
Poisson's Ratio, ν'	0.333
Coefficient of Permeability, k (m/day)	$1*10^{-5}$
Fluidity Parameter, $\bar{\gamma}$ (day^{-1})	$8.64*10^{-8}$
Initial Void Ratio, e_o	1.5
Slope of Normal Consolidation Line, λ	0.231
Slope of Swelling Line, κ	0.05
Angle of Internal Friction, ϕ'	22
Initial Vertical Effective Stress, σ'_{yo} (kN/m^2)	100

Figure (9), shows the settlement-time profile predicted at the center line elements and immediately below the embankment. Figures (10) and (11), show the distribution of excess pore pressure and lateral displacement versus depth build up during construction. Both lateral displacement and pore pressure are at maximum values between 2 to 5 m depth as shown.

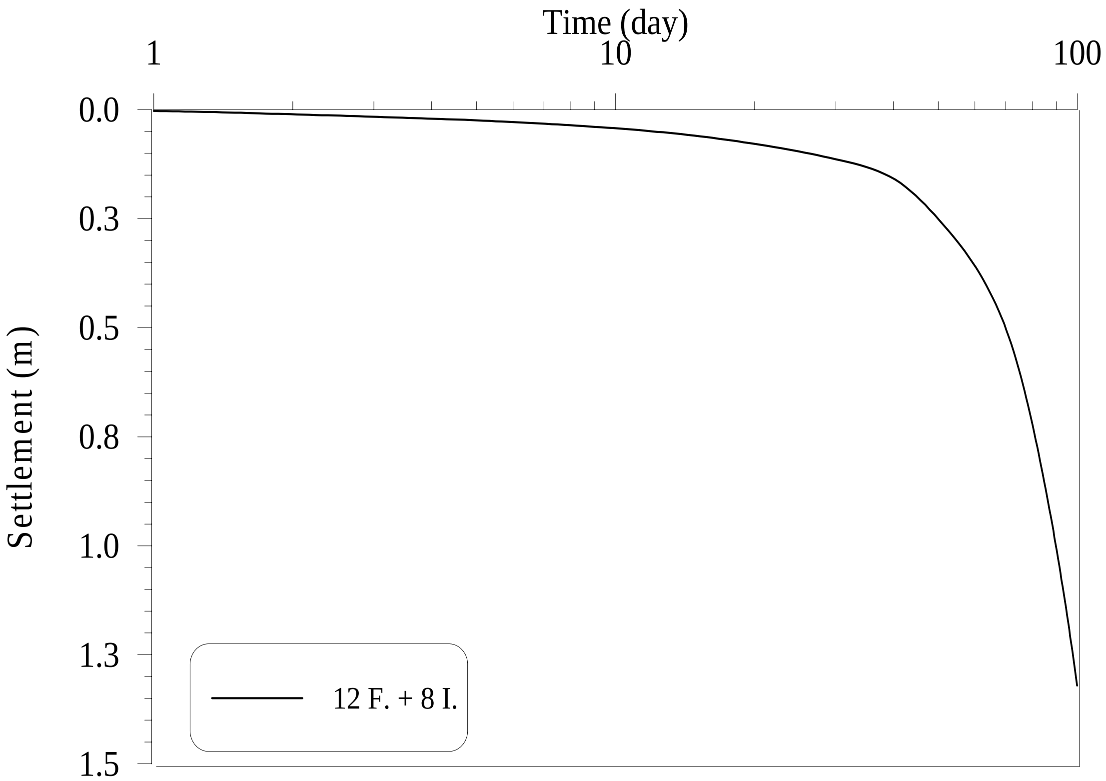

Fig. (9) Settlement-Time profile Center Line Elements.

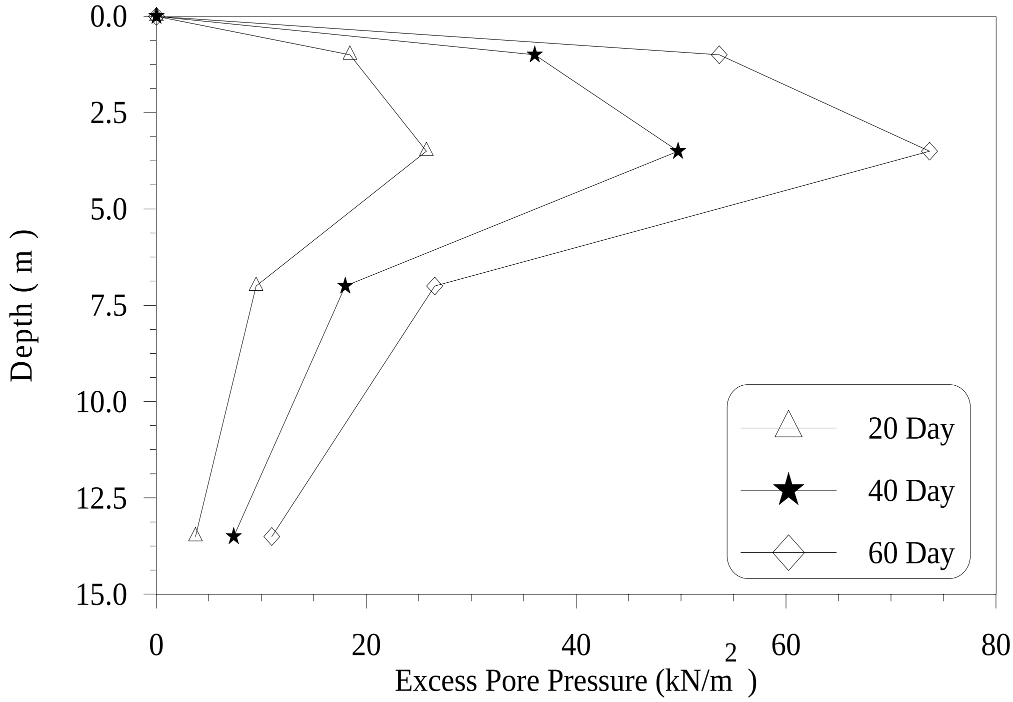

Fig. (10) Distribution of Excess Pore Pressure at Center of the First Column of the Elements versus Depth.

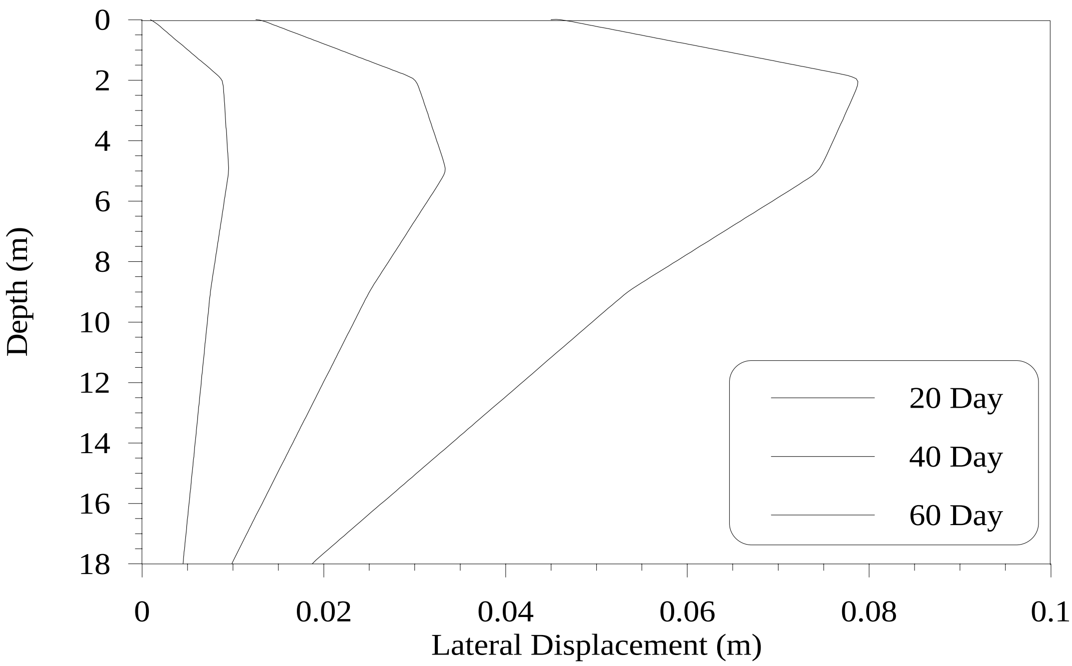

Fig. (11) Distribution of Lateral Displacement with Depth at Embankment Toe.

The relation between excess pore pressure and total vertical stress increment $\Delta\sigma_v$ of element 1 is shown in Fig. (12). As shown the ratio $p/\Delta\sigma_v$ is almost equal to 0.8 during early construction, thereafter the ratio increases to 1.0.

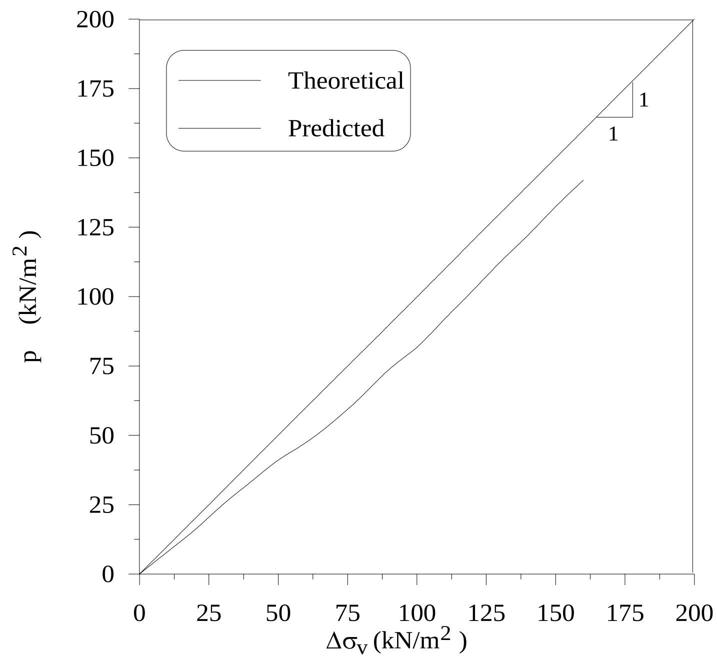

Fig. (12) Excess Pore Pressure versus Total Vertical Stress for Element1.

The trends of predicted behavior agree well with the field observation reported by Tavenas and Leroueil (1980). They related it to the fact that as natural clay is initially over consolidated, the rate of pore pressure dissipation is high at beginning of construction and $p/\Delta\sigma_v$ is smaller than unity, and as construction advances, pore pressure increment becomes equal to embankment load due to passage of clay to the state of normally consolidated, and the behavior is totally nonlinear.

SUMMARY AND CONCLUSIONS

The main conclusions obtained from these analyses can be summarized as follows:

1. The effect of using different over consolidation ratio (OCR) on the predicted clay behavior was examined, in which the elastic behavior was obtained when (OCR) equal 2, while the nonlinear behavior was predicted immediately upon loading when (OCR) is equal to 1.
2. The inclusion of viscoplastic flow decreases the amount of dissipation of pore pressure for the time factor Tv between 0.03 to 0.5.

3. The developed visco-plastic model can describe the effect of the length of the drainage path on the consolidation settlement. Where, the viscoplastic volumetric strain rate for the thick sample is found to be less than that of the thin sample.
4. The behavior of clay foundation during embankment construction was simulated using the developed model. From the distributions of excess pore pressure at center line element and lateral displacement at toe versus depth, maximum values are found at depths between 2 to 5 m. Also, the rate of pore pressure dissipation is high at beginning of construction and ($p/\Delta\sigma_v$) is smaller than unity, and as construction advances this rate becomes equal to embankment load. This behavior is related to the passage of clay to state of normally consolidated from the state of over consolidated.
5. A numerical difficulty encountered in all analyses carried is in the choice of the fluidity parameter $\bar{\gamma}$. The predicted results were very sensitive and a value between 10^{-8} - 10^{-12} per second was used.

APPENDIX A

The state of effective stress of a soil continuum needed for the modified Cam clay model are the mean effective stress and the deviatoric stress respectively:

$$\left.\begin{aligned} P &= \frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right) \\ q &= \sqrt{3}\left(\frac{1}{2}S_{ij}S_{ij}\right)^{\frac{1}{2}} \end{aligned}\right\} \qquad \text{(A-1)}$$

Where σ_1, σ_2 and σ_3 are the three principal stresses and S_{ij} is the deviatoric stress as:

$$S_{ij} = \sigma_{ij} - \frac{1}{3}.\delta_{ij}.\sigma_{kk} \qquad \text{(A-2)}$$

Here σ_{ij} is the stress tensor, δ_{ij} is the kronecker delta.

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