APPLICATION OF MATHEMATICAL DRILLING MODEL ON SOUTHERN IRAQI OIL FIELDS

Dr. Sameera M. Hamad-Allah **Ali A. Ismael**
University of Baghdad/ College of Engineering
Petroleum Engineering Department

ABSTRACT

Minimum-cost well drilling demands the best use of controllable drilling variables for each formation to be drilled. To reach this aim, this study was divided into two main parts:

The first part deals with applying a mathematical drilling model to field data of forty wells drilled at three major oil fields (RU, R, and Z). Bourgoyne & Young (1974) drilling model has been modified to take into consideration the combined effect of weight on bit, rotary speed, bit type, bit size, flow rate, drilling fluid density, drilling fluid viscosity, oil content, bit-nozzle size, formation drillability, formation abrasiveness, bit bearing constant, formation hardness, formation compressive strength, differential pressure between mud column pressure and formation pressure, and bit dullness on drilling rate at these fields. The measurements of formation compressive strength have been achieved using 34 core plugs. These plugs were cut and prepared for soft, medium, and hard formations under study. The drilling model was fitted to Field data by using multiple regression analysis technique. The results of analysis gave low standard deviation, high correlation coefficient, and good matching between measured and calculated drilling rate. The validity of modeling process has been verified by applying the proposed drilling model on other wells that have not been included in the main analysis.

The second part deals with using the drilling model together with non-linear optimization technique to determine the optimum values of the controllable drilling variables. These variables are: weight on bit, rotary speed, flow rate, drilling fluid density, drilling fluid viscosity, oil content, bit-nozzle size. Using the proposed mathematical drilling model together with the Constrained Rosenbrock optimization technique achieved a marked reduction in drilling cost about 60%, 75%, 80% in soft, medium, and hard formations respectively. The results of optimization were used to construct optimum bit record for the next wells to be drilled. For comparison purpose, this optimum bit record has been used to reduce the drilling cost for well RU263 and saved about 295000 $ in the total cost of this well which is equal to 394176 $.

الخلاصة

ان حفر الابار باقل كلفة يتطلب افضل استخدام لمتغيرات الحفر المسيطر عليها في كل تكوين مراد حفره. لتحقيق هذا الهدف، قسمت هذه الدراسة الى جزئين اساسيين:

الجزء الأول يتناول تطبيق نموذج حفر رياضي على بيانات حقلية تم الحصول عليها من أربعين بئر محفورة في ثلاث حقول نفط رئيسية(RU, R و Z). تم تطوير نموذج الحفر الرياضي لبرجوين و يونك كي يأخذ بنظر الاعتبار التأثير المشترك لكل من الوزن المسلط على الحافرة، سرعة الدوران، نوع الحافرة، حجم

الحـافرة، معـدل الجريـان، كثافـة سـائل الحفـر، لزوجـة سـائل الحفـر، المحتـوى النفطـي، حجـم منافـث الحـافرة، انضغاطية التكوين، قابلية حفر التكوين، قابلية كشط التكوين، ثابـت محـور الحـافرة، صـلابة التكـوين، انضـغاطية التكوين، الضغط التفاضلي بين ضغط عمود الطين و ضغط التكوين، و تلف الحـافرة علـى معـدل الحفـر فـي هـذه الحقول. تم إنجاز قياسـات انضـغاطية التكـوين باسـتخدام(٣٤) سـدادة صـخرية. تـم قطـع و تحضـير هـذه السـدادات للتكوينات الرخوة والمتوسطة الصلابة و الصلبة تحت الدراسة. لقد تـم ضـبط هـذا النمـوذج علـى البيانـات الحقليـة باستخدام تقنيـة التحليل التراجعي المتعدد. نتـائج التحليـل أعطـت انخفـاض فـي الانحـراف معيـاري و زيـادة فـي معامل الارتباط بالإضافة إلـى تطـابق جيد بين قيم معدل الحفـر المقاسـة حقليـا و المحسـوبة. فحصـت دقـة عمليـة النمذجة عن طريق تطبيق نموذج الحفر الرياضي المقترح على آبار أخرى لم تدخل في التحليل الرئيسي.

الجزء الثاني يتناول استخدام نموذج الحفر الرياضي مع طريقة مناسبة من طرق الحل الامثل غير الخطية لتحديد القيم المثلى لمتغيرات الحفر المُتحكم بها. هذه المتغيرات تشمل الوزن المسلط على الحافرة، سرعة الدوران، معدل الجريان، كثافة سائل الحفر، لزوجة سائل الحفر، المحتوى النفطي،و حجم منافث الحافرة. استخدام نموذج الحفر الرياضي المقترح مع تقنيةروزنبروك المقيدة أعطى انخفاض ملحوظ في كلفة الحفر مقداره ٦٠٪ و٧٥٪ و٨٠٪ للتكوينات الرخوة والمتوسطة الصلابة و الصلبة على التوالي. استخدمت النتائج المثلى لإنشاء امثل سجل حافرة للآبار القادمة المراد حفرها. لأغراض المقارنة، تم استخدام سجل الحافرة الامثل لتقليل كلفة الحفر للبئر(RU263) حيث وفر ما يقارب ٢٩٥٠٠٠ دولار من الكلفة الكلية لهذه البئرالبالغة ٣٩٤١٧٦ دولار.

INTRODUCTION

Minimizing the drilling cost can be achieved by successful modeling of the drilling process and full understanding of the major factors that affecting directly or indirectly drilling rate. Through the work of numerous investigators, the most important drilling factors have been identified as controllable and uncontrollable factors. The controllable factors are; bit weight or drilling force, bit rotational speed, bit type and size, hydraulics, and drilling fluid type and properties. While the most important uncontrollable factors are; weather and location, water availability, rig conditions and flexibility, round trip time, rock properties, depth, bottom hole temperature, hole problems, and crew efficiency. The lowest drilling cost doesn’t result from increasing penetration rate alone, but also equipment life and wellbore stability. With a basic understanding of the principal mechanisms and the physical processes involved in the drilling operation, theoretical relationships and empirical correlations based on both field and laboratory measurements materialized out of 1950′s.

The basic drilling rate-weight-rotary speed relationships (the so-called R- W-N equations) were essentially the earliest form of drilling models. According to these basic forms, drilling rate was equated to the product of weight on bit and rotary speed, each raised to an empirically derived exponent, and multiplied by proportionality constant to take into account the formation characteristics. This elementary form of the penetration rate equation has basically remained unchanged over the years although modifications have been incorporated to include the effects of hydraulic system, drilling fluids, and bit dullness.

Results from a successful modeling effort can be used for optimization of drilling operations to achieve conditions of minimum cost. Drilling optimization is defined as a technique for pre-selecting the magnitude of drilling variables in order to maximize the drilled footage and reduce the total drilling time. The basic idea of optimized drilling is to use the recorded data of the first well as a basis for calculations and to apply optimum techniques to the second and third wells in order to improve the drilling efficiency.

There have been many drilling models presented before to predict the physical processes during drilling operations (*Galle et al. (1960), Young (1969), Bourgoyne et al. (1974), Moore (1974) Cunningham (1978)*). These mathematical models are used also with optimization techniques to provide a method for selecting optimum drilling variables with more cost savings.

PREVIOUS DRILLING MODELS

In general, a mathematical drilling model provides a method to predict and control drilling process and minimize drilling cost. Drilling models also provide a mean for recognizing unusual effects when the observed bit performance deviates from predictions. For successful application, any drilling model must have the following three basic equations: rate of penetration equation, rate of bit tooth-wear equation, and rate of bit bearing-wear equation. These equations can be used to find the values of rotating time (T_f) and footage drilled (F_f) during bit runs. Thus, they represent the heart of bit performance prediction and minimizing drilling cost.

The general form of the cost per foot (*CPF*) equation is given by:

$$CPF = \frac{CB + CR(T_f + T_t)}{F_f} \tag{1}$$

Minimum drilling cost per foot can be achieved by selecting the best available values of the controllable drilling variables that gives optimum values for final drilled footage (F_f) and rotating time (T_f).

Moore Drilling Model

Moore (1974) suggested a mathematical drilling model that reveals the effect of bit weight, rotary speed and bit dullness on drilling rate. He presented the following two fundamental equations:

1- Rate of Penetration Equation:

$$\frac{dF}{dT} = \frac{KN^l W}{1 + K'H} \tag{2}$$

Where the constants K' and l would have to be determined from field operations.

2-Bit life equation:

$$L_i = \frac{K''}{NW^b} \tag{3}$$

The exponent (b) is a function of drilling fluid type and will vary between (1.0 and 3.0) depending on the abrasive characteristics of the fluid in contact with the bearings.

Galle & Woods Drilling Model

Galle & Woods (1960) presented an empirical drilling model that shows the effect of weight on bit, rotary speed, and bit tooth dullness on drilling rate. They also presented the concept of bit dullness by developing two other equations for tooth wear rate and bearing wear rate.

- Rate of Penetration Equation:

$$\frac{dF}{dT} = C_f \frac{rW^k}{a^P} \tag{4}$$

Where:

$$a = 0.928125H^2 + 6.0H + 1.0$$

$$P = \begin{cases} 1.0 \text{ for flat-crested bit tooth wear.} \\ 0.5 \text{ for self-sharpening or chipping bit tooth wear.} \\ 0.0 \text{ for insert (button) bits.} \end{cases}$$

$$r = \left[e^{-100/N^2} N^{0.75} + 0.5N(1 - e^{-100/N^2}) \right] \quad \text{(For soft formations)}$$

$$r = \left[e^{-100/N^2} N^{0.428} + 0.2N(1 - e^{-100/N^2}) \right] \quad \text{(For hard formations)}$$

$$k = \begin{cases} 1.0 & (\text{For hard formations}) \\ 0.6 & (\text{For soft formations}) \end{cases}$$

- Rate of Bit Tooth-Wear Equation:

$$\frac{dH}{dT} = \frac{i}{A_f am} \tag{5}$$

Galle & Woods (1960) defined (i) and (m) as follows:

$$i = N + 4.348 \times 10^{-5} N^3$$

$$m = 1359.1 - 714.19 Log(W_n)$$

For calculation purposes, all functions of bit weight are normalized to 7 7/8 inch bit size:

$$W_n = \frac{7.875 \times W}{d}$$

$$m_n = \frac{m}{714.19}$$

- Rate of Bit Bearing-Wear Equation:

$$\frac{dB}{dT} = \frac{N}{SL} \tag{6}$$

where the symbol (L) is tabulated as a decreasing function with increasing bit weight (Moore, 1974).

Young (1969) derived a mathematical drilling model that described the rate of penetration in terms of weight on bit, rotary speed, and the degree of bit tooth dullness.

a-.Rate of Penetration Equation:

$$\frac{dF}{dT} = \frac{K(W-M)N^l}{1 + C_2 H} \tag{7}$$

Where the constants K, M, C_2, and λ must be determined experimentally in the formation drilled by using *Five-Spot* drilling rate tests.

b- Rate of Bit Tooth-Wear Equation:

$$\frac{dH}{dT} = \frac{A_f (P'N + QN^3)}{(D_2 - D_1 W)(1 + C_1 H)} \tag{8}$$

Where the tooth wear and size parameters P', Q, C_1, D_1, and D_2 are listed according to bit type and size (Barragan, 1997).

c- Rate of Bit Bearing-Wear Equation:

$$\frac{dB}{dT} = \frac{1}{b_r} NW^d \tag{9}$$

The weight exponent, δ, relates bearing wear rate to bit weight. A value of 1.5 was observed for common drilling fluids.

Bourgoyne & Young Drilling Model

Bourgoyne & Young (1974) developed a mathematical drilling model to show the effects of formation strength, formation compaction, formation depth, pressure differential a cross the hole bottom, bit weight and diameter, rotary speed, bit wear, and bit hydraulics on penetration rate.

- Rate of Penetration Equation:

This equation predicts the effect of various drilling variables, X_j, on rate of penetration, which is given by:

$$\frac{dF}{dT} = EXP\left[a_1 + \sum_{j=2}^{8} a_j X_j\right] \tag{10}$$

Modeling of the drilling process is accomplished by determining the constants (a_1 through a_8) in the above equation from a multiple regression analysis of field data. Thus, the eight drilling variables are defined as follows:

Effect of formation strength; The constant (a_1) represents the effect of formation strength and drillability on penetration rate. It also includes the effects of drilling parameters that have not been mathematically modeled.

Effect of compaction; The terms (a_2x_2) and (a_3x_3) model the effect of compaction on penetration rate. X_2 is defined by:

$$X_2 = 10{,}000 - D \tag{11}$$

And this assumes an exponential decrease penetration rate with depth (D) in a normally compacted formation .X_3 is defined by:

$$X_3 = D^{0.69}\left(Gp' - 9.0\right) \tag{12}$$

And this assumes an exponential increase in penetration rate with pore pressure gradient (Gp').

Effect of Differential Pressure; The term (a_4X_4) models the effect of differential pressure across the hole bottom on penetration rate. X_4 is defined by:

$$X_4 = D\left(Gp' - r\right) \tag{13}$$

And this assumes an exponential decrease in penetration rate with excess bottom hole pressure.

Effect of Bit Weight and Bit Diameter; The term (a_5X_5) models the effect of bit weight and diameter on penetration rate. X_5 is defined by:

$$X_5 = Ln\left(\frac{W/d - (W/d)_t}{4.0 - (W/d)_t}\right) \tag{14}$$

And this assumes that the penetration rate is directly proportional to $(W/d)^{a5}$.

Effect of Rotary Speed; The term (a_6X_6) models the effect of rotary speed on penetration rate. X_6 is defined by:

$$X_6 = Ln\left(\frac{N}{100}\right) \tag{15}$$

And this assumes that the penetration rate is directly proportional to N^{a6}.

Effect of Bit Tooth-Wear; The term (a_7X_7) models the effect of tooth-wear on penetration rate. X_7 is defined by:

$$X_7 = -H \tag{16}$$

And this assumes an exponential decrease in penetration rate with increasing tooth wear.

Effect of Bit Hydraulics; The term (a_8X_8) models the effect of bit hydraulics on penetration rate. X_8 is defined by:

$$X_8 = \frac{rq}{350 m d_n} \tag{17}$$

The constants a_1 through a_8 can be determined by using multiple regression analysis of drilling data. This statistical technique is used to model sets of data points by a suitable equation with the best possible accuracy. At first, the parameters X_2 through X_8 must be calculated with Eq.(11) through Eq. (17) for each data points, then multiple regression analysis can be applied to determine these constants.

- Rate of Bit Tooth-Wear Equation:

$$\frac{dH}{dT} = \frac{H_3}{A_f}\left[\frac{N}{100}\right]^{H_1}\left[\frac{(W/d)_{\max} - 4.0}{(W/d)_{\max} - (W/d)_t}\right]\left[\frac{1 + H_2/2}{1 + H_2 H}\right] \tag{18}$$

Where the constants H_1, H_2, H_3, and $(W/d)_{max}$ depend on bit type and size.

3- Rate of Bit Bearing-Wear Equation:

$$\frac{dB}{dT} = \frac{1}{b_r}\left[\frac{N}{100}\right]\left[\frac{W}{4d}\right]^{d} \tag{19}$$

Where the weight exponent (δ) is tabulated depending on bearing type and drilling fluid type (Cunningham, 1978).

APPLICATION OF PREVIOUS MODELS

Moore (1974), Galle&Woods (1960), and Young (1969) Drilling Models failed to represent drilling process at the selected oil fields. This can be attributed to the following main limitations:

1. These drilling models assumed that the bit type, hydraulic system, drilling fluid properties, and differential pressure are adequate and don't affect drilling rate.
2. Drilling optimization process has been restricted to find optimum bit weight and rotary speed only.
3. For Moore drilling Model, no equation has been given for tooth wear rate. This means that the tooth wear has not been considered to limit bit life

APPLICATION OF BOURGOYNE & YOUNG DRILLING MODEL

Several studies (AL-Betairi et al. (1988), Barragan et al. (1997), Wee et al. (1989) confirmed the validity and successful application of Bourgoyne and Young drilling model (1974) at different locations. This may be attributed to its ability in modeling drilling process using field data. It is still considered to be one of the most comprehensive drilling models available for rotary drill bits. Consequently; this section tests the application of this mathematical drilling model on the selected field data. The validity of regression results is evaluated using several statistical techniques.

EVALUATION METHODOLOGY

Data Requirements

In this study, three of the most important southern Iraqi oil fields have been selected as a case study. These fields are: RU, R, and Z. These three fields have relatively similar stratigraphic and formation properties. Since the bed sections can be classified according to their hardness into three main different types (soft, medium, and hard), data of forty wells located at these fields has been divided into three parts according to formation hardness and then each part has been subjected to a single regression analysis.

- Statistical Analysis

In order to calculate the best values of the regression constants a_1 through a_8 for each formation type, the parameters X_2 through X_8 must be calculated with equation (11) through equation (17) for each data point.

Since no information have been recorded about the threshold bit weight per inch of bit diameter $(W/d)_t$, and due to the fact that its value is too small as compared with the applied bit weight per inch of bit diameter (W/d), it has been assumed to be equal zero. This assumption agrees with the works of many authors (Wee et al., 1989). During this study, a statistical package has been used to calculate the eight unknowns through the multiple regression analysis technique. Evaluation of the validity of Bourgoyne and Young drilling model has been accomplished by using several statistical techniques which are: correlation coefficient (R), standard deviation (S.Dev.), and predicted versus observed data plots.

- Regression Results

The multiple regressions modeling procedure has been achieved by using a statistical package. Regression results for soft, medium, and hard formations are summarized in table (1) and Fig. (1) through Fig.(3). For each formation type, the following quantities are obtained: the number of data points included in a particular analysis, drilling constants a_1 through a_8, correlation of data points included in a particular analysis, drilling constants a_1 through a_8, correlation coefficient, and standard deviation.

For soft formation results, regression constants a_3, a_4, and a_6 have been found to be negative. These negative values are unreliable and don't represent the drilling behavior. *AL-Betairi et al. (1988)* attributed the presence of negative regression constants to the multicollinearity (linear dependence) problem between drilling variables.

The negative values of a_3 and a_4 are caused by the dependence between X_2, X_3, and X_4. As the depth increases, X_2 and X_4decrease while X_3 increase. The low correlation coefficient, scattered data of observed versus predicted drilling rate plot, and large values of standard deviation corresponding to these constants are all confirm their unreliability.

The negative value of a_6 caused by another dependence between X_5 and X_6. *Barragan et al.(1997)* mentioned that drilling variables like bit weight and rotary speed are not independent due to operational needs at the field area. When bit weight is reduced, rotary speed is normally increased.

For medium and hard formations results, although good correlation coefficients have been obtained, linear dependence has been also found between the drilling parameters X_2, X_3, and X_4, which are depth correlated. The large values of standard deviation and the negative values of a_2 and a_3 in medium formations and a_3 in hard formations confirm this linear dependence. *Barragan et al.(1997)* and *AL-Betairi et al.(1988)* mentioned that these parameters are actually correlated with depth and required very large number of data points to get reliable results.

POSSIBLE MODIFICATION

-Rate of Penetration Equation:

Bourgoyne and Young modeling procedure has failed to simulate drilling behavior and to give meaningful values of the model constants from the available field data. Regression results for soft, medium, and hard formations indicate that this failure can be attributed to the following limitations:

Table (1): *Regression Results, Bourgoyne & Young*

Drilling Coefficient	Soft Formations R=0.55 Data points=110 Intercept (a_1)=6.44		Medium Formations R=0.73 Data Point=55 Intercept (a_1)=4.24		Hard Formations R=0.625 Data Point=89 Intercept (a_1)=4.75	
	Regression Value	Standard Deviation	Regression Value	Standard Deviation	Regression Value	Standard Deviation
a_2	0.00044	3488.7	-3.989	12022.8	0.00034	407.34
a_3	-0.16286	51.3	-2.857	31.3	-0.00532	12.67
a_4	-0.000422	1423.7	1.344	6266.2	0.00006	2925.15
a_5	0.236689	0.388	0.294	0.22	1.57115	0.097
a_6	-0.376089	0.184	0.1939	0.17	0.37772	0.096
a_7	1.0743	0.198	0.293	0.24	0.28315	0.206
a_8	1.8716	0.162	0.306	0.11	0.42513	0.109

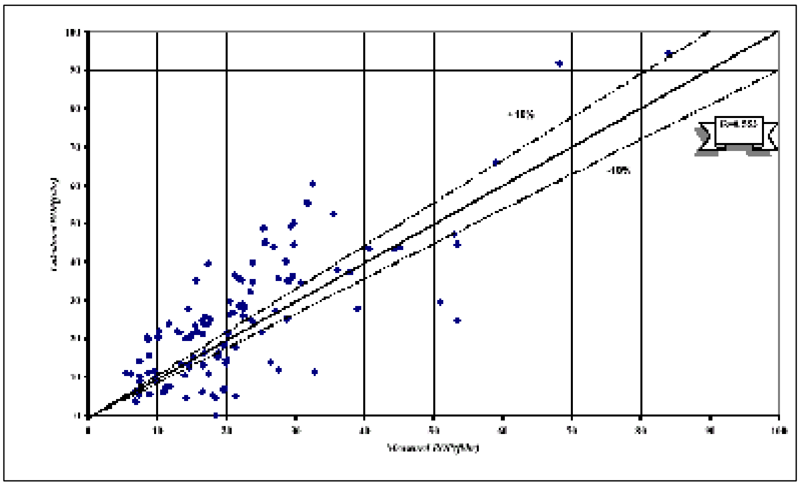

Fig. (1): *Calculated Vs. Measured ROP of Soft formations, Bourgoyne & Young model*

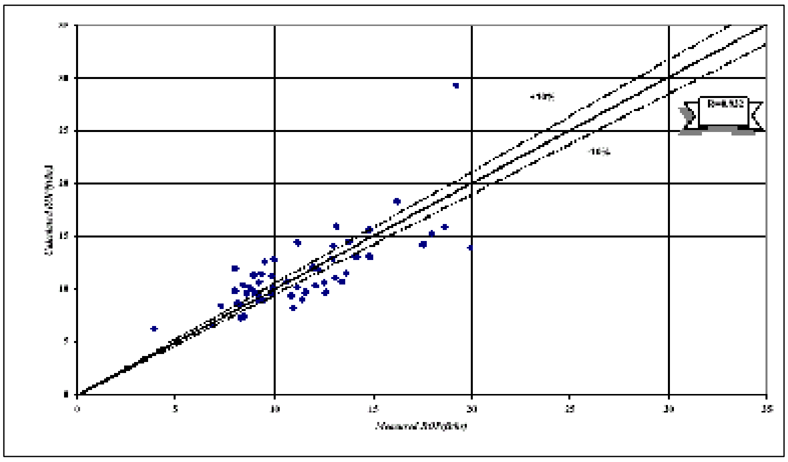

Fig. (2): *Calculated Vs. Measured ROP of Medium formations, Bourgoyne & Young model*

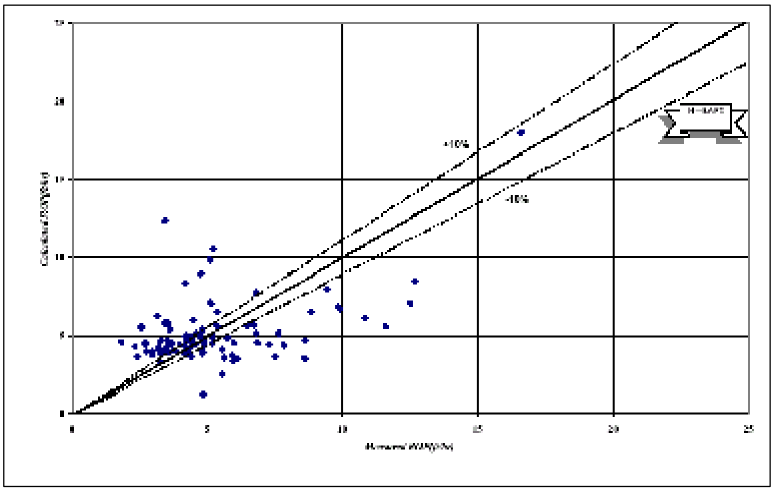

Fig. (3): *Calculated Vs. Measured ROP of Hard formations, Bourgoyne & Young model*

· The presence of linear dependence between drilling variables like X_2, $X3$, and X_4, which are normally, correlated with depth, and between the mechanical drilling variables X_5 and X_6. The large number of constants to be determined in drilling rate equation. This number of constants makes the results very sensitive to the number and accuracy of data points included.

· The effect of formation compressive strength has not been considered.

· Bit type effect on drilling rate has not been considered.

Thus, the following modifications have been achieved on Bourgoyne and Young drilling model to make it more general, and represent the drilling process in the selected fields with higher accuracy:

· In order to increase the accuracy of the drilling model and cancel the effect of linear dependence between drilling variables (X_2, X_3, and X_4), field data have been collected from each depth interval separately instead of from all intervals. Consequently, the drilling parameters correlated with depth X_2 and X_3 would have constant effect on drilling rate for the same depth interval and can be grouped into a single value included in the term of formation strength factor.

· Different bits have been used during the drilling process, therefore, the effect of bit type on drilling rate has been included in the modified model by further collection of data into groups depending on bit class (IADC Code) and depth interval, and subjecting each group to a single analysis. Thus, the term $EXP(a_1)$ will be more representative to the formation drillability factor.

· Effects of weight on bit, rotary speed, formation hardness, formation compressive strength, and differential pressure have been included in the modified model by the form presented in Cunningham Drilling model (Cunningham, 1978).

All the previous mentioned modifications would reduce the number of constants to be determined during the regression analysis and cancel the effect of linear dependence on the results. As the number of drilling constants to be determined in a certain analysis is decreased, the number of data points required to calculate the remaining regression constants is also decreased. As a result, the accuracy of these constants will increase.

- According to the experience and drilling results in the field area, it has been noticed that the drilling rate is affected by oil content in drilling fluids. There have been several studies [10,11] which emphasis this effect. Thus, it is necessary to include it in the modified model and increase the accuracy of drilling rate prediction. Oil content effect has been considered in the drilling model as stated by *Allen* (Chilingarian and Vorabutr, 1981).

- After including the proposed modifications, drilling rate equation is given by:

$$\frac{dF}{dT} = \frac{N^n (W/d)^w \, OfEXP[a_1 + a_7 X_7 + a_8 X_8]}{0.424 s^{1.5} + \sqrt{N^n (W/d)^w} \, \Delta P^{0.75}}$$

Where *Of* is the oil content function, and given by:

$$Of = Sin(10.6 OIL\% - 48.33) + 10.66 \qquad (20)$$

For simplification purpose, formation drillability factor $EXP(a_1)$ can be written as follows:

$$D_f = EXP(a_1)$$

Thus, the final form of drilling rate equation is defined by:

$$\frac{dF}{dT} = \frac{DfN^n (W/d)^w \, OfEXP[a_7 X_7 + a_8 X_8]}{0.424 s^{1.5} + \sqrt{N^n (W/d)^w} \, \Delta P^{0.75}} \qquad (21)$$

Where the constants D_f, a_7, and a_8 can be determined through a multiple regression analysis. Rotary speed exponents (n), depends on formation hardness and equal to (0.6, 0.7, 0.85) for soft, medium, and hard formations, respectively (Cunningham, 1978). Weight on bit exponent (ω) is calculated from the following equation according to formation compressive strength values that have been measured for each formation type.

$$w = 0.178254 Ln(s) + 1.09793$$

-Rate of Bit Tooth-Wear Equation:

In the general form of this equation, tooth wear rate increases with bit weight, rotary speed, and formation abrasiveness, and decreases with bit tooth dullness as can be shown in Eq.(5), Eq.(8), and Eq.(18).

Tooth-wear rate equation presented by Bourgoyne and Young includes tooth-wear parameters, which are restricted to limited bit types and sizes. However, the most general equation that represents tooth wear rate is that presented by *Galle and Woods (1960)*. Therefore, it has been considered in the modified drilling model:

$$\frac{dH}{dT} = \frac{i}{A_f \, am} \qquad (22)$$

-Rate of Bit Bearing-Wear Equation:

In the general form of this equation, bearing life decreases with increasing weight on bit and rotary speed, and increases with increasing bearing constant as can be shown in Eq.(3), Eq.(6), Eq.(9), and Eq.(19). The same equation of Bourgoyne and Young (1974) has been considered in the modified model by selecting a general value for bit weight exponent (δ). Both *Moore (1974)* and *Young* (1969) suggested a value of 1.5 for common drilling fluids. Thus, the rate of bearing wear equation is defined by:

$$\frac{dB}{dT} = \frac{1}{b_r} N (W/d)^{1.5} \qquad (23)$$

REGRESSION RESULTS OF THE MODEFIED DRILLING MODEL

Drilling variables required for modeling procedure are: depth, drilled footage, rotating time, drilling rate, bit type, bit size, weight on bit, rotary speed, flow rate, drilling fluid density and viscosity, oil content, nozzle sizes, bit dullness, formation type, formation compressive strength, and formation pore pressure gradient.

Field data points have been divided into groups according to bit class (IADC Code) and depth interval. The same statistical package and techniques have been used to determine the model constants D_f, a_7, and a_8 and to evaluate the validity of the regression analysis.

Regression results for soft, medium, and hard formations have been summarized in table (2) and Fig.(4) through Fig.(6).

It is obvious that there is no effect of the linear dependence problem on the final results. Positive values of the constants a_7 and a_8 and low standard deviation confirm the success of modeling process. Another noticeable improvements have been obtained in the modeling process by increasing the correlation coefficient for the combined data to equal (0.943, 0.883, 0.911) for soft, medium, and hard formations respectively. Furthermore, a marked reduction in the scattered data about the 45-degree line for these formations is more significant.

From table (2), it is obvious that the values of tooth wear constant (a_7) and hydraulics constant (a_8) are relatively identical for each regression analysis. Therefore, average values of 0.5 and 0.6 have been selected as common values in the drilling model for (a_7) and (a_8) respectively.

VERIFICATION OF DRILLING MODEL

Once modeling procedure is completed, a verification test is added to check that drilling model is a valid representation of the drilling process. Commonly, model verification involves using the drilling rate equation to calculate drilling rates of another set of data points that have not been considered in the multiple regression analysis and compare these values with observed drilling rates. If there are no great differences and the drilling model successfully pass this last test, then it can be used for predicting drilling rate and then minimizing drilling cost. Table (3) represents model verification results for four new wells which are: RU244, R488, Z128, and Z157.

DERIVATION OF ROTATING TIME AND FOOTAGE EQUATIONS

In order to determine the minimum drilling cost per foot and associated drilling variables, the previous three differential equations have been integrated and solved for the final drilled footage (F_f) and rotating time (T_f). However, bit life is limited by either bearing failure or tooth wear and the following procedure is considered to determine whichever take place first:

A- Bearing-Wear Limits Bit Life:

Assume the bit life is limited by bearing failure {i.e., B_f =1.0 and $H_f \leq 1.0$}, and the total drilled footage is calculated by the integrated form of Eq. (21) as follows:

$$\frac{dF}{dT}=\frac{D_f N^n (W/d)^w OfEXP[a_7 X_7 + a_8 X_8]}{0.424 s^{1.5} + \sqrt{N^n (W/d)^w}\, \Delta P^{0.75}}$$

$$dF=\frac{D_f N^n (W/d)^w OfEXP[a_7 X_7 + a_8 X_8]}{0.424 s^{1.5} + \sqrt{N^n (W/d)^w}\, \Delta P^{0.75}} dT \tag{24}$$

From Eq. (22):

$$dT=\frac{A_f am}{i} dH$$

After substituting in Eq.(24), we get:

$$dF=\frac{D_f N^n (W/d)^w OfEXP[a_7 X_7 + a_8 X_8] A_f madH}{0.424 s^{1.5} + \sqrt{N^n (W/d)^w}\, \Delta P^{0.75} i}$$

Table (2): *Regression Results, Modified Bourgoyne& Young Model*

No.	Depth Interval	Bit (IADC) Code	Bit Type	Bit Size (inch)	Df	a_7		a_8		Data Points	Correlation Coefficient (R)
						Regression Value	S.Dev.	Regression Value	S.Dev		
1	Dibdiba+LowerFaris+Ghar	1-1-1	R1,TS2,OSC-3A	17.5	0.18058	0.51	0.18	0.63	0.16	34	0.927
2	Dammam +Rus	1-1-4	X3A,SDS, ES2	12.25	0.12133	0.53	0.21	0.68	0.15	13	0.943
3	Dammam +Rus	1-2-4	X3, SDT	12.25	0.10584	0.38	0.19	0.71	0.16	17	0.911
4	Shiranish+Hartha	2-1-5	SDV, M44NG	12.25	0.05906	0.58	0.16	0.62	0.11	8	0.785
5	Shiranish+Hartha	5-3-7	J33, S86F, F3	12.25	0.08462	0.55	0.18	0.58	0.23	8	0.798
6	Hartha+Sa'adi	2-1-4	XVM44N, EM2	8.5	0.16359	0.51	0.18	0.59	0.16	8	0.915
7	Sa'adi	1-3-6	J4, S44	8.5	0.07993	0.53	0.15	0.61	0.16	18	0.852
8	Um AlRadoma+Tayarat	5-3-7	J33, S86F, F3	12.25	0.08862	0.47	0.33	0.56	0.34	11	0.709
9	Tan.+Ksb.+Msh.+Ru.+Ah.	5-3-7	J33, S86F, F3	8.5	0.08423	0.49	0.38	0.58	0.50	19	0.871
10	Moudod + Nhr-Omar	5-3-7	J33, S86F, F3	8.5	0.07888	0.51	0.30	0.58	0.27	19	0.809
11	Zubair	6-3-7	J55, FP63	8.5	0.01128	0.45	0.23	0.67	0.11	10	0.788
12	Zubair	5-3-7	J33, S86F, F3	8.5	0.01708	0.55	0.18	0.62	0.15	17	0.756
13	Zubair	5-1-7	J22, S84F	8.5	0.01763	0.52	0.14	0.61	0.29	8	0.879
14	Shiaba + Zubair	6-1-7	J44, M84F, F4	8.5	0.02059	0.55	0.55	0.48	0.36	9	0.786
15	Zubair	6-1-7	J44, M84F, F4	8.5	0.01463	0.50	0.19	0.61	0.26	24	0.852
16	Shiaba + Zubair	5-3-7	J33, S86F, F3	8.5	0.03096	0.47	0.58	0.65	0.17	14	0.929

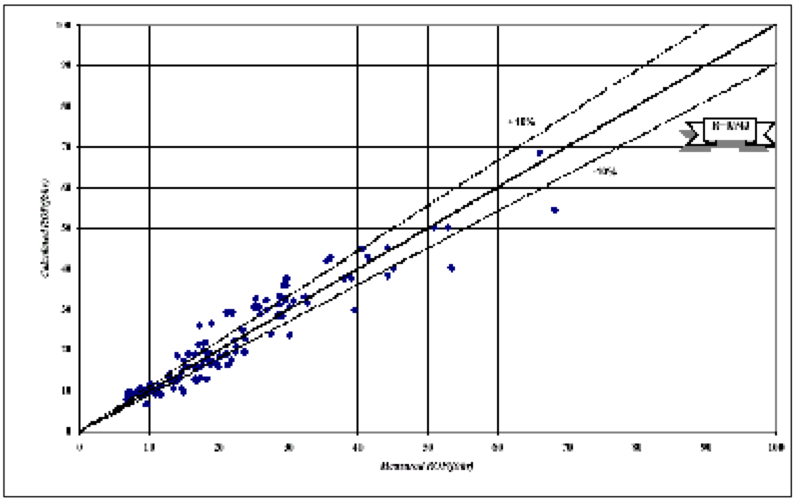

Fig.(4):*Calculated Vs. Measured ROP of Soft formations, Modified Bourgoyne & Young model*

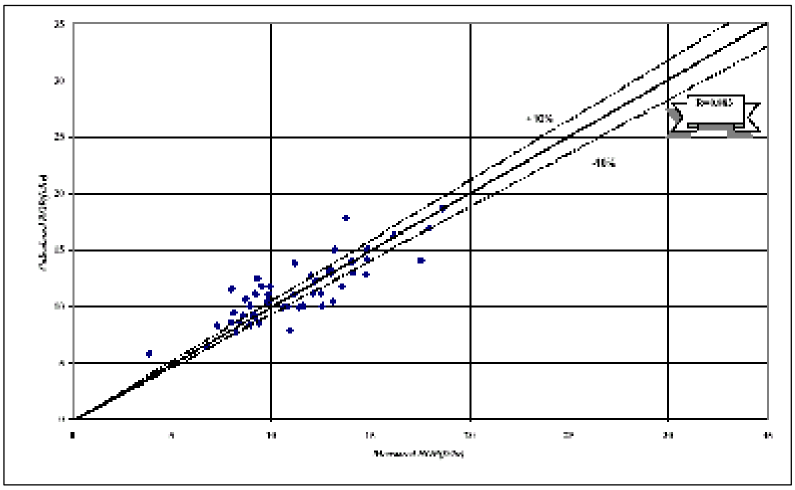

Fig.(5):*Calculated Vs. Measured ROP of Medium formations, Modified Bourgoyne & Young model*

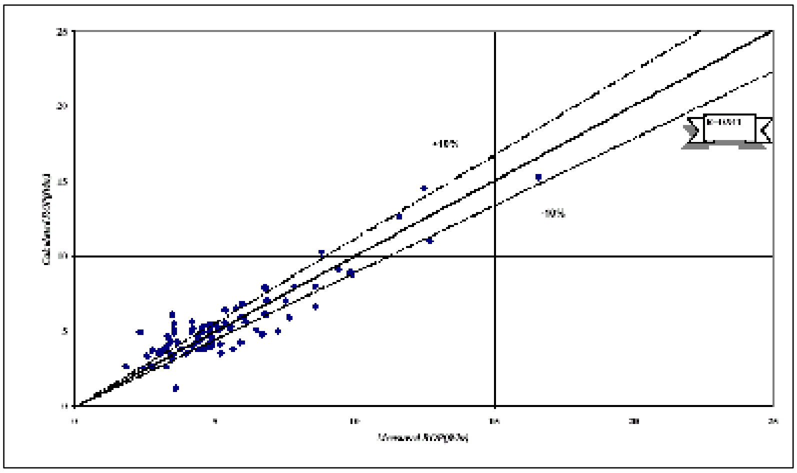

Fig.(6): *Calculated Vs. Measured ROP of Hard formations, Modified Bourgoyne & Young model*

$$\int_{o}^{F_f} dF = \frac{D_f A_f N^n (W/d)^w \, mOfEXP[a_8 X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w} \Delta P^{0.75}\right)} \int_{0}^{H_f} aEXP[-a_7 H] dH$$

Let: $$J_3 = \frac{D_f A_f N^n (W/d)^w \, mOfEXP[a_8 X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w} \Delta P^{0.75}\right)}$$

$$\int_{0}^{F_f} dF = J_3 \int_{0}^{H_f} \left(0.928125H^2 + 6H + 1.0\right) EXP[-a_7 H] dH$$

$$F_f = J_3 \int_{0}^{H_f} \left(0.928125H^2 e^{-a_7 H} + 6He^{-a_7 H} + e^{-a_7 H}\right) dH$$

Let: $L_1 = 0.928125 \int_{0}^{H_f} H^2 e^{-a_7 H} dH$ which can be integrated by parts as follows:

For $U=H^2$ $dU=2HdH$ and $dV = e^{-a_7 H} dH$ $\therefore V = \frac{-1}{a_7} e^{-a_7 H}$

$$L_1 = 0.928125\left[\frac{-1}{a_7} H^2 e^{-a_7 H} + \frac{2}{a_7} \int_{0}^{H_f} He^{-a_7 H} dH\right]$$

Another integration for $U=H$, $dU=dH$ and $dV = e^{-a_7 H} dH$ $\therefore V = \frac{-1}{a_7} e^{-a_7 H}$

$$L_1 = 0.928125\left[\frac{-1}{a_7} H^2 e^{-a_7 H} - \frac{2}{a_7}\left(\frac{-1}{a_7} H^2 e^{-a_7 H} + \frac{1}{a_7^2} e^{-a_7 H}\right)\right]_0^{H_f}$$

Thus;

$$L_1 = \left[\frac{1.856}{a_7^3} - e^{-a_7 H_f}\left(\frac{0.928125}{a_7} H_f^2 + \frac{1.856}{a_7^2} H_f + \frac{1.856}{a_7^3}\right)\right]$$

Let: $$L_2 = 6\int_0^{H_f} He^{-a_7 H}\, dH$$

Integration (L_2) by parts yields:

$$L_2 = \frac{6}{a_7^2} - e^{-a_7 H_f}\left(\frac{6}{a_7} H_f + \frac{6}{a_7^2}\right)$$

Let: $$L_3 = \int_0^{H_f} e^{-a_7 H}\, dH$$

$$\therefore L_3 = \frac{1}{a_7} - \frac{1}{a_7} e^{-a_7 H_f}$$

$$\therefore F_f = J_3\left[L_1 + L_2 + L_3\right]$$

Thus, the general expression of the final drilled footage is given by:

Table(3): Model Verification Results

No	Field &well No.	Depth Interval	Bit No.	IADC. Code	Bit type	WOB 1000 Lb	N rpm	Oil Content %	ΔP 1000 psi	X_7	X_8	ROP_m (ft/hr)	ROP_c (ft/hr)
2	Z157	Dibdiba+Lower.Faris+Ghar	2	1-1-1	R1	33.069	100	1	0.075	-0.25	0.258	27.00	27.24
3	Z128	Dibdiba+Lower.Faris+Ghar	1	1-1-1	R1	33.069	100	2	0.05	-0.625	0.401	30.60	32.31
4	R488	Dibdiba+Lower.Faris+Ghar	1	1-1-1	R1	22.045	100	3	0.042	-0.375	0.646	31.70	32.99
5	Z157	Dammam +Rus	3	1-1-4	X3A	33.069	80	2	0.119	-0.5	0.271	16.75	16.17
6	RU244	Dammam +Rus	2	1-2-4	X3	44.092	100	2	0.282	-0.375	0.303	22.36	23.64
7	R488	Dammam +Rus	2	1-2-4	X3	33.069	100	2	0.424	-0.625	0.455	20.50	22.77
8	RU244	UmAlRadoma+Tayarat	4	5-3-7	S86F	33.598	70	6	0.138	-0.75	0.588	14.58	11.36
9	Z128	Um AlRadoma+Tayarat	5	5-3-7	S86F	39.685	50	7	0.179	-0.375	0.485	14.06	13.95
10	Z157	Um AlRadoma+Tayarat	4	5-3-7	S86F	33.069	55	5	0.086	-0.5	0.442	14.85	15.04
11	RU244	Sa'adi	6	1-3-6	J4	39.685	60	5	0.191	-0.25	0.783	14.64	16.64
12	R488	Sa'adi	6	1-3-6	J4	39.685	80	8	0.280	-0.5	0.423	7.00	10.01
13	Z128	Sa'adi	8	1-3-6	J4	39.685	55	6	0.210	-0.625	0.569	8.50	11.43
14	Z157	Sa'adi	7	1-3-6	J4	33.069	60	7	0.226	-0.5	0.454	10.80	9.94
15	Z157	Tan.+Ksb.+Msh.+Ru.+Ah	8	5-3-7	S86F	39.685	50	7	0.425	-0.25	0.663	14.20	12.94
16	Z128	Tan.+Ksb.+Msh.+Ru.+Ah	9	5-3-7	S86F	39.685	50	8	0.585	-0.25	0.378	8.21	9.04
17	R488	Moudod + Nhr-Omar	10	5-3-7	J33	39.685	55	4	0.399	-0.5	0.534	8.40	9.78
18	RU244	Moudod + Nhr-Omar	9	5-3-7	S86F	33.069	50	8	0.096	-0.625	0.597	10.80	8.86
19	Z128	Shiaba + Zubair	12	5-3-7	J33	33.069	45	7	0.219	-0.625	0.437	4.90	5.72
20	RU244	Zubair	11	5-3-7	S86F	33.069	50	6	0.103	-0.75	0.584	4.30	4.34
21	Z128	Zubair	13	5-3-7	J33	35.273	45	8	0.230	-0.5	0.592	4.86	4.01
22	Z128	Zubair	14	5-3-7	J33	33.069	55	7	0.234	-0.25	0.613	5.58	4.79
23	RU244	Zubatr	10	6-1-7	M84F	33.069	50	7	0.100	-0.375	0.583	4.40	4.59

$$F_f = \frac{D_f A_f N^n (W/d)^w mOfEXP[a_8 X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w}\,\Delta P^{0.75}\right)}[(\frac{a_7^2 + 6a_7 + 1.856}{a_7^3}) - e^{-a_7 H_f}(\frac{0.928125}{a_7} H_f^2 +$$

$$\frac{6a_7 + 1.856}{a_7^2} H_f + \frac{a_7^2 + 6a_7 + 1.856}{a_7^3})] \qquad (25)$$

Since the values of a_7 and a_8 from the regression analysis results are 0.5 and 0.6 respectively, the final drilled footage that has been considered in this study is defined by:

$$F_f = \frac{D_f A_f N^n (W/d)^w mOfEXP[0.6X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w}\,\Delta P^{0.75}\right)}\left[40.85 - e^{-0.5H_f}\left(1.856H_f^2 + 19.425H_f + 40.85\right)\right] \qquad (26)$$

The rotating time is governed by the solution of the integrated form of equation (23) as follows:

$$\frac{dB}{dT} = \frac{1}{b_r} N (W/d)^{1.5}$$

$$T_f = \frac{b_r}{N (W/d)^{1.5}} \qquad (27)$$

In this case, the final bit tooth dullness (H_f) should be less or equal 1.0 and determined by the integrated form of Eq.(22), as follows:

$$\frac{dH}{dT} = \frac{i}{A_f am}$$

$$\int_0^{H_f} \left(0.928125H^2 + 6.0H + 1.0\right) dH = \frac{i}{A_f m}\int_0^{T_f} dT$$

$$0.309H_f^3 + 3.0H_f^2 + H_f = \frac{iT_f}{A_f m} \qquad (28)$$

If the value of tooth wear (H_f) that has been calculated at the rotating time (T_f) from Eq.(27) exceeds one [i.e., H_f >1.0], this gives an indication that tooth wear governed bit life and the following assumption must be considered.

B- Tooth-Wear limits Bit Life:

If the bit life is limited by tooth-wear [i.e. H_f =1.0 and B_f <1.0], the general form of the final drilled footage is obtained from Eq.(25) for [H_f =1.0], as follows:

$$F_f = \frac{D_f A_f N^n (W/d)^w mOfEXP[a_8 X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w}\,\Delta P^{0.75}\right)}[(\frac{a_7^2 + 6a_7 + 1.856}{a_7^3}) - e^{-a_7}(\frac{0.928125}{a_7} +$$

$$\frac{6a_7 + 1.856}{a_7^2} + \frac{a_7^2 + 6a_7 + 1.856}{a_7^3})] \qquad (29)$$

For a_7 =0.5 and a_8 =0.6 :

$$F_f = \frac{3.165 D_f A_f N^n (W/d)^w mOfEXP[0.6X_8]}{i\left(0.424s^{1.5} + \sqrt{N^n (W/d)^w}\,\Delta P^{0.75}\right)} \qquad (30)$$

The rotating time is governed by the solution of the integrated form of Eq.(22) for [H_f =1.0]:

$$T_f = \frac{4.309 A_f m}{i} \tag{31}$$

In this case, the final bit bearing wear (B_f) should be less 1.0 and calculated by integrating Eq.(23), as follows:

$$B_f = \frac{N(W/d)^{1.5} T_f}{b_r} \tag{32}$$

RESULTS OF OPTIMIZATION

The Constrained Rosenbrock optimization technique (Hiller (2001), James (1973)) has been used together with the modified Bourgoyne & Young drilling model to find optimum values for weight on bit, rotary speed, flow rate, drilling fluid density, drilling fluid viscosity, oil content, and nozzle size for each formation type. According to the results of optimization, the following bit types and corresponding drilling variables are recommended in table (4) for each depth interval.

CONSTRUCTION OF OPTIMUM BIT RECORD

The results of optimization have been used to design optimum bit record that can be used to drill the next wells in the area under study. For comparison purpose, it has been used to optimize the drilling cost for well RU263. It is obvious that the optimum bit record saved about 857 hours in total drilling time. In addition, the reduction in the total number of bits saved about 57.5 hours in the trip and connection time. Thus, the optimum solution saved about 294915 $ from the total drilling cost which is equal to 394176$. Furthermore, it is also possible to repeat utilization of the used bits to drill the same intervals in other wells if the total bit life has not been totally used. Fig.(7) shows the comparison in drilling cost between the optimized well and the non-optimized well to reach the same depth.

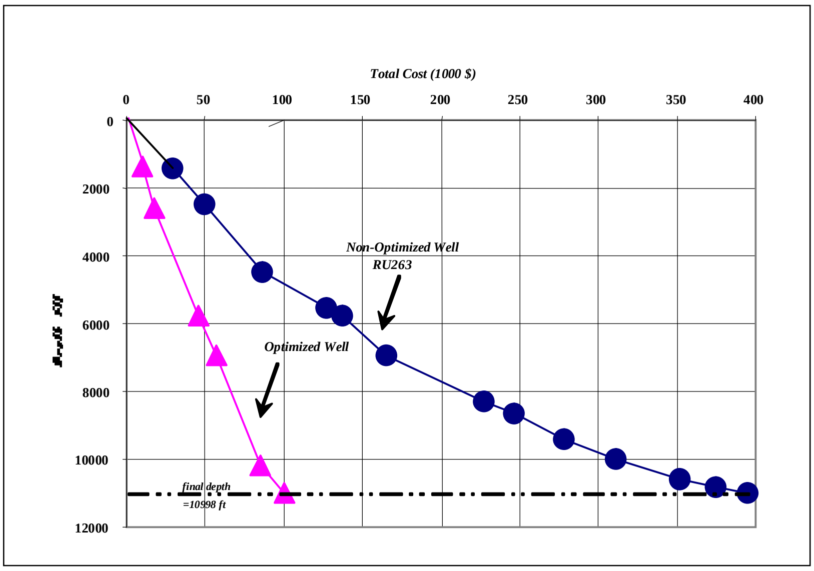

Fig.(7): *Drilling Cost Vs. Depth for Optimized and Non-Optimized Wells*

CONCLUSIONS

1- Bourgoyne & Young drilling model has been modified successfully using other drilling models to include the effects of weight on bit, rotary speed, bit type, bit size, flow rate, drilling fluid density, drilling fluid viscosity, nozzle size, oil content, formation drillability, formation abrasiveness, formation compressive strength, formation hardness, bit bearing constant, differential pressure between mud column pressure and formation pressure, and bit dullness on drilling rate. This modified model gave reasonable results in simulating the drilling process for the fields under study.

2- Modified Bourgoyne and Young drilling model can be used to predict bit performance for a certain formation to be drilled. Accordingly, it provides a method for recognizing any anomaly in the field bit performance when it deviates from predictions.

3- It has been noted that different bit types of the same IADC Code have the same bit performance and drillability through a certain formation.

NOMENCLATURE

a_j :Drilling constant for drilling variable, j (to be determined)

A_f :Formation abrasiveness factor

b :Bit weight exponent in bearing- wear rate equation

B :Bearing wear, 1/8 of bearing life

b_r :Bit bearing constant

CB :Bit cost,$

C_f :Formation drillability factor

CPF :Cost per foot, $/ft

CR :Rig cost, $/ft

d :Bit size, inch

D :Depth, ft

D_f :Formation drillability factor

d_n :Bit nozzle size, inch

N :Rotary speed, rpm

Of :Oil content Function

ΔP :Differential pressure between drilling mud and pore pressure at the bit, psi

q :Flow rate, gal/min

$(W/d)_{\max}$: Bit weight per inch of bit size at which bit teeth would fail, 1000 lb/in

$(W/d)_t$:Threshold bit weight per inch of bit size at which bit begins to dill, 1000 lb/in

r :Drilling fluid density, lb/gal

w :Weight on bit exponent dependent upon formation compressive strength

s :Compressive strength, 1000 psi

Table (4): *Recommended Bit Types and Drilling Variables*

No.	Depth Interval	Bit IADC Code	Bit Size In.	WOB 1000 lb	N rpm	ρ lb/gal	Q gal/min	µ cp	dn 1/32 in.	Oil %	*Footage* ft	*Time* hrs.	Rop ft/hr
1	Dibdiba+Lower.Faris+Ghar	1-1-1	17.5	40.851	113	8.72	682	15.5	2×15 1×16	6.7	6267	73.56	85.2
2	Dammam +Rus	1-1-4	12.25	40.352	96	8.73	588	15.2	Without Nozzles	6.2	3026	47.20	64.1
3	Um AlRadoma+Tayarat	5-3-7	12.25	44.838	87	9.00	575	14.3	2×10 1×11	9.9	4847	82.88	58.5
4	Shiranish+Hartha	5-3-7	12.25	40.683	99	9.00	565	14.6	2×11 1×10	9.9	3352	102.36	32.8
5	Sa'adi	1-3-6	8.5	41.077	98	9.00	565	14.6	2×11 1×12	9.8	1736	46.93	37.0
6	Tan.+Ksb.+Msh.+Ru.+Ah.	5-3-7	8.5	44.835	86	9.10	528	14.7	2×9 1×10	9.9	6153	95.65	64.3
7	Moudod + Nhr-Omar	5-3-7	8.5	44.605	85	9.20	565	14.8	2×9 1×10	9.9	2732	59.85	45.6
8	Shiaba + Top of Zubair	5-3-7	8.5	47.810	64	9.9	568	13.3	2×9 1×10	13.2	2478	65.66	37.7
9	Zubair	5-3-7	8.5	47.991	64	9.9	573	13.2	3×9	13.4	2110	65.73	32.1

REFERENCES

AL- Betairi, E.A., Moussa, M.M., and AL-Otaibi, S.S., “ Multiple Regression Approach to Optimize Drilling Operations in the Arabian Gulf Area ”, *Soc. Pet. Eng. J., No.13694, pp.83-88, Mar. 1988.*

Barragan, R.V., Santos, O.L., and Maidia, E.E.,“ Optimization of Multiple Bit Runs ”, *Soc. Pet. Eng. J., No.37644, pp.579-589, Mar. 1997.*

Bourgoyne, A.T. and Young, F.S.,“ A Multiple Regression Approach to Optimal Drilling and Abnormal Pressure Detection ”, *Soc. Pet. Eng. J., pp.371-384, Aug. 1974.*

Chilingarian, G.V. and Vorabutr, D.,“ Drilling and Drilling Fluids ”, *Elsevier Publishing Co., Development in Petroleum Science, (1981).*

Cunningham, R.A.,“ An Empirical Approach for Relating Drilling Parameters ”, *J. Pet. Tech., pp. 987-991, Jul. 1978.*

Galle, E.M., and Woods, H.B., “ How to Calculate Bit Wight and Rotary Speed for Lowest-Cost Drilling ”, *Oil & Cas J., Nov. 14,1960.*

Hiller, F.S.,“ Introduction to Operations Research ”, *7th Edition, McGraw-Hill Higher Education, (2001).*

James, L.K. and Joe, H.M.,“ Optimization Techniques With FORTRAN ”, *New York, McGraw-Hill, (1973).*

Moore, P.L., “ Drilling Practice Manual ”, *Petroleum Publishing Co., Tulsa, Oklahoma (1974).*

Murphy, D., “ What Factors Affect Drilling Rate? ”, *Oil & Cas J., Feb. 17,1969.*

Simpson, M.A., “ Calculator Program Optimizes Bit Weight, Rotary Speed, Reducing Drilling Cost ”, *Oil & Cas J., Apr. 23,1984.*

Warren, T.M., “ Drilling Model for Soft-Formation Bits ”, *J. Pet. Tech., pp.963-970, Jun. 1981.*

Wee, W., and Kalogerakis, N., “ Modelling of Drilling Rate for Canadian Offshore Well Data ”, *J.Can. Pet. Tech., pp.33-48, Nov.-Dec. 1989.*

Young, F.S., “ Computerized Drilling Control ”, *J. Pet. Tech., pp.483-496, Apr. 1969.*