A FULL MISSILE HOMING SYSTEM DESIGN BASED ON PROPORTIONAL NAVIGATION GUIDANCE LAW AND ELECTRO-OPTICAL TRACKING SYSTEM

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ABSTRACT

Generally, the homing systems are majorly constructing from three components: the guidance law, the target tracking system, and the missile flight control system**.** Therefore, in this paper, we construct our homing system from the following components: the proportional navigation guidance law which is considered as the guidance scheme for most homing missile systems, the electro-optical tracking system, and a tail controlled missile. And subsequently complete mathematical derivations and the demand transfer function formulations of all these three components have been introduced. The proposed homing system is capable to pursuit and hit any target just by specifying the required missile flight time. A SIMULINK software program has been built mainly from four subsystems to simulate the operation of this homing system, and the simulation results show clearly the efficient performance of the proposed homing system under any probable disturbance.

الخلاصة:

بصورة عامة ان اي نظام لتوجيه صاروخ يتكون بشكل اساسي من ثلاثة اجزاء وهي: قانون الملاحة، و منظومة تتبع الهدف، و منظومة السيطرة على الصاروخ. ولذلك في هذا البحث، سنقوم ببناء منظومة من الاجزاء التالية: قانون الملاحة التناسبية والذي يعتبر القانون الاساسي لكل الصواريخ تقريباً ، و منظومة تتبع كهروبصرية، و صاروخ مسيطر عليه بواسطة الذنب.وبشكل متتابع سنقوم بأشتقاق المعدلات الرياضية ودوال التحويل الخاصة بهذه الاجزاء. ان منظومة التوجيه المقترحة في هذا البحث تستطيع متابعة واصابة اي هدف من خلال تحديد الزمن المطلوب لطيران الصاروخ فقط. لقد تم بناء برنامج بأستخدام السيمولينك متكون من اربعة اجزاء فرعية لمحاكاة عمل تلك المنظومة وان نتائج هده المحاكاة اظهرت بوضوح فاعلية ودقة اداء منظومة المصممة في هذا البحث للتتبع اي هدف لاي حالة..

Keywords: Proportional Navigation Guidance Law, Linearization, Flight Control system, Target Tracking System, and Linearized Proportional Navigation Guidance Block Diagram

INTRODUCTION

Generally, homing systems are consist of three components: the guidance law, the target tracking system, and the missile flight control, the control system may be surface or thrust vector control in the plane of the velocity vector i.e. the homing system is responsible for a pitch-controlling and not responsible for roll-controlling.

Many guidance laws have been developed for decades. Although their mathematical forms may have differed, the basic concepts of the developed guidance laws could be classified into three categories. The first category includes the guidance laws based on the line-of-sight (LOS) vector and their objective is to maintain the missile position on the LOS vector throughout engagement **(Lin, 1991; Lin and Mon, 2001)**. The pursuit guidance law eliminates the difference between the missile velocity vector direction and the LOS vector direction, and the command to LOS (CLOS) guidance law tries to place the missile on the LOS vector. Therefore, the classical pursuit guidance law and the CLOS guidance law can be classified into this category.

The second category includes the guidance laws that are based on the constant bearing course guidance method **(Ha, Hur, Ko, and song; Rajasekhar, and Sreenatha, 2000; Moon, Kim, and Kim, 2001)**. The laws in this category try to make the heading angle error zero. The missile velocity vector direction is on the collision triangle when the heading angle is zero. The well known proportional navigation (PN) guidance law and its variations such as the augmented proportional navigation (APN) guidance law **(Babu, Sarma et al Swamy, 1994)**, the modified proportional navigation (MPN) guidance law **(Song and Ha, 1994)** belong to this second category.

The last category includes the guidance laws that guide the missile into the predicted engagement course **(Alamir, 2001; Cho, Ryoo et al Tahk, 1999; Gurfil, 2001; Ben Asher and Ben Yaesh, 1997)**. The performance of the guidance laws in this category is greatly affected by the time to go estimation and the update rules of engagement. The guidance laws in this category are usually implemented by applying the optimal control method the predictive control method. In the predictive control method, the engagement point is estimated using information on current and past data, whereas in the optimal control method, the engagement condition is imposed as the hard constraint or included into the performance index.

The general formulation of a nonlinear three-dimensional PNG interception problem is complicated. However by assuming that the lateral and longitudinal maneuver planes are decoupled by means of roll-control, one can deal with the equivalent two-dimensional problem in quite a realistic manner **(Shinar & Steinberg, 1977)**. Furthermore, a linearized model of the two-dimensional PNG about the collision course can be developed. This model has been widely used **(Zarchan, 1990)**, and it has been shown to faithfully approximate the full nonlinear guidance dynamics **(Shinar & Steinberg, 1977)**.

A block diagram describing the linear model based on PN guidance law is given in **Fig.(1) (Zarchan, 1990 and Asher & Yaesh 1998)**. In this paper, a complete derivation and modification of this model have been adopted. Where, we will add to this model an electro-optical tracking system and a tail control system as seen later in this paper.

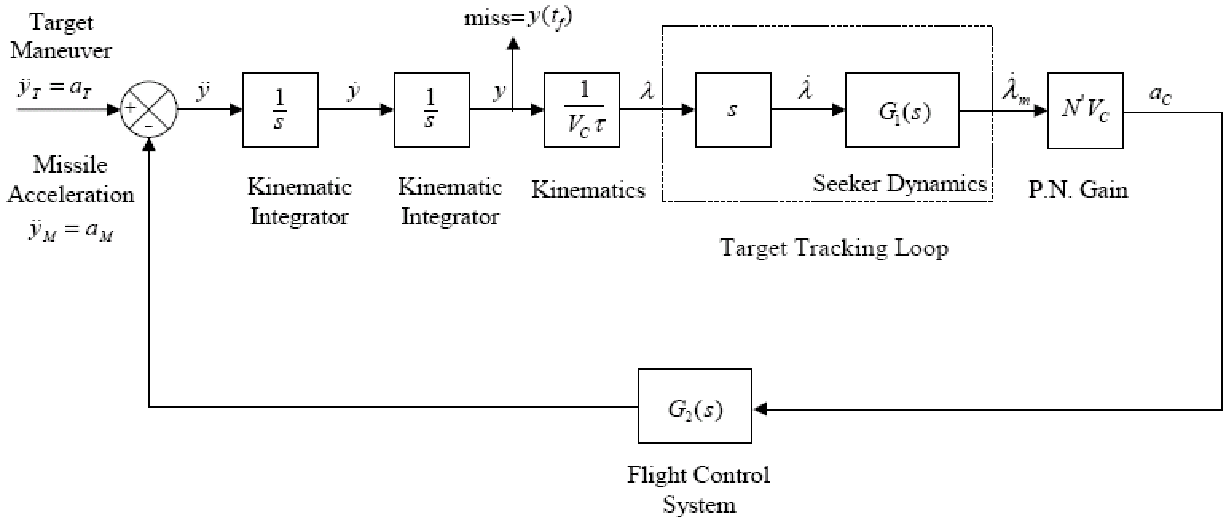

Fig.(1) Linearized Proportional Navigation Guidance Block Diagram

PROPORTIONAL NAVIGATION

Theoretically, the proportional navigation guidance (PNG) law issues acceleration commands, perpendicular to the instantaneous missile-target LOS, which are proportional to the LOS rate and closing velocity.

Mathematically, the guidance law can be stated as **(Zarchan, 1990 and Asher & Yaesh 1998)**

$$a_c = N' V_c \dot{\lambda}_m \tag{1}$$

In tactical radar homing missiles using PNG the seeker provides an effective measurement of the LOS rate, and a Doppler radar provides closing velocity information. In tactical IR missile applications of PNG, the LOS rate is measured, whereas the closing velocity required by the guidance law is guesstimated.

In tactical missiles within the Earth atmosphere, PNG commands are usually implemented by moving fins or other control surfaces to obtain the required lift. Outside the Earth atmosphere strategic interceptors use thrust vector control, lateral divert engines, or squibs to achieve the desired acceleration levels **(Zarchan, 1990 and Asher & Yaesh 1998)**.

PROPORTIONAL NAVIGATION IN TWO DIMENSIONS

In this paper, an inertial coordinate system fixed to the surface of a flat-Earth model (i.e., the axis 1 is downrange and the axis 2 can either be altitude or crossrange) has been adopted. Using the inertial coordinate system of **Fig.(2)** means that we can integrate components of the acceleration and velocities along 1 and 2 directions without having to worry about additional terms due to Coriolis effect. In this model it is assumed that both the missile and target travel at constant velocity. In addition, gravitational and drag effects have been neglected for simplicity **(Zarchan, 1990 and Ben Asher & Ben Yaesh 1998)**.

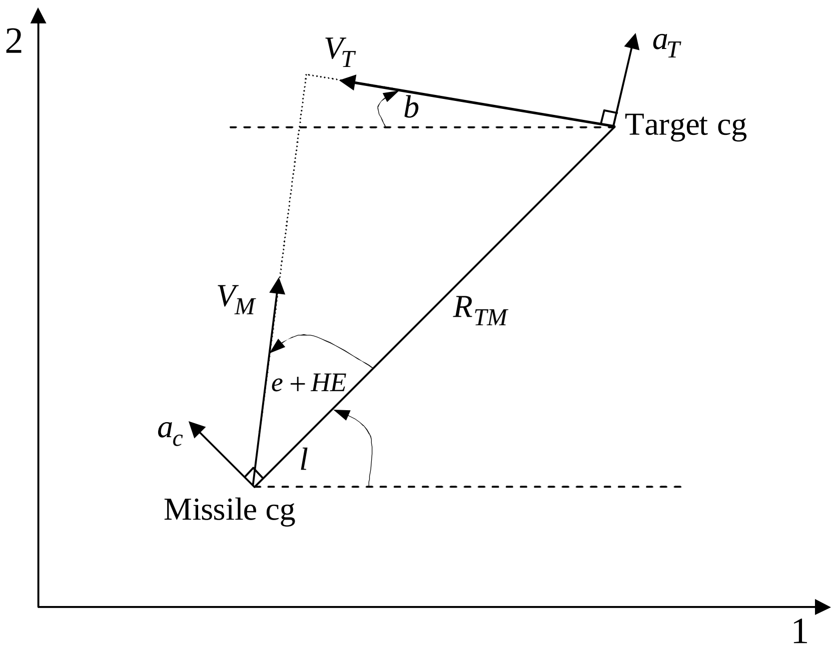

Fig.(2) Missile-Target Engagement Geometry

It can be seen from **Fig.(2)** that the missile, with velocity magnitude V_M is heading at an angle of $e + HE$ with respect to the LOS. The angle e is known as the missile lead angle. The lead angle is theoretically correct angle for the missile to be on a collision triangle, no further acceleration commands are required for the missile to hit the target. The angle *HE* is known as the heading error. This angle represents the initial deviation of the missile from the collision triangle.

In **Fig.(2)** the imaginary line connecting the missile and the target is known as the LOS. The LOS makes angle l with respect to the fixed reference, and the length of the LOS (instantaneous separation between missile and target) is a range denoted R_{TM} . From a guidance point of view, it desired to make the range between missile and target at the expected intercept time as small as possible (hopefully zero). The point of closest approach of the missile and target is known as the miss distance.

The closing velocity V_c is defined as the negative rate of change of the distance from the missile to the target, or

$$V_c = -\dot{R}_{TM} \tag{2}$$

Therefore, at the end of the engagement, when the missile and target are in closest proximity the sign of V_c will change. In other words, it can be concluded that the closing velocity will be zero when R_{TM} is a minimum (i.e. the function is either minimum or maximum when its derivative is zero). The desired acceleration command a_c, which is derived from the PNG law, is perpendicular to the instantaneous LOS.

In our engagement model of **Fig.(2)** the target can maneuver evasively with acceleration magnitude a_T. Since target acceleration a_T in the preceding model is perpendicular to the target velocity vector, the angular velocity of the target can be expressed as **(Zarchan, 1990)**

$$\dot{b} = \frac{a_T}{V_T} \tag{3}$$

Where V_T is the magnitude of the target velocity. The components of the target velocity vector in the Earth or inertial coordinate system can be found by integrating **Eq.(3)**, and substituting in

$$V_{T1} = -V_T \cos b \tag{4a}$$
$$V_{T2} = V_T \sin b \tag{4b}$$

Target position components in the Earth fixed coordinate system can be found by directly integrating the target velocity components. Therefore, the differential equations for the components of the target position are given by

$$\dot{R}_{T1} = V_{T1} \tag{5a}$$
$$\dot{R}_{T2} = V_{T2} \tag{5b}$$

Similarly, the missile velocity and position differential equations are given by

$$\dot{V}_{M1} = a_{M1} \tag{6a}$$
$$\dot{V}_{M2} = a_{M2} \tag{6b}$$
$$\dot{R}_{M1} = V_{M1} \tag{6c}$$
$$\dot{R}_{M2} = V_{M2} \tag{6d}$$

Where a_{M1} and a_{M2} are the missile acceleration components in the Earth coordinate system. In order to find the missile acceleration components, the components of the relative missile-target separation must be found. This is accomplished by first finding the components of the relative missile-target separation by

$$R_{TM1} = R_{T1} - R_{M1} \tag{7a}$$
$$R_{TM2} = R_{T2} - R_{M2} \tag{7b}$$

It can be seen from **Fig.(2)** that the LOS angle can be found, using trigonometry, in terms of the relative separation components as

$$l = \tan^{-1}\frac{R_{TM2}}{R_{TM1}} \tag{8}$$

if the relative velocity components in Earth coordinates are

$$V_{TM1} = V_{T1} - V_{M1} \tag{9a}$$
$$V_{TM2} = V_{T2} - V_{M2} \tag{9b}$$

the LOS rate can be calculated by direct differentiation of **Eq.(8)** as

$$\frac{d}{dt}\left[\tan^{-1}\left(\frac{R_{TM2}}{R_{TM1}}\right)\right] = \left[\frac{1}{1+\left(\frac{R_{TM2}}{R_{TM1}}\right)^2}\right]\frac{d}{dt}\left(\frac{R_{TM2}}{R_{TM1}}\right)$$

and by using the quotient rule **(Finny and Thomas 1990)** will have

$$\frac{dl}{dt} = \frac{1}{1+\left(\frac{R_{TM2}}{R_{TM1}}\right)^2}\left[\frac{R_{TM1}V_{TM2} - R_{TM2}V_{TM1}}{R_{TM1}^2}\right]$$

more simplifying will give

$$\dot{l} = \left[\frac{R_{TM1}V_{TM2} - R_{TM2}V_{TM1}}{R_{TM}^2}\right] \tag{10}$$

The relative separation between missile and target R_{TM} , can be expressed in terms of its inertial components by application of the distance formula as

$$R_{TM} = \sqrt{R_{TMx}^2 + R_{TMy}^2} \tag{11}$$

Since the closing velocity is defined as the negative rate of change of the missile target separation. It can be obtained by differentiating **Eq.(11)**, yielding

$$V_c = -\dot{R}_{TM} = \frac{-\left(R_{TM1} V_{TM1} + R_{TM2} V_{TM2}\right)}{R_{TM}} \quad (12)$$

The magnitude of the missile guidance command n_c can then be found by substituting **Eq.(11)** and **Eq.(12)** into **Eq.(1)**, after some algebra will have

$$a_c = N' \left[\frac{R_{TM1} R_{TM2} \left(V_{TM1}^2 - V_{TM2}^2\right) + V_{TM1} V_{TM2} \left(R_{TM2}^2 - R_{TM1}^2\right)}{R_{TM}^3} \right] \quad (13)$$

Since the acceleration command is perpendicular to the instantaneous LOS, the missile acceleration components in Earth coordinates can be found by trigonometry using the angular definitions from **Fig.(1)**. The missile acceleration components are

$$a_{M1} = -a_c \sin l \quad (14a)$$

$$a_{M2} = a_c \cos l \quad (14b)$$

Now, a set of all the differential equations required to model a complete missile-target engagement in two dimensions have been listed. However, some additional equations are required for the initial conditions on the differential equations in order to complete the engagement model.

A missile employing PNG is not fired at the target but is fired in a direction to lead the target. The initial angle of the missile velocity vector with respect to the LOS is known as the missile lead angle e . In essence the missile is firing at the expected intercept point. It can be seen from **Fig.(2)** that for the missile to be on a collision triangle (missile will hit target if both continue to fly along a straight line path at constant velocities), the theoretical missile lead angle can be found by application of the sine law, yielding

$$e = \sin^{-1} \left[\frac{V_T \sin\left(b + l\right)}{V_M} \right] \quad (15)$$

In practice, the missile is usually not launched exactly on a collision triangle, since the expected intercept point can only be approximated because we don't know in advance what the target will do in the future. In fact, that is why a guidance system is required. Any initial angular deviation of the missile from the collision triangle is known as a heading error **(HE)**. The initial missile velocity components can therefore be expressed in terms of the theoretical lead angle and actual heading error as

$$V_{M1}(0) = V_M \cos\left(e + HE + l\right) \quad (16a)$$

$$V_{M2}(0) = V_M \sin\left(e + HE + l\right) \quad (16b)$$

LINEARIZATION OF PROPORTIONAL NAVIGATION GUIDANCE LAW

The linearization of the missile-target geometry can easily be accomplished if some new relative quantities have been defined as shown in **Fig.(3)**. Here y is the relative separation between the missile and the target perpendicular to the fixed reference.

The relative acceleration (difference between missile and target acceleration) can be written by inspection of **Fig.(3)** as

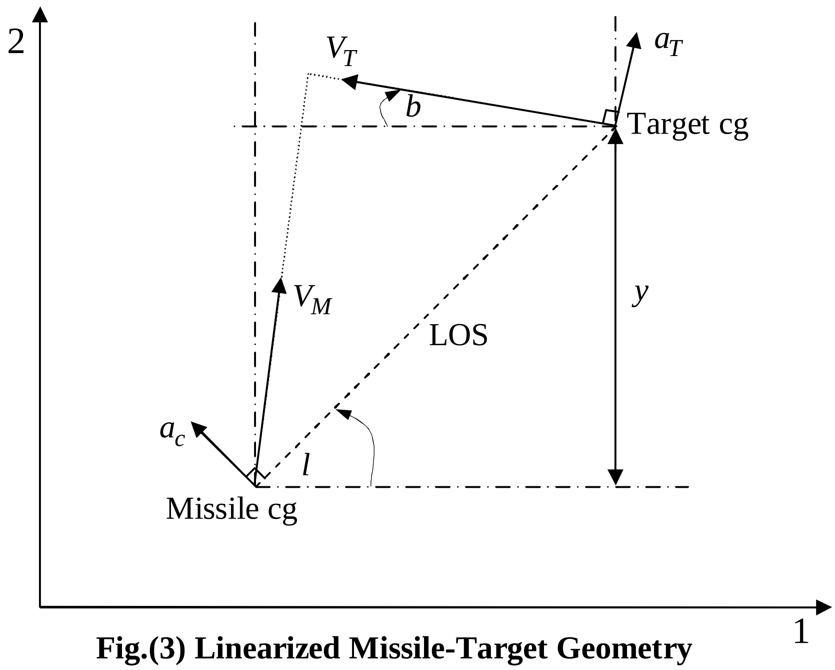

Fig.(3) Linearized Missile-Target Geometry

$$\ddot{y} = a_T \cos b - a_c \cos l \qquad (17)$$

If the flight-path angles are small (near head-on or tail chase case), the cosine terms approximately unity, and **Eq.(17)** becomes **(Zarchan, 1990 and Asher & Yaesh 1998)**.

$$\ddot{y} = a_T - a_c \qquad (18)$$

Similarly, the expression of the LOS angle can also be linearized using the small angle approximation, yielding

$$l = \frac{y}{R_{TM}} \qquad (19)$$

For a head-on case the closing velocity can approximated as

$$V_c = V_M + V_T \qquad (20)$$

Whereas in a tail chase case the closing velocity can be approximated as

$$V_c = V_M - V_T \qquad (20)$$

Therefore, in a linearized analysis the closing velocity will be treated as a positive constant. Since closing velocity has also been previously defined as the negative derivative of the range from the missile to target, and since the range must go to zero at the end of the flight, it can also linearize the range equation with the time varying relationship

$$R_{TM} = V_c\,(t_F - t) = V_c\, t \qquad (21)$$

$$l = \frac{y}{V_c t} \tag{22}$$

Where t is the current time and t_F is the total flight time of the engagement. Note that t_F is also now a constant. The quantity $(t_F - t)$ or t is the time to go until the end of flight. Therefore, the range from the missile to the target is also the closing velocity multiplied by the time to go until intercept.
Since range goes to zero at the end of the flight by definition, the definition of miss distance must be reexamine. The linearized miss distance is taken to be the relative separation between the missile and target, y at the end of the flight, or

$$Miss = y(t_F) \tag{23}$$

HOMING SYSTEM SYNTHESIS

In order to design a missile homing system based on PNG law, transfer functions of flight control system, $G_2(s) = a_M(s)/a_c(s)$, and the tracking loop, $G_1 = \dot{\lambda}_m / \dot{\lambda}$, are required. These transfer functions can be found in two steps. First, the nonlinear terms are left out. Second, the resulting high order linear models are reduced using state truncation method such as balanced realization. It is important to stress that this procedure is used for the guidance design only, not for overall performance evaluation of the missile, where the complete, detailed nonlinear stochastic models are used.

The flight control system used in this paper was adopted from **(Nesline and Nesline 1984)** and is depicted in **Fig.(4)**. This pitch-plane three-loop control system comprises a rate loop, a synthetic stability loop and an accelerometer feedback loop.

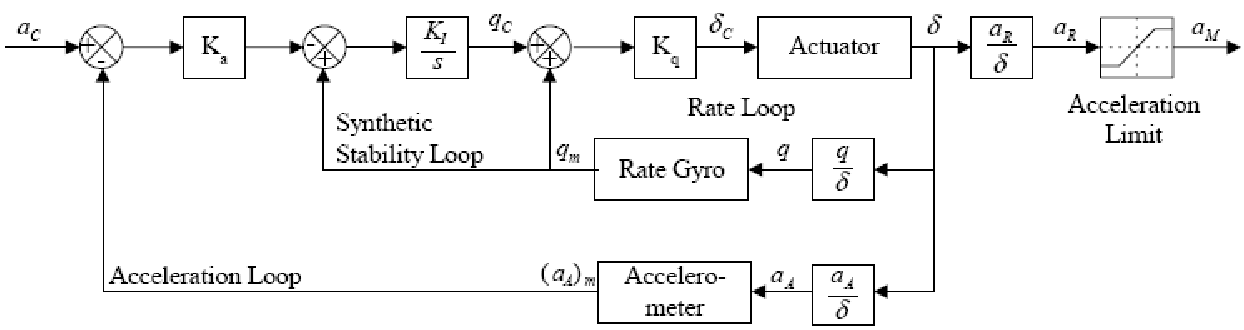

Fig.(4) The Missile Flight Control System

The input to the accelerometer feedback loop is the command acceleration a_c, which is generated by the guidance law. The output is the required acceleration a_R, which is limited due to aerodynamic or structural constraints, to yield the actual acceleration a_M. The autopilot of this loop is the gain K_a the feedback signal $(a_A)_m$ is generated by an accelerometer, which is located at the point X_{ACC}. The signal a_A is the output of the aerodynamic transfer function a_A/d, with d being the fin deflection angle. d is the output of the body pitch rate control loop, whose input is the commanded pitch rate q_C, generated by the synthetic stability loop. The autopilot of the pitch rate loop is the gain K_q. This gain generates a commanded fines deflection angle d_c, which constitutes an input to the fin actuators. The aerodynamic transfer function a_R/d then yields the required output acceleration a_R. The complete derivation of the flight control system can be obtained from returning to **(Nesline and Nesline 1984)**.

We start with model reduction of a complex flight control system, described above, which has 9 zeros and 13 poles Using balance realization state truncation which performed by **(Gurfil, 2002)** , and the parameter values given in **Table (1)**, we will get the following reduced order transfer function is obtained:

$$G_2(s) = \frac{\left(\frac{-s}{40.3}+1\right)}{\left(\frac{s}{23.3}+1\right)\left(\frac{s}{1.93}+1\right)} \tag{24}$$

Obviously, $G_2(s)$ is nonminimum phase, due to the fact that the missile is tail controlled. If the approximation that addressed by **(Gurfil, 2002)** is used**,** it is evident that the right half plane zero is ‘‘fast’’. Hence, an additional state truncation yields

$$G_2(s) = \frac{1}{0.56s+1} \tag{25}$$

The simplified model **Eq.(25)** constitutes an adequate approximation to the overall flight control system dynamics, both in the frequency and time domains. It is subsequently used for homing system design.

Also in this paper, the electro-optical target tracking system that introduced by **(shneydor, 1998)** is adopted. The purpose of the target tracking loop of an electro-optical missile is to maintain the target within field-of-view (FOV) of a stabilized imaging device, such as a CCD camera. The general layout of a such tracking loop, depicted in **Fig(5)**, was adopted from Shneydor. This tracking loop is based upon a rate-gyro stabilized platform, where the camera is mounted on gimbals, whose movement is (ideally) isolated from the motion of the missile. The location of the target within FOV limits is measured by an electro-optical tracker, which is an implementation of a correlation algorithm that utilizes the sequence of images generated by the visual motion (the so called "optical flow").

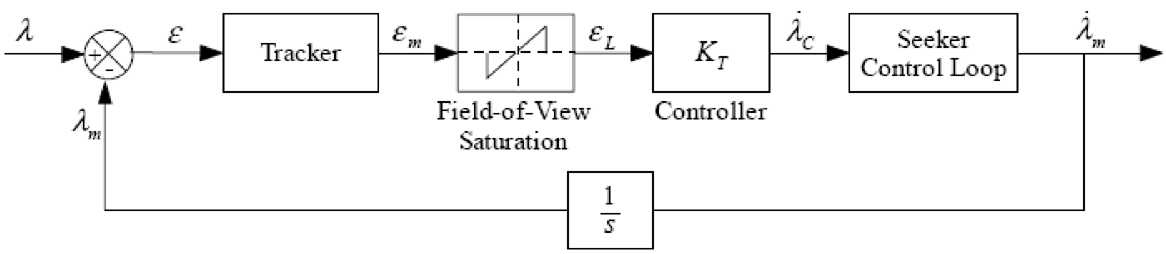

Fig.(5) The Electro-Optical Target Tracking Loop

The tracking loop overall transfer function, $G_1(s)$, is obtained in a similar manner. Using the numerical values of **Table (1)**, neglecting the FOV saturation and the pure tracking delay, will have

$$G_1(s) = \frac{1}{0.1s+1} \tag{26}$$

RESULTS AND DISCCUSSION

A MATLAB / SIMULINK software program has been constructed to simulate the missile homing system operation. **Fig.(6)** shows an outline of this program which contains a four major subsystems and this program start working from input the missile specifications and the following initial measurements (target acceleration, and target velocity)

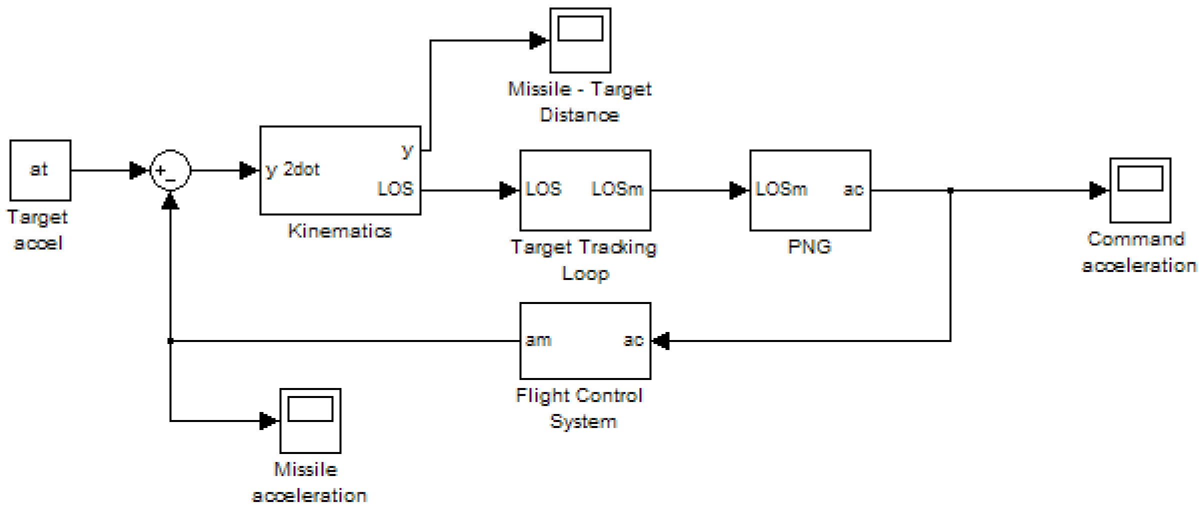

Fig.(6) The SIMULINK Software Program Outline

In this paper, many complicated cases have been studied, where in these cases the initial LOS angle is varied and study the missile response for the following scenarios (without any change in target situation, target maneuvering with 3g acceleration, missile is launching with 20^{o} initial heading error).

All the results from **Fig.(7)** to **Fig.(18)** show the efficient behavior of the homing system and the ability of the missile to hit its target and treat any probable disturbance from it. Also, the results show that the peak acceleration of the missile heading error case is the maximum acceleration that the missile owns in comparing with the other disturbance (the missile try fast to adjust its direction) but this acceleration is rapidly decreased to zero at the end of flight time. And is true that the peak acceleration for target maneuvering case is less than the peak acceleration case, but it's clearly from the results that the missile acceleration for the target maneuvering case is much higher than any acceleration at the end of flight time.

Its obvious from results of homing system simulation **Fig.(7)** to **Fig.(18)** especially for heading error and target maneuvering cases that at the beginning of the missile guidance operation the relative distance between the target and the missile is increased due to the fact that the homing system dose not correct the missile direction to be in a collision triangle with the target yet, but with guidance operation progressing this relative distance is decreased rapidly to be zero at the end of flight.

CONCULSIONS

In this paper, a full missile homing system is proposed. The proposed homing system simulation results show a rigid response for any probable disturbance such as launching heading error or missile maneuvering

The peak acceleration for heading error case is higher than any other disturbance case but this peak acceleration is rapidly decreased to be zero at the end of the missile flight. The peak missile acceleration for target maneuvering case is lower than the heading error case but this acceleration is approximately constant to the end of flight time.

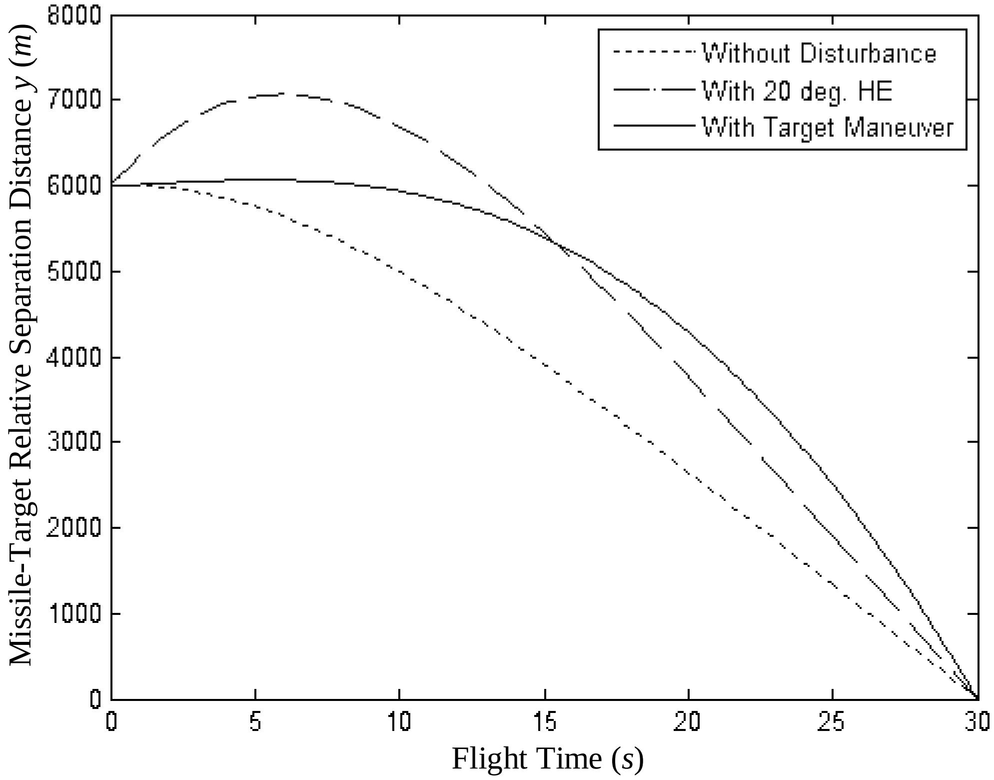

Fig.(7) Missile-Target Relative Separation Distance ($N' = 4$) and initial $l = 0^{\mathbf{o}}$

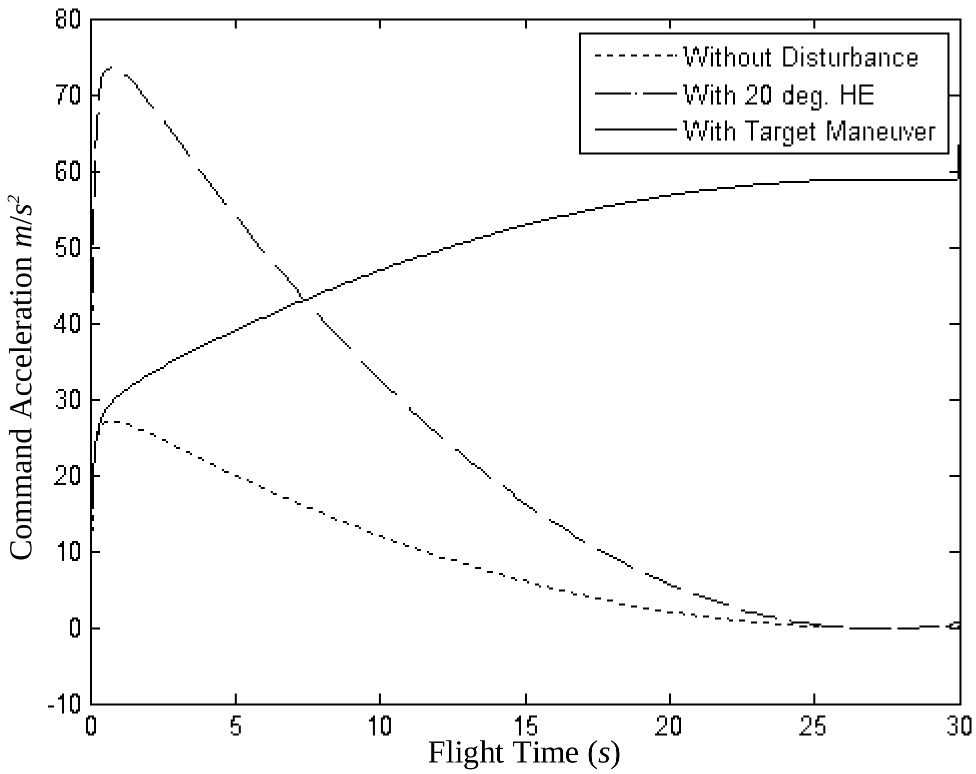

Fig.(8) Guidance Command Acceleration ($N' = 4$) and initial $l = 0^{\mathbf{o}}$

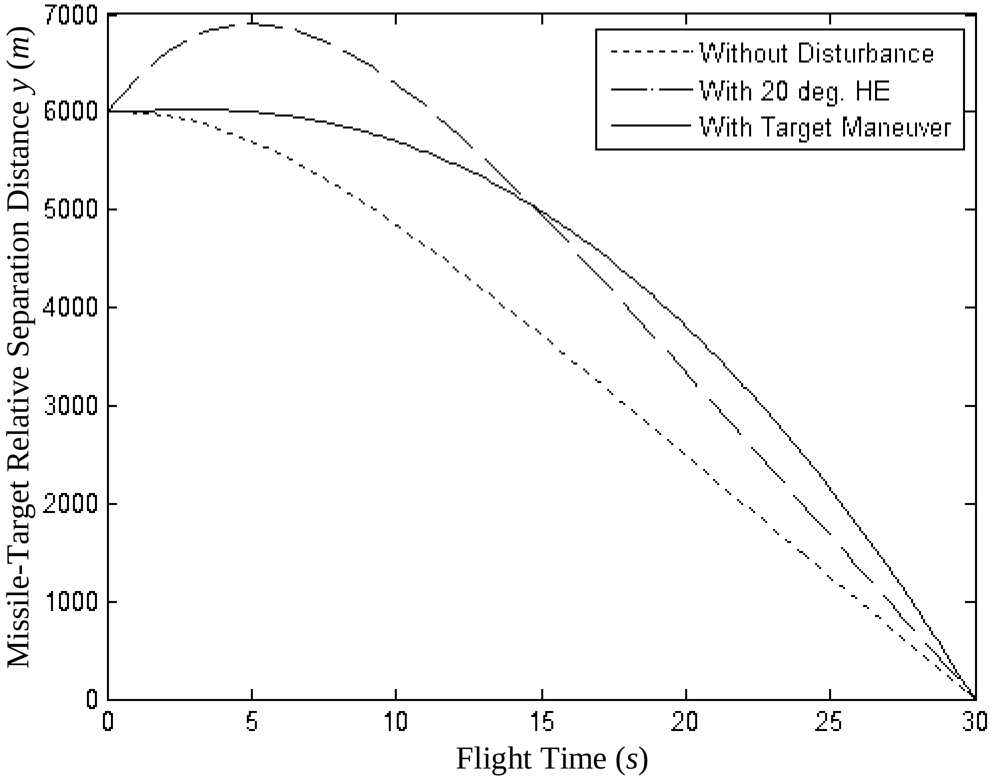

Fig.(9) Missile-Target Relative Separation Distance ($N' = 5$) and initial $l = 0^{\circ}$

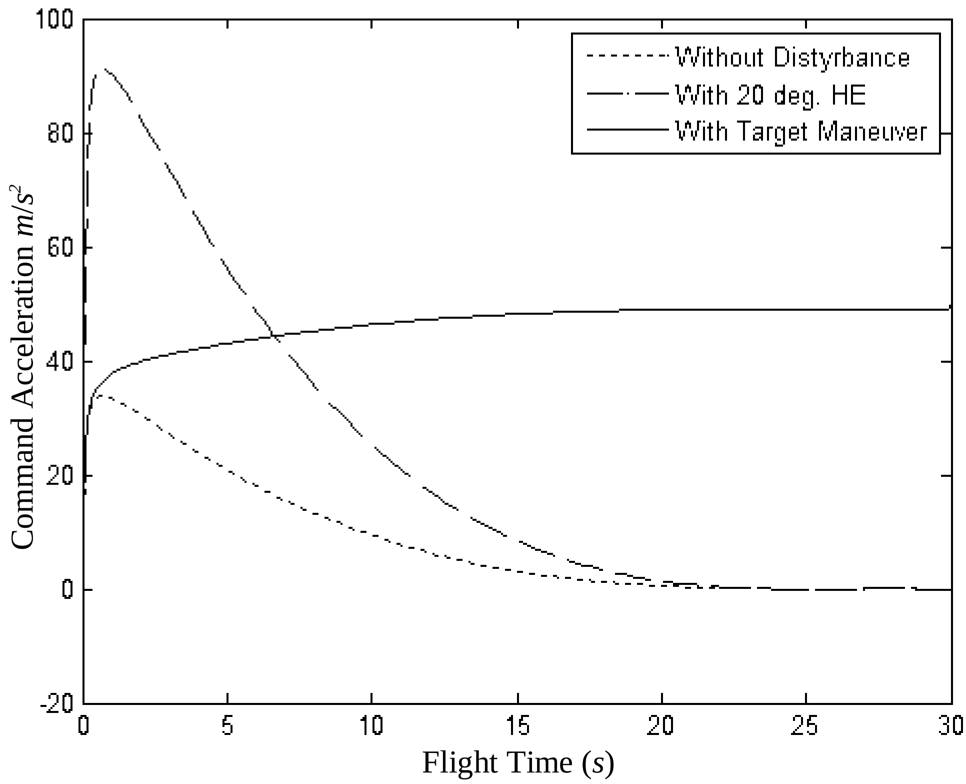

Fig.(10) Guidance Command Acceleration ($N' = 5$) and initial $l = 0^{\circ}$

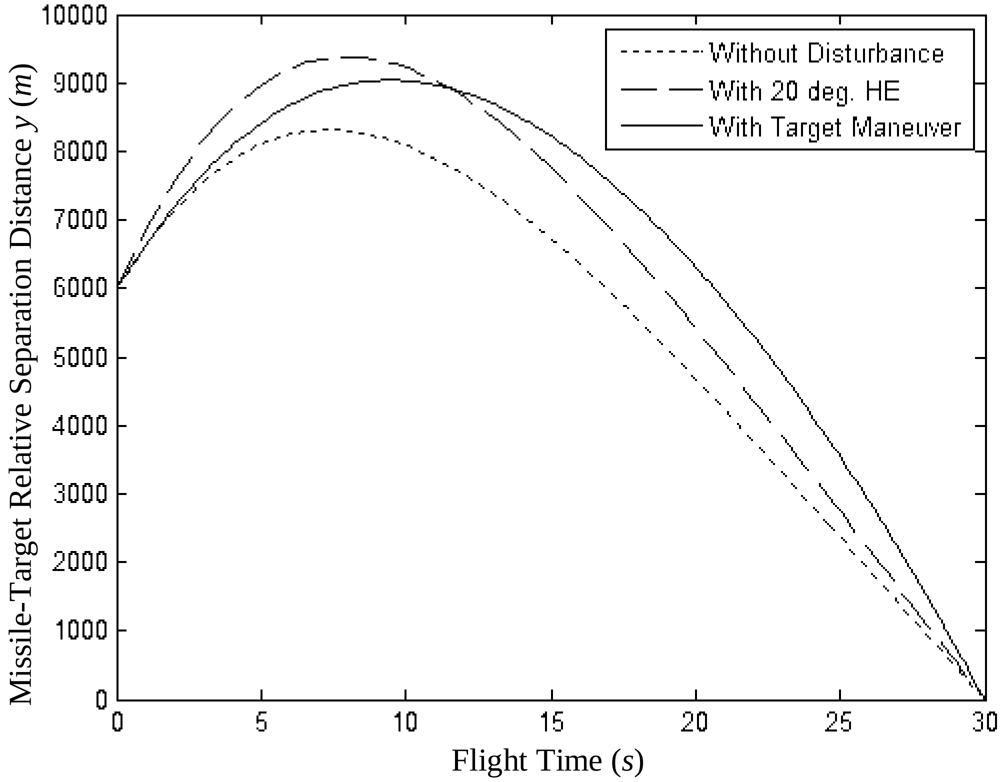

Fig.(11) Missile-Target Relative Separation Distance ($N' = 4$) and initial $l = 30^{\circ}$

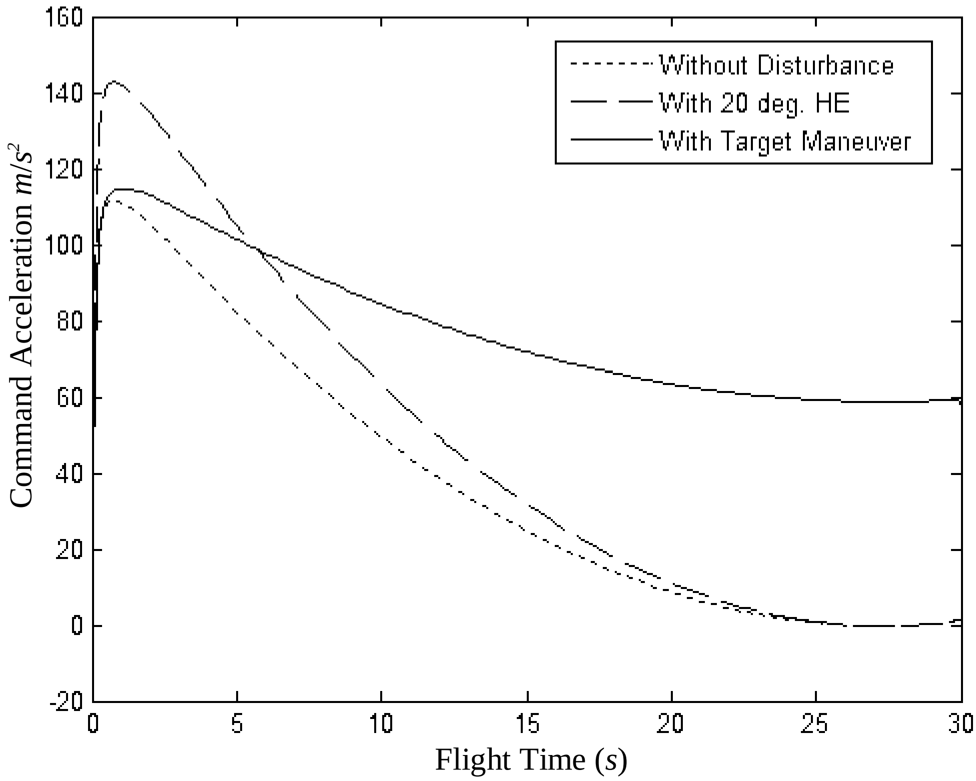

Fig.(12) Guidance Command Acceleration ($N' = 4$) and initial $l = 30^{\circ}$

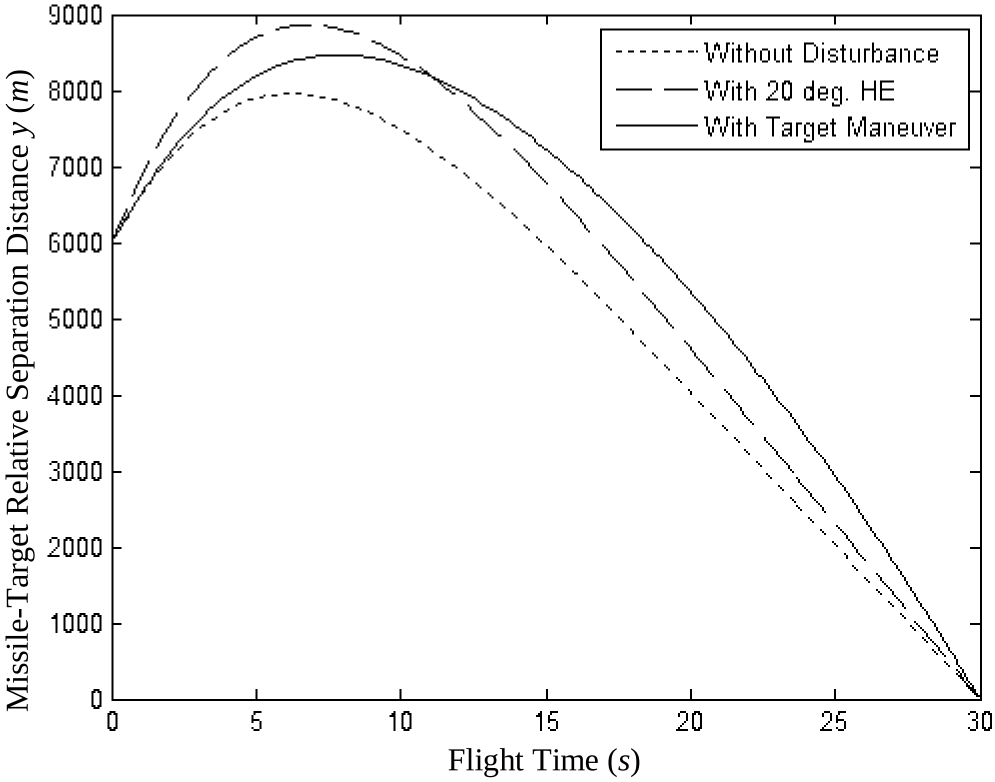

Fig.(13) Missile-Target Relative Separation Distance ($N' = 5$) and initial $l = 30^{\mathbf{o}}$

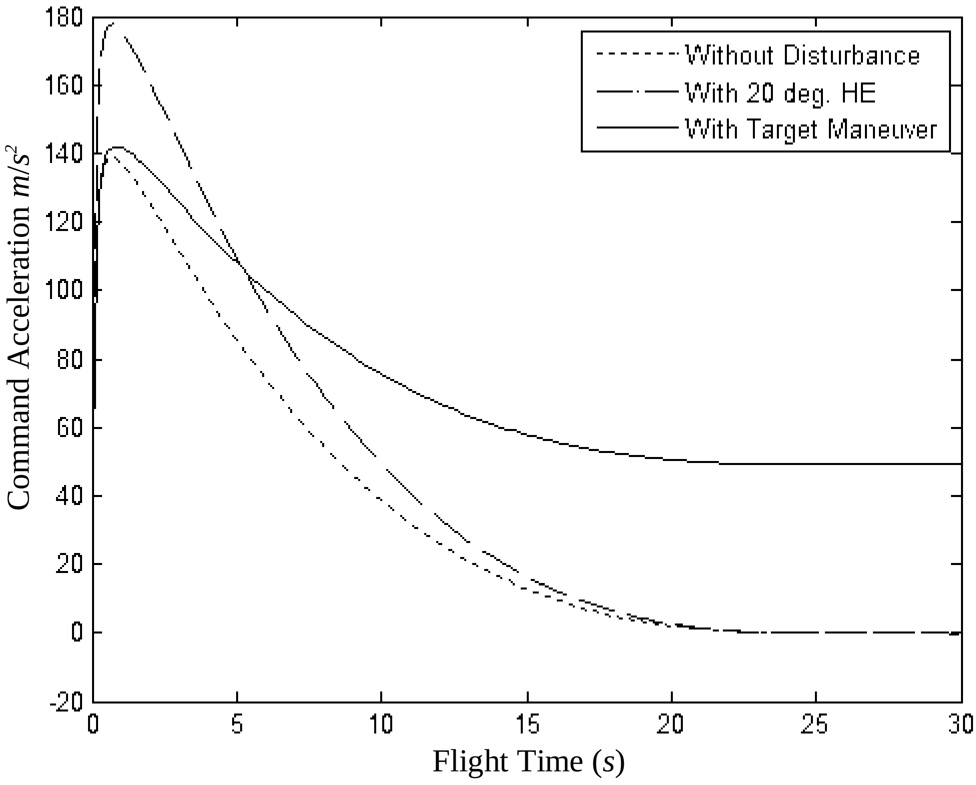

Fig.(14) Guidance Command Acceleration ($N' = 5$) and initial $l = 30^{\mathbf{o}}$

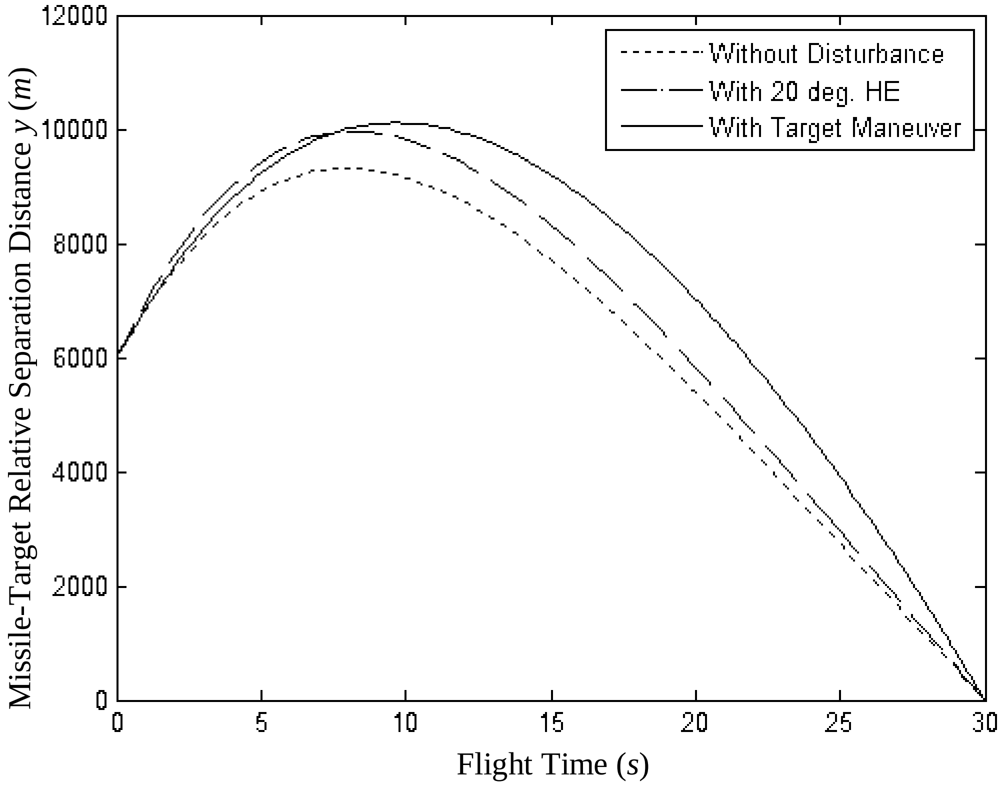

Fig.(15) Missile-Target Relative Separation Distance ($N' = 4$) and initial $l = 45^{o}$

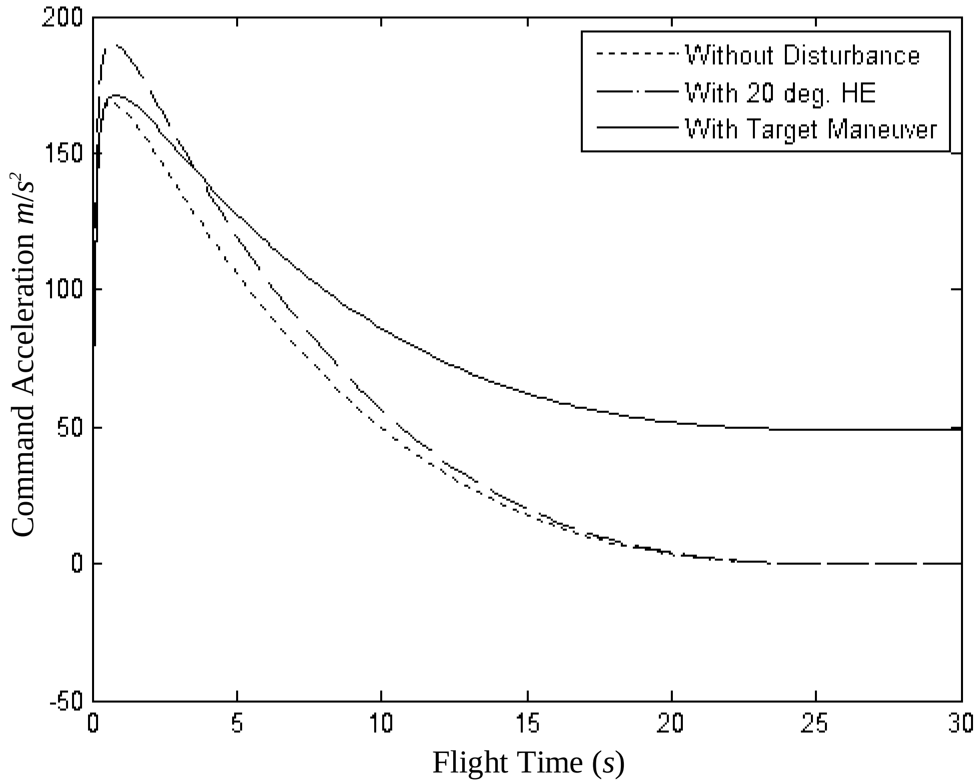

Fig.(16) Guidance Command Acceleration ($N' = 4$) and initial $l = 45^{o}$

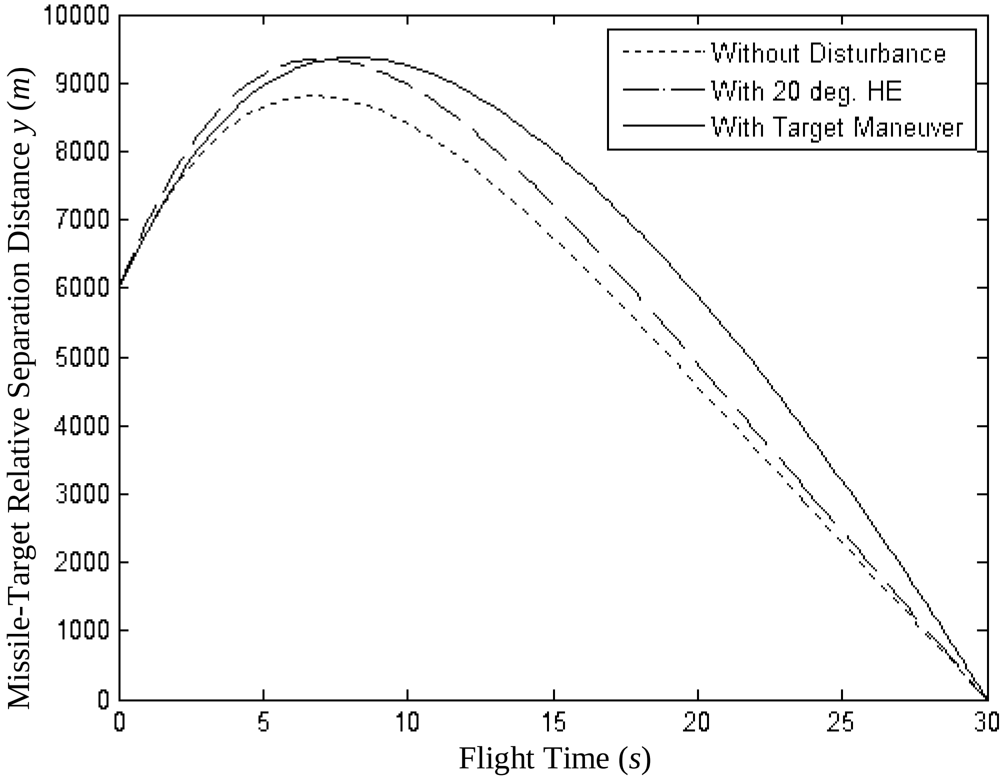

Fig.(17) Missile-Target Relative Separation Distance ($N' = 5$) and initial $l = 45^{\mathbf{o}}$

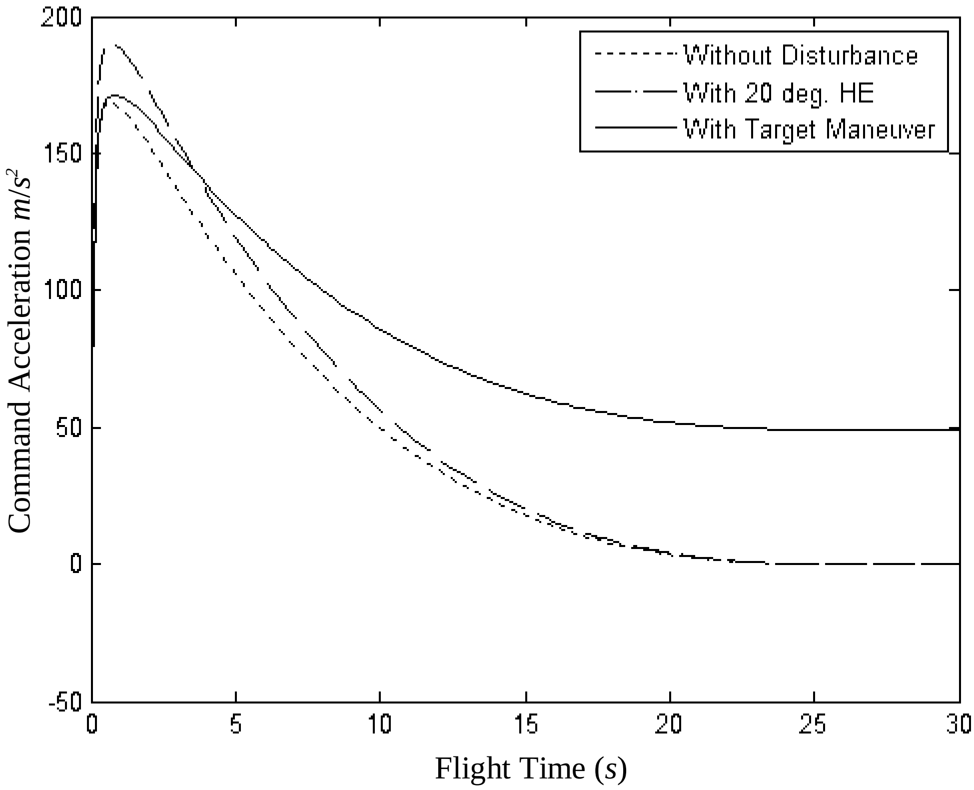

Fig.(18) Guidance Command Acceleration ($N' = 5$) and initial $l = 45^{\mathbf{o}}$

Table (1) contains parameter values that were used in the illustrative example

Parameter values for the illustrative example

	Parameter/ disturbance	Units	Mean value (nominal)	Standard deviation
Aerodynamics	M_α	$1/s^2$	−250	16.7
	M_δ	$1/s^2$	280	18.7
	Z_α	1/s	−1.6	0.107
	Z_δ	1/s	0.23	0.0153
	M_q	1/s	−1.5	—
	ΔX	m	0.7	—
	V_M	m/s	500	—
	V_C	m/s	1000	—
	μ_0	—	2	—
Autopilot	K_q	s	0.1534	—
	K_a	deg s/m	0.0007804	—
	K_I	rad/s	13.55	—
Fin actuators	$\omega_{n_{servo}}$	rad/s	200	2
	ζ_{servo}	—	0.6	0.01
	K_{servo}	—	1	0.0167
	δ_B	deg	0	0.167
	n_δ	deg	0	0.0167
	δ_{max}	deg	20	—
	$\dot{\delta}_{max}$	deg/s	230	—
	$\ddot{\delta}_{max}$	deg/s^2	17000	—
Rate gyro	$\omega_{n_{RG}}$	rad/s	300	3
	ζ_{RG}	—	0.65	0.0108
	K_{RG}	—	1	0.0267
	q_B	deg/s	0	0.0667
	n_q	deg/s	0	0.0167
Accelerometer	$\omega_{n_{ACC}}$	rad/s	300	3
	ζ_{ACC}	—	0.65	0.0108
	K_{AC}	—	1	0.0167
	$(a_A)_B$	mg	0	1
	n_a	mg	0	1
Target tracking loop	τ_T	ms	25	—
	n_ε	mrad	0	0.05
	ε_{max}	deg	1.5	—
	K_T	1/s	10	—
Seeker	$\omega_{n_{seeker}}$	rad/s	150	1.5
	ζ_{seeker}	—	0.7	0.0117
	$\dot{\lambda}_B$	deg/s	0	0.0667
	n_λ	deg/s	0	0.0167

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NOMENCLATURE

(SI units are used, unless otherwise stated)

a_c	Command acceleration
a_M	The actual acceleration.
a_R	the required acceleration
a_T	The target acceleration
g	Local Earth gravitational acceleration
HE	The heading error
K_a	The autopilot gain
K_q	The autopilot of the pitch rate loop gain
N'	Proportional navigation constant
q_C	commanded pitch rate,
R_{TM}	Target-Missile separation distance
t_F	Final flight time
t	Time to go
t	The instantaneous flight time
V_c	Closing velocity between the missile and the target
V_M	Missile vehicle velocity
V_T	The target velocity
y	The relative separation between the missile and the target
g	Flight path angle
e	The missile lead angle
b	The target flight path angle
d	The fin deflection angle.
l	Local line of sight angle
l_m	Actual missile local line of sight angle