



NUMERICAL STUDY OF NATURAL CONVECTION IN A CAVITY WITH WAVY VERTICAL WALLS

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ABSTRACT

This paper describes a numerical study of natural convection heat transfer and fluid flow characteristics inside a cavity with wavy vertical walls. The bottom wall is heated by spatially varying temperature and other three walls are kept at cooled temperature. Governing equation was discretized using the finite volume-method with staggered variables arrangement in curvilinear coordinates. Two geometrical configurations were used in this study for symmetrical and unsymmetrical wavy vertical walls (total of 132 cases) for range of $Ra=10^0$ to 10^6 and fixed Prandtl number (0.71). The effects of the wave geometry, wave amplitude, number of undulation, and Rayleigh number on flow behavior, thermal field, local Nusselt number and Nusselt number ratio (NNR) factor have been studied. Streamline, velocity vector, and isothermal contour are used to present the corresponding flow and thermal field inside the cavity. The Results show that the enhanced of heat transfer rate seems to depend on geometrical configuration.

KEYWORDS

Natural convection, Numerical, Curvilinear coordinate, Wavy wall

الخلاصة:

البحث الحالي يصف دراسة عددية لأنتقال الحرارة بالحمل الطبيعي وسمات جريان المائع داخل تجويف ذا جدران عمودية متموجة. الجدار السفلي مسخن بتغير حيزي لدرجة الحرارة ($T=F(x)$) بينما بقية الجدران الثلاثة حفظت بدرجة حرارة ثابتة. المعادلات الحاكمة حُلّت بطريقة الحجوم الثابتة مع ترتيب متخالف للمتغيرات المدروسة بأحداثيات مطابقة الجسم. تم أخذ حالتان لتركيب الشكل الهندسي للتجويف، حالة التماثل، وحالة عدم التماثل للجدران العمودية المتموجة (كلياً ١٣٢ حالة) لمدى من $Ra=10^0$ الى 10^6 وثبتت $Pr=0.71$ ، تم دراسة تأثير كل من الشكل الهندسي المتموج، مدى الموجة، عدد الموجات، Ra على تصرف الجريان والمجال الحراري داخل التجويف، Nu الموقعي، العامل NNR ، تم الاعتماد على خطوط الانسياب، متجه السرعة، خطوط ثبوت درجة الحرارة لأظهار الشكل المتكون للجريان والمجال الحراري داخل التجويف. النتائج أظهرت هناك تحسس لمعدل انتقال الحرارة يعتمد على تركيب الشكل الهندسي للتجويف.

INTRODUCTION

Heat transfer and flow behavior inside wavy-walled cavity has not been investigated widely due to geometric complexity. Numerous references deal of cavities with flat walls due to its huge applications in engineering and geophysical systems like solar-collectors, double-wall insulation, electric machinery, cooling system of electronic devices, natural circulation in the atmosphere etc. These are always complex interactions between the finite fluid content inside the cavity with the cavity walls. This complexity increases when the wall becomes wavy or with the change of orientation of the cavity. **(Yao, 2006)** has studied natural convection for more complex surface and he found that the heat transfer rates for complex surface are greater than that of a flat plate, and the results show the local Nusslet number depends on the ratio of amplitude and wavelength of the surface. **(Adjout et al., 2002)** studied the effect of a hot wavy wall of a laminar natural convection in an inclined square cavity. One of their findings was the decrease of heat transfer with the surface waviness when compared with flat wall cavity. **(Mahmud et al. , 2002)** studied flow and heat transfer characteristics inside an enclosure bounded by two isothermal wavy wall and two adiabatic straight walls at different Grashof number.

(Das and Mahmud , 2003) investigated buoyancy induced flow and heat transfer inside a wavy enclosure. They reported that the amplitude-wavelength ratio affected local heat transfer rate, but it had no significant influence on average heat transfer rate. **(Jang et al. , 2003)** investigated the effects of the amplitude-wavelength ratio, buoyancy ratio, and Schmidt number on momentum and energy equations, moreover to study the skin friction coefficient and Nusselt number on wavy walls under these parameters. They found that for higher amplitude-wavelength ratio increase the fluctuation of velocity, temperature and concentration. **(Jang and Yan, 2004)** studied the transient behaviors of natural convection heat and mass transfer along a vertical wavy surface subjected to step changes of wall temperature and wall concentration. They found that wave geometry is an important factor in this problem moreover to buoyancy ratio, and Schmidt number.

(Dalal and Das , 2003) have considered a case of heating from the top surface with a sinusoidal varying temperature and cooling from the other three surfaces. The right vertical surface was undulated having one and three numbers. The effect of the number and the amplitude of undulation were studied. In another study, **(Dalal and Das , 2005)** have made a detailed study by considering the same geometry as of **(Dalal and Das , 2003)**. The study was conducted at different inclination of the enclosure from 0 to 360 deg in steps of 30 deg. They concluded that the maximum and minimum average Nusslet number occurs at certain orientation angles. **(Dalal, and Das, 2006)** studied natural convection inside cavity with right wavy wall only and heated from below while other walls are kept at cooled temperature. They found that, the presence of undulation in the right wall affects in both local Nusselt number and flow and thermal field. The results of them were applied in valuated case with the result of the present code. **(Rathish Kumar et al., 1997)** have reported the effect of sinusoidal surface imperfections on the free convection in a porous enclosure heated from the side. The observations reveal that the heat transfer decreases as the amplitude of the wave increases. Also, the total heat transfer rates less when compared with the heat transfer in an enclosure with plane walls. **(Rathish Kumar and Gupta, 2005)** have analyzed the combined influence of mass and thermal stratification on non-Darcian double-diffusive natural convection from a wavy vertical wall to analyze the influence of various parameters. It is observed that the presence of surface waviness brings in a wavy pattern in the local heat fluxes.

Most of the previous researches investigated the natural convection with either uniform wall temperature or wall heat flux thermal boundary condition. However, these imposed thermal boundary conditions are not suitable in many practical applications such as heat exchangers, inject mold, transient setup and shutdown processes and non-equilibrium solidification processes. Furthermore, to meet the industrial requirements, a non-uniform thermal boundary is necessary. For example, some



researcher utilized a non-uniform temperature distribution to obtain a uniform thickness substance film in chemical deposition process. Therefore it is necessary to discover the influences of the non-uniform thermal boundary conditions on the heat transfer and flow characteristics in natural convection flow.

In the present investigation, a numerical analysis of natural convection in a two-dimensional cavity heated from below surface and uniformly cooled from the top surface and both vertical sides is conducted. The cavity is having two flat walls and the two vertical wavy walls consisting of one, two, three and four undulations. The amplitude of undulations is varied from 0.00 to 0.10. The two vertical wavy and top walls are cooled with a fixed temperature (isothermal) whereas the bottom wall is heated non-uniformly with a sinusoidal temperature distribution in space coordinate. Air has been taken as the working fluid with $Pr=0.71$. The flow structure type and heat transfer rate are analyzed and discussed for a wide range of Rayleigh number 10^0 to 10^6 in this study.

GEOMETRICAL DESCRIPTION

The proposed physical model for a two-dimensional cavity (height H , and length L) with wavy vertical walls filled with viscous fluid shown in **Fig. 1** for two cases of wavy vertical walls; symmetrical and unsymmetrical. In present study it is assumed that ($H=L$) square cavity, the vertical wavy walls is taken as sinusoidal varying as the expression below:

$$f(y) = 1 - l + l * \cos(2p \cdot n \cdot y) \quad (1)$$

Where n is the number of undulations. Four different values of $n=1, 2, 3,$ and 4 are studies. The wave amplitude λ changed for 0 to 0.1 in all cases. The flow in a cavity is air ($Pr=0.71$) and Rayleigh number varied from 10^0 to 10^6 . The heated wall (bottom wall) considered to be spatially varying with sinusoidal temperature T_h as the expression below:

$$T_h(x) = 0.5 * [1 - \cos(2p \cdot x)] \quad (2)$$

While the other walls kept at cooled temperature T_c .

GOVERNING EQUATIONS

The governing equations for natural convection laminar two-dimensional incompressible steady flow in dimensionless form using the following dimensionless variables (**Xundan, 2003**) are:

$$x = \frac{x^*}{H}, \quad y = \frac{y^*}{H}, \quad u = \frac{u^* \cdot H}{a_f}, \quad v = \frac{v^* \cdot H}{a_f}$$

$$p = \frac{p^* \cdot H^2}{r \cdot a_f}, \quad T = \frac{T^* - T_c^*}{T_h^* - T_c^*}$$

The governing equations of continuity, momentum, and thermal energy become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \text{Pr} \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \text{Pr} \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + Ra \text{Pr} T$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$$
(3)

The fluid properties assumed constant except for variation of density in the buoyancy force term of momentum in Y-direction which is approximated by the Boussinesq assumption. Boundary conditions are specified as shown in **Fig. 1**.

GRID GENERATION

It is of great importance to implement the surrounding boundaries of arbitrary curvature in General partial differential equations (GPDE) and to become a part of solution. The proper choice of the used technique to transfer the physical domain into computational domain has a great influence on the solution. Elliptical Partial differential equations (PDE) method is the most general, applicable and programmable method. There are two types of generating system, Laplace equation type and Poisson equation type. The second type was used in this study.

The transformation function $\mathbf{x} = \mathbf{x}(x, y)$, $\mathbf{h} = \mathbf{h}(x, y)$ is individually obtained by solving the following two elliptic Poisson equations:

$$\mathbf{x}_{xx} + \mathbf{x}_{yy} = P(x, y)$$

$$\mathbf{h}_{xx} + \mathbf{h}_{yy} = Q(x, y)$$
(4)

Where P and Q are two arbitrary function specified to adjust the local density of the grids. Meanwhile, the orthogonality of the generated grids system can be improved by carefully setting the boundary conditions. **Fig. 2** show symbol cases of curvilinear grid system applied in this study.

TRANSFORMATION OF THE GOVERNING EQUATIONS

The governing equations mass, momentum and energy transformed from the Cartesian coordinates (x,y) to the curvilinear coordinates (ξ, η) can be derived as:

Continuity equation:

$$U_x + Vh = 0$$
(5)

The general transport equation becomes:



$$(U\Phi)_x + (V\Phi)_h = S_\Phi + \left[\frac{\Gamma_\Phi}{J} (g \cdot \Phi_x - b \cdot \Phi_h) \right]_x + \left[\frac{\Gamma_\Phi}{J} (a \cdot \Phi_h - b \cdot \Phi_x) \right]_h \quad (6)$$

Where the source terms S_Φ is defined in **Table 1** as below:

Table 1 Source terms for general transport equations

Equation	Φ	Γ_Φ	S_Φ
Momentum	U	Pr	$p_h \cdot y_x - p_x \cdot y_h$
	V	Pr	$p_x \cdot x_h - p_h \cdot x_x + J \cdot Ra \cdot Pr \cdot T$
Energy	T	1	0

Where

$$g = x_{hh} + y_{hh} \quad , \quad a = x_{xx} + y_{xx} \quad , \quad b = x_x \cdot x_h - y_x \cdot y_h$$

And the relation between the Cartesian and contravariant velocity components is:

$$U = u \cdot y_h - v \cdot x_h \quad (7)$$

$$V = v \cdot x_x - u \cdot y_x$$

MODEL VALIDATION

The code was tested under two cases; **case one** described the buoyancy driven laminar heat transfer in a square cavity with differentially heated sidewall. The left wall is maintained hot while the right wall is cooled. The top and bottom wall are insulated. **Table 2** shows the comparison of average Nusselt number on the hot wall with numerical results of (**De Vahl Davis, 1983**), (**Markatos, 1984**), and (**Xundan, 2003**).

Table 2 Comparison of the predicted mean Nusselt number on the hot wall in a square cavity.

References	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
De Vahl Davis (1983)	2.243	4.519	8.800
Markatos (1984)	2.240	4.510	8.820
Xundan (2003)	2.247	4.532	8.893
Present Study	2.245	4.540	8.901

Case two of validate code represent of case study of (**Dalal, and Das, 2006**) for natural convection inside cavity with right wavy wall only and heated from below while other walls are kept at cooled temperature. **Fig. 3** shows the comparison of numerical results with results of (**Dalal, and Das, 2006**) for average Nusselt number on right wavy wall. The influence of the wall undulations is clearly seen in the results; where for all cases the average Nusselt number increase with an increase in Rayleigh

number (negative sine means that wavy wall is cold wall). The results applied of wide range of Ra and number of undulation. The results for two cases are showing a good agreement with the other results.

NUSSLT NUMBER CALCULATION

In order to evaluate how the presence of the wavy vertical walls affect the heat transfer rate along the wall according to the parameters Rayleigh number, wave amplitude, and number of undulation it is necessary to observe the variation of the local Nusselt number on these walls. In generalized coordinate the local Nusslet number defined as (Dalal, and Das, 2006):

$$\begin{aligned} \text{Right wall } Nu_l &= \frac{1}{J \cdot \sqrt{g}} [g \cdot T_x - b \cdot T_h] \\ \text{Left wall } Nu_l &= \frac{-1}{J \cdot \sqrt{g}} [g \cdot T_x - b \cdot T_h] \end{aligned} \quad (8)$$

While the average Nusselt number is calculated by the following expression:

$$Nu_{ava} = \frac{1}{L} \int_0^L Nu_l \cdot dl \quad (9)$$

To show the effect of the wavy vertical walls on heat transfer rate, we introduce a variable called Nusselt number ratio (NNR) with its definition given as:

$$NNR = \frac{Nu_{ave}|_{with \ wavy \ wall}}{Nu_{ave}|_{without \ wavy \ wall}} \quad (10)$$

If the value of NNR greater than 1 indicated that the heat transfer rate is enhanced on that surface, whereas reduction of heat transfer is indicated when NNR is less than 1.

COMPUTATIONAL DETAILS

The solution of the governing equations can lead to a complete understanding of the streamline, velocity vector and isothermal contours field for natural convection in a cavity with wavy vertical walls. The steady state governing equations were iteratively solved by the finite volume method using SIMPLE algorithm in curvilinear coordinates. A two dimensional uniformly spaced staggered grid was used; with power law scheme was utilized for the convection terms, whereas the central difference scheme was used for the diffusion terms. The residual level at each iteration must be less than or equal 10^{-6} to convergence and stability of the solution.

RESULTS AND DISCUSSION:

In order to understand the flow field and heat transfer characteristics of this problem, a total of 132 cases were considered, 90 cases for symmetrical wavy vertical walls and 42 cases for unsymmetrical wavy vertical walls. Rayleigh number was varied from 10^0 to 10^6 and number of undulation changed from $n=1$ to 3 for asymmetrical case and $n=1$ to 4 for unsymmetrical case. Wave amplitude changed from 0 to 0.1 for all cases. The flow is air considered, and the results show the streamline, velocity vector, and isothermal contour moreover to local Nusselt number distribution on the wavy walls.



Flow and Thermal Fields

For symmetrical cases; **Fig. 4, and 5** show the flow and thermal field for streamline, velocity vector, and isothermal contour for Ra equal 10^5 with two value of wave amplitude $\lambda=0.05$ and 0.1 at three cases of number of undulation $n=1, 2,$ and 3 . From the result, the flow heated from bottom wall and moves up near the vertical midline of the cavity. Then the flow impinges near the middle of the top wall and moves horizontally to ward corners losing heat to the top wall. Finally it descends along the cold wavy sidewalls. The important processes occur here is the combination between undulation and convection strength. From the figures we could notes that increase of wave amplitude leads to more deformation in flow behavior and thermal fields and increase the velocity in the middle of the cavity.

The same process we could notes in **Fig. 6, and 7** but the intensity of the recirculation pattern increase with increase Ra (10^6) because the convection becomes stronger. Also the deformation in flow and thermal field will become very clear with increase the value of Ra. For Ra 10^3 and 10^4 the isothermal contours are distributed uniformly in the cavity because the convection is weak in this case. On the other hand the isothermal contour are swirl at Ra= 10^5 and 10^6 due to influence of increased convection current.

For unsymmetrical cases; **Fig. 8, and 9** show the flow and thermal field for streamline, velocity vector, and isothermal contour for Ra 10^5 and 10^6 at different configuration of undulation in left and right wavy vertical walls at fixed wavy amplitude $\lambda=0.05$. The flow behavior deform in left wall more than right wall because the different of number of undulation in both walls. The recirculating flow becomes stronger in **Fig. 9** according to increase the value of Ra, and undulation.

Local Nusselt Number

Fig. 10, 11, and 12 show the variation of local Nusselt number along wavy vertical walls at different values of Ra, number of undulations, and wave amplitude. From **Fig. 10**, for law Ra the max. Nu_1 is located near the bottom heated wall, if Ra is increased, the location gradually rises up because of the increasing convection strength. Increase the values of number of undulation lead to decrease of max. Nu_1 on the left and right walls. Moreover the distribution will becomes more fluctuation with the wavy surface. **Fig. 11** show the variation of Nu_1 along the wavy walls for different wave amplitude value at Ra= 10^5 . For $n=1$ and $\lambda \leq 0.75$ it is observed that there is not much variation in the max. Nu_1 , but for $\lambda=0.1$ there is a change in value and locations of max. Nu_1 . For $n=2,$ and 3 the variation in Nu_1 will become more wavy according to the wavy surface.

For unsymmetrical case the **Fig. 12** described the variation of Nu_1 along wavy walls. Nu_1 distribution in the left wavy wall could be changed more than in the right wall according to the number of undulations For small Ra, max. Nu_1 in the left wavy wall increase with increase the NL/NR ratio. If Ra increases leads that to decrease the value of max. Nu_1 according to that ratio. Max. Nu_1 in the right wall will is have small effect with the ratio NL/NR, but it romaine with the same behavior of the left wall.

Average Nusselt Number

For Ra $\leq 10^5$, increase the value of number of undulation leads that to decrease the value of Nu_{ave} in both side walls, but in Ra = 10^6 the Nu_{ave} behavior is not clear as shown in **Table 3**. Also increase the wave amplitude leads that to decrease of Nu_{ave} too. The same sequence repeated for unsymmetrical case at $\lambda=0.05$ and $\lambda=0.1$ but in $\lambda=0.1$ and Ra $\geq 10^5$, the change in the value of Nu_{ave} will become large as shown in **Table 4**.

Table 3 Average Nusselt number on the right and left wavy walls (symmetrical case) in a cavity with $\lambda = 0.05$ and $\lambda = 0.1$.

NL	NR	$\lambda = 0.05$	Rayleigh number				NL	NR	$\lambda = 0.1$	Rayleigh number			
			10^3	10^4	10^5	10^6				10^3	10^4	10^5	10^6
0	0	R.W	-0.567	-0.616	-1.030	-1.780	0	0	R.W	-0.567	-0.616	-1.030	-1.780
1	1	R.W	-0.631	-0.733	-1.050	-1.670	1	1	R.W	-0.681	-0.772	-1.440	-1.727
1	1	L.W	-0.631	-0.733	-1.050	-1.660	1	1	L.W	-0.681	-0.772	-1.090	-1.730
2	2	R.W	-0.689	-0.754	-1.115	-1.560	2	2	R.W	-0.830	-0.858	-1.174	-1.710
2	2	L.W	-0.689	-0.754	-1.113	-1.550	2	2	L.W	-0.831	-0.859	-1.175	-2.150
3	3	R.W	-0.747	-0.818	-1.115	-1.700	3	3	R.W	-0.995	-1.020	-1.270	-1.820
3	3	L.W	-0.748	-0.819	-1.115	-1.700	3	3	L.W	-0.995	-1.020	-1.380	-1.980

Table 4 Average Nusselt number on the right and left wavy walls (unsymmetrical case) in a cavity with $\lambda = 0.05$ and $\lambda = 0.1$.

NL	NR	$\lambda = 0.05$	Rayleigh number				NL	NR	$\lambda = 0.1$	Rayleigh number			
			10^3	10^4	10^5	10^6				10^3	10^4	10^5	10^6
0	0	R.W	-0.567	-0.616	-1.030	-1.780	0	0	R.W	-0.567	-0.616	-1.030	-1.780
2	1	R.W	-0.637	-0.773	-1.030	-1.600	2	1	R.W	-0.689	-0.802	-1.043	-2.563
2	1	L.W	-0.680	-0.714	-1.138	-1.630	2	1	L.W	-0.797	-0.792	-1.248	-1.440
3	1	R.W	-0.641	-0.756	-1.027	-1.749	3	1	R.W	-0.689	-0.792	-0.856	-1.860
3	1	L.W	-0.729	-0.786	-1.197	-1.625	3	1	L.W	-0.922	-0.929	-1.373	-1.678
4	1	R.W	-0.635	-0.746	-1.080	-1.728	4	1	R.W	-0.658	-0.761	-0.855	-1.898
4	1	L.W	-0.772	-0.838	-1.140	-1.759	4	1	L.W	-1.011	-1.037	-1.358	-2.194

NNR Distribution

Fig. 13 shows NNR distribution with Ra. The results show that for small Ra, NNR is increase with increase the number of undulation but it will become reduction in its value if Ra is increase. Also if wave amplitude increases that leads to increase the NNR too. This relation also studied for unsymmetrical case as shown in **Fig. 14** For left wavy wall and higher ratio of NL/NR, NNR increase with increase the number of undulation to $Ra=10^4$ and will be decrease. But for small ratio the relations decrease from the first. For right wavy wall the same sequence repeated with non uniformly distributed in the figure.

CONCLUSIONS

Natural convection heat transfer in a cavity with wavy vertical walls has been analyzes. The effects of wave amplitude, number of undulation, Rayleigh number for two cases, symmetrical and unsymmetrical wavy walls have been studied in detail. The results show that, for symmetrical case increase of number of undulation leads to decrease of $\max. Nu_1, Nu_{ave}$ and increase the NNR factor. While increase the wave amplitude leads to wavy distributed of Nu_1 and decrease the value of Nu_{ave} to $Ra \leq 10^5$ with increase of NNR factor. Also increasing Ra the convection strength become stronger and leads to decrease of $\max. Nu_1, Nu_{ave}$ and increase of NNR. The same sequence repeated for unsymmetrical case with more deformation in the flow and thermal field, also increase the Ra leads to increase the $\max. Nu_1$ in this case.

**NOMENCLATURE:**

SYMBOLS	TITLES	UNITS
B	Thermal expansion coefficient	1/k
H	Height of the cavity	m
J	Jacobian	----
L	Length of the cavity	m
L.W	Left wavy wall	----
n	Number of undulation	----
NL	Number of undulation in left wavy wall	----
NR	Number of undulation in right wavy wall	----
Nu	Nusslet number	----
NNR	Nusslet number ratio	----
P	Dimensionless pressure	----
Pr	Prandtl number (ν/α_f)	----
Ra	Rayleigh number ($g.B.(T_h-T_c).L^3/\nu\alpha_f$)	----
R.W	Right wavy wall	----
S	Source term	----
T	Dimensionless temperature	----
u,v	Dimensionless Cartesian velocity components in x, and y coordinates	----
U,V	Dimensionless contravariant velocity components in ξ , and η coordinates	----
x, y	Dimensionless Cartesian coordinates	----

GREEK SYMBOLS

α_f	Thermal diffusivity of fluid	m ² /sec
α, β, γ	Transformation functions	----
ξ, η	Dimensionless curvilinear coordinates	----
λ	Wave amplitude	----
ν	Kinematics viscosity	m ² /sec
Φ	General variables representing U, V, and T.	----

SUBSCRIPTS

ave	Average
c	Cold wall
h	Hot wall
l	Local
max	Maximum
x, y	Derivative relative to x, y, ξ, η respectively.
ξ, η	

SUPERSCRIPT

- Dimensional form

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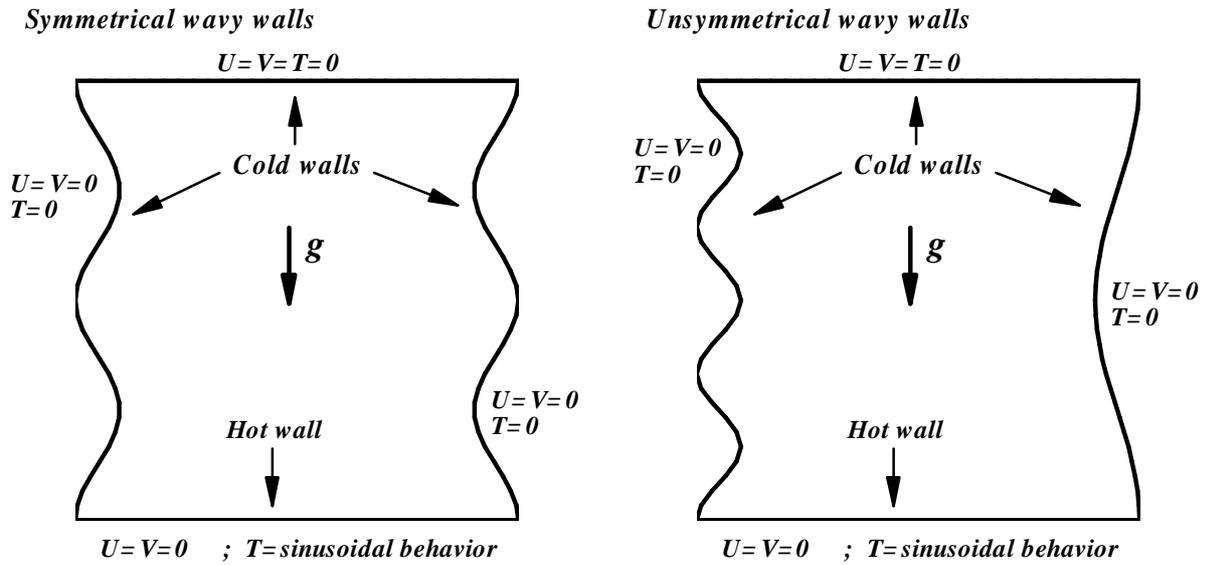


Fig. 1 Symbol Case of Geometrical Description and Boundary Conditions Applied in this study for $\lambda = 0.05$
 Left: Symmetrical wavy walls (NL=NR=2) - Right: unsymmetrical wavy walls (NL=3, NR=1)

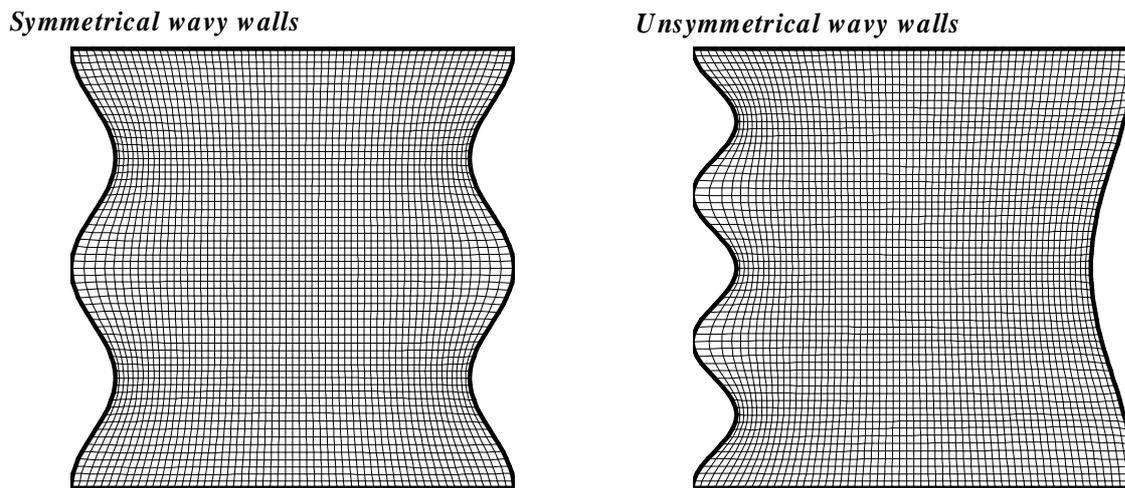


Fig. 2 Grid Generation Applied in this study for Two Cases.

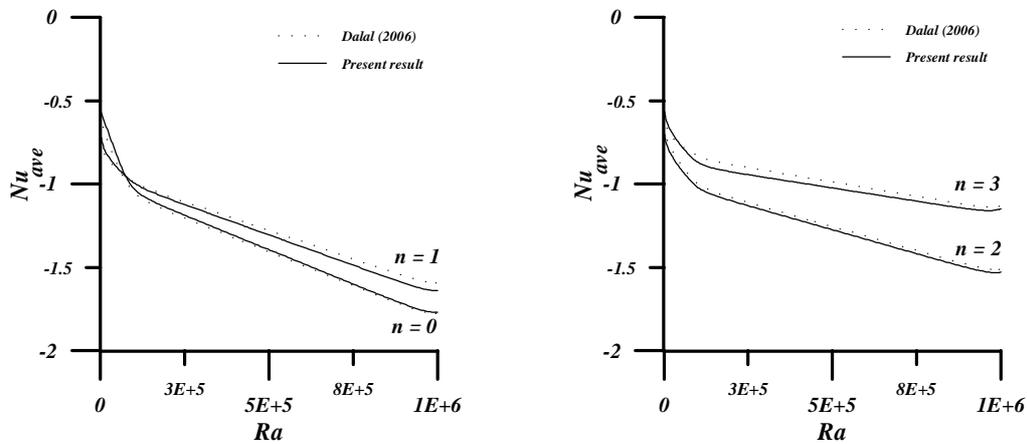


Fig. 3 Comparison of the Predicted Average Nusselt Number on the Right Wavy Wall Only with

Results of Dalal, and Das (2006) at Wave Amplitude $\lambda=0.05$.

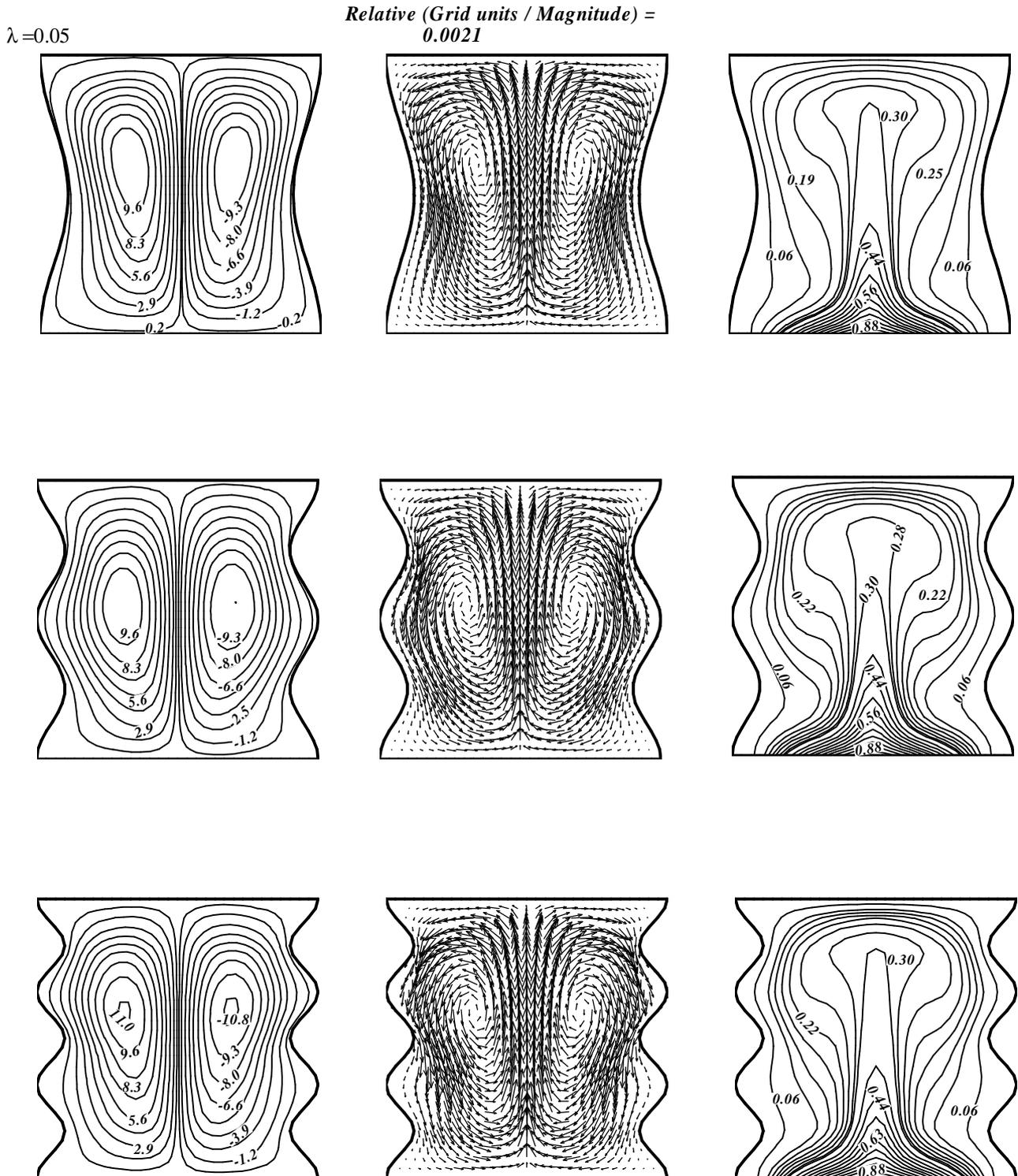


Fig. 4 Streamline, Velocity Vector and Isotherm Contour for $Ra = 10^5$ in Symmetrical Case for Number of Undulation Equal 1, 2, and 3 Respectively, at Wave Amplitude 0.05.



$\lambda = 0.1$

Relative(Grid units/Magnitude)=
0.0023

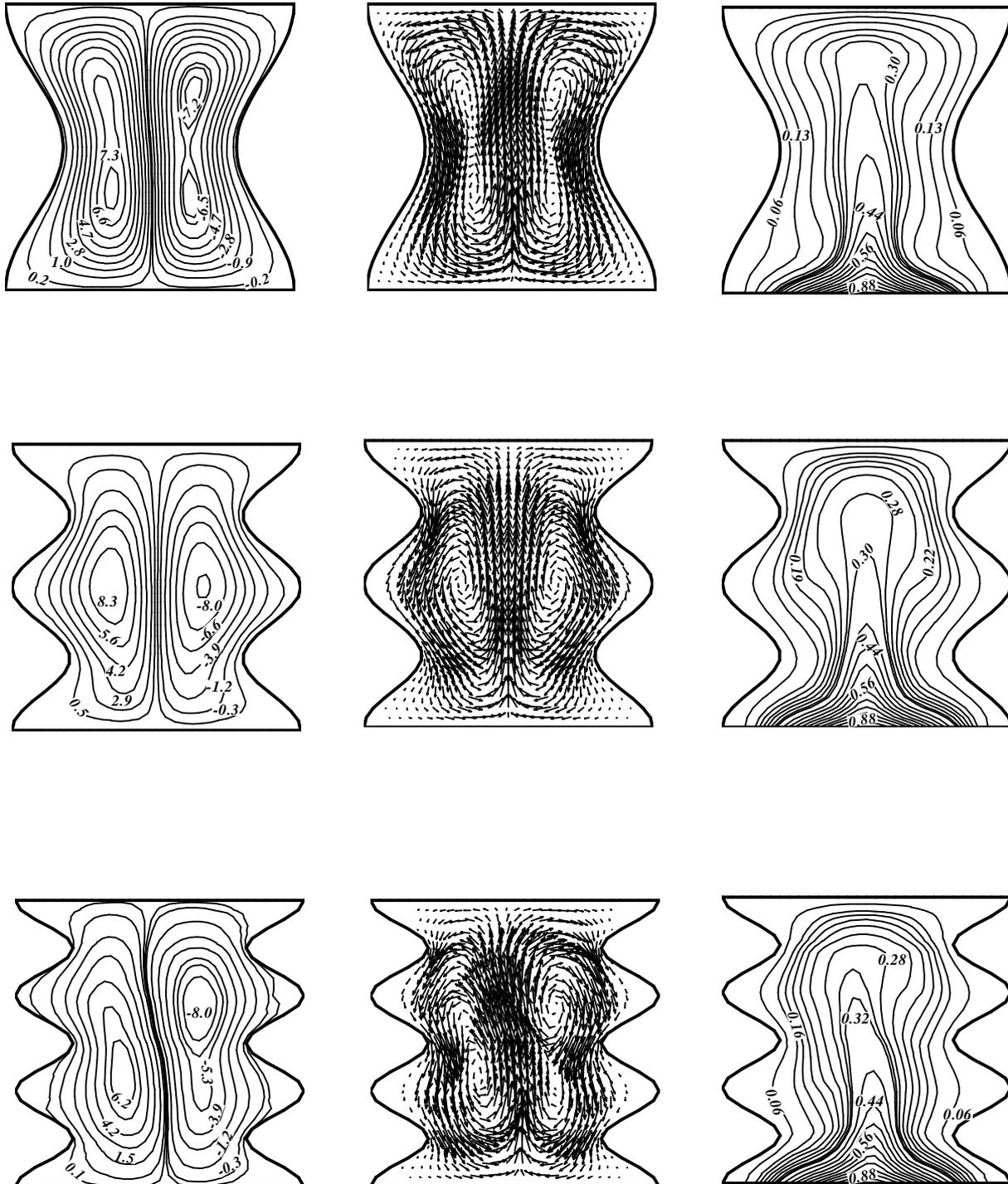


Fig. 5 Streamline, Velocity Vector and Isotherm Contour for $Ra = 10^5$ in Symmetrical Case for Number of Undulation Equal 1, 2, and 3 Respectively, at Wave Amplitude 0.1.

$\lambda = 0.05$

Relative (Grid units / Magnitude) = 0.0005

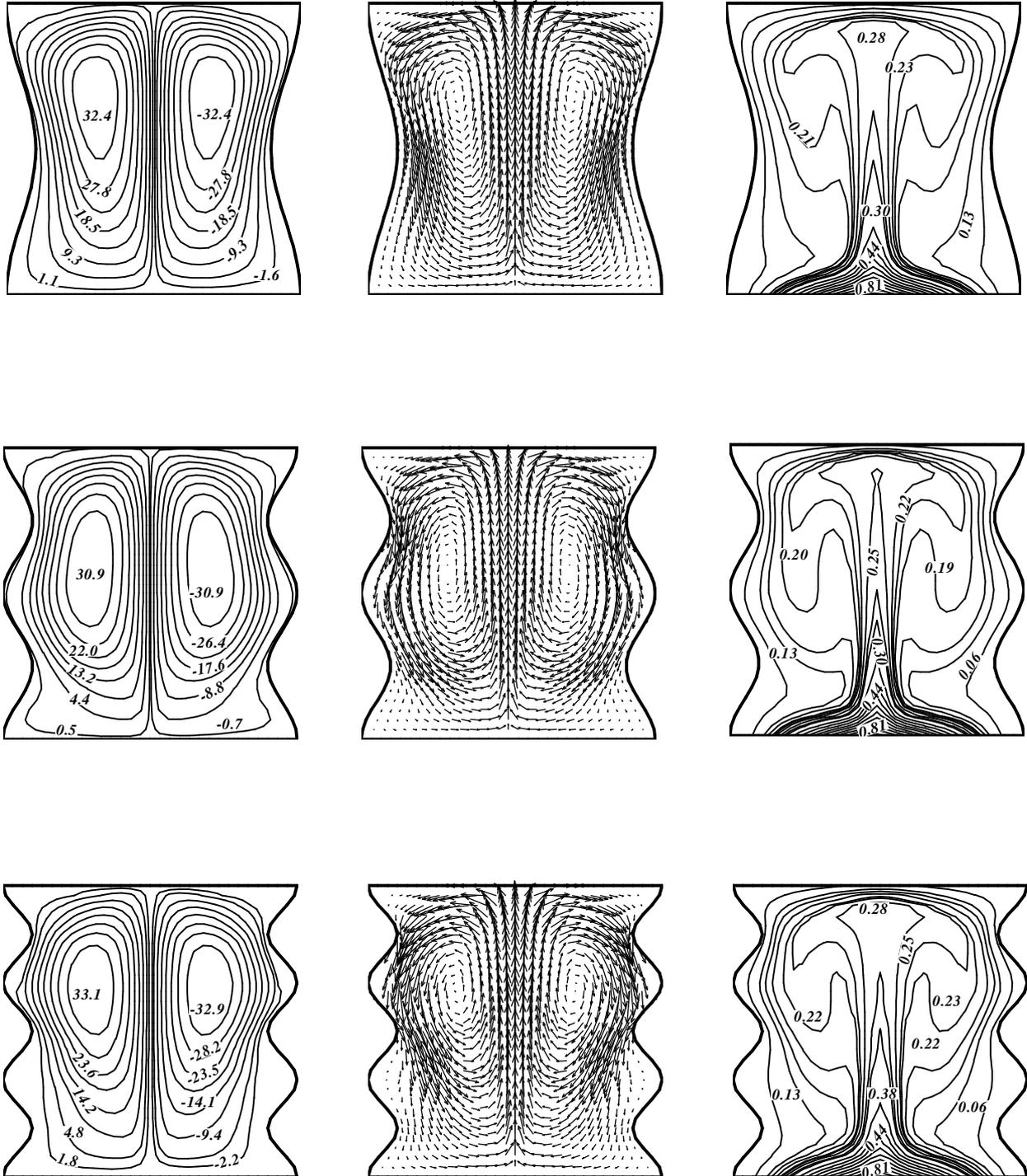


Fig. 6 Streamline, Velocity Vector and Isotherm Contour for $Ra = 10^6$ in Symmetrical Case and Number of Undulation Equal 1, 2, and 3 Respectively, at Wave Amplitude 0.05.



$\lambda = 0.1$

Relative (Grid units / Magnitude) = 0.0005

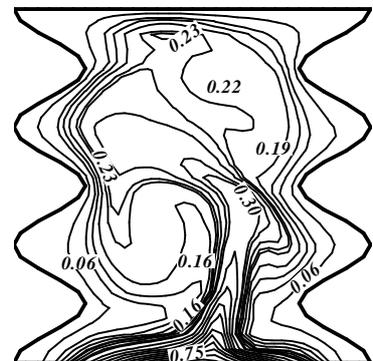
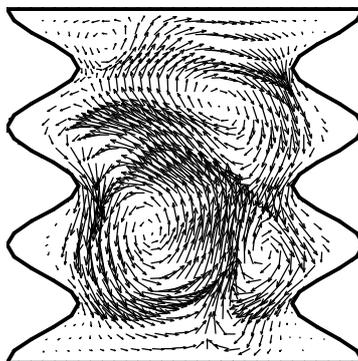
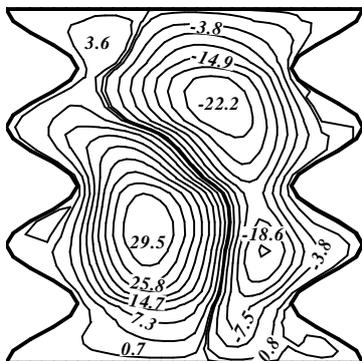
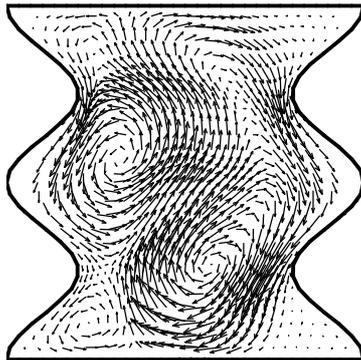
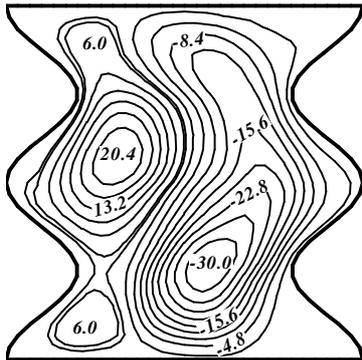
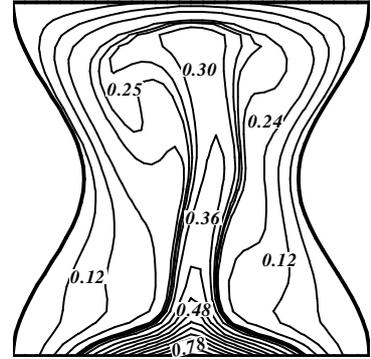
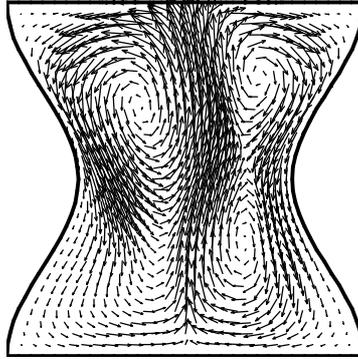
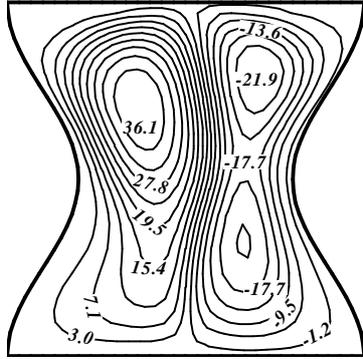


Fig. 7 Streamline, Velocity Vector and Isotherm Contour for $Ra = 10^6$ in Symmetrical Case and Number of Undulation Equal 1, 2, and 3 Respectively, at Wave Amplitude 0.1.

$\lambda = 0.05$ (NL = 2, NR = 1)

lative (Grid units / Magnitude) = 0.0025

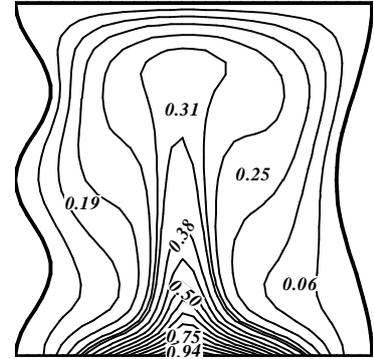
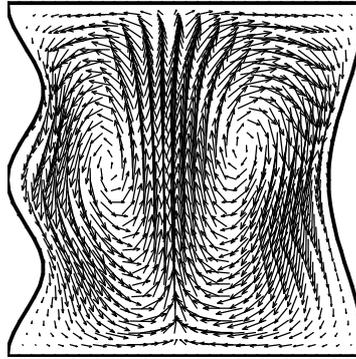
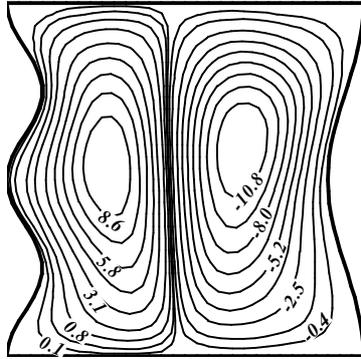
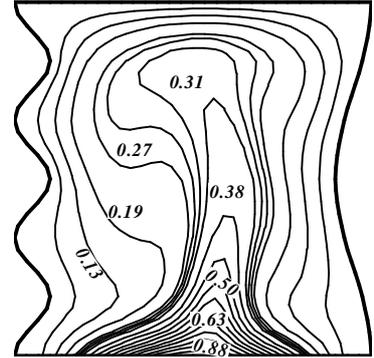
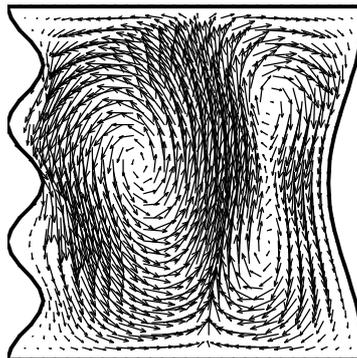
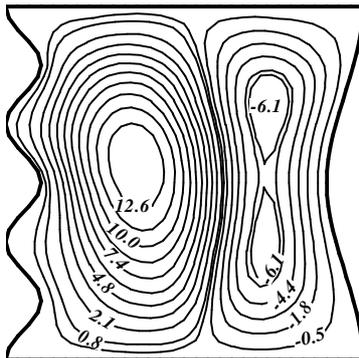


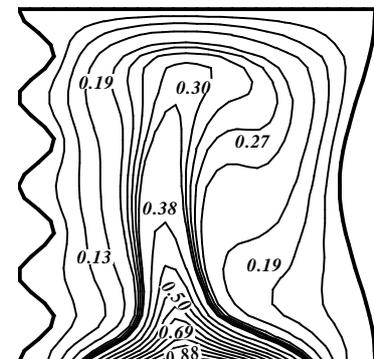
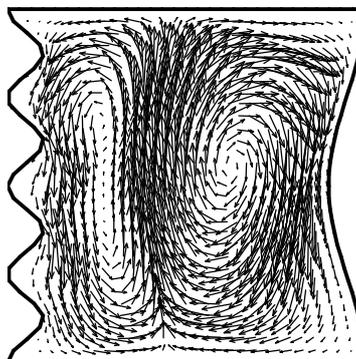
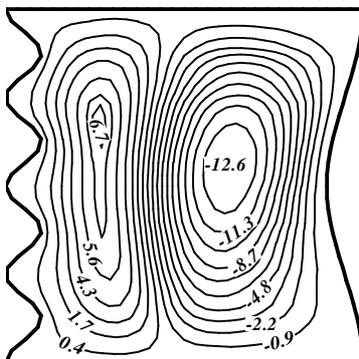
Fig. 8 Streamline, Velocity

Vector and

$\lambda = 0.05$ (NL = 3, NR = 1)



$\lambda = 0.05$ (NL = 4, NR = 1)

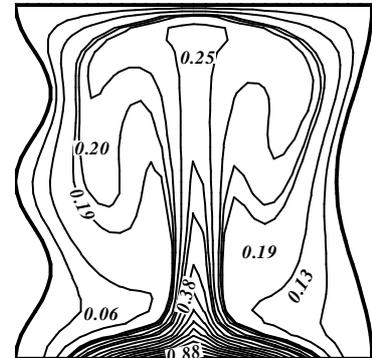
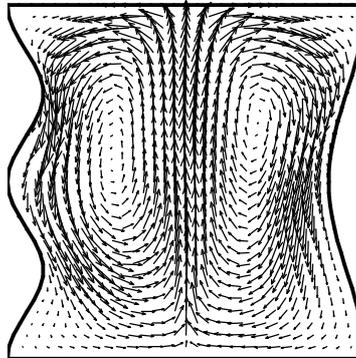
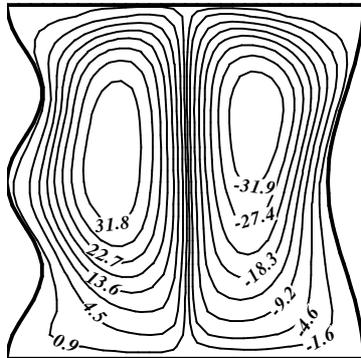


Isotherm Contour for $Ra = 10^5$ in Unsymmetrical Case for NL=2, 3, and 4, respectively and NR=1 at Wave Amplitude 0.05.

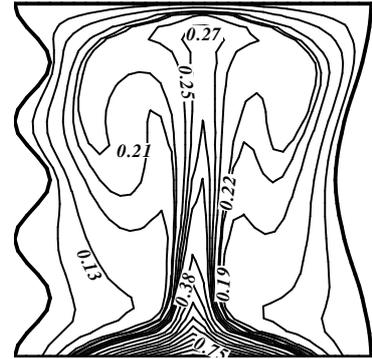
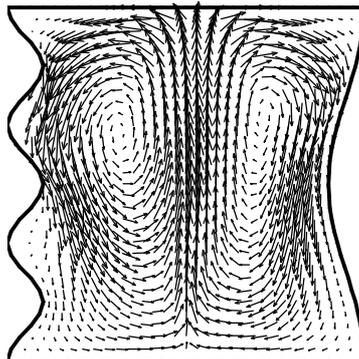
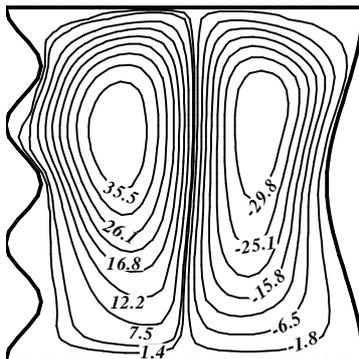


$\lambda = 0.05$ (NL = 2 , NR = 1)

Relative (Grid units / Magnitude) = 0.0005



$\lambda = 0.05$ (NL = 3 , NR = 1)



$\lambda = 0.05$ (NL = 4 , NR = 1)

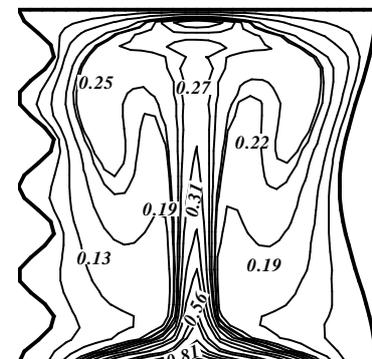
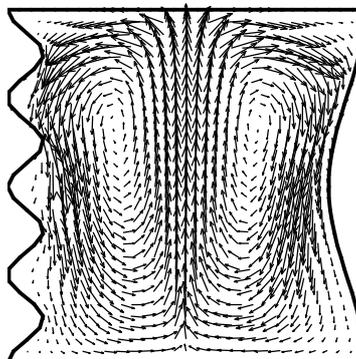
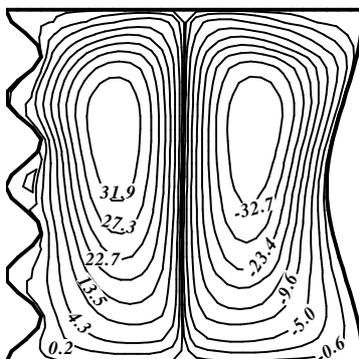


Fig. 9 Streamline, Velocity Vector and Isotherm Contour for $Ra = 10^6$ in Unsymmetrical Case for NL=2, 3, and 4, respectively and NR=1 at Wave Amplitude 0.05.

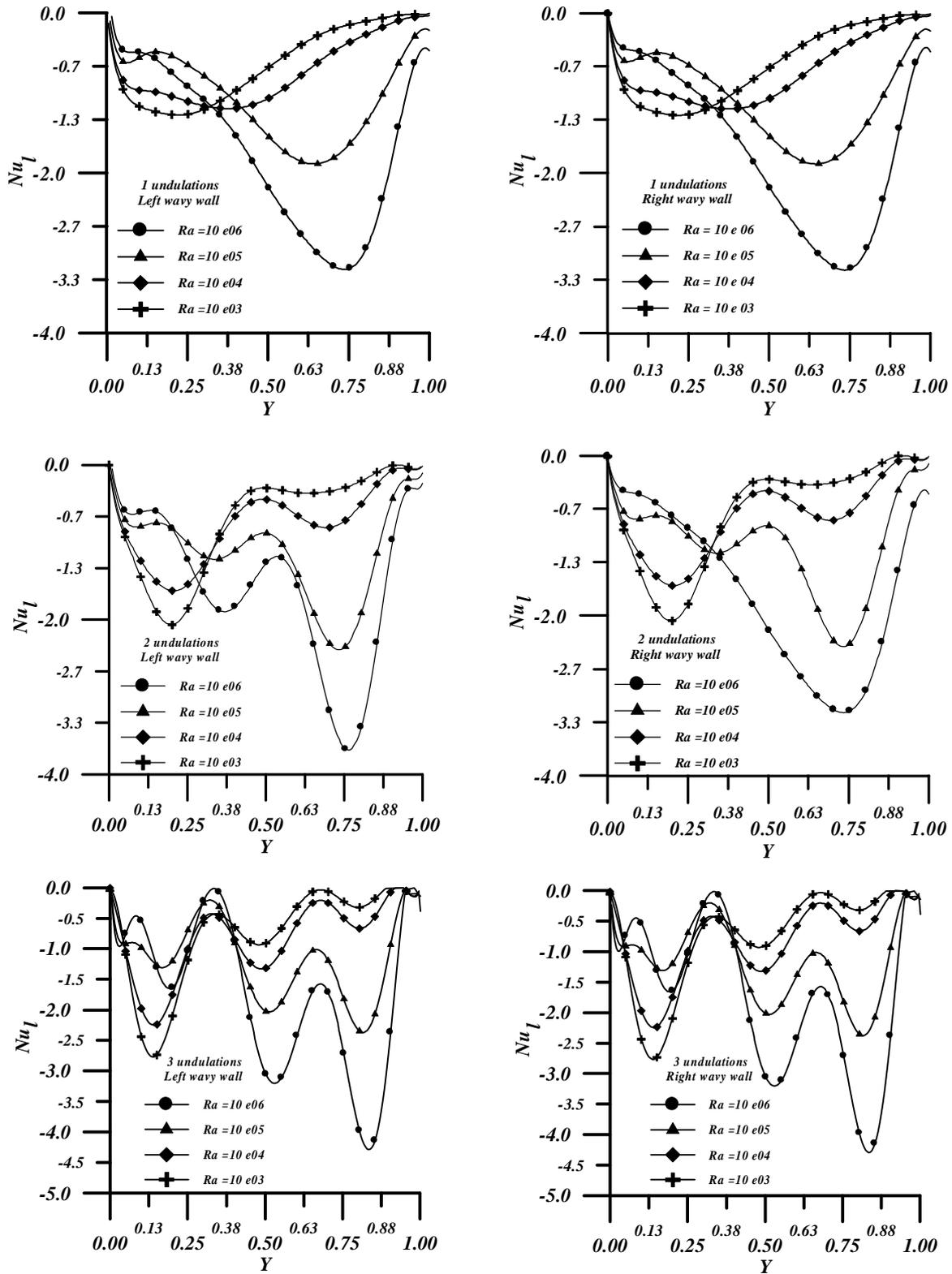


Fig. 10 Local Nusselt Number Distribution in Symmetrical Case and Number of Undulation Equal 1, 2, and 3 Respectively, at Wave Amplitude 0.05.

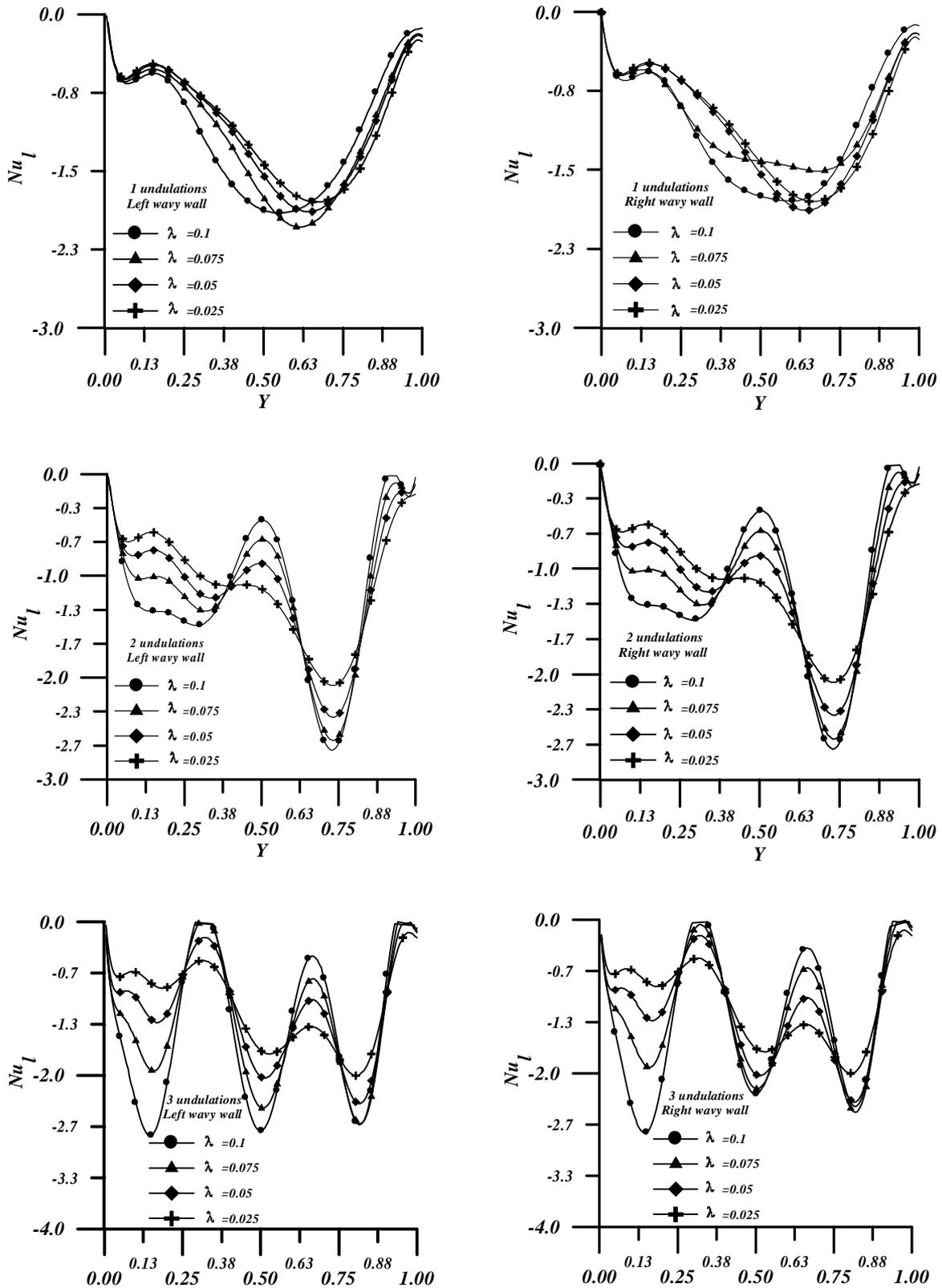


Fig. 11 Local Nusselt Number Distribution in Symmetrical Case at $Ra = 10^5$ and Number of Undulation Equal 1, 2, and 3 Respectively for Different value of Wave Amplitude.

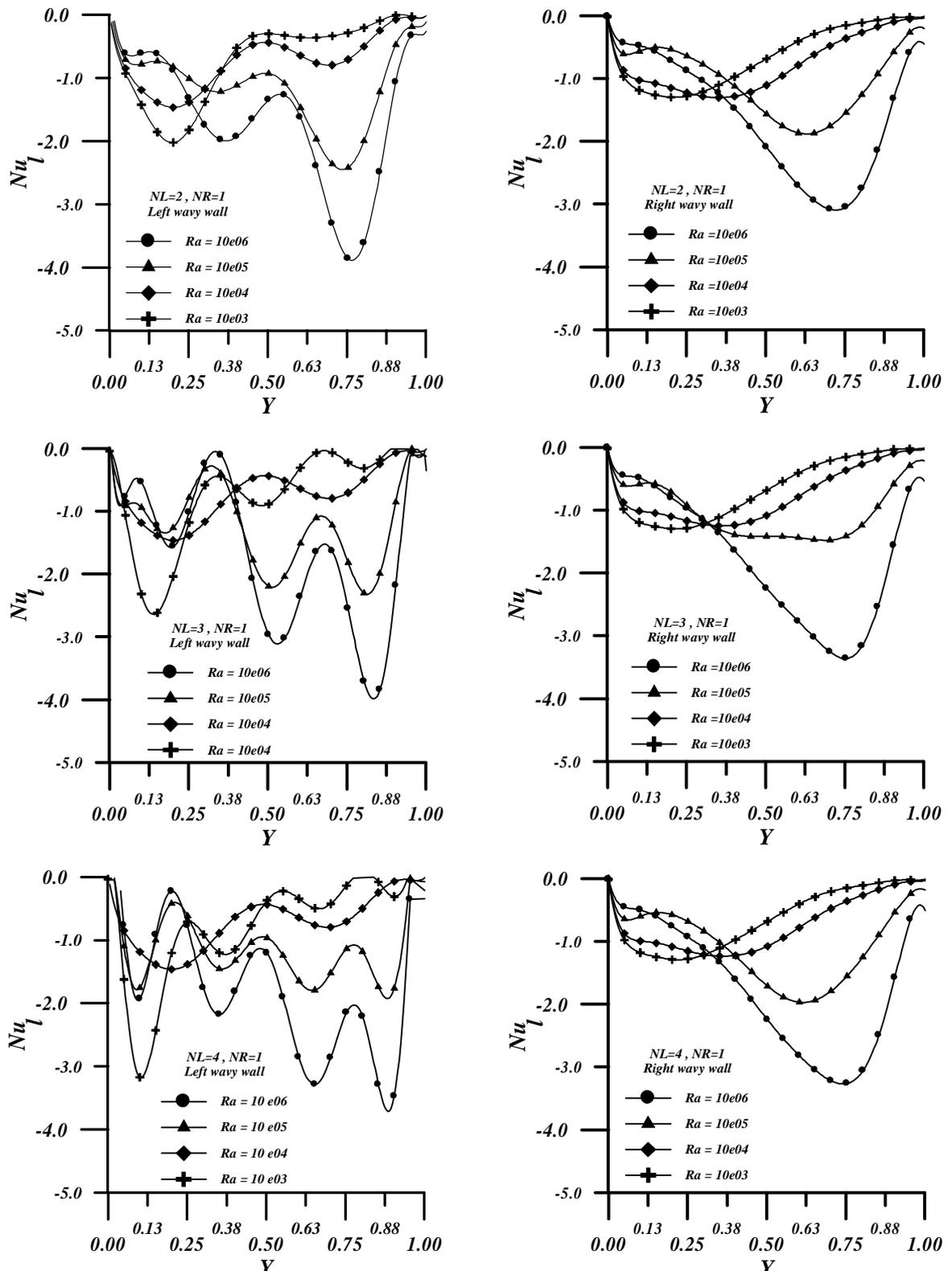


Fig. 12 Local Nusselt Number Distribution in Unsymmetrical Case for Number of Undulation NL= 2, 3, and 4, NR=1, at Wave Amplitude 0.05.

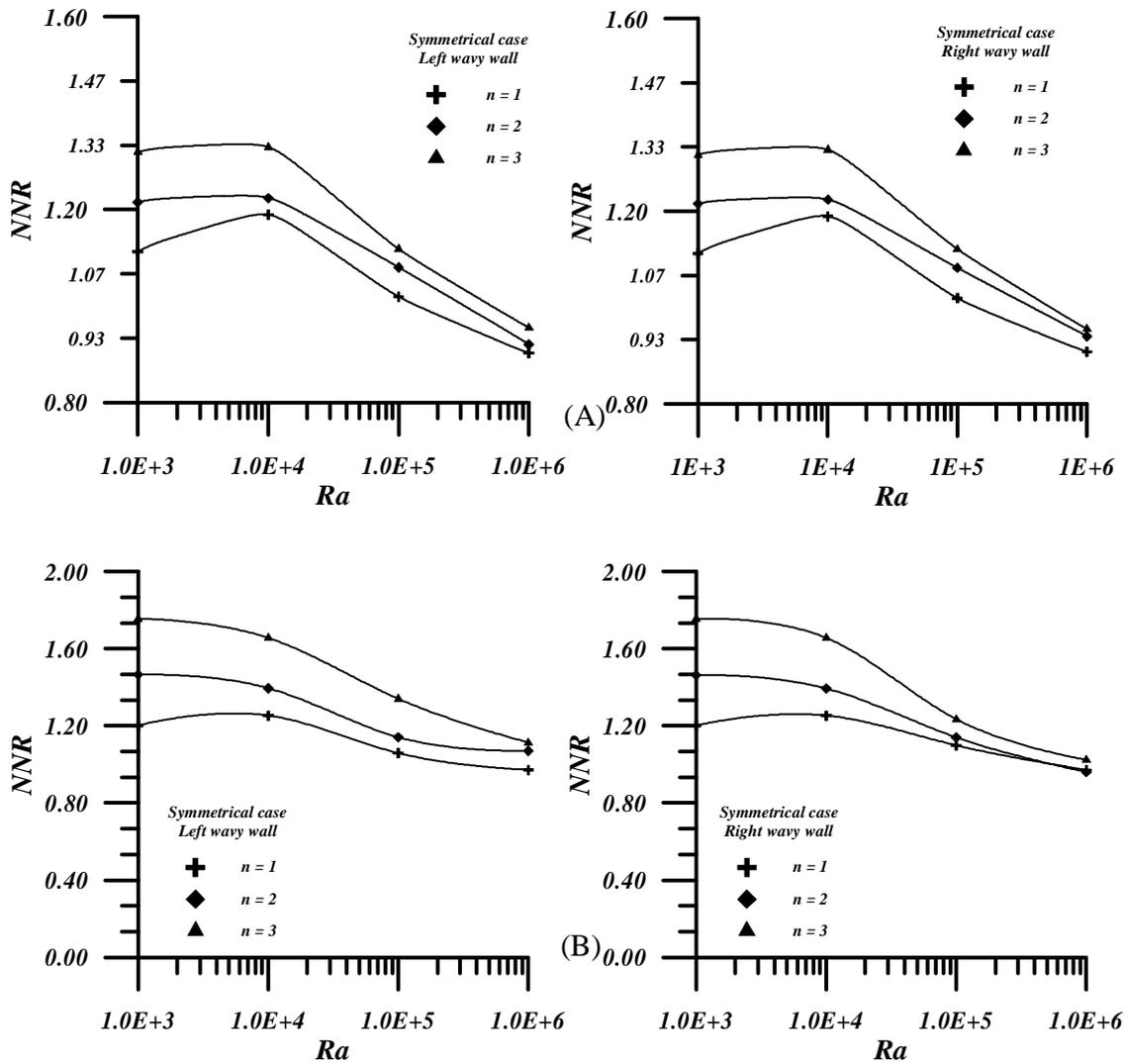


Fig. 13 NNR Distribution in Symmetrical Case at Number of Undulation $n=1, 2,$ and 3 and Wave Amplitude A. $\lambda = 0.05$, B. $\lambda = 0.1$.

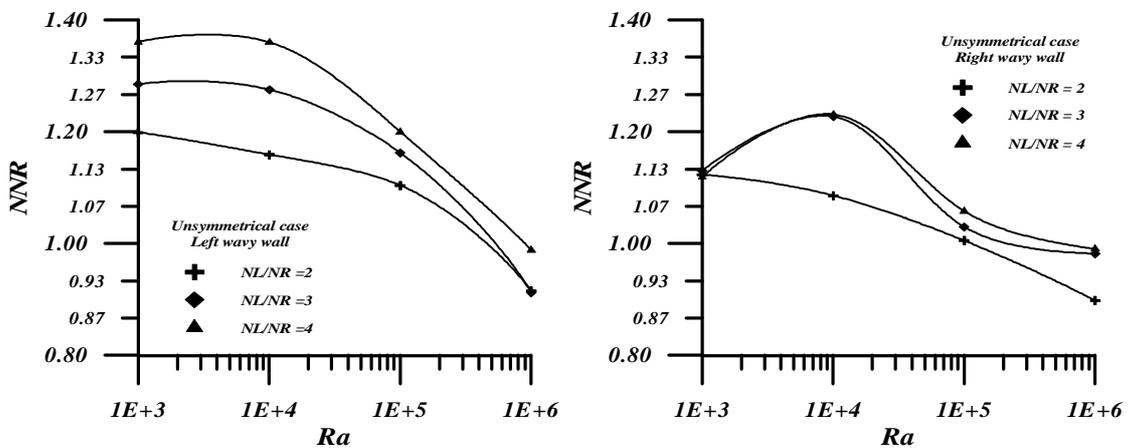


Fig. 14 NNR Distribution in Unsymmetrical Case and at Wave Amplitude $\lambda = 0.05$.