Analysis and Optimum Design of Self Supporting Steel Communication Tower

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ABSTRACT

The present study deals with the optimum design of self supporting steel communication towers. A special technique is used to represent the tower as an equivalent hollow tapered beam with variable cross section. Then this method is employed to find the best layout of the tower among pre-specified configurations. The formulation of the problem is applied to four types of tower layout with K and X brace, with equal and unequal panels. The objective function is the total weight of the tower. The variables are the base and the top dimensions, the number of panels for the tower and member's cross section areas. The formulations of design constraints are based on the requirements of EIA and ANSI codes for allowable stresses in the members and the allowable displacement at antenna position. The Sequential Unconstrained Minimization Technique (SUMT) is used to perform the process.
Direct stiffness method is used for the analysis of the structure, with beam elements. The strain energy is used to derive the stiffness matrix for members of unsymmetrical cross section. A computer program in FORTRAN is developed to represent the tower as an equivalent beam, and generate the tower nodes and members, analysis, design and to find the optimum design.
Four types of tower are studied with different load cases. The effects of earthquake and wind loadings are taken in two directions and two positions of antenna are considered in the process to seek the optimum design. The tower type of X-brace with unequal panels has the minimum weight compared with other types of tower and the optimum design is satisfied when the angle of main leg is equal to (87^{O}).

KEYWORDS: Angle, Beam, Brace, Communication, Design, Optimum, Self supporting, Steel, SUMT, Tower, Wind.

التحليل والتصميم الامثل لابراج الاتصالات الحديدية المسندة تلقائيا

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الخلاصة

يتعلق البحث بدراسة التحليل والتصميم الامثل لابراج الاتصالات الحديدية المسندة تلقائيا. تم تمثيل البرج بعتب مكافئ وناتئ ومجوف ومتغير المقطع. تم ايجاد التصميم الامثل لهذا العتب واستغلال هذا التصميم لايجاد الابعاد الخارجية للبرج وعدد الخانات. اختيرت اربعة انواع من التشكيل والربط لاعضاء البرج هي (K and X brace) بارتفاع خانات متساوية واخرى غير متساوية.

دالة الهدف هي الوزن الكلي للبرج ومتغيرات التصميم هي ابعاد القاعدة السفلى والعليا للبرج وعدد الخانات وابعاد مقاطع الاعضاء الانشائية. محددات التصميم معتمدة على مقدار الازاحات المسموح بها عند مواقع المرسلات والمستقبلات وحدود الاجهادات المسموح بها لاعضاء البرج وفقا للمعادلات التصميمية الخاصة بالمعهد الامريكي للابراج الحديدية (ANSI CODE) ومواصفة جمعية الصناعات الالكترونية (EIA). استخدمت طريقة البرمجة غير المتعاقبة وغير المقيدة (SUMT) لايجاد التصميم الامثل.

استخدمت طريقة الصلادة في التحليل الانشائي للبرج واستعمال طاقة الاجهاد لاشتقاق مصفوفة الصلادة لاعضاء انشائية ذات مقطع غير متناظر حول المحاور الرئيسية. تم كتابة برنامج بلغة فورتران ليقوم بتمثيل البرج بعتب ثم ايجاد التصميم الامثل للابعاد الخارجية واستحداث ملف يحتوي البيانات اللازمة للتحليل واخيرا يقوم البرنامج بايجاد المقاطع اللازمة للاعضاء الانشائية.
تمت دراسة الانواع الاربعة من الابراج لحالات تحميل مختلفة وهي تأثير الهزة الارضية وتأثير الرياح بزاويتين مختلفتين ولمواقع مرسلات ومستقبلات مختلفة. لقد وجد ان اقل وزن للبرج يتحقق بتشكيل ربط من نوع (X brace) ولارتفاع خانات غير متساوية. كما وجد ايضا ان التصميم الامثل يتحقق عندما تكون زاوية ميل البرج مساوية الى (87) درجة.

الكلمات الرئيسية: برج، زاوية، عتب، ربط، اتصالات، تصميم، امثل، مسند تلقائيا، حديد، البرمجة غير المتعاقبة وغير المقيدة.

1. STRUCTURAL ANALYSIS

The structure of a communication tower consists of a large number of straight steel bars with constant cross section. Here the cross sections of the bars are equal leg angles. The self supporting communication tower is a large latticed steel structure and it should be analyzed as an indeterminate space structure.

In this work, the analysis of the tower is made by using the linear elastic and standard stiffness method. For each member the equilibrium equations can be represented by the form:

$$\{P\}= [K]\{U\} \quad (1)$$

Where
{P} is the applied nodal load vector, which is for the space beam element
$\{P\}^T=\{F_{xl}\ F_{y1}\ F_{z1}\ M_{x1}\ M_{y1}\ M_{z1}\ F_{x2}\ F_{y2}\ F_{z2}\ M_{x2}\ M_{y2}\ M_{z2}\}$
[K] is the stiffness matrix of the member.
{U} is the displacement vector, which is for beam element
$\{U\}^T= [u_1\ v_1\ w_1\ \theta_{x1}\ \theta_{y1}\ \theta_{z1}\ u_2\ v_2\ w_2\ \theta_{x2}\ \theta_{y2}\ \theta_{z2}]$

By assembling these equations for all members, the whole structure equilibrium equations are obtained. The equations can be solved for the unknown displacements, from which the internal forces can be obtained. A computer program is used to solve these equations.

1.1 Three-Dimensional Straight Beam Element

A three dimensional prismatic beam element is considered, the beam is subjected to a general system of transverse loads and it is assumed to be linearly elastic. The element orientation with respect to local coordinate system and the nomenclature are illustrated in Fig.(1).

The local *x-y-z* axes coincide with the centroidal axis of the element. Positive signs are in the directions indicated. The element has 12 degrees of freedom, six at each node. The forces and displacements at the two nodes are taken to be positive if their vectors points are in the directions of coordinates. The right-hand rule is used for moments and rotational displacements. Since the cross section of equal leg angles is not bisymmetrical the shear center dose not coincide with the centroid and consequently the stiffness matrix is somewhat different.

1.2 Effect of Axial, Torsion and Bending

The total strain energy for axial, torsion and bending about two centroidal axes is:

$$U = \int_0^L \{[\frac{1}{2E}\left(\frac{Fx}{A}+\frac{MyZ}{Iy}+\frac{MzY}{Iz}\right)^2+\frac{Mx^2}{2Gj}]\}dx \quad (2)$$

The relationships between internal forces and the displacements of a point on the centroid of an unsymmetrical cross section of a straight member as given by Oden [1] are:

$F_x = EA\, u'$ (3)

$M_y = -EI_{yz}\, v'' - EI_y\, w''$ (4)

$M_z = EI_z\, v'' + EI_{yz}\, w''$ (5)

$M_x = GJ\, \theta_x / L$ (6)

From the above equations and by neglecting the small terms, the total bending strain energy U_b can be obtained as:

$$U_b = \frac{E}{2}\int_0^L (Iz.v" + Iy.w"^2 + 2Iyz.w".v")\, dx \quad (7)$$

where: Fx is the axial force, u is the axial deformation in the direction of x, My and Mz are the bending moments about y and z axes, v and w are the displacements in the directions of the principal axes y and z respectively, Mx is the torque, θx is the angle of rotation about x-axes, A is the cross sectional area, Iy and Iz are the moments of inertia, Iyz, is the product of inertia G is the modulus of rigidity and E is the modulus of elasticity, and Ix or J is the torsional constant which can be obtained as follow:
For thin-walled open sections $J = \frac{1}{3}\sum b.t^3$ and for equal leg angle with cross section as shown in Fig.(2), where $J = \frac{2bt^3}{3}$ (approximately). The effect of each of these forces, which are axial, bending and torsion, is each separated from the others (i.e. uncoupled) when the stiffness coefficients are derived.

1.3 Shear Deformations

The transverse shearing stress makes the cross section to warp in the longitudinal direction. The shear deformation is neglected because it is usually very small compared with those deformations due to bending. The convential beam theory is employed which assumes that plane sections remain plane in flexure, except in deep beams that have a usually large depth to span ratio (greater than 0.2) [1]. Due to the fact that all members in the steel tower are not deep beams so the shear deformation is neglected

1.4 Warp Effect

Torsional moments may cause longitudinal displacements resulting from the out of plane warping of the cross section. Torsional moments may cause longitudinal displacements resulting from the out of plane warping of the cross section.

The equal leg angles twist without warping [1,2,3]. The shear center of the angle lays at the intersection of the centerlines of the thin rectangular strips. Because of this, the resultant of shear flows produced by any type of loading must pass through this point, hence no warping torque can be developed and no longitudinal stresses are produced by torsional loads.

But, when the wall thickness t, is not extremely small compared with the other dimensions, a secondary stress system can be developed perpendicular to the contour line of the section. Normal stresses then vary linearly over *(t)* and secondary-warping torque is developed.

However the largest value of *thickness/width ratio* for cross section of a commercial angle is close to (0.2), and this makes all effects from warping removed [2,3].

1.5 Complete Element Stiffness Matrix

The element stiffness matrix is found by using the approach of strain energy in terms of the nodal displacement of the element. By using Eq.7, the complete beam element stiffness matrix *[K]* in local coordinate system with 12 degrees of freedom, is derived as presented in Eq.8.

To obtain the stiffness matrix for the entire structure, the usual transformation of each individual member stiffness matrix from the local to the global coordinates is needed.

1.6 Loading Cases

In general there are two types of loading on the tower, dead load and live load. The live load consists of wind load and earthquake load. The dead load and wind load are calculated according to (EIA) standards [4], the wind load can be divided into two groups:

(A): The wind load on the steel tower structure.
(B): The wind load on the antennas.

The full description of wind load calculation can be found in (EIA) Standard. While the earthquake loads are calculated according to Iraqi seismic code requirements for building (ISD) [5].
It is being noted that wind load is not considered in combination with seismic actions.

$$[K]=\begin{bmatrix} \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ & \frac{12EI_z}{L^3} & \frac{12EI_{yz}}{L^3} & 0 & \frac{-6EI_{yz}}{L^2} & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & \frac{-12EI_{yz}}{L^3} & 0 & \frac{-6EI_{yz}}{L^2} & \frac{6EI_z}{L^2} \\ & & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & \frac{6EI_{yz}}{L^2} & 0 & \frac{-12EI_{yz}}{L^3} & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & \frac{6EI_{yz}}{L^2} \\ & & & \frac{GI_x}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GI_x}{L} & 0 & 0 \\ & & & & \frac{4EI_y}{L} & \frac{-4EI_{yz}}{L} & 0 & \frac{6EI_z}{L^2} & \frac{6EI_{yz}}{L^2} & 0 & \frac{2EI_y}{L} & \frac{-2EI_{yz}}{L} \\ & & & & & \frac{4EI_z}{L} & 0 & \frac{-6EI_{yz}}{L^2} & \frac{-6EI_z}{L^2} & 0 & \frac{-2EI_{yz}}{L} & \frac{2EI_z}{L} \\ & & & & & & \frac{AE}{L} & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & \frac{12EI_z}{L^3} & \frac{12EI_{yz}}{L^3} & 0 & \frac{6EI_{yz}}{L^2} & \frac{-6EI_z}{L^2} \\ & & \text{Symm.} & & & & & & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & \frac{-6EI_{yz}}{L^2} \\ & & & & & & & & & \frac{GI_x}{L} & 0 & 0 \\ & & & & & & & & & & \frac{4EI_y}{L} & \frac{-4EI_{yz}}{L} \\ & & & & & & & & & & & \frac{4EL_z}{L} \end{bmatrix} \quad (8)$$

2 Formulation of optimum design

The purpose of this study is to develop an optimum design of a steel communication tower which is defined as the structure of least weight, subject to a prescribed set of constraints on the design and behavior variables. The problem is to design both the shape and the member sizes and locations. The problem can lead to mixed design variables, which, in turn, can have a wide range of sensitivities.
The optimum design of a tower consists of shape and layout of tower, and the areas of the cross sections of the members. The optimum shape of the tower includes the horizontal dimensions along the height of the tower and the number of panels and their height, where the configuration of the members is same at each panel.
The optimum layout of a tower can be obtained by removing some members and not allowing re-entering the design problem. The member's having areas reduced to zero are removed during the process of optimization, while joints are moved until an optimum geometry for the given structure of the tower is found.
In the case of removed members, there is no mathematical justification for the removal of these members and no proof exists that they would not subsequently help to reduce the weight of the structure. Furthermore, if the buckling stress constraint is critical for a particular member then its area would not reduce to zero in the design process and the member is not deleted.
While in the case of moving the joints, some reasonable initial geometry is specified and it is difficult to estimate the best number of panels in a tower, since the joint move is simple, always in horizontal dimension.
Most latticed towers may be built-up with angles at the corners and lacing in the faces as shown in Fig.(3). One alternative is to model the tower from three-dimensional truss or beam system to one or several equivalent beams is possible.
The equivalent beam to a latticed tower is a beam having properties which give the same deflected shape to the tower and same axial stress in leg members under the same load conditions.
The equivalent beam has the same base and top dimensions x_2 and x_1 respectively for the tower and the length of beam L represent the height of the tower. The equivalent beam and the tower have the same material properties.

2.1 Properties of Equivalent Beam

In this study approximation concepts are made for the design of the tower to find the optimum layout, by modeling the tower as a tapering thin hollow square cantilever, whose section varies continuously from one end to the other. The beam has constant ratio of width/thickness, as shown in Fig. (4).The Y-axis coincides with the centroidal axis of the beam.

The concept of constant CB ratio along the beam is due to the fact that the size of the member is reduced towards the top of the tower, also the dimension of the cross section of the tower.

Writing the ratio of $\frac{x1}{t1} = \frac{x2}{t2} = \frac{x(y)}{t} = CB$, then

$$x(y) = x_1(1 + R.y) \quad (9)$$

Where:

$$R = \frac{1}{L}\left(\frac{x2}{x1} - 1\right) \quad (10)$$

Then the area at any section $A = \frac{4x(y)^2}{CB}$, and the moment of inertia can be given as:

$$Ix(y) = Iz(y) = I_o(1 + B.y) \quad (11)$$

Where

I_0= moment of inertia at y= 0 (top of tower).

$$B = \frac{1}{L}\left(\frac{x_2^4}{x_1^4} - 1\right) \quad (12)$$

Every tower structure has a point of optimum economy, which depends primarily upon the dimensions at the base and top, and number of panels and number of members.

The ratio CB is an indicator to the total volume of material of members used in the tower. In addition to that, the number of panels affects on the value of CB, and this effect is different when the panels are equal or not equal (having an algorithm relationship for height of panel as shown in Fig.(5).

Writing TV be equal to the total volume of material in the actual tower ($\sum A_i l_i$) and the CT to represent the material volume ratio, thus

$$CT = \frac{2L(x_2^2 + x_1^2)}{TV} \quad (13)$$

Equation 13 is the same for any configuration system, for any number of panels of tower with square bases. Then CT for the equivalent beam can be represented in terms of CB:

$$CT = f(CB, n) \quad (14)$$

Where n is the number of Panels.

Equation 14 is different for equal or unequal panels and configuration system. It can be obtained by analyzing a number of examples for each type. Then,

$TV = f(CT, x_2, x_1)$

$TV = f(CB\ x_2, x_1, n)$

Also there is a relation between the axial force *FT* in leg members in the actual tower and the force calculated due to allowable stress at the bottom of the equivalent beam *FB*.

2.2 Displacements in an Equivalent Beam

The most important displacements in the tower are the axial (vertical) and transverse (horizontal) displacements at the top of the tower. The following sections explain the derivation of the displacements of the tapering hollow square beam:

2.2.1 Axial Displacement

Mainly the axial (vertical) displacement *(v)* is caused by the se1f-weight of the structure. The vertical displacement at any distance *(y)* is due to the weight of the segment from origin (top of the tower) to distance *(y)* and it can be given as follows:

$$fy = \frac{2\gamma s}{CB}\left(x_1^2 + xy^2\right)y \quad (15)$$

Where:

γs: the weight density of steel.

fy: is the vertical force in *y*-direction which is equal to the weight from the top to distance *y*.

The differential equation of axial displacement of a straight bar is:

$$\frac{dv}{dy} = \frac{fy}{A.E} \quad (16)$$

Where *E*: modulus of elasticity

$$v(y) = \int \frac{fy}{A.E}\, dy \quad (17)$$

The boundary condition is *v=0 at y= L*. By integrating Eq.16, the axial displacement can represented as follows:

$$v(y)=\frac{\gamma s}{2E}\left[\frac{1}{R^2}\left\{\begin{array}{c}\ln(1+R.y)+\\ \ln(1+R.L)+\\ \dfrac{1}{(1+R.y)}+\\ \dfrac{1}{(1+R.L)}\end{array}\right\}+\frac{y^2}{2}+\frac{L^2}{2}\right] \quad (18)$$

2.2.2 Transverse Displacement

The transverse (horizontal) displacement in beam is u in x direction and w in z direction, which are the same for a symmetrical square tower. The differential equation of transverse displacement *(u)* or *(w)* is:

$$\frac{d^2\,u}{dy^2}=\frac{-M_z}{EI_z(y)} \quad (19)$$

$$M_z=-EI_z(y)\,\frac{d^2\,u}{dy^2} \quad (20)$$

The horizontal loads on the tower are concentrated loads (load on antenna) or uniform loads (wind loads), thus the derivation of displacement for concentrated and uniform loads are separately obtained.

2.2.2.1 Transverse Displacement under Uniform Load

The differential equation of transverse displacement under uniform load is:

$$\frac{d^2\,M_z}{dy^2}=\frac{dV_x}{dy}=-P_x$$

$$\frac{d^2}{dy^2}\left(-EI_z(y)\,\frac{d^2\,u}{dy^2}\right)=-P_x \quad (21)$$

Where Vx shear force, & Px load intensity per unit length

$$Px=Wx.\,x(y) \quad (22)$$

Where Wx load intensity per area in x direction.

$$\frac{d^2}{dy^2}\left(-EI_z(y)\,\frac{d^2\,u}{dy^2}\right)=-W_x.\,x(y) \quad (23)$$

The boundary conditions are:

$$\frac{d^3\,u}{dy^3}=\frac{d^2\,u}{dy^2}=0\ at\ y=0$$

(zero shear and moment), and

$$\frac{du}{dy}=u=0\ at\ y=L$$

(zero slope and displacement)

By integrating Eq.21, the transverse displacement is represented as follows:

$$u(y)=\frac{W_x.x_1}{EI}\left[\begin{array}{c}\dfrac{R.y^4}{72B}+y^3\left\{\dfrac{-R}{36B^2}-\dfrac{1}{12B}\right\}+\\ y^2\left\{\dfrac{-1}{4B^2}+\dfrac{R}{12B^3}\right\}+\\ y\left\{\begin{array}{c}\dfrac{-1}{2B^3}+\dfrac{R}{6B^2}-\dfrac{L^2}{4B}+\dfrac{L}{2B^2}-\\ \dfrac{R.L^3}{18B}+\dfrac{R.L^2}{12B^2}-\dfrac{R.L}{6B^3}\end{array}\right\}+\\ \{ln(1+BL)-ln(1+B.y)\}.\\ \left\{\dfrac{R.y}{6B^4}-\dfrac{y}{2B^3}-\dfrac{1}{2B^4}-\dfrac{R}{6B^5}\right\}+\\ \dfrac{L^3}{6B}-\dfrac{L^2}{4B^2}+\dfrac{L}{2B^3}+\dfrac{R.L^4}{24B}-\\ \dfrac{R.L^3}{18B^2}+\dfrac{R.L^2}{12B^3}+\dfrac{R.L}{6B^4}\end{array}\right] \quad (24)$$

2.2.2.2 Transverse Displacement under Concentrated Load

The differential equation of transverse displacement under a concentrated load is:

$$\frac{d^2\,u}{dy^2}=\frac{f_x(y-a)}{EI_z(y)} \quad (25)$$

Where
fx is the concentrated load at distance *(a)* from the origin, and
(y-a) is Macaulay's brackets: *(y-a) =(y-a) for y >a, and (y-a)= zero for y < a.*

The boundary conditions are:

$$\frac{du}{dy} = u = 0 \; at \; y = L$$

(zero slope and displacement)

By integrating Eq.25 the transverse displacement can be represented as follows:

$$u(y) = \frac{f_x}{EI_z}\left[\frac{y^2}{2B} - \left(\frac{1}{B^2} + \frac{a}{B}\right)\left\{\begin{matrix} y.ln(1+By) - \\ y + \frac{ln(1+B.y)}{B} - \\ y.ln(1+BL) + \\ L + \frac{ln(1+B.L)}{B} \end{matrix}\right\} - \frac{L.y}{B} + \frac{L^2}{2B}\right] \quad (26)$$

The final equation for displacement in other direction *(w)* has the same statements for displacement *(u)* in uniform and concentrated load by replacing *w*, and *fx* by *wx* and *fx* respectively, since $Ix = Iz$ for the square box section.

2.3 Representation of a Tower by an Equivalent Beam

To find the properties of the equivalent beam which represents the tower, the relationship between *CT* with *CB* and *FT* with *FB* must be known for each type of configuration. These relations can be known by analyzing the tower with specific heights, x_2, x_1 layout, n, and cross section of the members. Then analyzing the hollow tapered beam with the same height of this tower, and same x_2, and x_1, to find the value CB, which gives the same deflected shape of the tower under the same load conditions. Here the tolerance for displacement should not exceed (0.5 %). This procedure is repeated for the number of panels and x_2, and x_1 for each type of configuration.

The analysis of the actual tower is done by the use of beam elements. The analysis includes the leg members and main brace members (excluding the redundant members). The same things are used in the calculation of the value of *TV*.

Because of the large number of analysis procedure and the number of input data files needed, four programs in FORTRAN-77 language using Fortran Power Station 4.0 [7] are built to generate these data files.

These programs just need dimensions of base and top and number of panels to generate the node numbers, the coordinates, the number of members, and the connectivity nodes. Also these programs can distribute the load on the tower according to the equations provided.

Considering the number of examples with different dimensions at base and top and different number of panels with specific height, the relation between *CT* and *CB* and the relation between *FT* and *FB* for X-brace (Fig (6-a)) with equal panels are obtained, and they are shown in Fig.(7) and Fig.(8) respectively. These relations can be represented by the following equations:

$$CT=136.5 -20.484\, n +1.68\, n^2 -0.044\, n^3 + (0.153 + 0.03\, n -0.0025\, n2)\, CB$$

$$FT=39.6 -9.24\, n+ 0.7n^2 -0.017n^3+ (0.0106 - 0.00262\, n + 0.0002\, n^2)\, FB-0.0006\, FB^2 \quad (27)$$

The relation between *CT* and *CB* and the relation between *FT* and *FB* for X-brace with unequal panels, are shown in Fig.(9) and Fig.(l0) respectively. *CT* and *FT* can be represented by the following equations:

$$CT=133 -13.56\, n +1.123\, n^2 -0.03\, n^3 + (0.451 + 0.04\, n -0.003\, n^2)\, CB$$

$$FT=9.34-2.306n+0.2006n^2-0.00573n^3 +0.81\, FB+(-0.0405 +0.01n-0.00085\, n^2)FB^2 \quad (28)$$

For the tower of type K-brace (Fig. (6-b)) with equal panels, Fig. (11) and Fig. (12), explain the relation between *CT* and *CB* and the relation between *FT* and *FB*. The following equations represent the graph relationship:

$$CT= 112.93 -12.83\, n +1.09\, n^2 -0.031\, n^3 + (0.3 - 0.014\, n -0.00033\, n^2)\, CB$$

$$FT= -8 +1.808n-0.15n^2+0.004n^3+ (1.245+$$

$0.073n- 0.006n^2 -0.00015n^3) FB + (-0.0016 + 0.00024n) FB^2$ (29)

For the tower of type K-brace and unequal panels configuration, Fig. (13) and Fig. (14) explain the relation between CT and CB and the relation between FT and FB, which can be represented by the following equations:

$$CT=104 -11.67 n + 0.976 n^2 -0.027 n^3 + (0.064 + 0.06n^2 -0.0054 n^3) CB$$

$$FT= -3.65 -0.98 n+ 0.08 n^2 -0.0022 n^3 + 0.9 FB + 1-0.0068 + 0.0015 n -O.000l2 n^2) FB^2 \quad (30)$$

2.3.1 Verification Problems

A tower of type X-brace with four equal panels which shown in Fig. (15 b) is subject to a number of load cases. The tower has dimensions (L=20m), (XB=6m), (XT=2m), and cross section area for leg members equal to (A=51.5 cm^2) and, (A=23.5 cm^2) for redundant members. The *TV* for this tower is (1.15m^3), according to Eq.13 and (1396.) for *CT*. The equivalent tapering hollow square beam shown in Fig. (15 a) has CB equal to (5647). The results of analysis and load cases for the tower and the equivalent beam are shown in Table(1).

The displacements at top and forces (at base) for the equivalent beam are calculated for the same CB so the tower can be represented by the equivalent beam.

Table (1) Analysis results and load cases for the tower and the equivalent beam.

Number of loading case in X-Dir. (kN)											
1				2				3			
Equiv. beam at point		Tower at node		Equiv. beam at point		Tower at node		Equiv. beam at point		Tower at node	
a	0	1	0	*a*	6	1	0	*a*	0	1	0
		2	0			2	0			2	0
b	0	3	0	*b*	6	3	3	*b*	4	3	2
		4	0			4	3			4	2
c	0	5	0	*c*	6	5	3	*c*	6	5	2
		6	0			6	3			6	2
d	10	7	5	*d*	6	7	3	*d*	8	7	2
		8	5			8	3			8	2

No. of load case	Structure Type	Horizontal displacement at top (mm)	Axial force (kN)
1	Equivalent beam	1.2930235	18.75664
	Tower	1.2929880	12.68638
2	Equivalent beam	1.1786454	25.32147
	Tower	1.1192827	13.070202
3	Equivalent beam	1.4513451	27.19713
	Tower	1.3785101	15.40162

2.4 Optimum Layout of Tower by Equivalent Beam

The approach presented in the formulation of optimum layout of the tower is based on finding the optimum design of the equivalent beam and using its layout dimension for the layout dimension of the tower. There are many available methods of optimization, here the Sequential Unconstrained Minimization Technique SUMT method, [8, 9], is used.

2.4.1 The Objective Function

The objective function to be minimized is the total weight of the equivalent beam W that is:

$$W = \gamma TV \quad (31)$$

Where γ is the density of member material, which used in the tower. Since γ is constant for all the tower elements, hence the objective function to be minimized is *TV*, $(TV=2L(x_2^2 + x_1^2)/CD$. There are three variables, direct variables (x_2, x_1) and indirect variable (n). *CT* is a function of *CB* and *n*.

2.4.2 Behavior Constraints

2.4.2.1 Displacement at Top

Since the variation of the allowable horizontal displacements along the height of the tower is linear, the displacement at top is considered as the maximum i.e. the displacement at *y=0*. Therefore the displacement constraint will be as follow:

$$\frac{\gamma s}{2E}\left[\frac{1}{R^2}\begin{Bmatrix}\ln(1+R.L)+1+\\ \ln(1+R.L)\end{Bmatrix}+\frac{L^2}{2}\right]\le$$

Allowable vertical displacement at top.

$$\frac{f_x}{EI_z}\begin{bmatrix}-\left(\frac{1}{B^2}+\frac{a}{B}\right)\left\{L-\frac{ln(1+B.L)}{B}\right\}\\ +\frac{L^2}{2B}\end{bmatrix}\le$$

Allowable horizontal displacement at top due to concentrated load.

$$\frac{W_x.x_1}{EI}\begin{bmatrix}ln(1+BL)\left\{-\frac{1}{2B^4}-\frac{R}{6B^5}\right\}+\\ \frac{L^3}{6B}-\frac{L^2}{4B^2}+\frac{L}{2B^3}+\frac{R.L^4}{24B}-\\ \frac{R.L^3}{18B^2}+\frac{R.L^2}{12B^3}+\frac{R.L}{6B^4}\end{bmatrix}\le$$

Allowable horizontal displacement at top due to uniform load.

2.4.2.2 Axial Force at Bottom Leg

Here the optimum design satisfies the maximum allowable force at the bottom of the equivalent beam *FB* that is equal to the computed force for the bottom leg member *FT*, because the maximum stresses occur in this member. The allowable compressive stress in a leg member is *Fa* on the gross sectional area of axially loaded compression member shall be calculated as follows [10, 11].

$$F_a=\left[1-\frac{1}{2}\left(\frac{kL_u/r}{Cc}\right)^2\right]F_y \quad for\ \frac{kL_u}{r}\le Cc \qquad (32)$$

And

$$F_a=\frac{\pi^2E}{\left(\frac{kL_u}{r}\right)^2} \qquad for\ \frac{kL_u}{r}\ge Cc \qquad (33)$$

$$Cc=\pi\sqrt{\frac{2E}{Fy}} \qquad (34)$$

Where
Fy: minimum yield stress
E: modulus of elasticity
L: un-braced length
r: radius of gyration
k: effective length coefficient

For a leg member bolted in both faces at connections:

$$\frac{kL_u}{r}=\frac{L_u}{r}\ and\ \frac{L_u}{r}\le 150$$

The compressive stress *Fa* to be calculated for maximum available size of cross section area and *Lu* can be calculated from the height of the bottom panel.

2.4.3 Side Constraints

The design variables must be positive in order to be realizable. In structural design, the design variables are normally bounded and depend on either availability or the handling restrictions. In this work, the variables should be greater than zero since some or all of the behavior constraints would be violated even if one variable is zero. The non-negativity restrictions on the design variables are

$$Xi > 0 \qquad (35)$$

2.5 Optimum Design of Member Cross Sectional

Area After finding the optimum layout and configuration system of the tower, the second stage for the optimum design, is the optimum cross sectional area of the members. The constraints are limited to be the maximum stress due to the combination of bending moment and axial force, taking the slenderness ratio of the members and buckling of flange into account, which are based on ANSI [10], EIA[4] and ISD [5] specifications and also the allowable displacement at antenna position according to the recommendation.

From the analysis of the tower the forces for the leg members and main brace are calculated. The magnitude of the load in a redundant member can vary from (0.5 -2.5%) of the load in the supported

members [10].
The objective goal to be reached is to minimize the total weight of the structure W consisting of a number of members m, the length of a member i is Li and cross sectional area is Ai.

This may be obtained by optimizing each member separately and then summing the results.

$$W = \sum_{i=1}^{m} \gamma_i L_i A_i \quad (36)$$

Where γi is the density of the member i and γi and Li are specified for the members. Minimizing the steel cross section area leads to minimum weight, so the objective function will be Ai. Members are selected with minimum cross section area from the list of multi-standard steel sections [AISC sections], which satisfy the allowable stress according to ANSI specification.

3. Applications

In order to find the optimum slope of the tower i.e. the angle of the main leg (α) as shown in Fig.(16), a tower with (7 and 10) number of panels and (1*m*) as the dimension of the top and base dimension changing with height (50*m*) is considered. The minimum weight of tower can be obtained when the angle of the main leg i.e. the angle of the outline of the tower is equal to (87^o) as shown in Fig.(17).

4. Conclusions

According to the present formulation the following conclusions can be given:

1. It is found that the SUMT is a proper method used for optimum design of large self supporting steel communication towers.

2. The present formulation of the problem has proved to be effective in solving the problem of optimum design of communication towers compared with the conventional case which involves a large number of design variables and constraints. Formulation of the optimum design problem by the present method has yielded good results with overall convergence behavior in relatively short time.

3. The tower of type X-brace with unequal panels has the minimum weight compared with other types of tower and the optimum design is satisfied when the angle of main leg is equal to (87^o).

4. The leg members of the lower panel generally control the design of other similar members due to the high stresses carried by these members.

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List of Symbols

A	Cross-sectional area.
B	Width of steel angle
CB	width /thickness ratio
E	Modulus of elasticity
Fx, Fy, Fz	Axial and shear forces
G	Shear modulus
Ix, or J	Torsional constant
Iy, Iz	Moments of inertia
Iyz	Product of inertia
[K]	Stiffness matrix of the member
L	Element length, height of the tower
My, Mz	Bending moments
Mx	Torque
n	Number of panels
{P}	Applied nodal load vector
TV	Total volume of the material in the actual tower
t	Thickness of steel angle
U	Strain energy
{U}	displacement vector
W	Wight of tower
x ,y, z	Local coordinates
x1,x2	Width of base and top of tower
γi	Density of the member
u,v,w	Displacements in direction of x, y, z
θx, θy, θz	Angle of rotation in direction of x, y, z

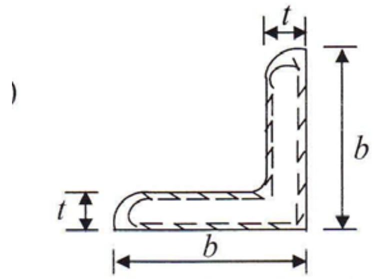


Figure (2): Cross section of member with shear

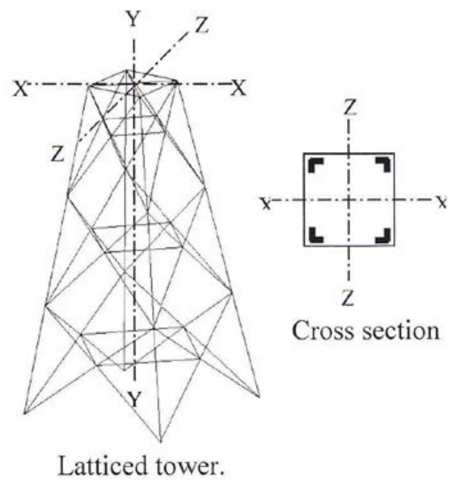


Figure (3): Segment of latticed tower.

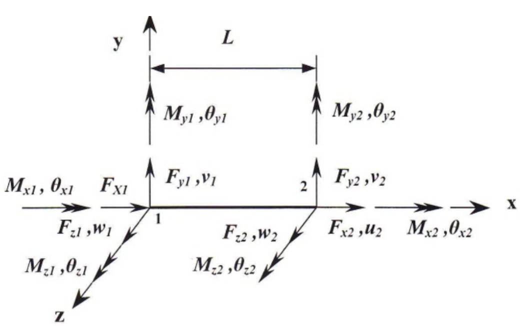


Figure (1): Three-dimensional beam under general loading.

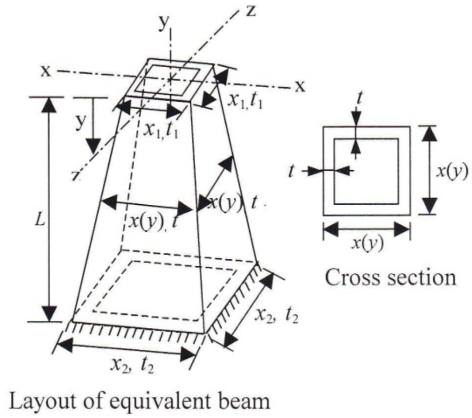


Figure (4): Communication tower idealized as an equivalent beam.

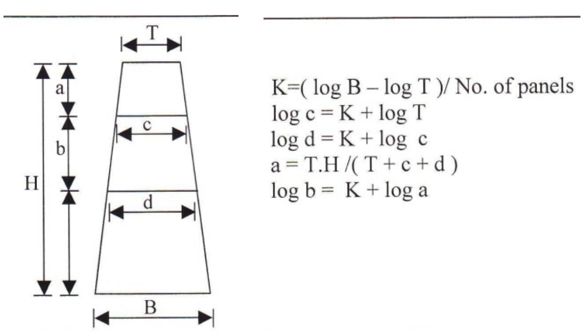


Figure (5): Unequal panels, which have algorithm relation for height of panel Ref.[6]

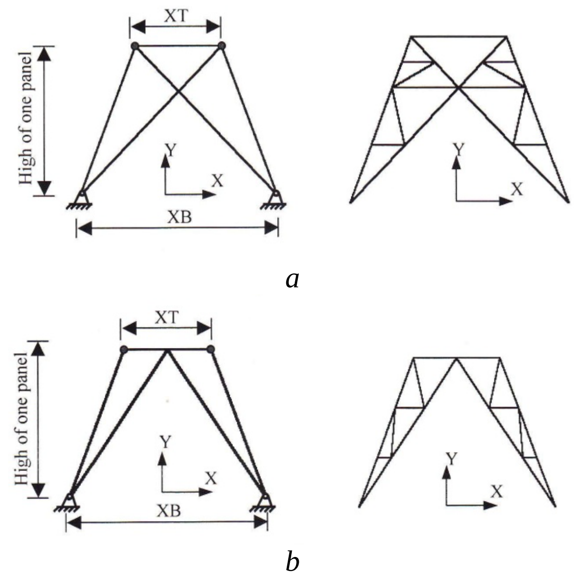


Figure (6): X-Y Plane of structure.
(a): Structure of type X-brace.
(b): Structure of type K-brace.

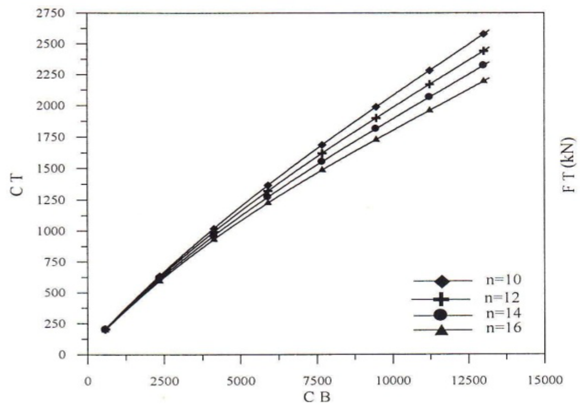


Figure (7): Relationship between CB and CT for a tower of type X-brace configuration with equal panels.

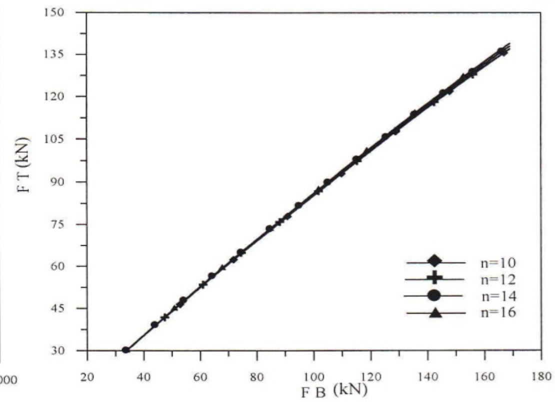


Figure (8): Relationship between FB and FT for a tower of type X brace configuration with equal panel.

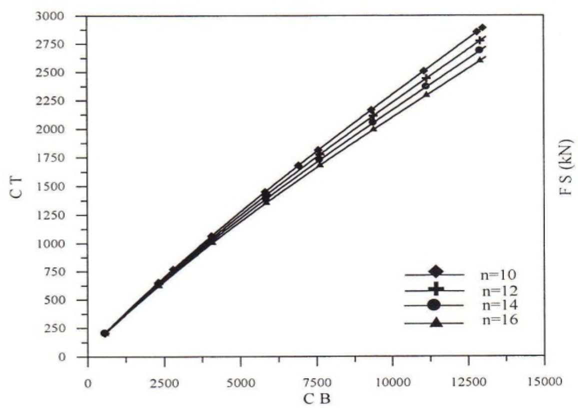


Figure (9): Relationship between CB and CT for a tower of type X-brace configuration with unequal panels.

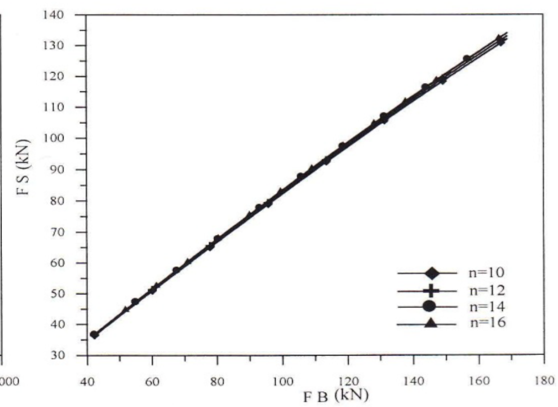


Figure (10): Relationship between FB and FT for a tower of type X-brace configuration with unequal panels.

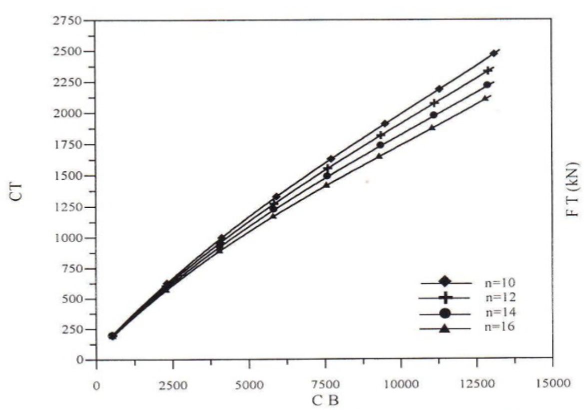


Figure (11): Relationship between CB and CT for a tower of type K-brace configuration with equal panels.

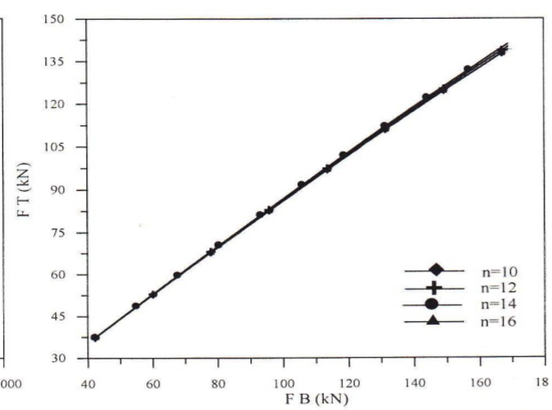


Figure (12): Relationship between FB and FT for a tower of type K-brace configuration with equal panels.

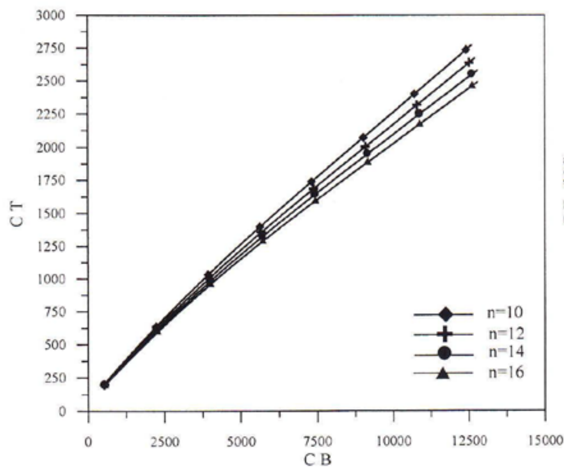


Figure (13): Relationship between CB and CT for a tower of type K-brace configuration with unequal panels.

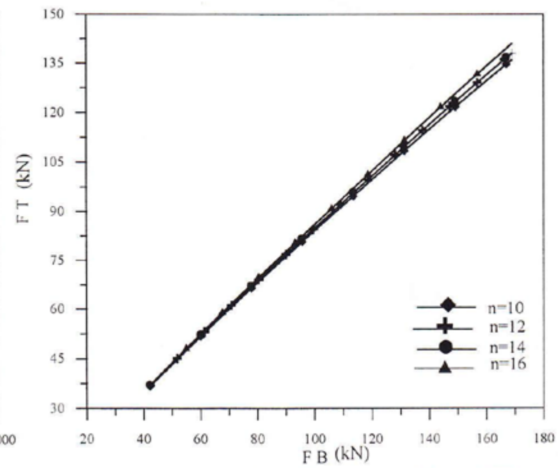
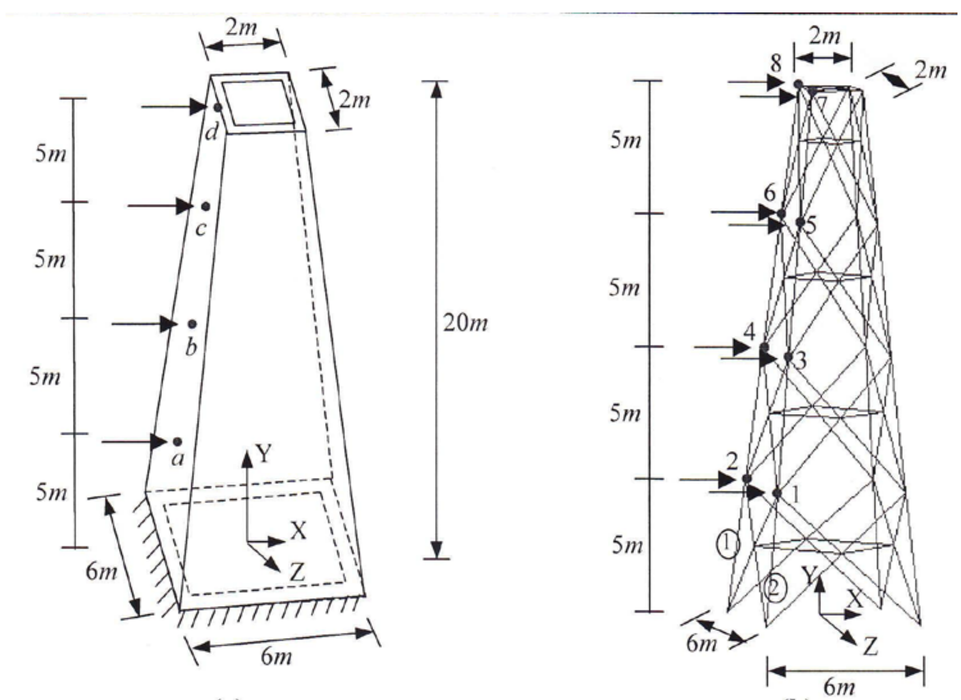


Figure (14): Relationship between FB and FT for a tower of type K-brace configuration with unequal panels.



(a) **(b)**

Figure (15): Communication tower idealized as a beam.
a: Equivalent beam **b**: Communication tower

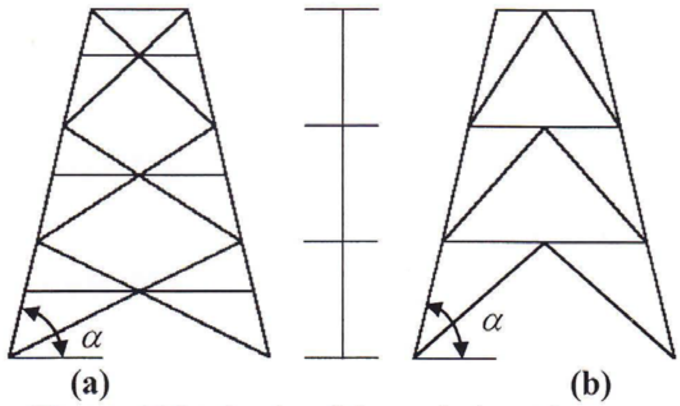
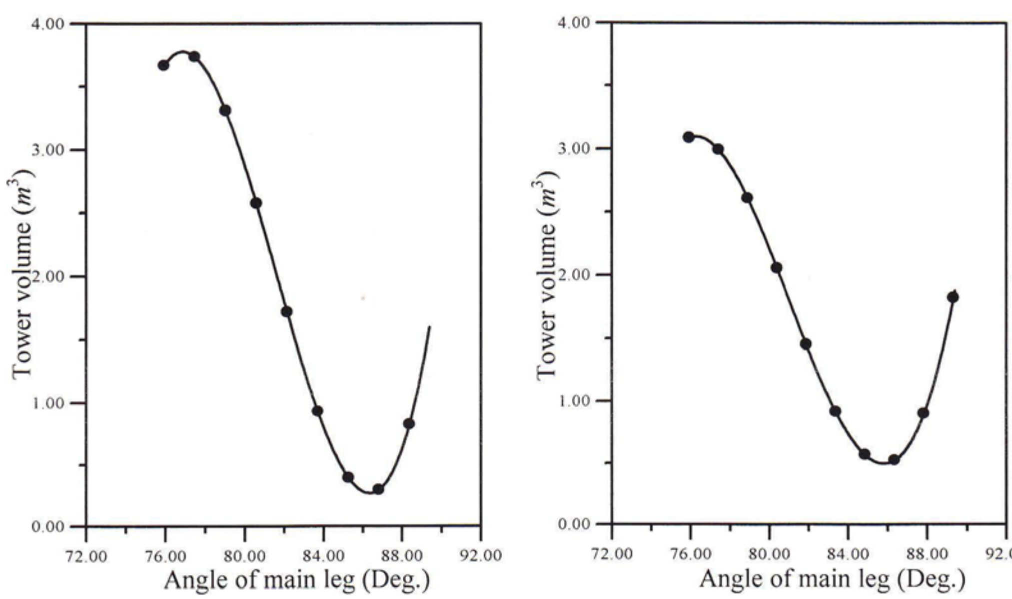


Figure (16): Angle of the main leg of tower.
a: Tower of type X-brace **b:** Tower of type K-brace



(a) Tower with seven panels. **(b)** Tower with ten panels.
Figure (17): Effect of the angle of main leg on tower volume.