



## SECTIONAL CURVATURE DUCTILITY OF REINFORCED CONCRETE COLUMNS UNDER LARGE INELASTIC DEFORMATION

Dr. Raad K. Al-Azzawi  
Lecturer. /Civil Eng. Dept.  
College of Eng. / University of Baghdad.

### ABSTRACT

A mathematical model is developed to express stress-strain relationship of normal-strength concrete confined by transverse reinforcement. The sectional ductility analysis on rectangular reinforced concrete columns is performed under axial load. Analytical models available in the literatures for confinement concrete by different types of transverse reinforcement are used. A parametric study is performed to study the influence of these parameters on the curvature ductility of reinforced concrete columns. The main parameters introduced are the amount of longitudinal reinforcement, amount of transverse reinforcement, tie spacing and the level of the compression axial force. The experimental and analytical results available in the literatures are being compared with present study.

### KEYWORDS:

Columns, Confined Concrete, Curvature Ductility, Stress-Strain Relationships.

### الخلاصة:

في هذا البحث تم تطوير التمثيل الرياضي لعلاقة الإجهاد مع الانفعال للخرسانة المسلحة الاعتيادية والمحصورة بالتسليح الجانبي . تم حساب مطيلية التقوس لمقاطع أعمدة ذات خرسانة مسلحة ومستطيلة الشكل تحت تأثير القوة المحورية المسلطة. تم استخدام عدد من علاقات الإجهاد مع الانفعال للخرسانة المحصورة والمتوفرة في المصادر . تم إجراء الدراسة بإدخال عدة متغيرات لمعرفة مدى تأثيرها على مطيلية التقوس. المتغيرات الرئيسية المستخدمة هي كمية الحديد الطولي و كمية الحديد الجانبي والمسافات بين روابط التسليح الجانبي ومقدار القوة المحورية المسلطة. تم مقارنة النتائج مع نتائج عملية متوفرة.

## INTRODUCTION

Columns subjected to lateral forces such as dynamic loads must be designed according to the displacement or recently performance based approach , Bousalem, Chikh and . Hachem (2004).

A significant amount of transverse reinforcement, which has a double function in resisting shear forces as well as providing confinement to the concrete core, must be provided to achieve a ductile behavior. The presence of transverse reinforcement to increase strength and ductility in these regions of the columns is a major consideration. Therefore, tests have shown that the confinement of concrete by adequate arrangement of transverse reinforcement will cause a significant enhancement in the flexural strength capacity of the reinforced concrete column.

A mathematical model is developed to express stress-strain relationship of normal-strength concrete confined by transverse reinforcement in order to perform a moment curvature analysis with quantity of the confining steel required in reinforced concrete columns sections to achieve the ultimate curvatures required for ductility demands. The predictions of the behavior of a four representative specimens tested by Sheikh and Yeh (1990) from the considered stress-strain models for confined concrete are compared with the test results. The unconfined moment capacity is calculated based on the equivalent rectangular concrete stress block and as stated by Bousalem, Chikh and Hachem (2004).

## ANALYTICAL STRESS-STRAIN MODELS:

The analytical models were developed in the light of the experimental data and various stress-strain models for the prediction of the confined concrete have been illustrated and a summary of mathematical expressions of some established models is shown in **Table (1)**.

## STRESS-STRAIN RELATIONSHIP MODEFIED FOR CONFINED CONCRETE:

The modefied model was developed on the basis of the observations derived from several experimental studies tested by Sheikh and Yeh (1990) and as indicated by Bousalem, Chikh and Hachem (2004).The experimental observations show that the stress strain curve of confined concrete is constantly characterized by three parts: the ascending branch, descending branch and the sustaining branch. The mathematical formulation of the stress-strain relations are shown in **Tables (1) and (2)**.

## ULTIMATE COMPRESSION STRAIN:

As confirmed by Bousalem, Chikh and Hachem (2004), Scott et al.(1982) observed that it is reasonably conservative to define the limit of useful concrete compressive strain  $\epsilon_{cu}$  where the first hoop fractures occurs. Recently, Priestley et al. (1996) proposed a value of  $\epsilon_{cu}$  based on energy balanced approach and is given by the following expression;

$$\epsilon_{cu} = 0.004 + 1.4 \frac{\rho_{sh} f_{yh} \epsilon_{sm}}{f_{cc}} \quad (1)$$

Equation 1 has been adopted in many updating models except for Hoshikuma model(1997) where the value of  $\varepsilon_{cu}$  was used as proposed by the author and is given by equation 2.

$$\varepsilon_{cu} = \varepsilon_{cc} + \frac{f_{cc}}{2E_{des}} \tag{2}$$

**Table (1) Summary of stress- strain model for confined concrete**

Stress- Strain Model for Concrete			
Author	Ascending region	Descending region	Softening rate
Modified  (2006)	$f_c = \frac{f_{cc} \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^r}{r - 1 + \left( \frac{\varepsilon_c}{\varepsilon_{cc}} \right)^r}$	$f_c = f_{cc} - E_{des}(\varepsilon_c - \varepsilon_{cc}) \geq 0.25f_{cc}$	$E_{des} = \frac{kf_{co}^2}{\rho_{sh}f_{yh}}$
B. Bousalem et al.(2004)	$f_c = \frac{f_{cc}xn}{n - 1 + x^n}$	$f_c = f_{cc} - E_{soft}(\varepsilon_c - \varepsilon_{cc}) \geq 0.3f_{cc}$	$E_{soft} = \frac{4f_{co}^2}{k_e\rho_{sh}f_{yh}}$
Hoshikuma et al. (1997)	$f_c = E_c\varepsilon_c \left[ 1 - \frac{1}{n}x^{n-1} \right]$	$f_c = f_{cc} - E_{des}(\varepsilon_c - \varepsilon_{cc}) \geq 0.51$	$E_{des} = \frac{11.2f_{co}^2}{\rho_{sh}f_{yh}}$
Mander et al.  (1988)		$f_c = \frac{f_{cc}mr}{r - 1 + m^r}$	

Note that:

$$r = \frac{E_c}{(E_c - E_{sec})}$$

$$E_{sec} = \frac{f_{cc}}{\varepsilon_{cc}}$$

$$E_c = 4730\sqrt{f_{cc}}$$

And the analytical expression of  $k$  is proposed to be equal to:

$$k = 6.25 + \frac{f_{co}}{\rho_{sh}f_{yh}}$$

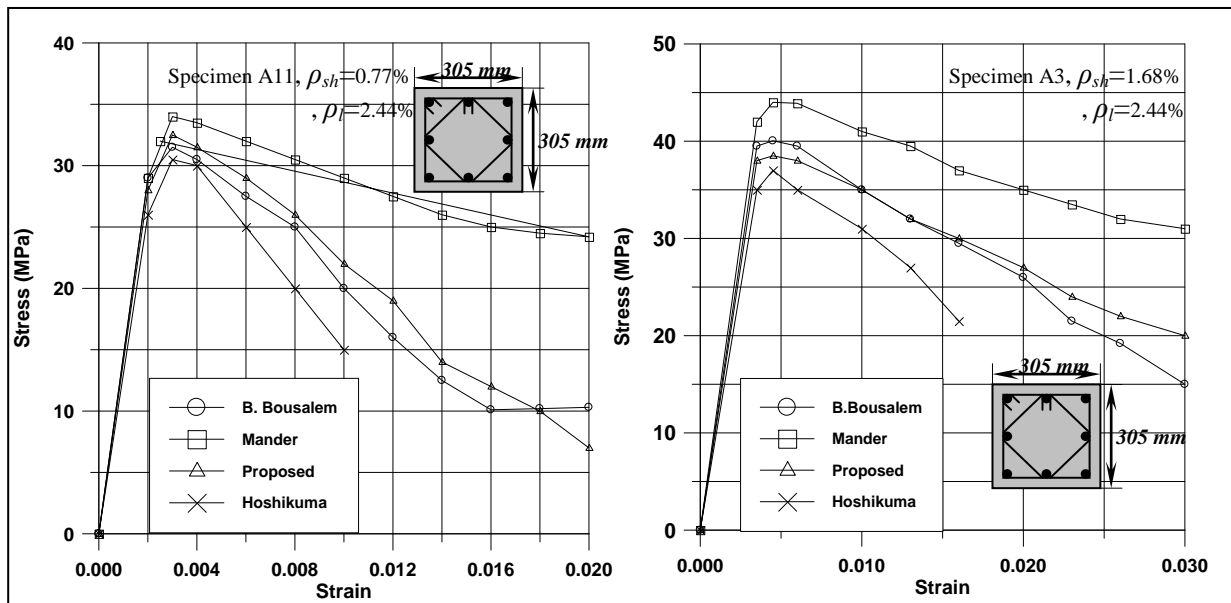
**Table (2) Analytical expressions of  $K_s$  et  $K_d$  values for rectangular section columns**

Authors	Proposed equation for $K_s = \frac{f_{cc}}{f_{co}}$	proposed equation for $K_d = \frac{\varepsilon_{cc}}{\varepsilon_{co}}$
Mander et al.	$-1.254 + 2.254\sqrt{1 + 7.94\frac{f'_1}{f_{co}}} - 2\frac{f'_1}{f_{co}}$	$1 + 5\left[ \frac{f'_{cc}}{f_{co}} - 1 \right]$

Hoshikuma et al.	$1 + 0.73 \frac{\rho_{sh} f_{yh}}{f_{co}}$	$1 + 4.98 \frac{\rho_{sh} f_{yh}}{f_{co}}$
B.Bousalem et al.	$1 + 0.4 \frac{k_e \rho_{sh} f_{yh}}{\sqrt{f_{co}}}$	$1 + 2.7 \frac{k_e \rho_{sh} f_{yh}}{\sqrt{f_{co}}}$

**MOMENT-CUTVATURE RELATIONSHIPS:**

Fig.(1) shows a typical stress-strain curves for two representative specimens, A3 and A11, as predicted from using different confinement models. It is clear that Mander model predict respectively higher strength and lower ductility of confined concrete compared with other models and also as indicated by Bousalem, Chikh and Hachem (2004). The stress strain curves for concrete and the stress strain model for longitudinal reinforcement proposed by Mander et al.(1984), a series of analytical moment curvature curves can be generated taking into account the different influencing parameters.



**Fig. (1) Analytical stress-strain curves for confined concrete**

As confirmed by Bousalem, Chikh and Hachem (2004) and other literatures listed through this study, the assumptions used in theoretical moment curvature analysis are as follows.

- Plane section before bending remain plane after bending,
- Tensile strength of concrete is ignored,
- Stresses in concrete are derived from the appropriate stress strain curves respectively for confined and unconfined concrete
- Stresses in longitudinal reinforcement are derived from the stress strain curve using Mander model and experimental data including strain hardening
- Perfect bond between steel and concrete
- In the application of the analytical models, the lateral steel stress is suggested to be equal to yield stress
- The ultimate state is reached when one of the three limit is attained:

1. The concrete strain at the extreme compression fiber reaches the specified value  $\epsilon_{cu}$ ,

2. The steel strain in the tension zone reaches the specified value  $\epsilon_{su}$ ,
3. The flexural moment drops to a value of  $0.8 M_{max}$ .

### PARAMETRIC STUDY USING IN THE MODEL:

As stated in previous studies and illustrated in this study, the effect of different variables is considered by comparing moment curvature relations of the column sections in which only one major variable differed significantly. These variables included the level of axial load, the amount of both transverse and longitudinal steel, and the tie spacing.

#### Level of Axial Load:

The effect of axial load on moment curvature relationship is shown in **Fig.(2)**. The level of axial load simulates the column axial load from medium to high stories. This level is defined as  $F$  equal to  $P/f'_c A_g$  where  $p$  is the compressive axial load on the columns. The moment curvature curves were derived using the experimental data of column A3 tested by Sheikh et al.(1990) where the confining and longitudinal steel ratios were 1.68 and 2.44% respectively. It is observed that the ductility reduces as a result of higher axial load levels, and the moment beyond the maximum point degrades more rapidly as the level of axial load increases and as confirmed by Bousalem, Chikh and Hachem (2004).

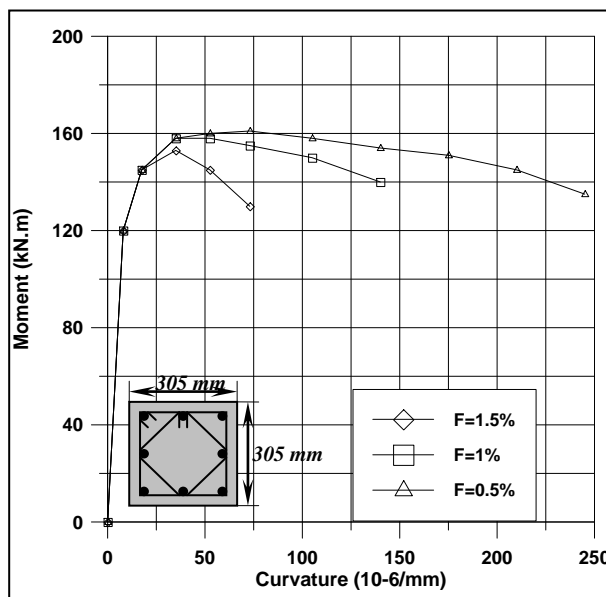


Fig. (2) Effect of axial load level

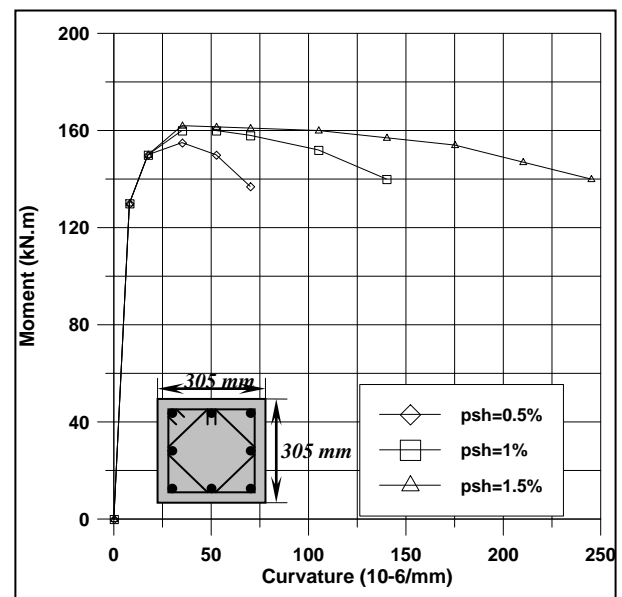


Fig. (3) Effect of amount of transverse steel

#### Amount of Lateral Reinforcement:

The effect of this variable is clearly evaluated by comparing the behaviour of the three curves shown in **Fig. (3)**. The identical ascending parts of the curves indicate that the quantity of transverse steel doesn't influence the section behaviour prior to the first cracking of unconfined concrete. It can be noticed that the ductility increases with the increase of transverse steel content. It is apparent that with lower lateral volumetric ratios, the confining pressure is not sufficient enough to maintain the moment capacity of the section and as confirmed by Bousalem, Chikh and Hachem (2004).

**Amount of the Longitudinal Reinforcement:**

Fig. (4) illustrates the effect of longitudinal steel content on the moment curvature curve for a specimen having the same details as specimen A11. It is demonstrated that the increase of the moment capacity of the section with the increase of the amount of longitudinal steel is small. The post peak behaviour is characterized by gradual drop in the moment capacity of the section where it reaches approximately a same value at ultimate curvatures. However a large longitudinal steel content means that less reliance is placed on the concrete capacity, and therefore the moment capacity can be better maintained at high curvatures and as confirmed by Bousalem, Chikh and Hachem (2004).

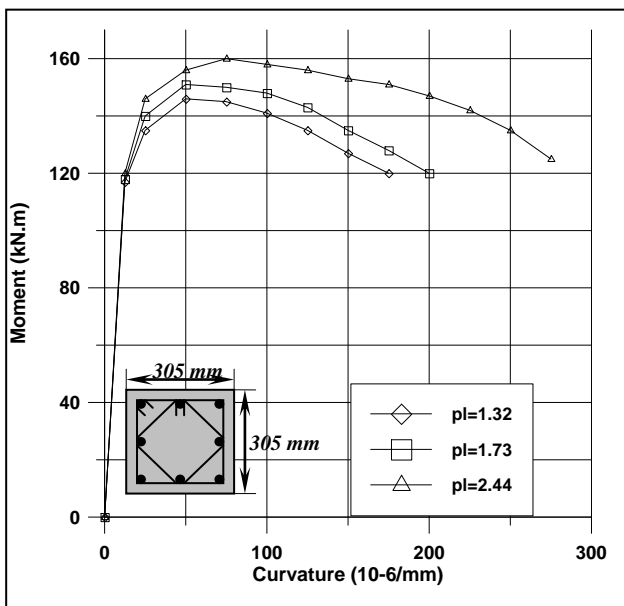


Fig. (4) Effect of the amount of longitudinal steel

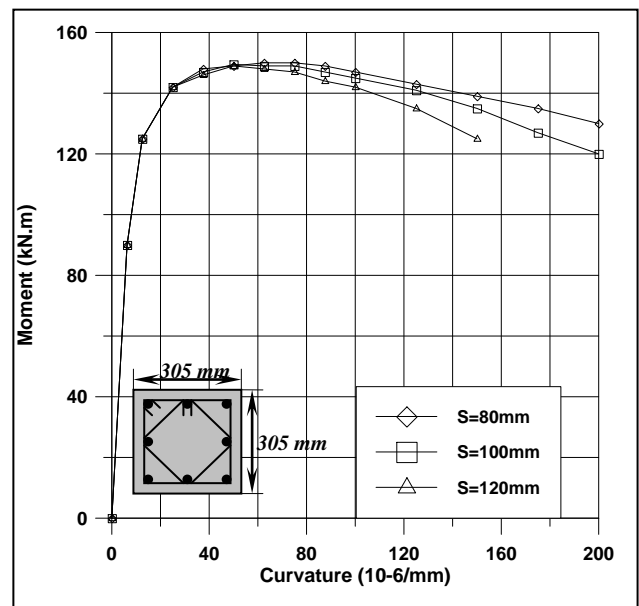


Fig. (5) Effect of tie spacing

**Tie Spacing:**

The effect of tie spacing on the moment curvature behaviour of the section is shown in Fig.(4) for specimen A11 and subjected to a load level of 0.5%. In general smaller tie spacing resulted in higher curvature ductility. The moment capacity in this case also increased by reducing of tie spacing and as confirmed by Bousalem, Chikh and Hachem (2004).

**MOMENT-CURVATURE CURVES:**

Figs.(6) and (7) show the comparisons between the moment curvature relations for four representative specimens (Sheikh and Yeh 1990) and those derived from numerical analysis from the four models discussed previously. Except for specimen D5,  $\rho = 2.58\%$ ,  $\rho_{sh} = 1.68\%$ , the model by Hoshikuma et al.(1994) constantly overestimated the moment capacity of the sections. The other models tend to underestimate the moment capacity of the sections and this underestimation is even larger for specimen D5 with higher volumetric ratio of longitudinal steel and lower level of axial load. Sheikh et al.(1990) reported in their experimental investigation that in the case of specimen D5, it has been observed that yielding of steel took place before a significant drop in the moment capacity, indicating that crushing of core had started after lateral steel became effective, this may explain the discordance of the theoretical results compared with the experimental ones in spite they are on the conservative side.

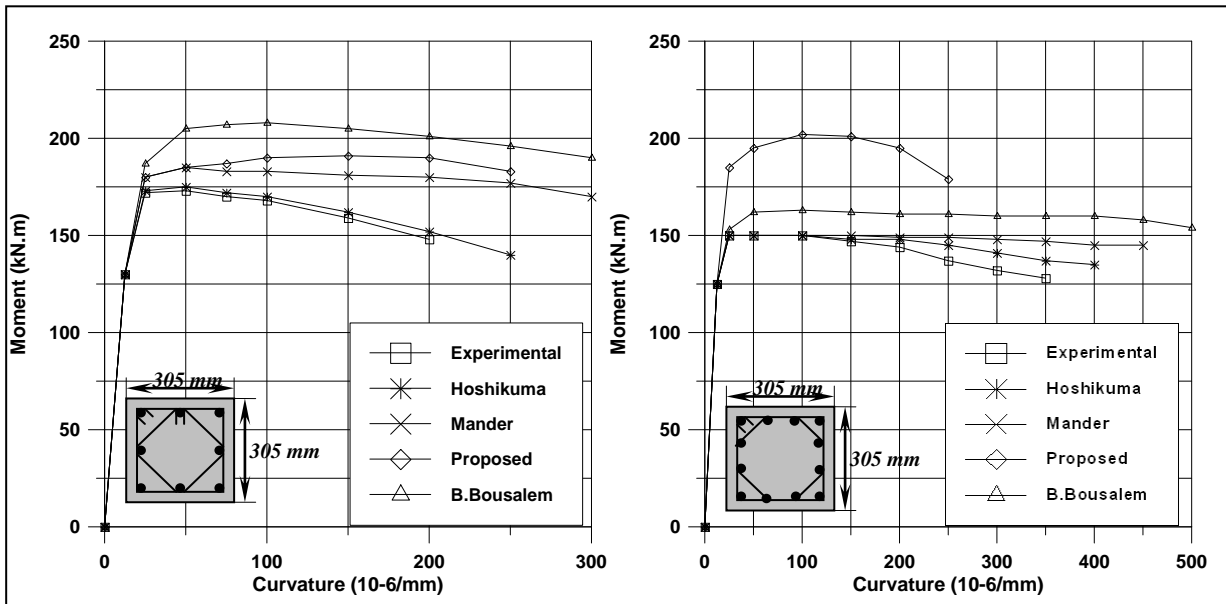


Fig. (6) Comparison of experimental and analytical curves for specimens A3 and D5

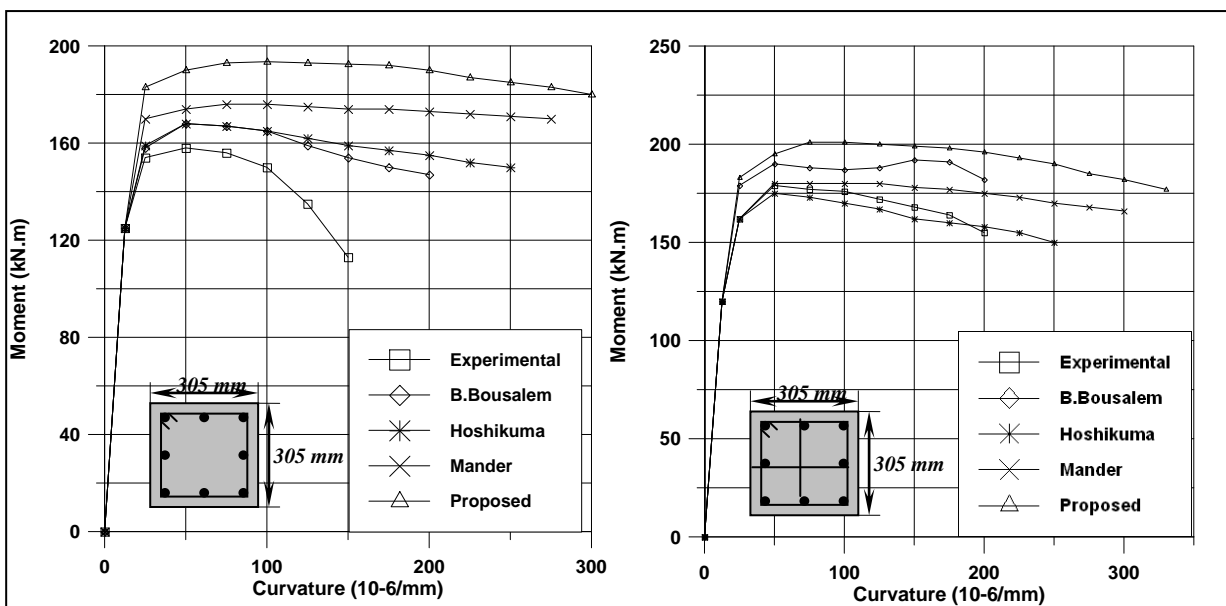


Fig. (7) Comparison of experimental and analytical curves for specimens E2 and F4

## CONCLUSIONS

The conclusions reached from this analysis for the range of variables considered in this study and as confirmed by Bousalem, Chikh and Hachem (2004) are as follows:

1. The large transverse steel content and reduced tie spacing increase the ductility and enable the moment capacity to be better maintained at higher curvatures.
2. The level of axial load reduces the ductility and the moment beyond the maximum point degrades more rapidly as the level of axial load increases.

3. The increase of the amount of longitudinal steel on the moment capacity of the section is small. However a large longitudinal steel content means that less reliance is placed on the concrete capacity, and therefore the moment capacity can be better maintained at high curvatures.
4. The proposed model reflects well the influence of the variables considered in this study, namely the axial load level, the transverse and longitudinal steel content and the tie spacing. The accuracy of the model in the representation of experimental results is acceptable.

### **NOTATIONS:**

$f_{cc}'$ ,  $f_{co}'$  = confined & unconfined concrete compressive strength in members.

$f_y$  = steel yield strength.

$f_{yt}$  = yield strength of transverse reinforcement.

$\varepsilon_c$  = concrete strain.

$\varepsilon_{cc}$  = strain corresponding to peak stress of confined concrete.

$\varepsilon_{01}$  = strain corresponding to peak stress of unconfined concrete.

$\varepsilon_{85}$  = strain corresponding to 85% of peak stress of confined concrete.

$\varepsilon_h$  = static strain hardening initiation strains of steel.

$\varepsilon_u$  = static ultimate strains of steel.

$E_c$  = modulus of elasticity for concrete.

$E_{sec}$  = secant modulus of elasticity for concrete.

$E_s$  = modulus of elasticity for steel.

$\rho_{sh}$  = total transverse steel area in two orthogonal directions divided by corresponding concrete area.

$S$  = spacing of transverse reinforcement.

$S_L$  = spacing of longitudinal reinforcement laterally supported by corner of hoop or hook of cross tie.

### **REFERENCES**

- B. Bousalem, N. Chikh and R. Hachem, Curvature Ductility of rectangular Reinforced Concrete Columns, ICCT 2004-I-(17).
- S.A. Sheikh and C. Yeh., Tied concrete columns under axial load and flexural. Journal of Structural Engineering, 116(10), October 1990, 2780-2800.
- J. B. Mander, M. J. N. Priestley and R. Park, Theoretical Stress-Strain Model for Confined Concrete, Journal of Structural Engineering, 114(8), August, 1988, 1804-1826.
- S. Murat and S. R. Razvi, Strength and ductility of confined concrete, Journal of Structural Engineering, 1992, 118(6), 1590-1607.





- J. Hoshikuma, K. Kawashima, K. Nayaga and A. W. Taylor, Stress –strain model for confined RC in bridge piers, journal of Structural Engineering, 1994, 123(5), 624-633.
- B. D. Scott, M. j. N. Priestley and R. Park, Stress- strain behavior of concrete confined by overlapping hoops at low and high strain rates, ACI Journal, Jan. Feb., 1982, 13-27.
- M. J. N. Priestley, F. Seible and G. m. Calvi, Seismic design and retrofit of bridges, John Wiley, 1996, 686 pp.
- J. B. Mander, M. J. N. Priestley and R. Park, Seismic design of bridge piers, Research report 84-2, Department of civil engineering, University of Canterbury, 1984, p. 483.