Design and Simulation of Sliding Mode Fuzzy Controller for Nonlinear System

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ABSTRUCT

Sliding Mode Controller (SMC) is a simple method and powerful technique to design a robust controller for nonlinear systems. It is an effective tool with acceptable performance. The major drawback is a classical Sliding Mode controller suffers from the chattering phenomenon which causes undesirable zigzag motion along the sliding surface. To overcome the snag of this classical approach, many methods were proposed and implemented. In this work, a Fuzzy controller was added to classical Sliding Mode controller in order to reduce the impact chattering problem. The new structure is called Sliding Mode Fuzzy controller (SMFC) which will also improve the properties and performance of the classical Sliding Mode controller. A single inverted pendulum has been utilized for testing the design of the proposed controller. Programming and Simulink by Matlab have been used for the simulation results.

Key words: sliding mode control, fuzzy logic control, sliding mode fuzzy control, chattering phenomenon.

تصميم وتنفيذ مسيطرات النمط الأنزلاقي الضبابي لنظام لا خطي

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الخلاصة

أن مسيطرات النمط الأنزلاقي هي مسيطرات بسيطة وفعالة للحصول على مسيطر متين وجيد للتعامل مع الأنظمة اللاخطية, وهي طريقة فعالة للحصول على مسيطر متين لا خطي يعطي نتائج ذات مواصفات جيدة ومقبولة. أن ظاهرة التذبذب هي أكثر ظاهرة سيئة تعاني منها أنظمة النمط الأنزلاقي. وتسبب هذه الظاهرة حركة متعرجة على أمتداد السطح الأنزلاقي. لتقليل ظاهرة التذبذب في أنظمة النمط الأنزلاقي تم أقتراح عدد من الطرق. في هذا البحث تم أضافة المسيطر الضبابي الى المسيطر الأنزلاقي من أجل التغلب على مشكلة ظاهرة التذبذب. الشكل الجديد للمسيطر يدعى مسيطر النمط الأنزلاقي الضبابي الذي سوف يحسن مواصفات وخصائص مسيطرات النمط الأنزلاقي. تم أستخدام البندول الأحادي المقلوب لأختبار مسيطر النمط الأنزلاقي الضبابي الذي تم أقتراحه. لقد تم أستخدام البرمجة بلغة ماتلاب لأيجاد النتائج.

ألكلمات الرئيسيه: سيطرة السطح الأنزلاقي, سيطرة السطح الضبابي, سيطرة السطح الأنزلاقي الضبابي, ظاهرة التذبذب.

1. INTRODUCTION

Most nonlinear systems suffer from uncertainty in their dynamic parameters which necessitates the design of high performance controllers. Today, many strong and new methods are used to design adaptive nonlinear robust controllers with acceptable performance. The Sliding mode controller (SMC) is one of the best nonlinear robust controllers that can be used in nonlinear systems that suffer from parameters uncertainty, **Utkin, 2009**. The sliding mode controller was first proposed in the 1950. It is consists of two phases; reaching and sliding phases. In reaching phase, the sliding mode control drives the state trajectory, from any initial point, toward the sliding surface in the state space by using a discontinuous control action. In sliding phase, it will force the state trajectory to stay on this sliding surface and to slide along this surface until reaching the origin. In this method, the control action was smoothed to reduce the chattering. The ultimate advantage of using sliding mode controller is achieved when the sliding surface becomes insensitive to parameters uncertainty or external disturbances inside a plant, **AL-Samarraie 2011.** The chattering phenomenon that results from the discontinuous control action is, however, a severe problem in SMC. In a method called modified sliding controller, the boundary layer was employed in order to reduce the chattering phenomenon, **Hamoudi, 2014** and **Piltan, et al., 2011.** The disadvantages of using pure sliding mode controller were resolved after adopting a modified sliding mode controller scheme. Some authors used Genetic algorithms to improve the classical sliding mode controller, **Wong, et al., 2001 and Lin, 2003.** Others combined fuzzy logic controller (FLC) with the sliding mode control method (SMC) to overpass the disadvantages of the pure sliding mode controllers, **Rahmdel, and Bairami, 2012.** Fuzzy logic controller is, however, weaker in testing the stability. Nevertheless, the stability can be ensured by combining together fuzzy and sliding mode controllers to get a new; more practical, structure called sliding mode fuzzy controller (SMFC). In the current study, a sliding mode fuzzy controller (SMFC) was used in order to reduce the chattering phenomenon; usually appears with pure sliding mode controllers.

2. SLIDING MODE CONTROL (SMC)

For nonlinear systems control, the most challenging problem in designing a control algorithm is to design a linear controller for nonlinear systems. This method, however, needs some stringent setup in which the controller must work near the system operating point. This is very difficult for large variations in dynamic system parameters and high nonlinearities **Lin, and Chen, 1994**. To solve the above problems in nonlinear systems, most researches went toward designing a nonlinear controller. SMC is one of the most powerful nonlinear techniques; first proposed in the 1950 and was used later in wide range applications due to its acceptable control performance. This controller ensures insensitive control systems to unpredicted disturbance and parameters uncertainty.

The sliding surface can be represented as;

$s(x,t)=\lambda x+\dot{x}=0$; Where λ is constant with positive value. (1)

Let us define that $x_1 = x \; and \; x_2 = \dot{x}$, so the sliding surface will be re-written as:

$$s=\lambda x_1 + x_2$$

And for $\lambda = 1$ the sliding surface will be as:

$$s = x_1 + x_2 \tag{2}$$

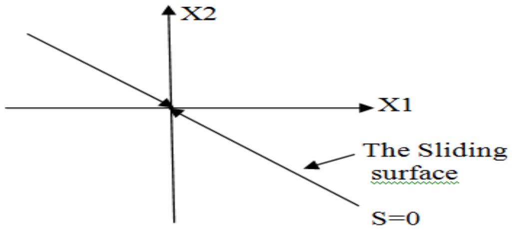

Figure 1. The sliding surface in state space for $\lambda = 1$.

The idea is to keep $s(x,t)$ near zero, in the phase plane, and to derive the system's state trajectory to sliding surface, $s(x,t) = 0$, if it is outside the sliding surface.

The control law of sliding mode controller can be described as:

$$u = u_{eq} + u_{dis} \tag{3}$$

Where, u_{eq} represent the equivalent part of SMC and u_{dis} is the discontinuous part.

The u_{dis} is defined as;

$$u_{dis} = -k.\ sign(s) \tag{4}$$

Where k is constant and its value is $k > 0$, where the $sign(s)$ function can be described and defined as;

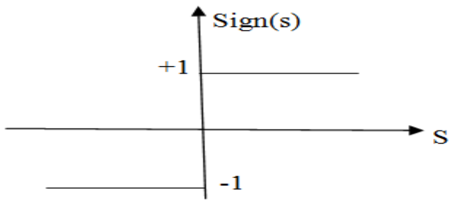

Figure 2. Sign(s) function.

$$sign(s) = \begin{cases} +1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \tag{5}$$

Therefore equation (3) can be re- written as

$$u = u_{eq} - k.\ sign(s) \tag{6}$$

3. MODIFIED SLIDING MODE CONTROL

To reduce the chattering phenomenon, the modified sliding mode controller will be used. The new controller is achieved when using the boundary layer function $sat(s)$ instead of the $sign(s)$ function. The $sat(s)$ function is described as;

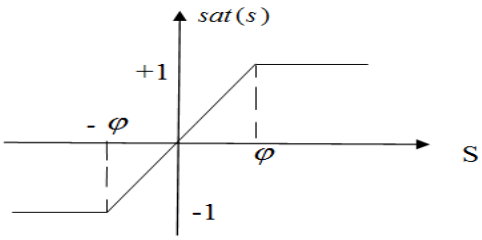

Figure 3. The $sat(s)$ **function**.

$$sat(s/\varphi) = \begin{cases} +1 & (s/\varphi > 0) \\ s/\varphi & (-1 < s/\varphi < 1) \\ -1 & (s/\varphi < 0) \end{cases} \tag{7}$$

Where φ is the thickness of the boundary layer

$$u_{dis} = u_{sat} = -k.\ sat(s/\varphi) \tag{8}$$

By substituting **Eq. (8)** in **Eq. (3),** the total control will be described as;

$$u = u_{eq} + u_{sat} \tag{9}$$

$$\therefore u = u_{eq} - k.sat(s/\varphi) \tag{10}$$

4. FUZZY LOGIC CONTROL

In 1960, the control science made use of the fuzzy logic theory to design a powerful fuzzy logic controller to control nonlinear systems which suffer from parameters uncertainty and nonlinearity**.** In many applications, when using pure fuzzy controller, the stability cannot be guarantee and the performance may be unacceptable. The fuzzy logic controller (FLC), used in this work, is based on Mamdani's method and consists of many stages as described below;

4.1 Fuzzification: In this stage the inputs and outputs must be determined firstly and then selecting the suitable membership function (MF) according to this input and output.

4.2 Fuzzy rule base: The rules in this stage consist from two parts, the antecedent and the consequent. The antecedent contains inequality or suitable relation that must be satisfied. Satisfying the antecedent will give the consequent. This point is exemplified as follows. If A is satisfied, then the output is B. Where; (A) and (B) are the antecedent and the consequent respectively.

4.3 Aggregation of the rules: It is the process of obtaining the total conclusion from the consequents that come from each rule.

4.4 Defuzzification: This is the final stage, where, the fuzzy output set is converted to crisp output value. There have been many defuzzification methods introduced; one such was used in this work, and is called the center of area (COA).

5. SLIDING MODE FUZZY CONTROL (SMFC)

SMC is a strong mathematical tool which can be considered as a robust nonlinear controller with acceptable performance. This controller can be used in nonlinear systems with parameters uncertainty. However, pure SMC is suffering from chattering problem which is undesired properties. For this reason, the present work focuses on combining fuzzy logic with sliding mode controllers to obtain a new structure; called SMFC of better performance (small settling time, fast response, and with no oscillation). Our main task is to find a suitable control law, for system's output, capable of tracking reference trajectories. The structure of the SMFC is consisting of two parts as explained bellow:

1: The SMC: This part has error (e) as its input and u_s as its output.

2: The fuzzy controller: This part has one input (s) and one output (u_{fuzzy}). The input, s , is come from the output of the sliding mode controller. The membership functions of the fuzzy controller are illustrated in **Fig. 4** bellow:

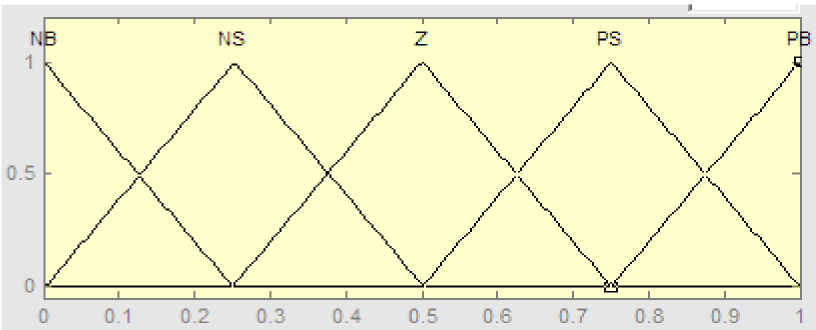

Figure 4. The membership function of the fuzzy controller

Where; *NB, NS, Z, PS, PB* are linguistic terms of antecedent fuzzy set. They mean negative big, negative small, zero, positive small, and positive big, respectively. A general form can be used to describe the fuzzy rules as it shown below:

$$if\ \ S\ is\ A_i,\ then\ U_f\ \ is\ B_i,\ i=1,........,\ 5 \tag{11}$$

Where A_i represent the fuzzy triangle-shaped number and B_i represents the fuzzy singleton.

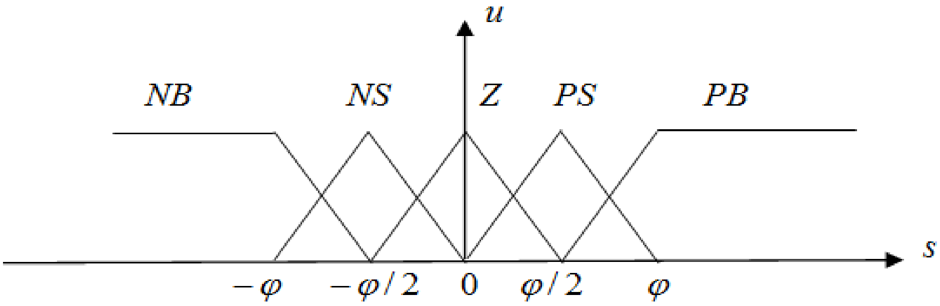

Figure 5. The input membership function of the sliding mode fuzzy controller

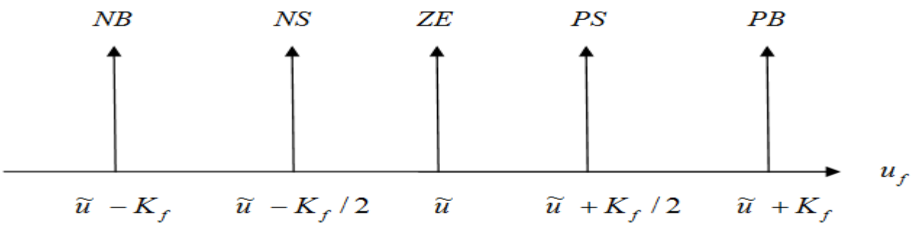

Figure 6. The output membership function of sliding mode fuzzy controller

From **Fig. 5 and Fig. 6,** it can be concluded that for the sliding mode fuzzy controller

$$u=\tilde{u}-k_f.sig(s/\varphi) \tag{12}$$

$$sig(a)=\begin{cases} +1\ if\ \ a\geq 1 \\ a\ \ if\ \ -1\langle a\langle 1 \\ -1\ if\ a\leq -1 \end{cases} \tag{13}$$

From above, it can be concluded that the control signal in the sliding mode fuzzy controller in **Eq. (12)** and the modified sliding mode controller in **Eq. (10)** are completely the same. In the design of SMFC, the membership function for the input and output of the fuzzy controller part can be found after making use of the modified sliding mode controller. In **Eq. (12)** for the SMFC, the center of the fuzzy output $\tilde{u}$ and the gain k_f can be substituted by u_{eq} and k respectively of **Eq. (10)** for the modified sliding mode controller. So, completely stability and robust can be ensured for the fuzzy controller part in SMFC. The total controller of the SMFC will, therefore, be described as:

$$u_{total} = u_{sliding} + u_{fuzzy} \tag{14}$$

Where $u_{sliding}$ is defined in **Eq. (10).** So **Eq. (14)** can be re-written as:

$$u_{total} = u_{eq} - k.sat(s/\varphi) + u_{fuzzy} \tag{15}$$

6. SINGLE INVERTED PENDULUM

The proposed Sliding Mode Fuzzy Control was implemented to single inverted pendulum systems. The position of such system is widely used in engineering systems. The main advantage of using this system is its ability for high tracking, fast response, no overshoot, and high robustness. The dynamic equation of single inverted pendulum can be given as in **Wang, Wu, 2009**.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g\sin x_1 - mlx_2^2\cos x_1 \sin x_1/(m_c+m)}{l(4/3 - m\cos^2 x_1/(m_c+m))} + \frac{\cos x_1/(m_c+m)}{l(4/3 - m\cos^2 x_1/(m_c+m))}u(t) + d(t)$$

Where x_1 is the angular position, x_2 is the velocity, $g = 9.81m/s^2, m_c$ is the mass and $m = 0.1kg,\ l = 0.5m$ is a half length, u is the control input and $d(t)$ is the external disturbance.

In this work, it is assumed that $d(t) = 0,$ and the initial condition is $x(0) = [pi/8 \quad 0]^T$

7. SIMULATION RESULTS

In this section two sets of results are obtained. The first is obtained by using the classical sliding mode controller described in **Eq. (6).** The second set is obtained by using the proposed approach "the sliding mode fuzzy controller" described in **Eq. (15).** In both cases, it is assumed that $k = 10$. The figures from **Fig. 7** to **Fig. 11** are belonging to the classical SMC, where the figures from **Fig. 12** to **Fig. 16** are belonging to the proposed SMFC. The results in SMFC are found by assuming $\varphi = 1.$

8. DISCUSSION

In this paper, two controllers are used for testing the single inverted pendulum. The first is the classical sliding mode controller; described in **Eq. (6)** and the second is the sliding mode fuzzy controller described by **Eq. (15)**; to reduce the chattering associating the classical SMC.

Fig. 9 shows the undesired chattering which appears clearly in the classical SMC. This undesired chattering was highly reduced after using the proposed SMFC method as it is shown clearly in **Fig. 14.** In classical SMC the chattering is illustrated because the system state hits the sliding surface vertically as it is shown in **Fig. 11**. When using the proposed method SMFC, the system state hits the sliding surface approximately in an arc shape as shown in **Fig. 16.** As a result, the chattering was reduced. Also in classical SMC, it is noticed clearly that the error in **Fig. 7** and the derivative of error in **Fig. 8** are reached zero value in steady state. This is also appeared clearly when plot the phase plane between x_1 and x_2 in **Fig. 11.** These zero values in error and derivative of error means that the system is asymptotically stable, and this is considered as important properties of the SMC. The same result is illustrated when using the proposed method SMFC as it is shown clearly in **Fig. 12, Fig. 13,** and **Fig. 16.** These results of both the classical SMC, and the proposed method SMFC, lead us to conclude the ability of both types of controllers to force the system to be asymptotically stable when they are used with it.

9. CONCLUSION

The obtained results show an improvement in the response of the proposed SMFC. The chattering usually appear in the classical SMC has been reduced as it is seen clearly when comparing between **Fig. 9 and Fig. 14.** Also from the above results, it can be seen clearly the ability of both controllers, the classical SMC and the proposed SMFC, to make the system in asymptotically stable case by making the error and the derivative of error at zero value as it shown clearly in **Fig. 7, Fig. 8,** and **Fig. 11** in classical SMC and **Fig. 12, Fig. 13,** and **Fig. 16** in proposed SMFC**.** This is an important property that is associated with the SMC and SMFC.

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Table 1. Table of fuzzy rules for **Fig. 5 and Fig. 6** in SMFC system

S	*NB*	*NS*	*Z*	*PS*	*PB*
U_f	*PB*	*PS*	*ZE*	*NS*	*NB*

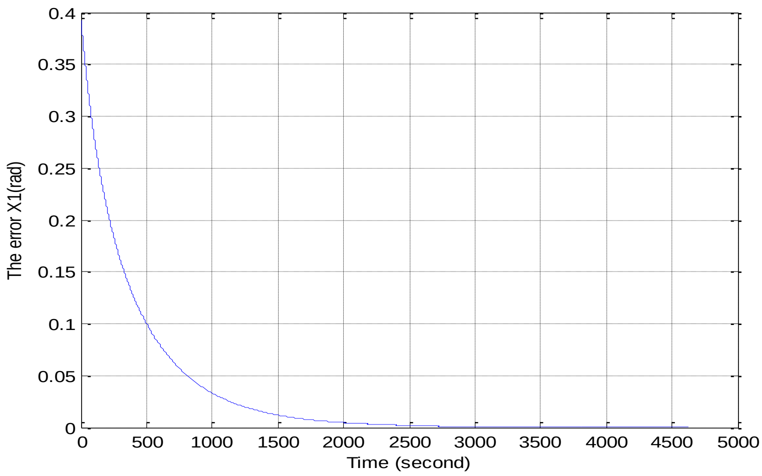

Figure 7. The error x_1 vs. time in classical SMC.

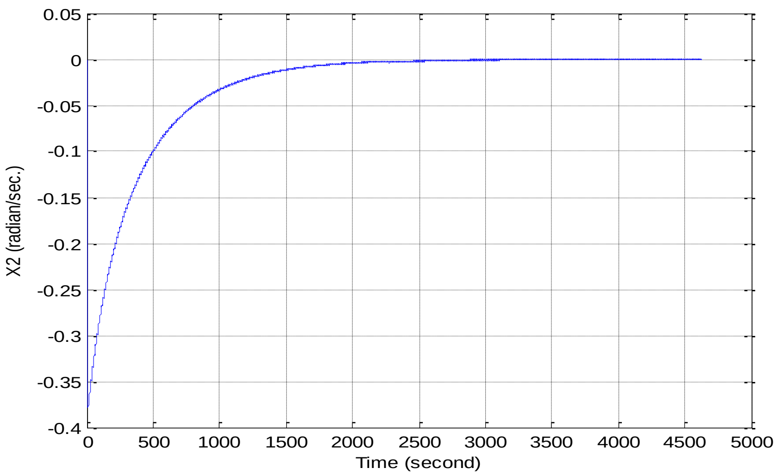

Figure 8. Plot of x_2 vs. time in classical SMC.

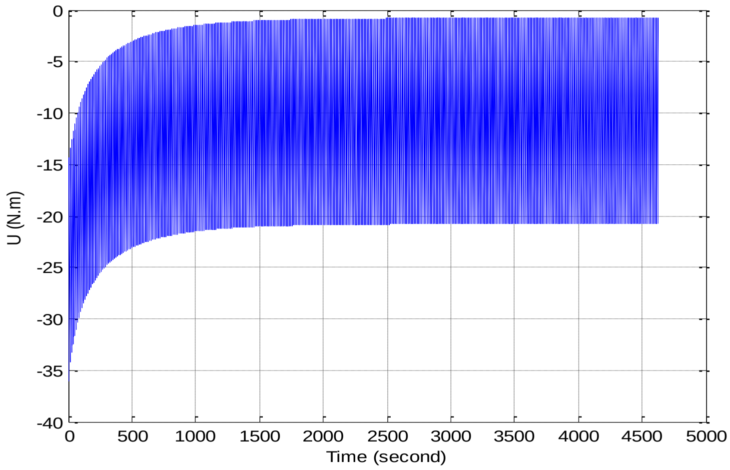

Figure 9. The control action U vs. time in classical SMC**.**

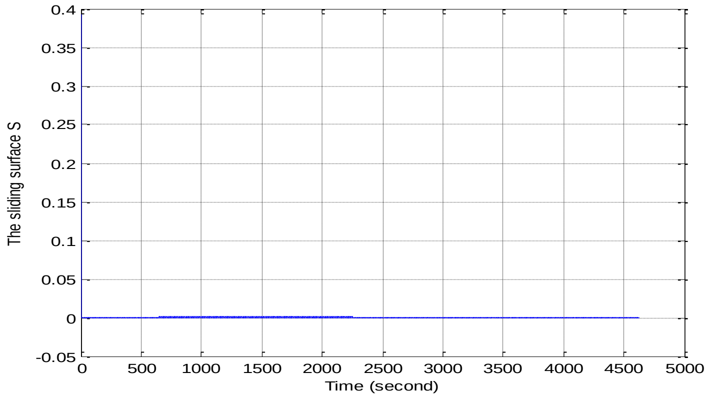

Figure 10. The sliding surface S vs. time in classical SMC**.**

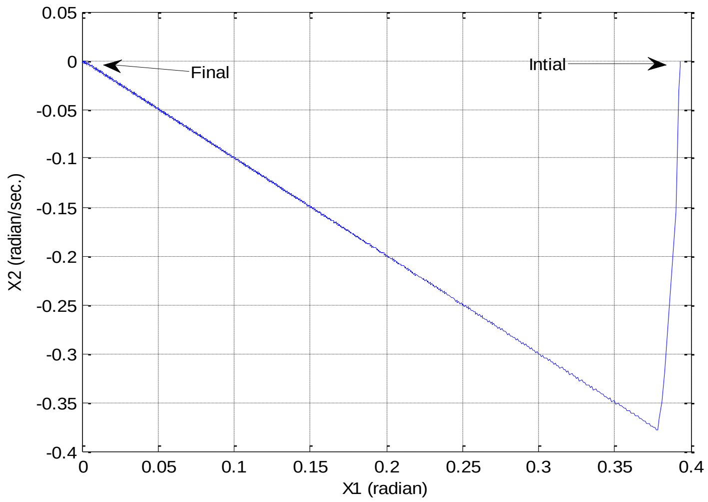

Figure 11. The phase plane between x_2 and x_1 in classical SMC.

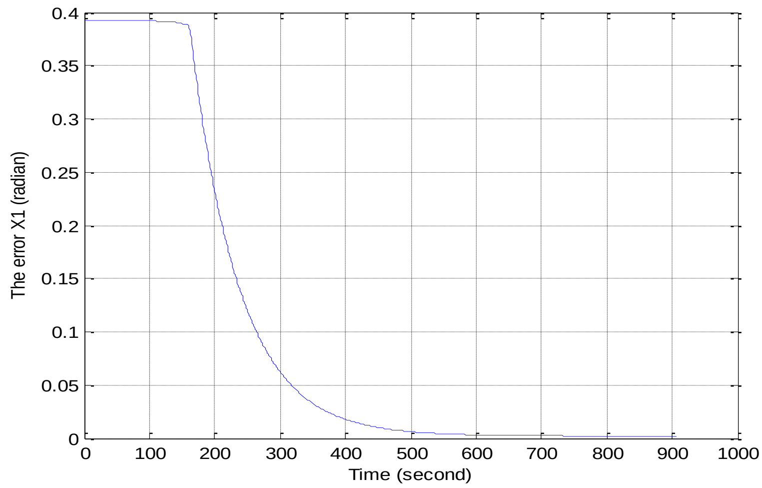

Figure 12. Plot of the error x_1 vs. time in SMFC.

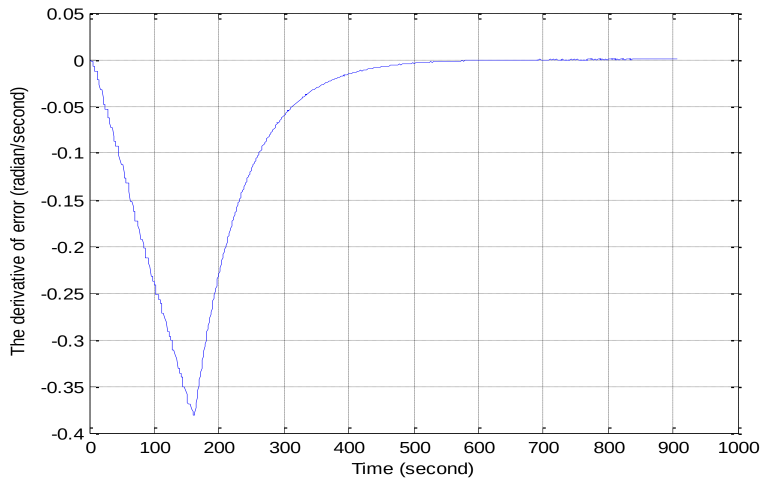

Figure 13. Plot of x_2 vs. time in SMFC.

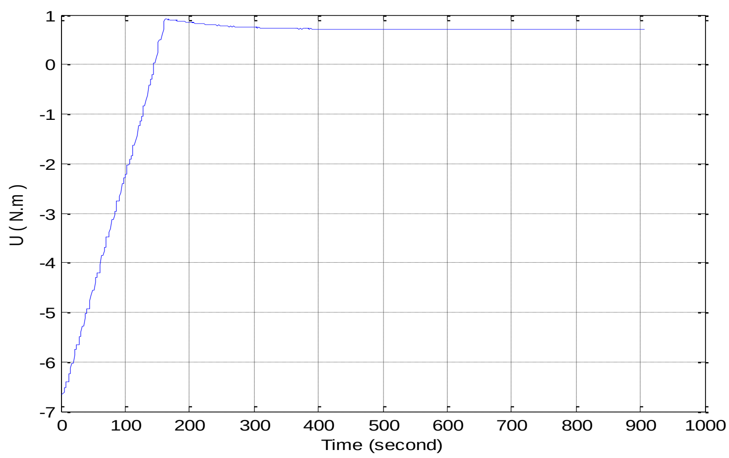

Figure 14. The control action U vs. time in SMFC.

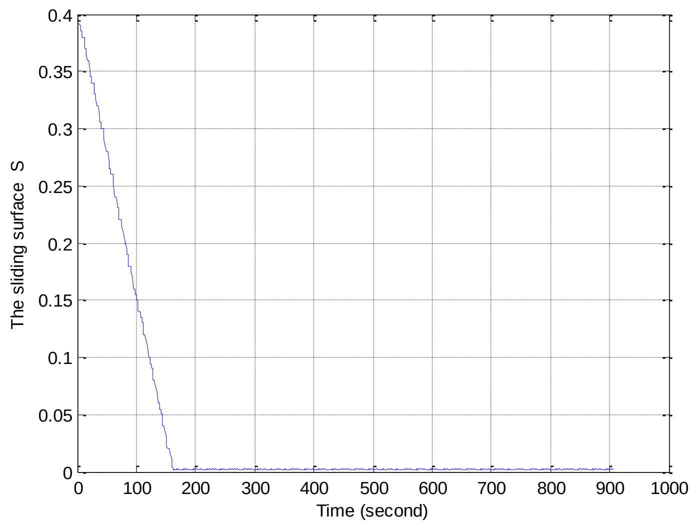

Figure 15. The sliding surface S vs. time in SMFC.

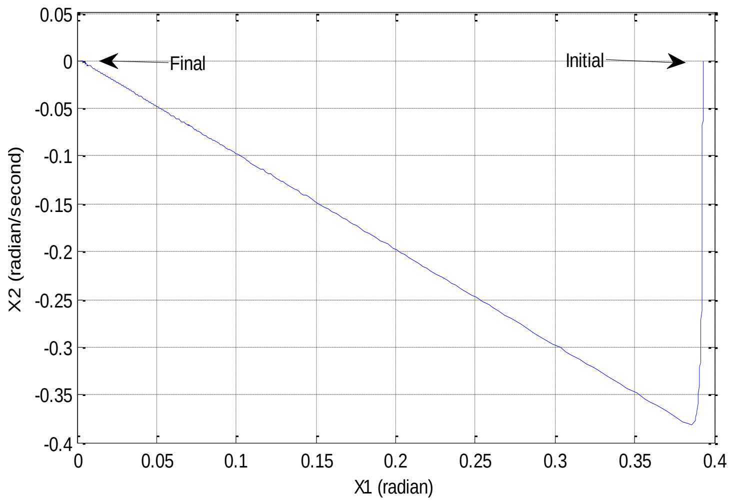

Figure 16. The phase plane between x_2 and x_1 in SMFC.