ANALYTICAL AND NUMERICAL STRESS ANALYSIS OF THICK CYLINDER SUBJECTED TO INTERNAL PRESSURE

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ABSTRACT

The present work is an attempt to investigate the vibrations characteristics and effect of static stresses and deformation in partially pressurized thick cylindrical shells, such as the gun barrels. The method used cover analytical investigation developed to determine static stresses and deformation along the thick cylindrical shell using LAME'S equation. The numerical investigation is developed using the finite element method with axisymmetric element (Plane 42) four nodes to determine the static response and solid element (Solid 45) eight nodes for vibration analysis by using the ANSYS package. The obtained results show a good agreement with the other investigators. It's found that the natural frequency of the selected models almost equal (150. Hz) and these results indicate that the frequency of powder gasses pressure more than (150 Hz) to be far away from resonance phenomena.

الخلاصة

أن العمل الحالي هو محاولة لمعرفة الخصائص الاهتزازية و تأثير الاجهادات و التشوهات الاستاتيكية في الاسطوانات سميكة الجدران كما هو الحال في سبطانة المدفع. وتشمل الطرق المستخدمة, التقنيات التحليلية و العددية, طور الحل التحليلي لإيجاد الاجهادات و التشوهات الاستاتيكية على طول النماذج المستخدمة باستعمال معادلة لامي (LAME'S EQUATION), تم انجاز الحل العددي باستخدام طريقة العناصر المحددة و اختيار العنصر المسطح (Plane 42) ذو الأربع عقدة لإيجاد الاستجابة الاستاتيكية, و العنصر الصلب (Solid 45) ذو ثمانية عقد لحساب الاهتزازات الحرة و باستخدام برنامج حاسبة (ANSYS). بينت النتائج التي تم الحصول عليها تطابقا جيدا مع نتائج الباحثين الآخرين. لقد وجد بأن التردد الطبيعي للنماذج المستخدمة (150 Hz) تقريبا وتبين هذه النتيجة بان تردد ضغط الغازات يجب أن يكون اكبر من (150 Hz) لتكون بعيدة عن ظاهرة الرنين.

KEY WORDS
Thick Cylinder, Barrels, FEM, Static Stresses, ANSYS Package, Vibration.

INTRODUCTION

The gun barrel loaded by the pressure of powder gasses has a dynamic characteristics deformations and stresses which do not depend on the value of the pressure of powder gasses only but also on the velocity of loading. This means that they depend on the jumps of pressure from the loaded points to unloaded points and on the barrel wall stiffness. The solution of barrel strength as a dynamic problem is a very complex problem and can be solved by a new progressive mathematical solution like finite element computations.

The static and dynamic behavior of the gun barrel are obtained by considering the barrel as a perfectly asymmetrical, and for the actual problem, the following assumptions are taken into consideration **(Mahmood, 1995).**

1. The barrels are two types.
a- Thin-walled pressure vessels.
b- Thick-walled pressure vessels.
2. Material of barrel wall is homogenous and isotropic.
3. Barrel is loaded by uniformly distributed pressure which is the pressure of powder gasses affecting the barrel as continuously uniformly distributed pressure on the whole barrel length.

There is a great amount of work has been down towards the investigation of the circular cylindrical shells under static load. The static stresses and deformation for a thick hollow cylinder under the affect of partially pressure are studied and reported in **(Mahmood, et al, 1990)**. The study of the static stresses and deformations a long the inner surface of (152 mm) gun-barrel are reported in **(Mahmood, 1995)** in which the effect of variation of powder gas pressure and driving band pressure a long the axial length of gun barrel were considered. The free vibration of the thick circular cylindrical shells and rings are discussed in **(Singal and Williams, 1992)**. The well knows energy method which is bases on the three-dimensional theory of elasticity was used in the derivation of the frequency equation of the shell. This yields resonant frequencies for all the circumferential modes of vibration including the breathing and beam-type modes. A semi analytical finite element was employed in **(Ganesan and Sivadas, 1994)** for the investigation of vibration behavior of cantilever homogeneous isotropic circular cylindrical shells with variable thickness, the thickness varies in the axial direction; the mass of the shell is made constant for particular length to radius ratio.

The dynamic behavior of steel cylindrical shell panels subjected to air-blast loading has been investigated in **(Redekop, 1990),** a combined theoretical and numerical solutions where obtained, results where computed for cases of rectangular and square panels having hinged and immovable boundary condition, a comparison of the study was presented for several panel rise cases, and conclusions are drowning. The work done in **(Weingarten and Fisher, 1982)** concerned the transient response of a homogeneous, hollow, conical frustum loaded by an ax symmetric time dependent lateral pressure, mode shapes and eigenvalues calculated in the free vibration analysis were used in a modal solution for the transfer displacement, stress resultant, stress couple, resultant and meridian stress of the forced vibration problem. Numerical results were presented for fully clamped frustum subjected to an instantaneous impulsive loading. A solution was presented in **(Wiliam, 1972)** for a semi-infinity cylinder shell subject to dynamic loading at one and using the method characteristics, explicit results were obtained for the propagation of discontinuities, these results were combined with a simple numerical reduce to obtain the solution in all region.

ANALYTICAL SOLUTION:

The problem of determining the tangential stress (σ_t) and the radial stress (σ_r) at any point of a thick walled cylinder in terms of the applied pressures and the dimensions was solved by the French electrician Gabriel Lame in (1833). The cylinder shown in **Fig. 1** has radii (a) and (b) subjected to both a uniformly distributed internal pressure of (P_i) and an external pressure of (P_o).

Select a thin shell of radius (r), the thickness (dr) and the length unity. The tangential stress in this shell is (σ_t), the radial stress on the inner surface is (σ_r) and that on the outer surface is $(\sigma_r + d\sigma_r)$, where $(d\sigma_r)$ is the increment in (σ_r) due to the variation of pressure across the cylinder wall. The radial stresses are assumed (in correctly) to be tensile, so a negative result for (σ_r) will denote compression. This shell may be treated as a thin cylinder, hence, for equilibrium, a vertical summation of forces must equal zero **(Nasman, 2005)**, **Fig. 2** shows a half-section of a typical shell:

$$(\sigma_r + d\,\sigma_r).2(r + d\,r) - \sigma_r\,(2r) - 2\sigma_t\,dr = 0 \tag{1}$$

The final equation obtained from solving Eq. (1) give the following general expression for (σ_r) and (σ_t) at any point **(Nasman, 2005):**

$$\sigma_r = \frac{a^2 P_i - b^2 P_o}{b^2 - a^2} - \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2},\ \sigma_t = \frac{a^2 P_i - b^2 P_o}{b^2 - a^2} + \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2} \tag{2}$$

Under the effect of internal pressure, the deformation of the cylinder walls takes place that results in stresses in the cylinder metal **(Nasman, 2005)**. Every wall element limited by adjacent radial and concentric circular sections is subjected to circular and axial extension and to radial compression as shown in the **Fig. 3**.

The wall deformation at r=a:

$$\Delta r_i = \frac{a}{E}\left(P.\frac{b^2 + a^2}{b^2 - a^2} + \upsilon \right) \tag{3}$$

Similarly at r=b;

$$\Delta r_o = \frac{b}{E}\left(P\frac{2a^2}{b^2 - a^2} \right) \tag{4}$$

THE FINITE ELEMENT SOLUTION:

The finite element method can be applied for any kind of problem including the investigated problem because it is a powerful method and it's almost easy to be implemented by the computer, therefore many packages may be found which deal with this kind of problem. One of these packages is the ANSYS which is a powerful package and can deal with the static and dynamic problems with a merely simple input data in files stored in files and from these data the results are obtained.

For the static analysis the element Plane 42 is used. This element used for 2-D modeling of solid structures. The element can be used either as a plane element (plane stress or plane strain) or as an axisymmetric element. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal X and Y directions as shown in **Fig. (4)**.
But for the free vibration analysis the Solid 45 element is used. This element used for the three-dimensional modeling of solid structures. The element is defined by eight nodes having three degrees of freedom at each node: translations in the X, Y and Z directions as shown in **Fig. (5)**.

Static Analysis

The system equations formed for static analysis include the system stiffness matrix and the system load vector. The equations may be written in matrix relations as follows **(Batha, 1976):**

$$[K]\{U\} = \{F\} \tag{5}$$

Eq. (5) solved by Gauss iteration method.

Normal Modes Analysis

The system equation for normal mode analysis includes the system stiffness and mass matrices may be written in matrix notation as follows **(Batha, 1976):**

$$[M]\{\ddot{U}\}+[K]\{U\}=0 \tag{6}$$

$$U_i=\Phi_i \sin(\omega_i t+\theta_i) \qquad i=1,2.......DOF \tag{7}$$

In this harmonic expression, Φ_i is a vector of nodal amplitudes (mode shape) for the ith mode of vibration. The symbol ω_i represents the angular frequency of mode i, and θ_i denotes the phase angle. By differentiating **Eq. (7)** twice with respect to time:

$$\ddot{U}_i=-\omega_i^2\Phi_i \sin(\omega_i t+\theta_i) \tag{8}$$

Substitution of **Eq. (8)** and **Eq. (7)** into **Eq. (6)** allows cancellation of the term sin (ωi t+θi), which leaves,

$$([K]-\omega_i^2[M])\Phi_i=0 \tag{9}$$

Eq. (9) has the form of the algebraic eigenvalue problem. **Eq. (9)** can be solved by subspace iteration method.

Verification Test for Static Analysis:

The following example is given to show applicability of the used program ANSYS to solve the cylindrical shells subjected to pressure loading,

$$R_i=a=37.5mm, R_o=b=62.5mm, P=60\frac{MN}{mm^2}, L=651mm$$

The output result are compared with numerical solution given by **(Cook, 1981)** are show in **Table. 1**. A good agreement is obtained which proves the applicability of ANSYS to deal with the static analysis of thick cylindrical shells.

Verification Test for Normal Mode Analysis The following example is given to show the applicability of the used program ANSYS to solve the cylindrical shells:

$$R_i=a=76mm, R_o=b=114.3mm, L=250.4mm, \rho=7860\frac{Kg}{m^3}, E=207Gpas, \upsilon=0.28$$

The values of the first three natural frequencies for the cylindrical shell were compared with numerical results given by **(Cook, 1981)** are show in **Table. 2**. A good agreement is obtained which proves the applicability of ANSYS to deal with the vibration of thick cylindrical shells.

GUN BARREL GEOMETRY

The geometry of the studied gun barrel is shown in **Fig. 6** where (t) is the thickness of the barrel and the chosen thickness is between (3-8 mm) (the internal diameter (caliber) of (60.7 mm), external diameter of (69.14 mm) and its length equal to (651 mm))(. The gun barrel is fixed from one end to the base and the other side is free where the projectile is fed and fired. The internal surface of the gun barrel is coated with chrome to decrease the friction and erosion between the projectile and the surface of the gun barrel.

The pressure-length curve and pressure-time curve are shown in **Figs.7 and 8** respectively after the ignition starts **(Mahmood, 1995)**, the pressure starts to increase until it reach (5.6 M Pa) which is able to carry the projectile and start moves, then the pressure keep on increasing until its reach the maximum value (56 M Pa) after (2.23 ms) when the projectile is (80 mm) away from the base of the barrier. Then the pressure began to decrease until the projectile leave the barrier after (6.33 ms).

Model No.	1	2	3
Thickness (mm)	3	4.2	8

RESULTS

Static Results:

Model-1

Figs. (9, 10, 11 and 12) show the static radial deflection and, the radial, tangential and equivalent static stress distribution along the cylinder at different time steps.

Model-2

Figs. (13, 14, 15 and 16) show the static radial deflection and, the radial, tangential and equivalent static stress distribution along the cylinder at different time steps.

Model-3

Figs. (17, 18, 19 and 20) show the static radial deflection and, the radial, tangential and equivalent static stress distribution along the cylinder at different time steps.

Vibration Analysis:

Fig. 21 shows the optimum mesh size for the first model. The first three natural frequencies for the first, second and third models are shown in **Table. 3.** **Figs. (22, 23 and 24)** show the first three modes shape of the first, second and third models respectively.

DISCUSSION:

The static deformations, stresses, natural frequencies and mode shapes are computed for thick cylinder (three models). It has been found that the maximum static radial deflections are (0.105mm), (0.074mm) and (0.042mm) for model (1), model (2) and model (3) respectively, and for the same models the maximum equivalent stresses along the cylinder are (640 N/mm^2), (460 N/mm^2) and (245 N/mm^2) respectively. From the static results, it can be observed the increases in of the cylinder thickness leads to decreases in the stresses and deformations and that due to the higher increases in the structural stiffness. It was observed that the equivalent static stresses induced at the cylinder wall for different time steps some positions the stresses reach a maximum values at a certain time.

Also it can be noted that when the thickness increases the natural frequency increases too. The percentage increases in the fundamental natural frequency when thickness variation from (3 mm to 8 mm) is found (7 %).

CONCLUSIONS

From this analysis can be concluded:-

1. It can be noted, the stresses and radial deformations in the first model larger than the second and third models.
2. The natural frequency decreases when the thickness of the cylindrical shell decreases because reduction in structural stiffness.
3. The maximum effect of the equivalent stresses occurs between (80 and 100 mm).
4. Were obtained a good agreement between the present numerical and the present analytical results.

Table. 1 Values of tangential stress and radial stress for cylindrical shell subjected to pressure loading.

Solution Type	$\sigma_r \ (N/mm^2)$	$\sigma_t (N/mm^2)$
FEM (Cook, 1981)(	-19	86.5
ANSYS (present)	-18.35	89.1
Error (%)	3.4	3

Table. 2 Values of the first three natural frequencies for the cylindrical shell.

Solution Type	**First Mode (Hz)**	**Second Mode(HZ)**	**Third Mode(HZ)**
FEM (Cook, 1981)(	1898.1	1873.52	2988.19
ANSYS (present)	1898.1	1898.2	3013.8
Error with Exact (%)	1.0	1.3	0.85

Table.3 The first three natural frequencies of thick cylinders models.

	First Mode (Hz)	**Second Mode(HZ)**	**Third Mode(HZ)**
Model-1	150.00	150.02	864.97
Model-2	152.90	152.94	879.52
Model-3	162.40	162.45	926.34

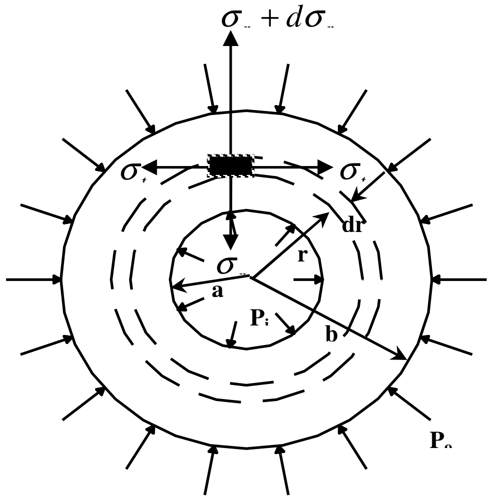


Fig. 1 Thick-walled cylinder subjected to uniform internal pressure (P_i) and external pressure (P_o).

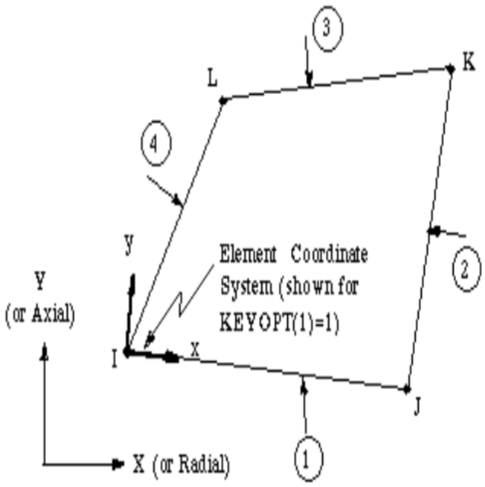


Fig. 4 Plane 42 structure solid element.

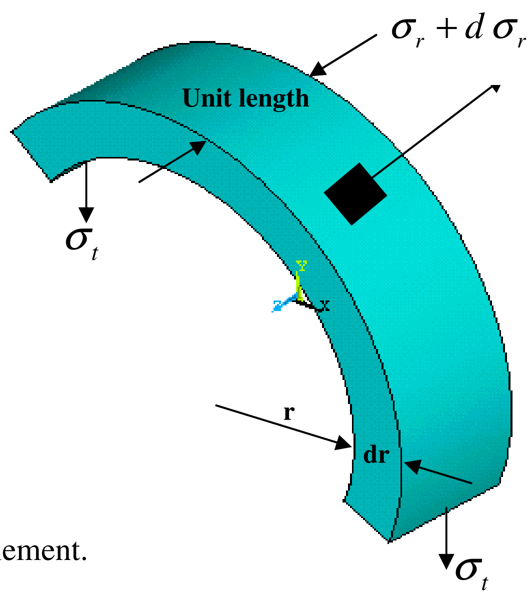
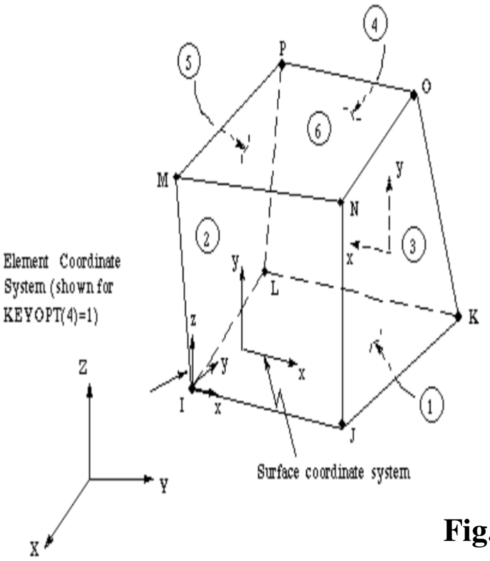


Fig. 2 Stress on half-shell cylinder.



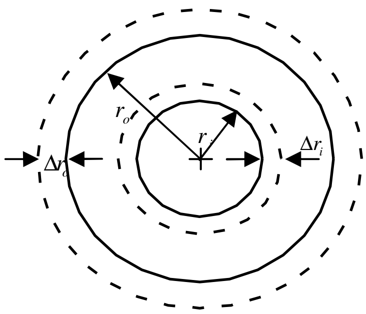


Fig. 3 Cylinder wall deformation.

Fig. 5 Solid 45 structure solid element.

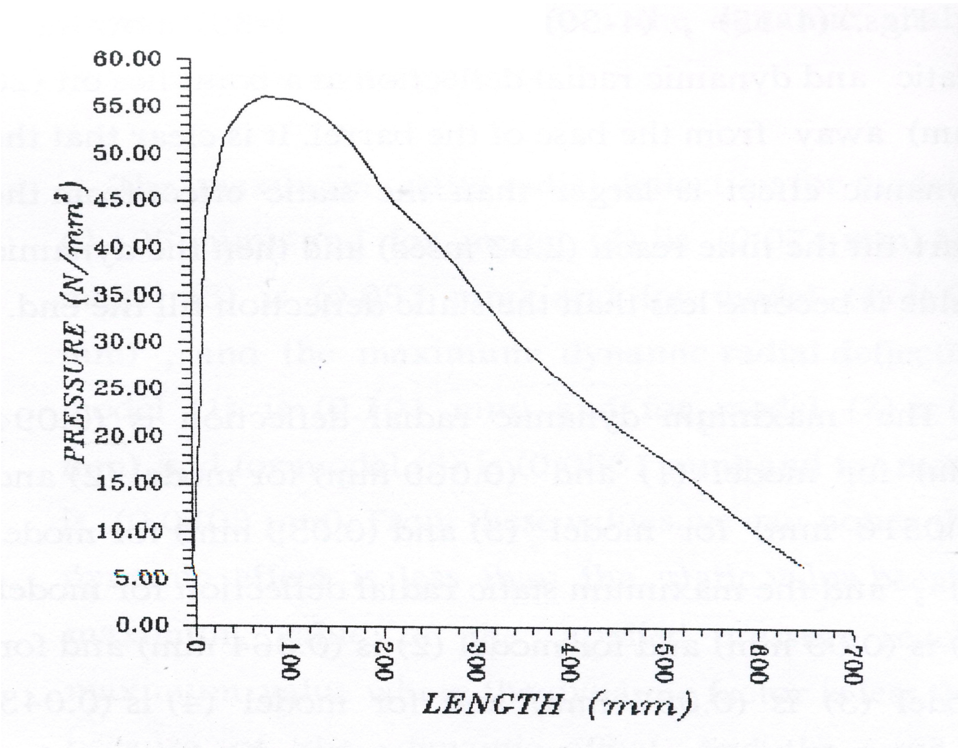


Fig. 7 Pressure Length Curve.

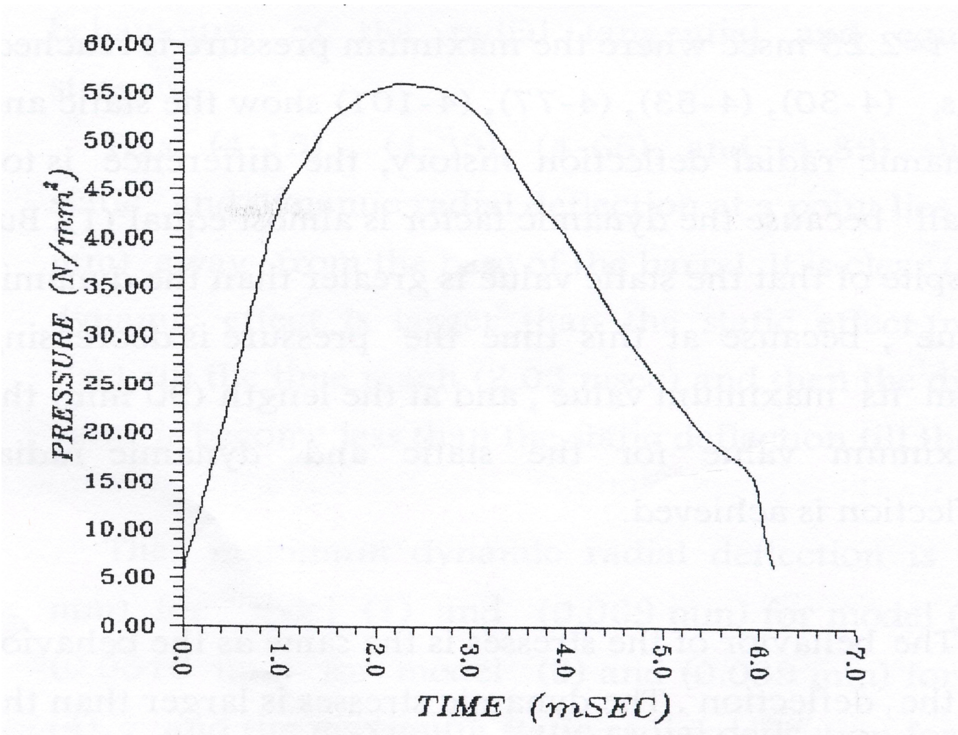


Fig. 8 Pressure Time Curve.

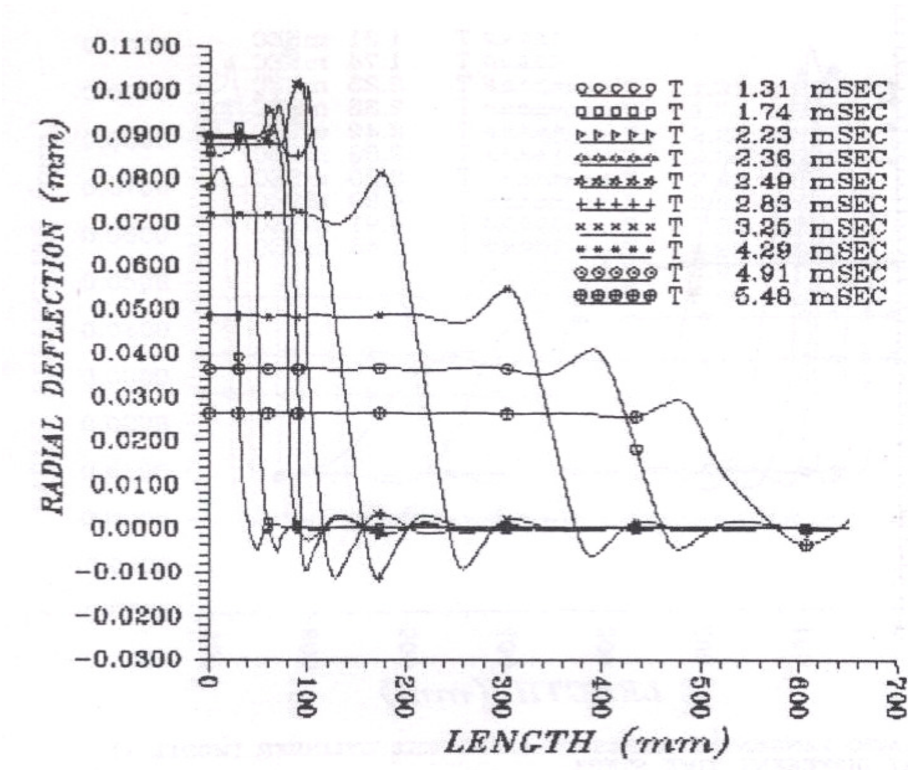


Fig.9 Static Radial Deformation along the Cylinder (Model-1) at Different Time Steps.

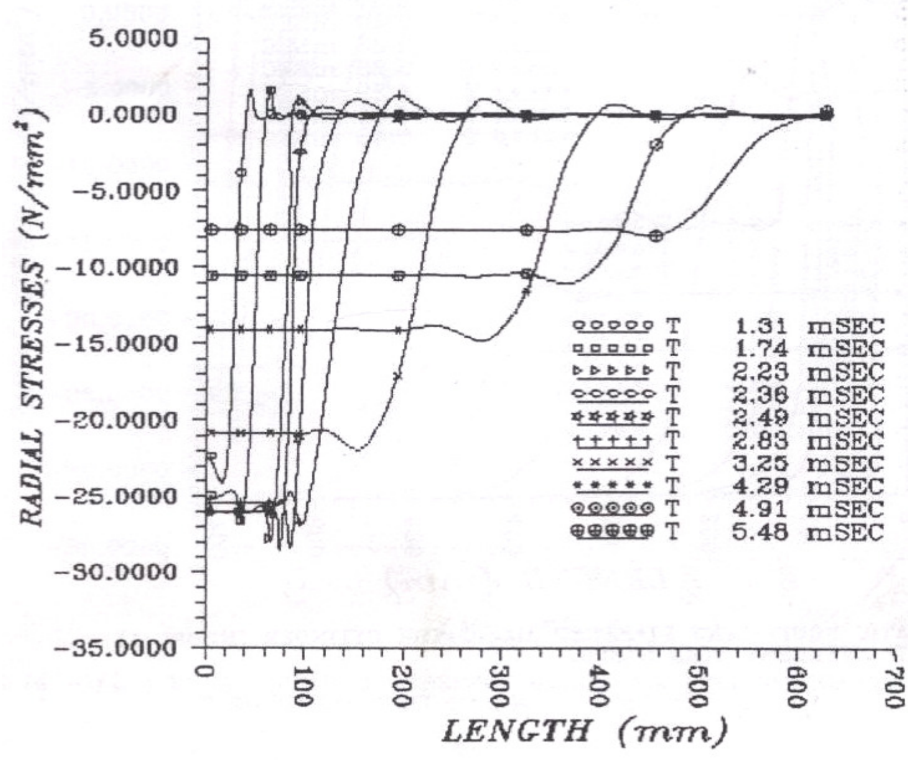


Fig. 10 Static Radial Stresses Along the Cylinder (Model-1) at Different Time Steps.

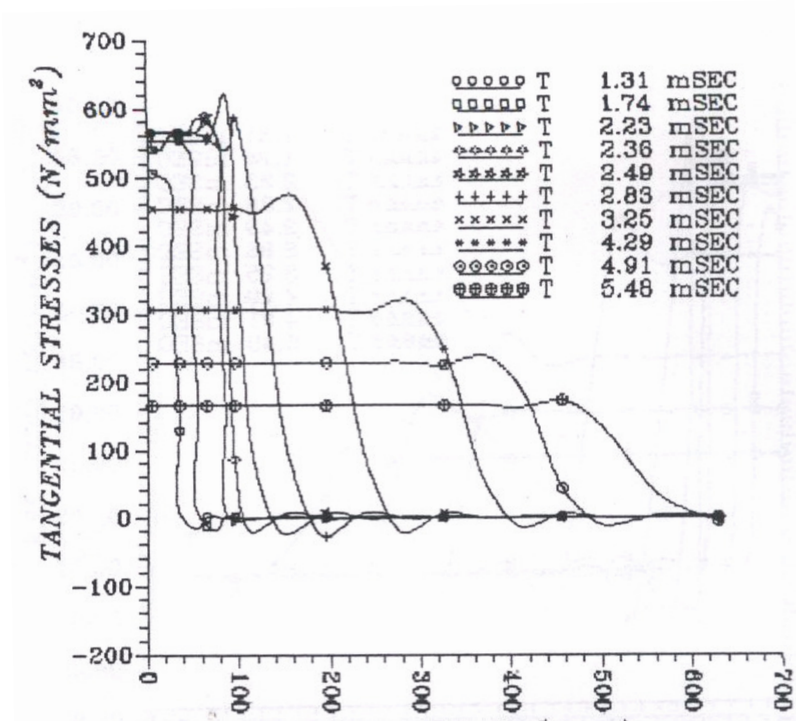


Fig. 11 Static Tangential Stresses Along the Cylinder (Model-1) at Different Time Steps.

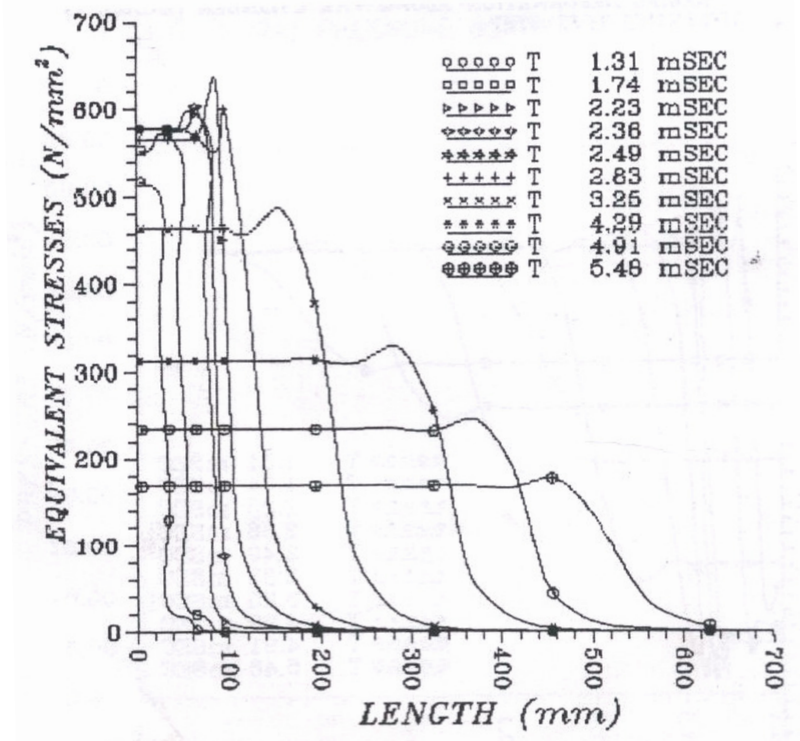


Fig. 12 Static Equivalent Stresses Along the Cylinder (Model-1) at Different Time Steps.

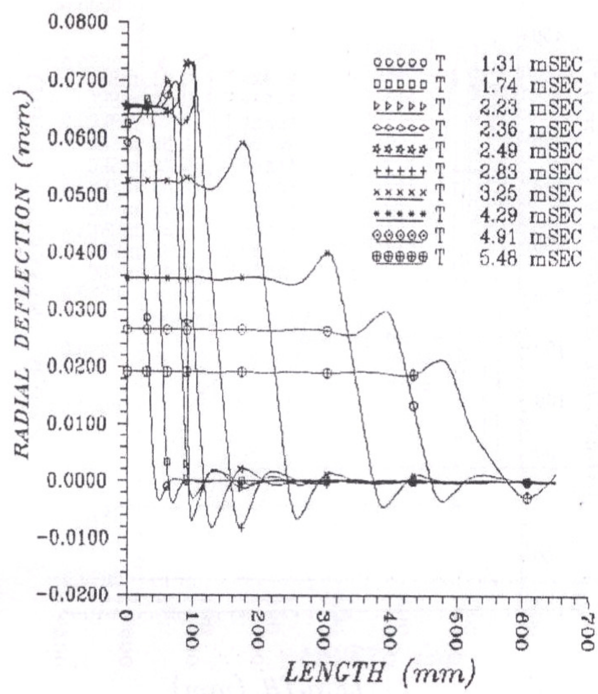


Fig. 13 Static Radial Deformation along the Cylinder

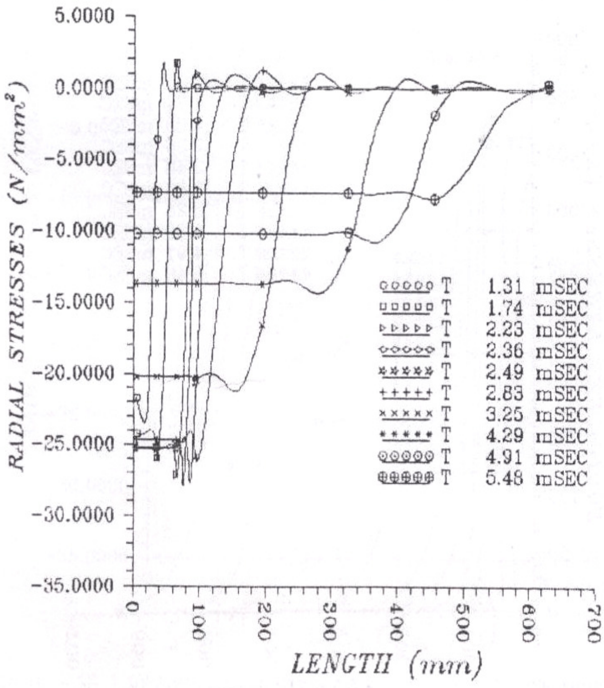


Fig. 14 Static Radial Stresses Along the Cylinder

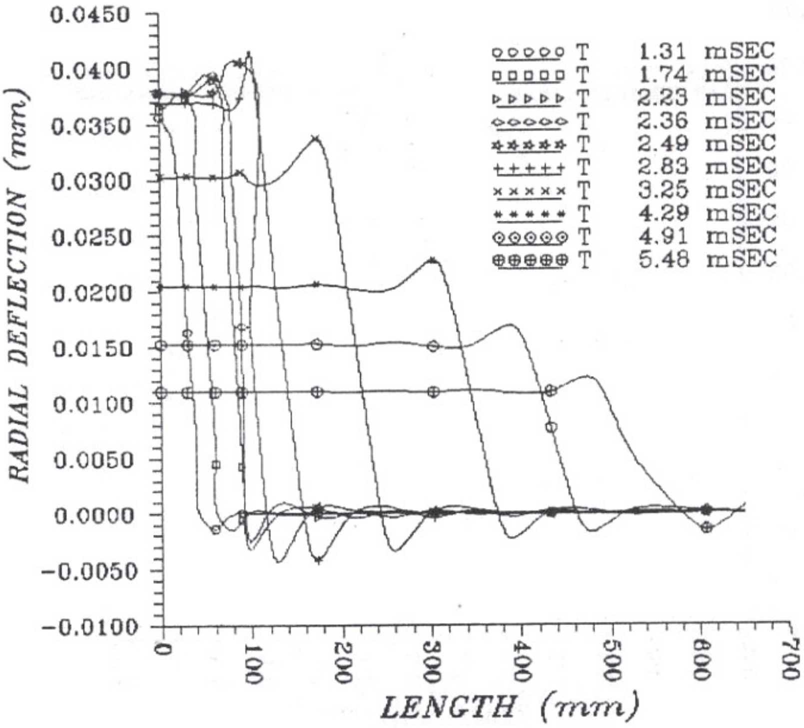


Fig. 17 Static Radial Deformation along the Cylinder (Model-3) at Different

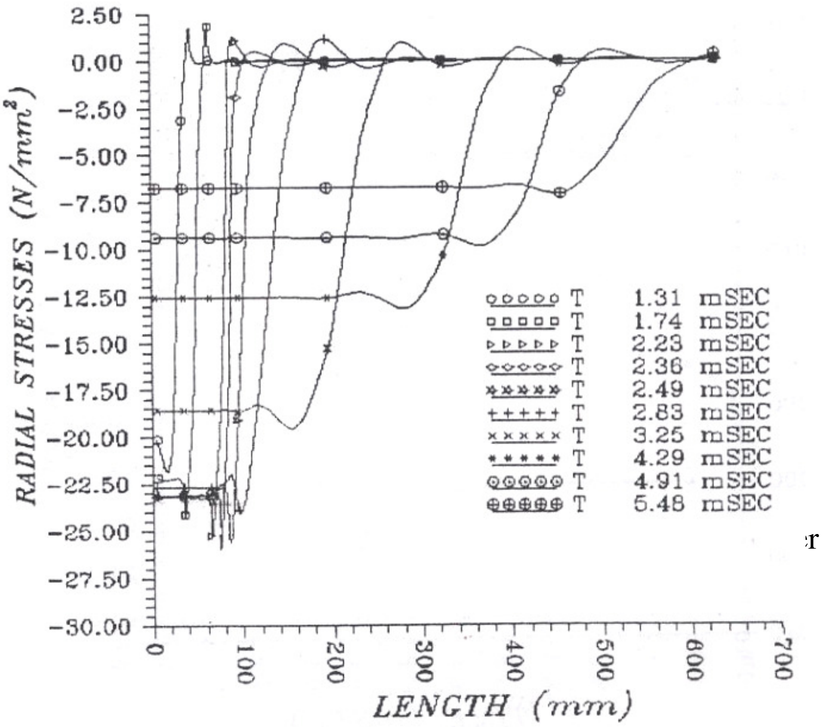
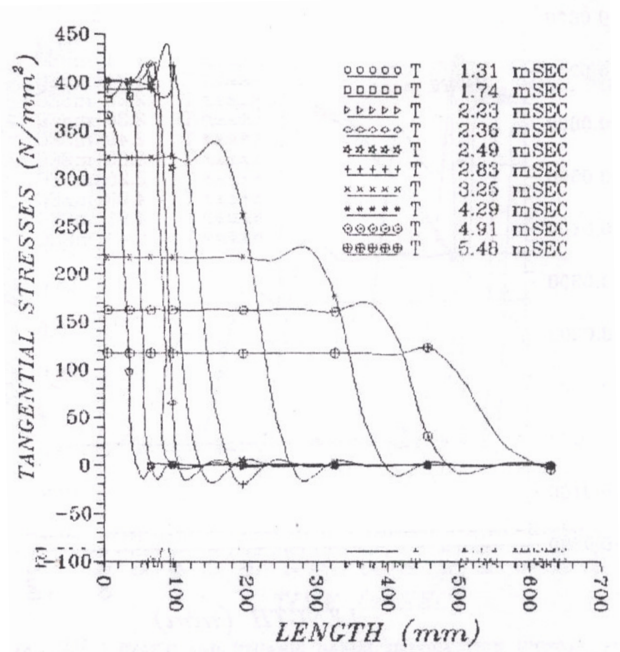
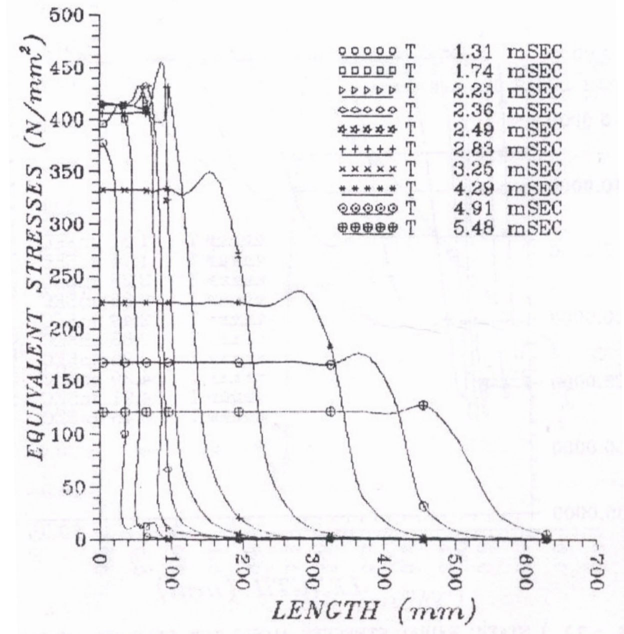


Fig. 18 Static Radial Stresses Along the Cylinder (Model-3) at Different



TANGENTIAL STRESSES (N/mm²)
LENGTH (mm)
T 1.31 mSEC
T 1.74 mSEC
T 2.23 mSEC
T 2.36 mSEC
T 2.49 mSEC
T 2.83 mSEC
T 3.25 mSEC
T 4.29 mSEC
T 4.91 mSEC
T 5.48 mSEC



EQUIVALENT STRESSES (N/mm²)
LENGTH (mm)
T 1.31 mSEC
T 1.74 mSEC
T 2.23 mSEC
T 2.36 mSEC
T 2.49 mSEC
T 2.83 mSEC
T 3.25 mSEC
T 4.29 mSEC
T 4.91 mSEC
T 5.48 mSEC

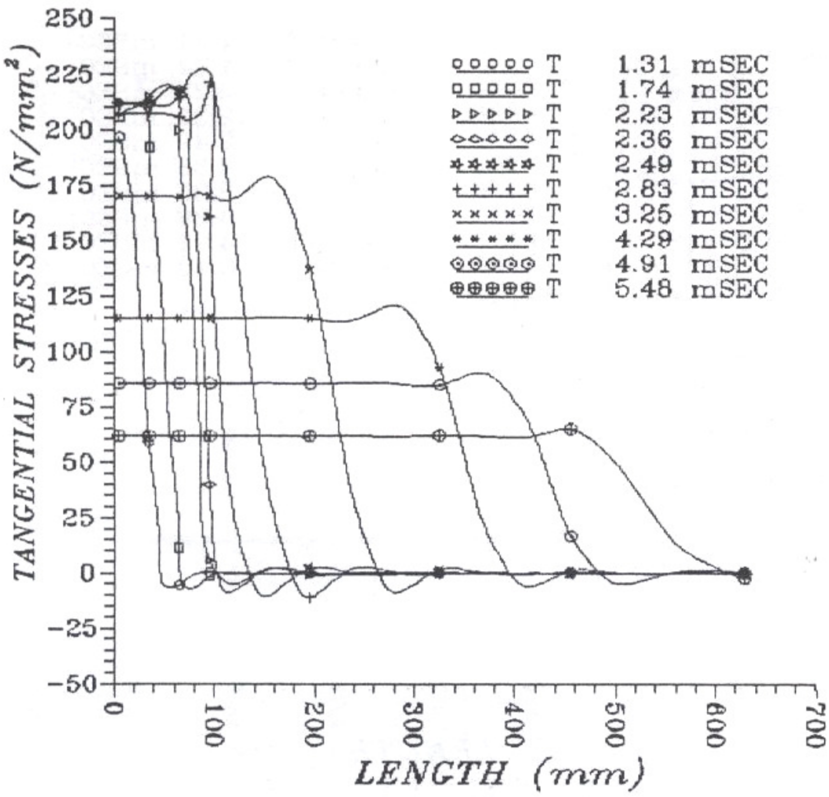


Fig. 19 Static Tangential Stresses Along the Cylinder (Model-3) at Different Time

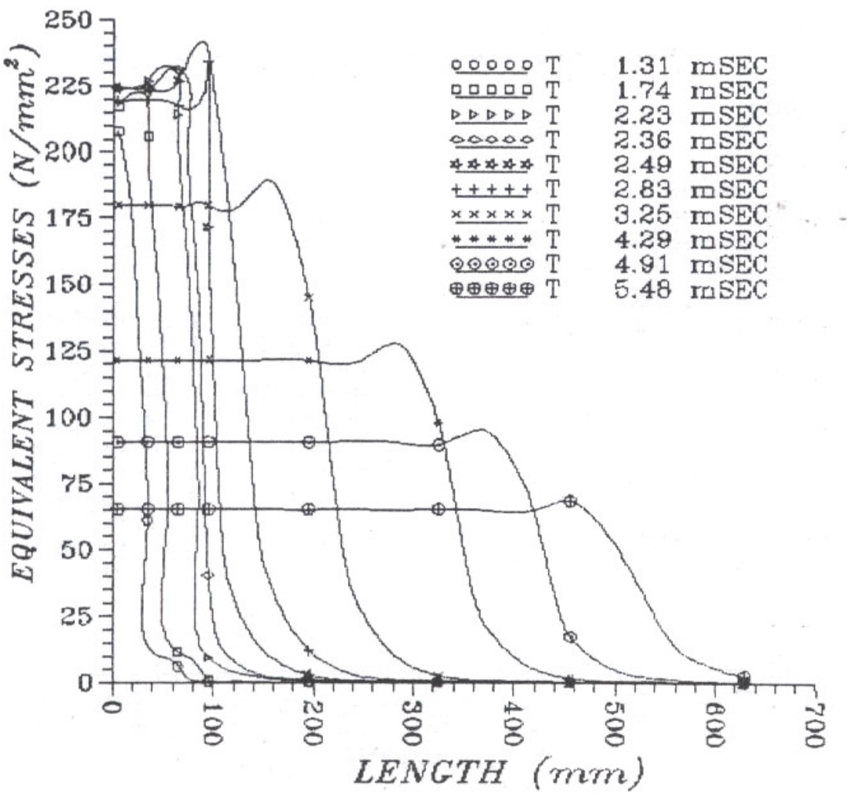


Fig. 20 Static Equivalent Stresses Along the Cylinder (Model-3) at Different Time Steps

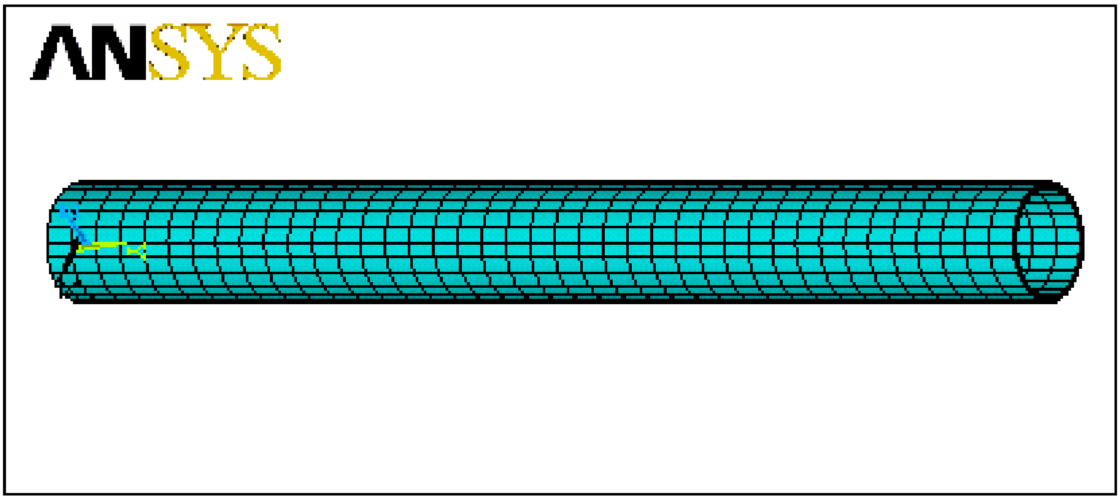


Fig. 21 Suitable mesh size for thick cylinder (Model-1).

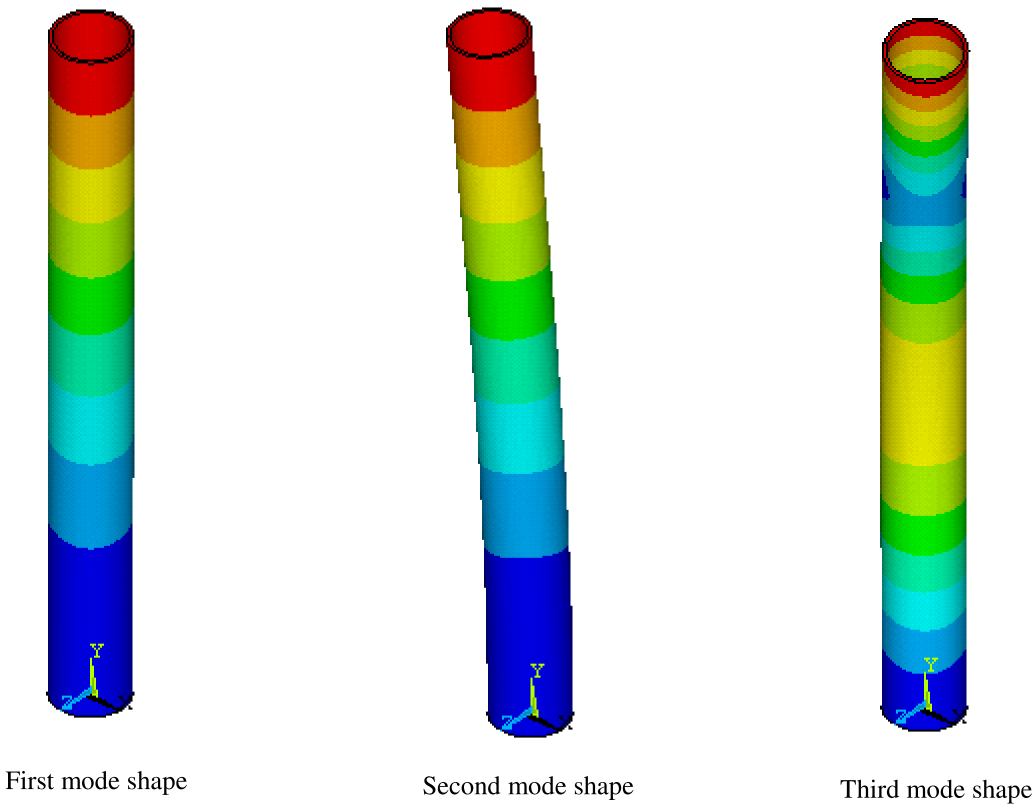
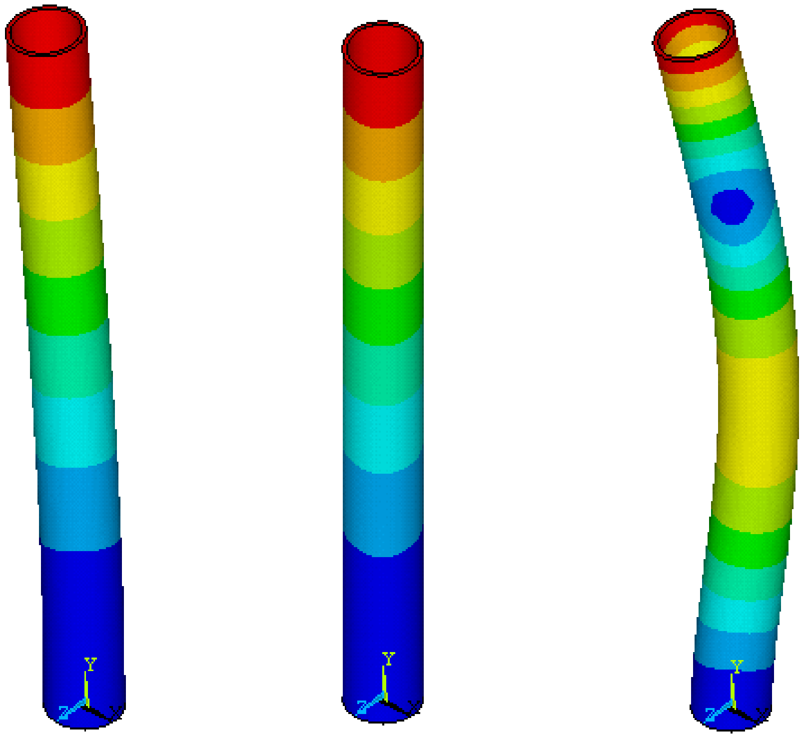
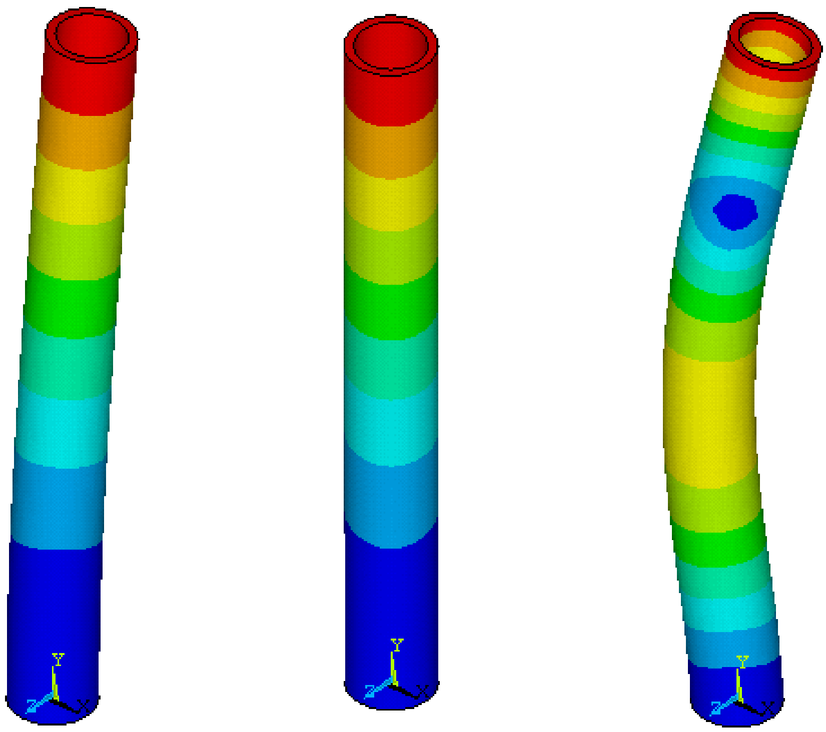


Fig 22 The first three mode shapes for the (Model-1).



First mode shape   Second mode shape   Third mode shape

Fig 23 The first three mode shapes for the (Model-2).



First mode shape   Second mode shape   Third mode shape

Fig 24 The first three mode shapes for the (Model-3).

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NOMENCLATURE:

a	Internal radius of thick cylinder (mm).
b	External radius of thick cylinder (mm).
L	Length of thick cylinder (mm).
r	Radius of thick cylinder (mm).
P_i	Distributed internal pressure (M Pa).
P_o	Distributed external pressure (M Pa).
(σ_t)	Tangential stress at any point of a thick walled cylinder N/mm^2.
(σ_r)	Radial stress at any point of a thick walled cylinder N/mm^2.
υ	Poisson's ratio.
E	Young's modulus.
Δr_i	Internal wall cylinder deformation (mm).
Δr_o	External wall cylinder deformation (mm).
$[K]$	Stiffness matrix.
$\{F\}$	Force load vector.
$\{U\}$	Displacement vector.
$\{U^{\bullet}\}$	Velocity vector.
$\{U^{\bullet\bullet}\}$	Acceleration vector.
$[M]$	Mass matrix.
Φ_i	Vector of nodal amplitudes (mode shape) for the ith mode of vibration.
ω_i	The angular frequency of mode I (rad /sec).
ρ	Density, Kg/m^3.