MATHEMATICAL SIMULATION OF FLOW THROUGH HOLLOW FIBRE MEMBRANE UNDER CONSTANT HYDRAULIC CONDUCTIVITY

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ABSTRACT

Distilled water flow through a virgin hollow fibre membrane, HFM is considered as steady nonuniform since the fibre's wall hydraulic conductivity coefficient is kept constant along the fibre and unchanged during the operation. Under these conditions, two well known laws were used to mathematically simulate the hydraulic flow through the HFM. These two laws are: the Darcy's Law, to simulate the flow thought the fibre wall, and the Hagen-Poiseuille's Law, to simulate the laminar flow through the fibre channel.

Laboratory measurements were carried out to provide necessary data for the calibration and verification of the mathematical model that was developed based on the Darcy's and Poiseuille's laws. A good agreement was obtained between the measured and predicted flowrate values under the same conditions.

The developed Mathematical model can be used as a tool to investigate the hydraulic performance of commercial HFM modules. A comparison was made between two commercially available of HFM modules of the same material but differ in the fibre sizes; it was found that there is a difference between its performance and the efficiency of the operation energy.

الخلاصة

عند استخدام ماء نقي فان الجريان في الأغشية الليفية المجوفة البكر يكون جريانا مستقرا لثبات الايصالية الهيدروليكية على امتداد الليف ولا تتغير إثناء التشغيل. في هذه الحالة أمكن استعمال قانونين معروفين لتمثيل الجريان خلال الأغشية الليفية المجوفة هما قانون دارسي لتمثيل الجريان خلال جدار الليف وقانون هايكن- بويزل لتمثيل الجريان الطباقي خلال قناة الليف.

أجريت قياسات مختبريه على الجريان خلال الأغشية الليفية المجوفة لتوفير البيانات الضرورية لمعايرة وبرهنة صحة عمل النموذج الرياضي المعد اعتمادا على قانوني دارسي وبويزل. كان التطابق جيد بين القياسات المختبرية وتلك المستنبطة من النموذج الرياضي تحت نفس الظروف.

يمكن استعمال النموذج الرياضي المعد كأداة لتحري الأداء الهيدروليكي للأغشية الليفية المجوفة التجارية. تمت مقارنة الأداء الهيدروليكي لمنتجين متوفرة تجاريا من الأغشية الليفيةمن نفس النوع مع اختلاف في ابعاد الليف ووجد إن هنالك اختلاف كبير في الأداء وفي كفاءة الطاقة اللازمة للتشغيل.

KEYWORDS : Hollow fibre membrane, Mathematical simulation, Mathematical Model

INTRODUCTION:

Hollow fibre membrane, HFM, shows a number of advantages over traditional water filtration technique which makes it attractive to potable water industry, with the high water quality produced using the HFM, which meets the stringent potable water regulations, makes the use of HFM to grow rapidly within the last decade.

Several different commercial HFM modules, used for microfiltration and ultrafiltration treatment of water, are available in the market. Even if they are made of the same material, they differ in their capacity, diameter and length of the fibre, number of fibres used, and pot length.

Many theoretical and experimental studies were carried out to evaluate the performance of the HFM and a number of mathematical models based on different hydraulic relations and simplification assumptions were developed.

The main objective of this study is to develop and verify a mathematical model to simulate the flow through the HFM under constant hydraulic conductivity based on the combination of Darcy's and Hagen-Poiseuille's Laws. The developed mathematical model with the solution procedure applied on computer is a useful tool for engineers to examine the performance of the HFM.

MATHEMATICAL SIMULATION OF THE FLOW THROUGH HFM

The mathematical simulation of steady flow through the HFM under constant hydraulic conductivity was developed based on two basic formulas governing the flow through the membrane wall and the fibre channel as described in the following sections.

Flow through HFM Wall

The flow through the HFM wall is a radial flow, **Fig. (1)**. An expression for the Darcy's law for redial flow through the wall of a fibre segment of ΔS length can be derived from the basic Darcy's formula by transforming its cartesian coordinate system into polar coordinate system, that is:

$$q = 2\pi \Delta S\, K \frac{TMB}{\ln(\frac{r_t}{r_n})} \tag{1}$$

Where
q = volumetric flowrate, L^3/T,
K= hydraulic conductivity of the porous media, L/T,
TMP = transmembrane pressure head, L,
K= hydraulic conductivity of the porous media, L/T,
r_t= fibre outer radius, L, and
r_n= fibre inner radius, L.

The hydraulic conductivity, K, is a measure of the ability of water to flow through a porous medium. It depends on the fluid properties and the porous medium properties through the intrinsic permeability.

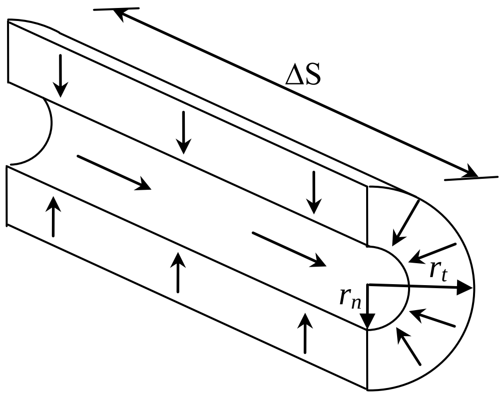

Fig. (1). Schematic diagrams of longitudinal cross sections along a HFM.

The Head Losses through HFM Channel

The head loss along the HFM channel segment can be estimated based on Poiseulle's Law for laminar flow. Poiseulle's Law in term of pressure head along a HFM channel segment of ΔS length may be written as:

$$h_l = \frac{8q\mu\Delta S}{\rho g \pi r_n^4} \tag{2}$$

In which:
h_l = head loss, L,
g = gravitational acceleration, L/T^2,
L = circular channel segment length, L,
μ= water viscosity, M/ (L.T), and
ρ = water density, M/L^3.

APPLICATION OF DARCY'S AND POISEULLE'S LAWS TO THE FLOW OF HFM

Fig. (2) shows a schematic diagram of a single HFM in an actual fibre module. The pot length, L_{pot}, is required to seal the fibres by using a special sealant of molding thermoplastic material so that all the flowrate of the module will be thought the fibre channels outlets only.

The fibre has two outlets at both ends that mean that flow of the fibre membrane module is symmetrical. Therefore, the flowrate calculations will be carried out on one half of the fibre and is doubled for actual fibre flowrate.

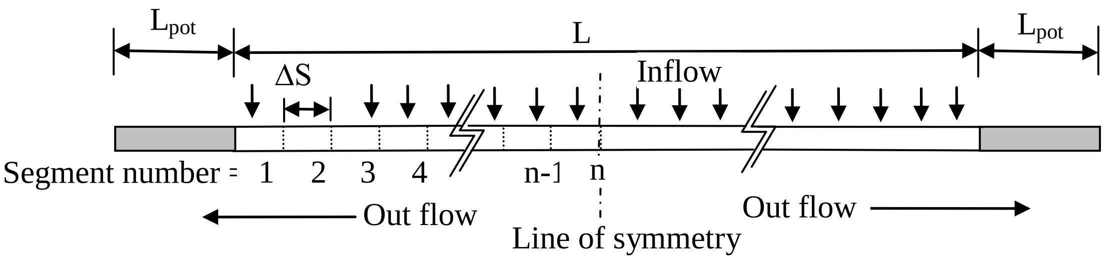

Fig.(2). A schematic diagram of a HFM in an actual module.

By dividing the fibre membrane length under consideration into n equal segments of a length equal to ΔS, then **Eq. (1)** is used to calculate the flowrate through segment as:

$$q_1 = 2\pi \Delta S\, K \frac{TMB_1}{\ln(\frac{r_t}{r_n})} \tag{3}$$

By assuming the fibre segments, ΔS, is short enough so that variation of transmembrane pressure, *TMP*, between the two ends of each segment is considered small and is neglected. The *TMP* throughout 1st segment may be written as:

$$TMP_1 = h_{ap} - hl_{pot} \tag{4}$$

In which:
h_{ap} = applied pressure head, L, and
hl_{pot}= head loss through the pot length, L.

The head loss throughout the fibre channel along the pot length may be calculated by applying Poiseulle's Law, **Eq. (2)**, that is:

$$hl_{pot} = \frac{8(q_1 + q_2 + \dots\dots\dots + q_n)\mu L_{pot}}{\rho g \pi r_n^4} \tag{5}$$

Where L_{pot} is the pot length, L.

Now, The expression for flowrate through 1st segment, **Eq. (3)**, may be written as:

$$q_1 = \frac{2\pi \Delta S\, K\, (h_{ap} - \frac{8(q_1 + q_2 + \dots\dots + q_n)\mu L_{pot}}{\rho g \pi r_n^4})}{\ln(\frac{r_t}{r_n})} \tag{6}$$

Rearranging and rewriting

$$(C_2 + 1)q_1 + C_2(q_2 + q_3 + \dots\dots + q_n) - C_1 = 0 \tag{7}$$

In which

$$C_1 = 2\pi \Delta S\, K \frac{h_{ap}}{\ln(\frac{r_t}{r_n})} \tag{8}$$

and

$$C_2 = 16 \Delta S\, K L_{pot} \frac{\mu}{\rho g r_n^4 \ \ln(\frac{r_t}{r_n})}$$

(9)

In general, **Eq. (7)** may be written as:

$$(C_2+1)q_1+C_2\sum_{i=2}^{n}q_i-C_1=0 \tag{10}$$

In which *i* is the segment number.

A similar expression can be obtained for the 2nd segment. The TMP along the segment can be written as:

$$TMP_1=h_{ap}-hl_{pot}-hl_1 \tag{12}$$

In which hl_1 is the head loss along the 1st segment, which may be obtained by using Poiseulle's Law, **Eq. (2)**, that is:

$$hl_1=\frac{8(0.5q_1+q_2+........+q_n)\mu\Delta S}{\rho g\pi r_n^4} \tag{13}$$

When calculating the head loss thought 1st segment, the flowrate thought is not fully developed it varies from o to q_1. Then it was assumed that the flowrate through the walls of this segment varies linearly along the segment and the average was taken to calculate the head loss. Then expression for flowrate through segment 1 may be written as:

$$q_2=-\frac{2\pi\Delta SK\,(h_{ap}-\dfrac{8(q_1+q_2+.......+q_n)\mu L_{pot}}{\rho g\pi r_n^4}-\dfrac{8(0.5q_1+q_2+q_3+......+q_n)\mu\Delta S}{\rho g\pi r_n^4})}{\ln(\dfrac{r_t}{r_n})} \tag{14}$$

By defining

$$C_3=16\Delta S^2K\frac{\mu}{\rho g r^4\ln(\dfrac{r_t}{r_n})} \tag{15}$$

and arranging and rewriting

$$(C_2+0.5)q_1+(C_2+C_3+1)q_2+(C_2+C_3)(q_3+q_4.......+q_n)+-C_1=0 \tag{16}$$

or

$$(C_2+0.5)q_1+(C_2+C3+1)q_2+(C_2+C_3)\sum_{j=3}^{n}q_j-C_1=0 \tag{17}$$

A similar equation may be obtained for the remaining segments, which may be written in general form as:

$$\sum_{j=1}^{i-1}(C_2+((j-1)+0.5)C_3))q_j+(C_2+(i-1)C3+1)q_i+(C_2+(i-1)C_3))\sum_{j=i+1}^{n}q_j-C_1=0$$
(18)

By applying **Eq. (10)** to the 1st segment and **Eq. (18)** to the 2nd segment through the nth segment, the resultant is a system of linear equations with n unknowns represents the flowrate throughout each segment, which may be written as shown in **Table 1**.

Table 1. The resulting system of linear equation.

Equation no.	q_1	q_2	q_3	q_4	--------	q_n	R.H.S
1	C_2+1	C_2	C_2	C_2	---------	C_2	C_1
2	$C_2+0.5\ C_3$	C_2+C_3+1	C_2+C_3	C_2+C_3	---------	C_2+C_3	C_1
3	$C_2+0.5\ C_3$	$C_2+1.5C_3$	C_2+2C_3+1	C_2+2C_3	--------	C_2+2C_3	C_1
4	$C_2+0.5\ C_3$	$C_2+1.5C_3$	$C_2+2.5C_3$	C_2+3C_3+1	--------	C_2+3C_3	C_1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$C_2+0.5\ C_3$	$C_2+1.5C_3$	$C_2+2.5C_3$	$C_2+3.5C_3$	--------	$C2+(n-1)C_3+1$	C_1

The above system of n simultaneous equations can be solved for the flowrate using any method for solving a system of linear equations. Having obtaining the values of the flowrate of each segment, the transmembrane pressure can be calculated for each segment.

MATHEMATICAL MODEL VERIFICATION

The developed mathematical model was calibrated and verified by using gathered laboratory experimental data to check the performance of the model and the validity of the assumptions made.

Laboratory experiments were carried out on polypropylene, PP, HFM with inner and outer diameters of 0.39mm and 0.65mm, respectively. The fibres were divided into four sets each set consist of ten fibres. The flowrate, under a constant head of 2m, of each set was measured with its initial length, and then the flowrate is measured each time after reducing the length as shown in **Table (2)**.

Table (2). Measured flowrate values.

Set no.	Length (cm)	Measured flowrate (ml/min)	Set no.	Length (cm)	Measured flowrate (ml/min)
1	50	18.1	3	50	18.6
	40	17.02		40	16.82
	30	14.84		30	14.84
	20	11.2		20	11.89
	10	5.9		10	6.48
2	45	18.1	4	45	18.14
	30	15.23		30	14.41
	15	8		15	7.84

The first step toward the mathematical model verification is the calibration of the hydraulic conductivity coefficient. The value of this coefficient was calibrated using the data of the first set with initial length of 50cm only. The conductivity coefficient is adjusted until the predicted flowrate value matches the experiment value. The conductivity coefficient value was found to be $4.64*10^{-7}$ cm/s.

The Mathematical model was used then to generate the flowrate values of the fibre sets by just changing the fibres length. **Fig. (3)** shows the measured and mathematical model predicted flowrate values. A segment length of 1cm was adopted in all calculations of the study. A good agreement between the measured and predicted flowrates values can be noticed with a correlation coefficient of 0.996.

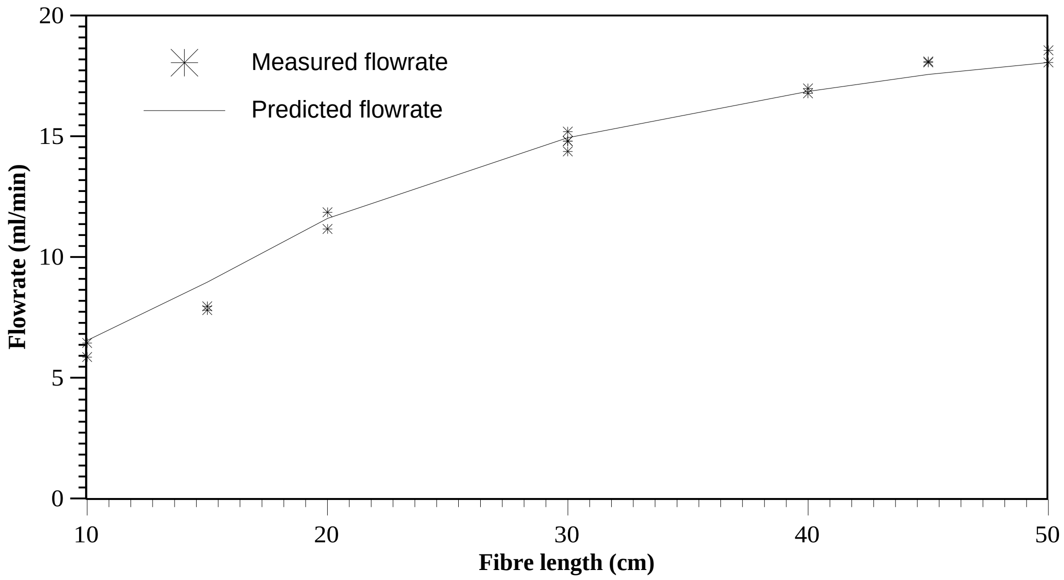

Fig. 3. Comparison between measured and predicted flowrate values.

APPLICATION OF THE MATHEMATICAL MODEL

The developed mathematical model being verified can be used to study the hydraulic performance of HFM rather than carrying out time consuming laboratory tests.

The mathematical model was applied to investigate the hydraulic performance of two types of virgin HFM modules.

First Type of HFM Module

Specifications of the first type HFM module are listed in **Table 3**.

Table 3. Specification of the first type HFM module.

Item	Value
Fibre inner diameter	0.25mm
Fibre outer diameter	0.55mm
Number of fibres per module	20 000
Effective fibre length	97cm
Pot length	10cm
Total effective area	33.5m^2
Normal Module operation flowrate	120-240 lmh/bar

In the factory, the measured flowrate of the fibres module with RO permeate is 2,000 lmh/bar at 20°C. This given permeability can be reached by the mathematical model under a hydraulic conductivity coefficient of $2.32x10^{-5}$ m/s.

The flowrate variation as a percentage of the total fibre flowrate a long a single fibre length is shown in **Fig. 4**. As it may be seen that more that 99.3% of the flowrate is just from the first 10cm of the fibre length.

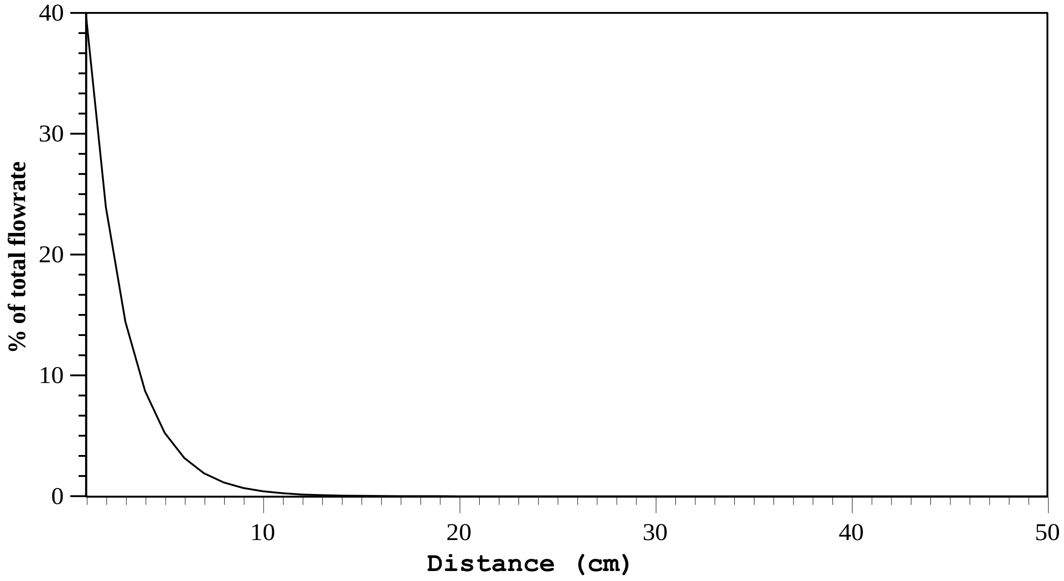

Fig. 4. The flowrate variation as a percentage of total flowrate along a single fibre length.

Fig. 5 shows the TMP variation as a percentage of the total applied pressure head along the single fibre length. It is clear that most of the applied head will be used to derive water from the first 10cm.

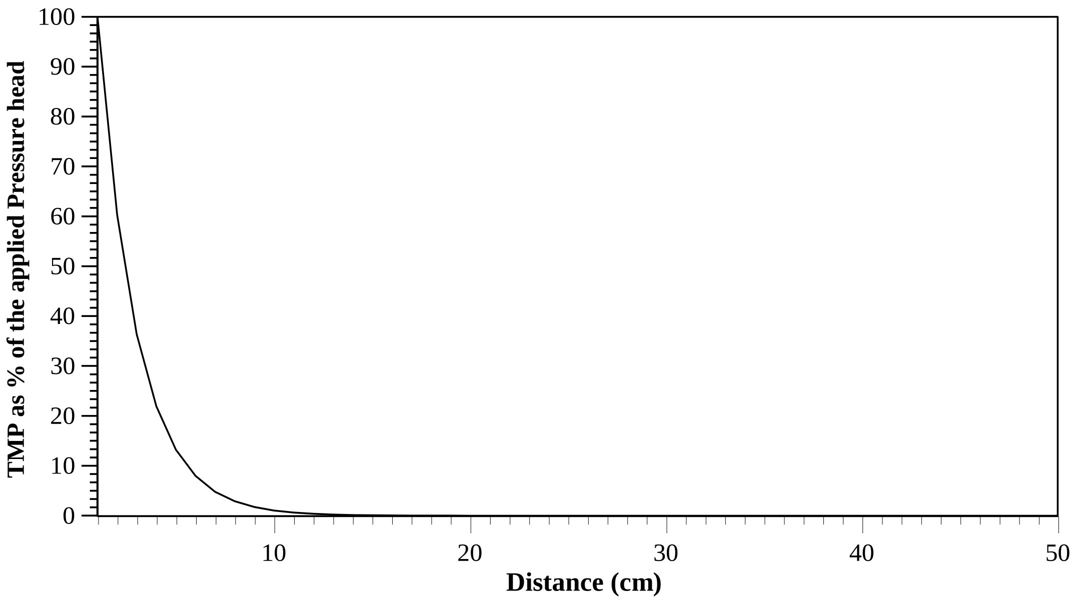

Fig 5. The TMP variation as a percentage of the total applied head along a single fibre length.

The flowrate of a single fibre is reduced when placed in actual fibre module because of the headloss through the module pot. The flowrate of the single virgin fibres will be reduced from 2000 lmh/bar down to 336.4 lmh/bar when placed in a full module.

The TMP variation as a percentage of the total applied pressure head and flowrate variation as a percentage of the total flowrate along the fibre length are independent of the applied pressure head. Thus, the flowrate variation along the fibre as a percentage of total flowrate will be the same as in **Fig. 4**.

Fig. 6. Shows the TMP variation along the fibre with a pot of a 10cm length. A great reduction in the TMP may be noticed when comparing the TMP variation of **Fig. 5,** of a single fibre, with that of **Fig. 6,** a fibre in an actual module. 83.2% of the total applied energy will be lost through the pot length.

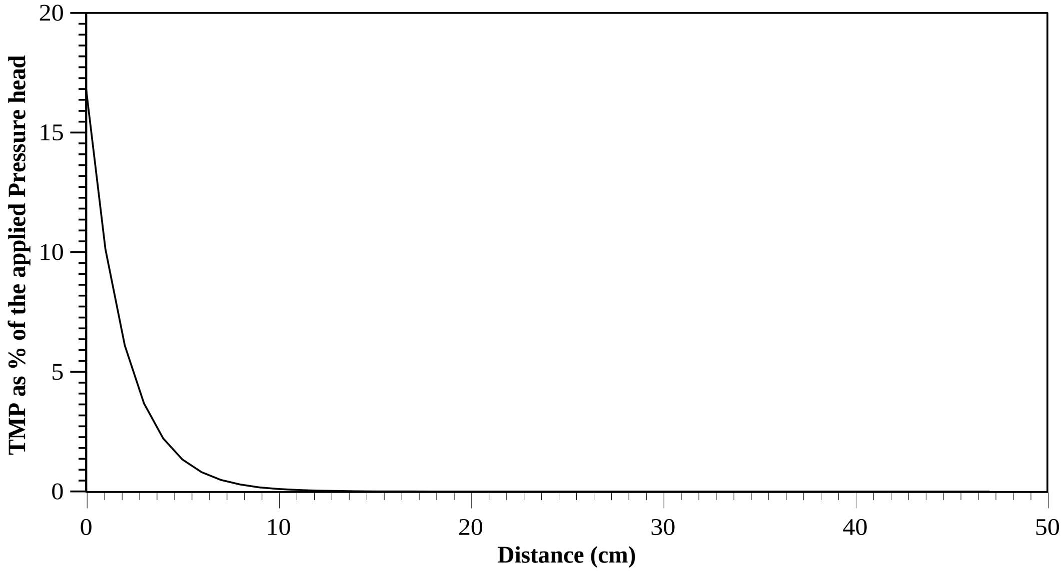

FIG 6. The TMP variation as a percentage of the total applied head along the fibre length.

Second Type of HFM Module

Specifications of the second type of HFM module are listed in **Table 4**.

Table 4. Table 3. Specification of the second type HFMe module.

Item	Value
Fibre inner diameter	0.8mm
Fibre outer diameter	1.2mm
Effective fibre length	144.75cm
Pot length	4cm
Total effective area	35m^2
Module Max operation flowrate	357 lmh/bar

The hydraulic conductivity coefficient was found to be 2.57*10^{-7}cm/s at which max operation permeability was reached.

The flowrate variation a long the fibre length as a percentage of the total fibre flowrate is shown in **Fig. 7**. As it may be seen that the ratio between the flowrate at the module outlet and that at its middle is about 1.1%.

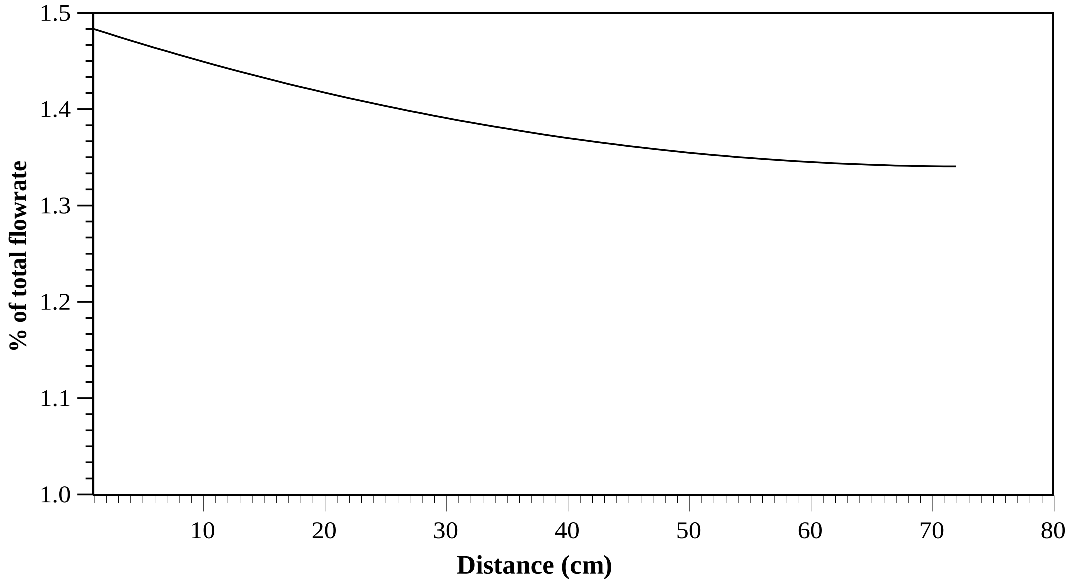

Fig. 7. The flowrate variation along the fibre length as a percentage of total flowrate.

Fig. 8 shows the TMP variation along the fibre length as a percentage of the total applied pressure head. Due to large fibre diameter and the short length of the pot the head losses through it is very small. The difference between the maximum and the minimum TMP along the fibre is less than 10%.

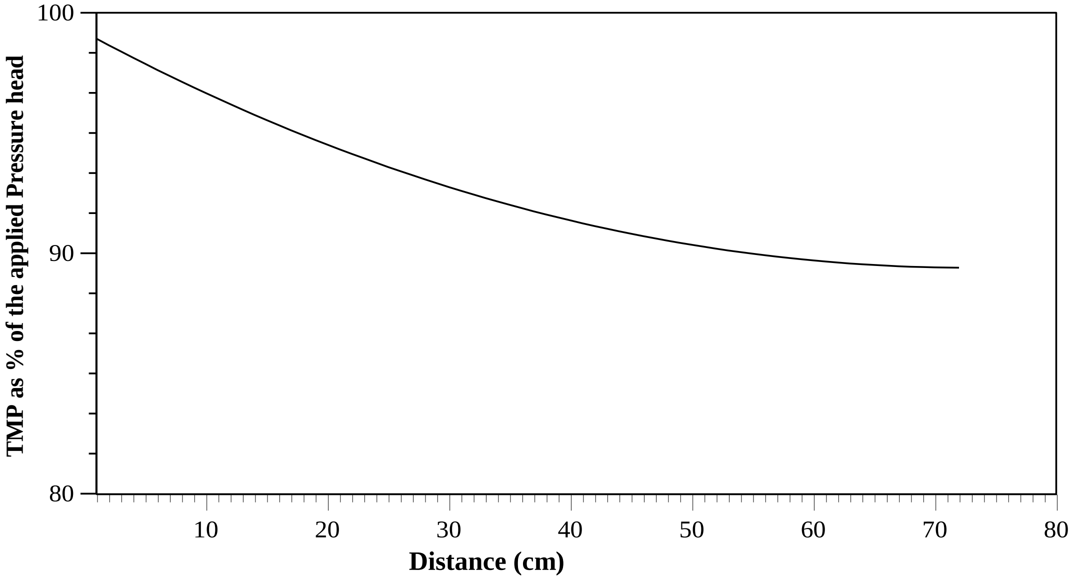

FIG 6. The TMP variation along the fibre length as a percentage of the total applied head.

CONCLUSIONS

- The flow though the HFM under the condition of constant hydraulic conductivity could be simulated mathematically by applying the Darcy's Law and Poiseulle's Laws. A Good agreement was found between the laboratory and predicted data under the same conditions.
- A great difference in the performance of two the commercial HFM was noticed.
- The TMP variation as a percentage of the total applied pressure head and flowrate variation as a percentage of the total flowrate along the HFM length are independent of the applied pressure head
- The head losses through the pot length could be so high and consumes 80% of the total applied pressure head.

RECOMMENDATIONS

The following recommendations were suggested to study:

- The variation of the hydraulic conductivity along the fibre length under normal operation conditions.
- The mathematical simulation of the hydraulic flow through the hollow fibre membrane under variable hydraulic conductivity.
- Optimizing the HFM module design.

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LIST OF SYMBOLS

ΔS= length of the fibre segment, L.
μ= water viscosity, L.T.
HFM= hollow fibre membrane.
h_{ap} = applied head, L.
h_l = head loss, L.
hl_{pot}= pressure head loss through the pot length, L.
K= hydraulic conductivity of the porous media, L/T.
L_{pot} = length HFM module pot, L
q = volumetric flowrate, L^3/T.
r_t= fibre outer radius, L, and
r_n= fibre inner radius, L.
TMP = transmembrane pressure, L.