Dynamic Stability Analysis and Critical Speed of Rotor supported by a Worn Fluid film Journal Bearings

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ABSTRACT

In this paper, the effect of wear in the fluid film journal bearings on the dynamic stability of rotor bearing system has been studied depending on the development of new analytical equations for motion, instability threshold speed and steady state harmonic response for rotor with offset disc supported by worn journal bearings. Finite element method had been used for modeling the rotor bearing system. The analytical model is verified by comparing its results with that obtained numerically for a rotor supported on the short bearings. The analytical and numerical results showed good agreement with about 8.5% percentage error in the value of critical speed and about 3.5% percentage error in the value of harmonic response. The results obtained show that the wear in journal bearing decrease the instability threshold speed by 2.5% for wear depth 0.02 mm and 12.5% for wear depth 0.04 mm as well as decrease critical speed by 4.2% and steady state harmonic response amplitude by 4.3% for wear depth 0.02 mm and decrease the critical speed by 7.1% and steady state harmonic response amplitude by 13.9% for wear depth 0.04 mm.

Key words: rotor, journal bearing, wear, instability, critical speed,

تحليل الاستقرارية الديناميكية والسرعة الحرجة لدوار مسنود بواسطة كراسي تحميل متآكلة

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الخلاصة

تم في هذا البحث دراسة تأثير ألتآكل في كراسي ألتحميل المقعدية ذات التزييت الهيدروديناميكي على ألاستقرار ألديناميكي لمحور دوار يستند على كراسي تحميل متاكلة بالاعتماد على تطوير معادلات جديدة للحركة واللااستقرارية والاستجابة التوافقية في حالة الاستقرار لمحور دوار مع قرص غير متمركز في الوسط. تم مقارنة النتائج النظرية والعددية لمحور دوار مستند على كراسي تحميل قصيرة. اظهرت النتائج التحليلية والعددية تطابقا جيدا مع نسبة خطا بحدود 8.5% بالنسبة للسرعة الحرجة و 3.5% بالنسبة للاستجابة التوافقية. أظهرت النتائج ان التآكل في كراسي التحميل يقلل سرعة اللااستقرارية بنسبة 2.5% لحالة التاكل بعمق 0.02 ملم و بنسبة 12.5% لحالة التاكل بعمق 0.04 ملم ويقلل كذلك السرعة الحرجة بنسبة 4.2% وسعة الاستجابة التوافقية العظمى بنسبة 4.3% لحالة التاكل بعمق 0.02 ملم اما لحالة التاكل بعمق 0.04 ملم فان السرعة الحرجة تنخفض بنسبة 7.1% وسعة الاستجابة التوافقية العظمى بنسبة 13.9%.

الكلمات ألرئيسية : ألدوار, كرسي تحميل, التآكل, اللااستقرارية, السرعة الحرجة

1. INTRODUCTION

Hydrodynamic fluid film journal bearings are frequently used in applications requiring high loads and high speeds. They usually exhibit lower friction and more damping compared to ball bearings. Hydrodynamic bearings, however, are susceptible to self excited instability known as oil whirl giving rise to large amplitude lateral vibrations and possibly a early wear. The prediction of the instability boundaries is an important step in the dynamic analysis of hydrodynamic bearings. Also the instability is of particular importance to the manufacturers and users of modern turbomachinery particularly with the present tend towards high speed and loading conditions.

Gunter, 1971, presented a survey of the various mechanisms that cause instability in rotor bearing system and the stability data on plain and multilobed journal bearings. He shows the effect of unbalance and external loading on the nonlinear rotor whirl .**Lund, 1975**, presented a method for calculating the threshold speed of instability and the damped critical speeds of a general flexible rotor in fluid film journal bearings. **Chauvin, 2003,** experimentally investigated the effect of lubricant temperature on the presence of whirl instability in journal bearings. Lubricant temperature, bearing temperature, frequency and amplitude of vibration, and rotational speed are monitored and analyzed in relation to presence of whirl instability. **Alsaeed, 2005**, studied the dynamic stability of an automotive turbocharger rotor-bearing system using both linear and nonlinear analyses. Several different hydrodynamic journal bearings were employed in the study of the turbocharger linearized dynamic stability. **Mancilla, et al., 2005**, presented new closed-form expressions for calculating the linear stability thresholds for rigid and flexible Jeffcott systems and the imbalance response for a rotor supported on a hybrid bearing. **Tuma and Bilos, 2007**, studied the instability of the rotor vibration in a journal bearing due to the oil whirl, they found that oil induced vibration, occurs when the rotor rotation speed crosses a certain threshold speed. **Miranda and Faria, 2014,** Presented a finite element procedure to perform the eigenvalue analysis of damped gyroscopic systems, represented by flexible rotors supported on fluid film journal bearings.

The present work focuses on the stability and unbalance response prediction for asymmetric rotor supported by two asymmetric fluid film bearings and one of them is worn

2. EQUATION OF MOTION OF ROTOR WITH NON CENTRAL DISK SUPORTED ON TWO ASYMMETRIC JOURNAL BEARINGS

For a one-mass flexible rotor with non central rigid disc (a ≠ b in this study) supported by two asymmetric bearings as shown in the **Fig 1**. A new equation of motion can be driven with the following assumptions

1- Small motion about equilibrium position of rotor bearing system
2- The difference between the eccentricity ratios of the two bearing is very small therefore it can be neglected.
3- Neglect the elastic coupling , the coupling between displacement and rotation because it has small effect on the system frequency , **Michael, et al., 2010**

There are three reference points were taken into consideration they were the journal's center, the bearings' center, and the rotor's center (which is located at the point of disc center). The first two equations of motion are found. **Rao, 1996**.

$$M\ddot{x}_r + K(x_r - x_j) = Mu\Omega^2 \cos \Omega t \quad \text{and} \quad M\ddot{y}_r + K(y_r - y_j) = Mu\Omega^2 \sin \Omega t \qquad (1)$$

Where the notation of (x_r) and (y_r) are the location of the rotor center, (x_j) and (y_j) are the location of the journal center and the terms $(\mathrm{Mu}\Omega^2 \cos \Omega \mathrm{t}\ \mathit{and}\ \mathrm{Mu} \sin \Omega \mathrm{t})$ are the unbalance forces due to unbalance mass in the *x* and *y* directions respectively The second two equations of motion are found by summing the forces on the fluid film journal bearing for a stable condition.

$$K(x_r - x_j) = F_{x1} + F_{x2} \quad \text{and} \quad K(y_r - y_j) = F_{y1} + F_{y2} \tag{2}$$

Where, subscript (1) and (2) are refer to bearing one and bearing two respectively, K is stiffness of shaft , F_{x1} , F_{x2} are the reaction forces of the bearing one and two in the x- direction and F_{y1}, F_{y2} are the reaction forces of the bearing one and two in the y- direction and the shaft damping is ignored because it has no effect on the unbalance response **Rao , 1996**. Also, the forces developed in the lubricating oil film of the bearings are

$$\begin{bmatrix} F_{x1} \\ F_{y1} \end{bmatrix} = \begin{bmatrix} K_{xx1} & K_{xy1} \\ K_{yx1} & K_{yy1} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} + \begin{bmatrix} C_{xx1} & C_{xy1} \\ C_{yx1} & C_{yy1} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{y}_j \end{Bmatrix} \tag{3}$$

$$\begin{bmatrix} F_{x2} \\ F_{y2} \end{bmatrix} = \begin{bmatrix} K_{xx2} & K_{xy2} \\ K_{yx2} & K_{yy2} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} + \begin{bmatrix} C_{xx2} & C_{xy2} \\ C_{yx2} & C_{yy2} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{y}_j \end{Bmatrix} \tag{4}$$

Substitute Eq. (3) and Eq. (4) into Eq. (2) get

$$\begin{bmatrix} K_{xx1} + K_{xx2} & K_{xy1} + K_{xy2} \\ K_{yx1} + K_{yx2} & K_{yy1} + K_{yy2} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} + \begin{bmatrix} C_{xx1} + C_{xx2} & C_{xy1+}C_{xy2} \\ C_{yx1} + C_{yx2} & C_{yy1} + C_{yy2} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{y}_j \end{Bmatrix} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} x_r - x_j \\ y_r - y_j \end{Bmatrix} \tag{5}$$

Where

M, is the mass of the rotor with offset disk and K the stiffness of the shaft with offset disk $K = 3EI\frac{a^3+b^3}{a^3b^3}$. , **Chen, et al., 2007**,

Above equations represent new equations of motion for rotor supported on asymmetric fluid film journal bearings (Asymmetry is a result of unequal load of bearings)

3. CRITICAL SPEED

Critical speeds are commonly defined as the rotational speeds at which vibration due to rotor unbalance is a local maximum. **Lee, 1993.** The critical speed of rotor supported on the fluid film journal bearings cannot be defined as in the case of a rigid bearing rotor, because the stiffness coefficients are functions of the speed of rotor. Consequently, it is always better to study the out-of- balance (unbalance) response to locate the critical speeds, **Rao, 1996.**

There are many types excitation forces that are encountered often in the rotating machinery, the mass unbalance is the most common source of excitation in the rotor bearing systems. The imbalance excitation is a harmonic excitation, which has an excitation frequency coincident with the rotor spin speed (rotational speed).

To find the critical speed, the harmonic steady state response due to unbalance mass can be calculated at disk location where the maximum response occurs and therefore the critical speed will be the rotor speed at the maximum response, **Rao , 1996**.. For harmonic motion Eq. (5) becomes

$$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} i\Omega x_j \\ i\Omega y_j \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} \begin{Bmatrix} x_r \\ y_r \end{Bmatrix} \tag{6}$$

Where $x_j = Be^{i\Omega t}$, $\dot{x}_j = i\Omega Be^{i\Omega t} = i\Omega x_j$, $y_j = De^{i\Omega t}$, $\dot{y}_j = i\Omega De^{i\Omega t} = i\Omega y_j$

$K_{xx} = (K_{xx1} + K_{xx2}),\ K_{xy} = (K_{xy1} + K_{xy2}),\ K_{yy} = (K_{yy1} + K_{yy2}), K_{yx} = (K_{yx1} + K_{yx2})$

$C_{xy} = (C_{xy1} + C_{xy2}), v\ C_{xx} = (C_{xx1} + C_{xx2})\ ,\ C_{yy} = (C_{yy1} + C_{yy2}),\ C_{yx} = (C_{yx1} + C_{yx2})$

Eq. (6) can be rearranged as following

$$\begin{bmatrix} K_{xx} + K + i\Omega C_{xx} & K_{xy} + i\Omega C_{xy} \\ K_{yx} + i\Omega C_{yx} & K_{yy} + K + i\Omega C_{yy} \end{bmatrix} \begin{Bmatrix} x_j \\ y_j \end{Bmatrix} = \begin{bmatrix} Kx_r \\ Ky_r \end{bmatrix} \tag{7}$$

Solve Eq. (6) to find x_j and, y_j In terms of x_r, y_r , yields

$$x_j = \frac{K[(K_{yy}+K+i\Omega C_{yy})x_r - (K_{xy}+i\Omega C_{xy})y_r]}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})} \tag{8}$$

$$y_j = \frac{K[(K_{xx}+K+i\Omega C_{xx})x_r - (K_{yx}+i\Omega C_{yx})y_r]}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})} \tag{9}$$

Substitute Eq. (8) and Eq. (9) in Eq. (1) get

$$M\ddot{x}_r + K_1 x_r + K_{12} y_r = Mu\Omega^2 \cos\Omega t$$
$$M\ddot{y}_r + K_2 y_r + K_{21} x_r = Mu\Omega^2 \sin\Omega t \tag{10}$$

Where

$$K_1 = \frac{K[(K_{xx}+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})]}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})}$$

$$K_2 = \frac{K[(K_{yy}+i\Omega C_{yy})(K_{xx}+K+i\Omega C_{xx}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})]}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})}$$

$$K_{12} = \frac{K^2(K_{xy}+i\Omega C_{xy})}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})}$$

$$K_{21} = \frac{K^2(K_{yx}+i\Omega C_{yx})}{(K_{xx}+K+i\Omega C_{xx})(K_{yy}+K+i\Omega C_{yy}) - (K_{xy}+i\Omega C_{xy})(K_{yx}+i\Omega C_{yx})}$$

The solution of Eq. (10) can be directly written as following, **Rao, 2011**

$$x_r = x_r^+ e^{i\Omega t} + x_r^- e^{-i\Omega t}\ \ ,\ y_r = y_r^+ e^{i\Omega t} + y_r^- e^{-i\Omega t} \tag{11}$$

Where x_r^+ and y_r^+ are the whirl radius of the forward precession components which are in the same direction of the rotor rotational speed while x_r^- and y_r^- are that of the backward precession components which are in the opposite direction of the rotor spin speed.
Substitute Eq. (11) in Eq. (10) yields

$$x_r^+ = \frac{(Mu\Omega^2/2)[(K_2 - M\Omega^2) + iK_{12}]}{(K_1 - M\Omega^2)(K_2 - M\Omega^2) - K_{12}K_{21}}\ ,\ y_r^+ = \frac{(-iMu\Omega^2/2)[(K_1 - M\Omega^2) + iK_{21}]}{(K_1 - M\Omega^2)(K_2 - M\Omega^2) - K_{12}K_{21}} \tag{12}$$

$$x_r^- = \frac{(Mu\Omega^2/2)[(K_2 - M\Omega^2) + iK_{12}]}{(K_1 - M\Omega^2)(K_2 - M\Omega^2) - K_{12}K_{21}}\ ,\ y_r^- = \frac{(-iMu\Omega^2/2)[(K_1 - M\Omega^2) + iK_{21}]}{(K_1 - M\Omega^2)(K_2 - M\Omega^2) - K_{12}K_{21}} \tag{13}$$

The out of balance response (unbalance response) can be written as following, **Rao, 1996**

$$r = \frac{x_r + i y_r}{u} \tag{14}$$

Substitute Eq. (12) and Eq. (13) in the Eq. (11) then substitute the resultant equation in the Eq. (14) get the ***a new equation*** for harmonic response of rotor supported by asymmetric bearings

$$r = r_f e^{i\Omega t} + r_b e^{-i\Omega t} \tag{15}$$

Where

$$r_f = \frac{\Omega^2\{(\Omega_1^2+\Omega_2^2-2\Omega^2)-i(\mu_2\Omega_2^2-\mu_1\Omega_1^2)\}}{2[(\Omega_1^2-\Omega^2)(\Omega_2^2-\Omega^2)-\mu_1\mu_2\Omega_1^2\Omega_2^2]} \ , \ r_b = \frac{\Omega^2\,(\Omega_1^2-\Omega_2^2)+i(\mu_2\Omega_2^2+\mu_1\Omega_1^2)}{2[(\Omega_1^2-\Omega^2)(\Omega_2^2-\Omega^2)-\mu_1\mu_2\Omega_1^2\Omega_2^2]}$$

$$\Omega_1^2 = \frac{K_1}{M} \ , \ \Omega_2^2 = \frac{K_2}{M} \ , \ \mu_1 = \frac{K_{12}}{K_1} \ , \mu_2 = \frac{K_{21}}{K_2}$$

Where r_f and r_b are the forward and backward components of unbalance response **Childs, 1993.**

The major and minor radii of the elliptic orbit of rotor at disk are

$$|r|_{maj} = |r_f| + |r_b| \quad |r|_{min} = |r_f| - |r_b| \tag{16}$$

The critical speed of the rotor supported by a symmetric journal bearings is the rotor speed when the maximum harmonic response at disk is equal to, $|r|_{maj}$.

4. INSTABILITY THRESHOLD SPEED

The following forms can be assumed as a solution to the Eq. (1) and Eq. (5) **Kramer, 1993.**

$$x_r = Ae^{\lambda t} \quad , \quad y_r = Be^{\lambda t} \quad , \quad x_j = Be^{\lambda t} \quad , \quad y_j = De^{\lambda t} \tag{17}$$

(The variable λ is the eigenvalue term) Substituting Eq. (17) into Eq. (1) and Eq. (5) and put unbalance forces equal to zero for free vibration yields,

$$M\lambda^2 A + K(A - B) = 0 \quad \text{and} \quad M\lambda^2 C + K(C - D) = 0 \tag{18}$$

$$K(A-B) - K_{xx}B - \lambda C_{xx}B - K_{xy}D - \lambda C_{xy}D = 0 \tag{19}$$

$$K(C-D) - K_{yx}B - \lambda C_{yx}B - K_{yy}D - \lambda C_{yy}D = 0 \tag{20}$$

The above system has four equations with four unknown constants. Putting these equations in matrix form and taking the determinant of the matrix and setting it equal to zero produces the values of A, B, C and D (the four unknowns) as following.

$$\begin{bmatrix} M\lambda^2 + K & -K & 0 & 0 \\ 0 & 0 & M\lambda^2 + K & -K \\ K & -(K + K_{xx} + \lambda C_{xx}) & 0 & -(K_{xy} + \lambda C_{xy}) \\ 0 & -(K_{yx} + \lambda C_{yx}) & K & -(K + K_{yy} + \lambda C_{yy}) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0 \tag{21}$$

The determinant of the matrix leads to the characteristic equation and when equating it to zero we get the non trivial solution. Making the characteristic equation dimensionless at this point will produce results of a general form. The dimensionless variables used to solve this problem are

$\bar{K} = K\frac{c}{Mg}$, $\bar{C} = C\frac{c\Omega}{Mg}$, $\bar{\lambda} = \lambda\sqrt{\frac{c}{g}}$, $\bar{\Omega} = \Omega\sqrt{\frac{c}{g}}$

The dimensionless characteristic equation will be as following.

$$\frac{G_1}{\bar{\Omega}^2}\bar{\lambda}^6 + \frac{G_2}{\bar{\Omega}}\bar{\lambda}^5 + (G_{31} + \frac{G_{32}}{\bar{\Omega}^2})\bar{\lambda}^4 + \frac{G_4}{\bar{\Omega}}\bar{\lambda}^3 + \left(G_{51} + \frac{G_{52}}{\bar{\Omega}^2}\right)\bar{\lambda}^2 + \frac{G_6}{\bar{\Omega}}\bar{\lambda} + G_7 = 0 \quad (22)$$

Where

$$G_1 = (\bar{C}_{xy1} + \bar{C}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) - (\bar{C}_{xx1} + \bar{C}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2})$$

$$G_2 = (\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) + (\bar{K}_{yx1} + \bar{K}_{yx2})(\bar{C}_{xy1} + \bar{C}_{xy2}) - \bar{K}(\bar{C}_{xx1} + \bar{C}_{xx2}) - \bar{K}(\bar{C}_{yy1} + \bar{C}_{yy2}) - (\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2}) - (\bar{K}_{yy1} + \bar{K}_{yy2})(\bar{C}_{xx1} + \bar{C}_{xx2})$$

$$G_{31} = (\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{K}_{yx1} + \bar{K}_{yx2}) - (\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{K}_{yy1} + \bar{K}_{yy2}) - \bar{K}(\bar{K}_{xx1} + \bar{K}_{xx2}) - \bar{K}(K_{yy1} + K_{yy2}) - \bar{K}^2$$

$$G_{32} = 2\bar{K}\{(\bar{C}_{xy1} + \bar{C}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) - (\bar{C}_{xx1} + \bar{C}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2})\}$$

$$G_4 = \bar{K}\{2(\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) + 2(\bar{K}_{yx1} + \bar{K}_{yx2})(\bar{C}_{xy1} + \bar{C}_{xy2}) - \bar{K}(\bar{C}_{xx1} + \bar{C}_{xx2}) - \bar{K}(\bar{C}_{yy1} + \bar{C}_{yy2})2(\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2}) - 2(\bar{K}_{yy1} + \bar{K}_{yy2})(\bar{C}_{xx1} + \bar{C}_{xx2})\}$$

$$G_{51} = 2\bar{K}(\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{K}_{yx1} + \bar{K}_{yx2}) - 2\bar{K}(\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{K}_{yy1} + \bar{K}_{yy2}) - \bar{K}^2(\bar{K}_{xx1} + \bar{K}_{xx2}) - \bar{K}^2(\bar{K}_{yy1} + \bar{K}_{yy2})$$

$$G_{52} = \bar{K}^2\{(\bar{C}_{xy1} + \bar{C}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) - (\bar{C}_{xx1} + \bar{C}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2})\}$$

$$G_6 = \bar{K}^2(\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{C}_{yx1} + \bar{C}_{yx2}) + \bar{K}^2(\bar{K}_{yx1} + \bar{K}_{yx2})(\bar{C}_{xy1} + \bar{C}_{xy2}) - \bar{K}^2(\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{C}_{yy1} + \bar{C}_{yy2}) - \bar{K}^2(\bar{K}_{yy1} + \bar{K}_{yy2})(\bar{C}_{xx1} + \bar{C}_{xx2})$$

$$G_7 = \bar{K}^2(\bar{K}_{xy1} + \bar{K}_{xy2})(\bar{K}_{yx1} + \bar{K}_{yx2}) - \bar{K}^2(\bar{K}_{xx1} + \bar{K}_{xx2})(\bar{K}_{yy1} + \bar{K}_{yy2})$$

The dynamic coefficients for worn and intact journal bearings are taken from, **Jameel et al., 2015**.

To find a solution to the Eq. (22), firstly $\bar{\lambda}$ must be found. The general form of $\bar{\lambda}$ is a complex form. It may be written as

$$\bar{\lambda} = m + is$$

Where (m) and (s) are the real and imaginary parts of the eigenvalue respectively. Observe that the equation of motion has the following form of a solution

$$x = Ae^{(m+is)t} = Ae^{mt}e^{ist}$$

If the real part of the eigenvalue is positive, then *x* goes to infinity, and if the real part is negative, then *x* goes to negative infinity. So for the rotor to be at a state that neither declines nor inclines, (m) must be equal to zero. Thus, real part (m) equal zero is a major stability limit (threshold) criteria and the eigenvalue takes the following form

$$\bar{\lambda} = is$$

The dimensionless new characteristic equation can be rewritten just a function of the eigenvalue as following

$$-\frac{G_1}{\bar{\Omega}^2}s^6 + i\frac{G_2}{\bar{\Omega}}s^5 + \left(G_{31} + \frac{G_{32}}{\bar{\Omega}^2}\right)s^4 - i\frac{G_4}{\bar{\Omega}}s^3 - \left(G_{51} + \frac{G_{52}}{\bar{\Omega}^2}\right)s^2 + i\frac{G_6}{\bar{\Omega}}s + G_7 = 0 \quad (23)$$

Since the characteristic equation, Eq. (23), equal zero, then the real part and the imaginary part must equal zero. Thus, there are two equations can be made as following.

$$-\frac{G_1}{\bar{\Omega}^2}s^6 + \left(G_{31} + \frac{G_{32}}{\bar{\Omega}^2}\right)s^4 - \left(G_{51} + \frac{G_{52}}{\bar{\Omega}^2}\right)s^2 + G_7 = 0 \quad (24)$$

$$i\frac{G_2}{\bar{\Omega}}s^5 - i\frac{G_4}{\bar{\Omega}}s^3 + i\frac{G_6}{\bar{\Omega}}s = 0 \quad (25)$$

Eq. (25) represents the imaginary part of the characteristic equation and it has the following solution.

$$s^2 = \frac{G_4 \pm \sqrt{(-G_4)^2 - 4G_2G_6}}{2G_2} \quad (26)$$

Since the two solutions of Eq. (26) satisfy the imaginary part of the characteristic equation, substitute the value of (s^2) into the real part of the characteristic equation, Eq. (24), and solve it for the dimensionless speed.

The dimensionless speed has two results one of them is usually near zero. While another gives a logical value. So only the second is valid. This value is known as the instability threshold speed.

$$\bar{\Omega}_{th} = \sqrt{\frac{G_1 s^6 - G_{32}s^4 + G_{52}s^2}{G_{31}s^4 - G_{51}s^2 + G_7}} \quad (27)$$

The instability threshold speed can be rewritten in term of Sommerfeld Number as following

$$\bar{\Omega}_{th} = S_s \frac{\pi M g c^2}{\mu \Omega L R^3}\sqrt{\frac{G_1 s^6 - G_{32}s^4 + G_{52}s^2}{G_{31}s^4 - G_{51}s^2 + G_7}} \quad (28)$$

5. ROTOR BEARING SYSTEM ANALYSIS USING ANSYS

3-D Solid model rotor bearing system gives more accurate results than in the case of one dimensional beam model as well as there are many advantages in adopting this model **Rao and Sreenivas, 2003.** Therefore it is used in this work; Solid187 element (4452 elements) has been used to model shaft and disk as shown in the **Fig.2. (a),** as well as COMBI214 element used to model journal bearing. All steps to model rotor bearing system can be found with details in **ANSYS Guide, 2012**. The eight dynamic coefficients of journal bearing are depending on the rotational speed therefore when these coefficients represent in ANSYS must be changed with rotor speed. The dimensions of rotor and bearings which used in this work are shown in **Fig.2. (b),** and **Table.1.**

6. RESULTS AND DISCUSSION

6.1 Effect of Wear on the Steady State Harmonic Response Amplitude and Critical Speed

. The present analytical has been validated with **Rao, 1996**. It is observed well agreement as shown in the **Fig.3**. Different combinations of dynamic coefficients have been studied separately to observe their effect on the harmonic response and critical speed. All results compared with case when all dynamic coefficients used in analysis **Fig.4.a.** Ignoring damping of journal bearings led to decreasing harmonic response and critical speed in the case of non-worn and worn journal bearing but the decreasing in the harmonic response higher in the case of non-worn Compared with the case of worn journal bearing as well as the harmonic response amplitude in the case of wear depth 0.04 mm becomes higher than in the case of wear depth 0.02 mm about 3.6% as shown in the **Fig .4.b.**ignoring cross coupled damping in bearings led to Slight decrease in the response amplitude and critical speed as shown in the **Fig.4.c**. Neglected direct damping in the bearings showed that increasing in the response amplitude and decreasing in the critical speed as shown in the **Fig.4.d**.Neglected the cross coupled stiffness led to increasing in the response amplitude and critical speed for all cases (worn and non-worn) as shown in the **Fig.5.a**. The last case when the cross coupled damping coefficients have been neglected led to increasing in the response amplitude and critical speed but the increase in the response amplitude and critical speed in the state of wear depth 0.04 mm higher than in the state of wear depth 0.02 mm by about 6.6% because the cross coupled damping coefficients in the case of wear depth 0.04 mm greater than in the case of 0.02 mm and when neglected it led to this state see **Fig.5.b.** In the **Fig.6**, **Fig.7** and **F.g.8**, all dynamic coefficients combinations have been plotted together showed that the major effect on the steady state harmonic response and critical speed is to the cross coupling stiffness and when neglected it led to increase in the response amplitude by 23% and critical speed by 7% because the cross coupled stiffness increasing with the increase in the rotor speed. The rotor orbit has been plotted as shown in the **Fig.9** for different rotational speed whereas the real value of Eq.(15) represent the x- coordinate of orbit and imaginary value of Eq.(15) represent the y- coordinate of orbit . The orbit radii increasing with the increase in rotor speed and with wear depth at the maximum response amplitude. The shape of rotor orbit approaching the shape of the circle at 9000 rpm because the eccentricity ratio becomes very small and real and imagery value of Eq. (15) become approximately equal for non-worn and worn journal bearing. Verification were done between analytical and numerical using ANSYS showed good agreement with error percentage about 8.5% for critical speed and about 3.5% error percentage for harmonic response as shown in the **Figs.10. 11 and 12** and **Table.2.**

6.2 Effect of Wear on the Instability Threshold Speed

The stability threshold speed represents the important parameter in the rotor dynamic for designers and operators. The stability decreasing with increase in the wear depth when the Sommerfeld number more than one as shown in the **Fig.13.d**, because the dynamic coefficients decreasing with increase in the rotor speed for worn journal bearing when compared with the dynamic coefficients of non worn bearing except for direct stiffness in the y-direction **Jameel et al., 2015,** also the cross coupled stiffness has little effect on the stability threshold speed for worn and non-worn journal bearing as shown in the **Figs.13.a, 13.b** and **13.d.**

7. CONCLUSIONS

1. Wear in journal bearing is generally decreasing the steady state harmonic response amplitude by 4.3% and critical speed by 4.2% for wear depth 0.02 mm and decreasing the steady state harmonic response amplitude by 13.9% and critical speed by 7.1% for wear depth 0.04 mm

2. Increase in wear depth lead to decrease the orbit dimension at speed near critical speed by the same percentage of harmonic response in the first conclusion.
3. Orbit shape approaching the shape of the circle at high speed greater than critical speed.
4. The wear in journal bearing decrease the instability threshold speed by 2.5% for wear depth 0.02 mm and 12.5% for wear depth 0.04 mm
5. Increasing in the wear depth in journal bearing increases damping in rotor bearing system but decreasing the stability of system.

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NOMENCLATURE

Cd = direct damping, Ns/m
Cc = cross coupled damping, Ns/m
c = radial clearance, m
E = modulus of elasticity, N/m^2
F_{x1} , F_{x2} = reaction forces of the bearing one and two in the x- direction, N
F_{y1}, F_{y2} = reaction forces of the bearing one and two in the y- direction, N
K = stiffness of rotor, N/m
Kd = direct stiffness, N/m
Kc = cross coupled stiffness, N/m
L = bearing length, m
M = rotor mass, Kg
r_f , r_b = forward and backward components of unbalance response respectively, m
R = journal radius, m
(x_r) , (y_r) = location of the rotor center
(x_j) , (y_j) = location of the journal center
u = mass eccentricity, m
μ = viscosity pa s
Ω = rotational speed of rotor, rad/s
$\bar{\Omega}_{th}$ = dimensionless instability threshold speed
λ, $\bar{\lambda}$ = eigenvalue term and dimensionless eigenvalue term

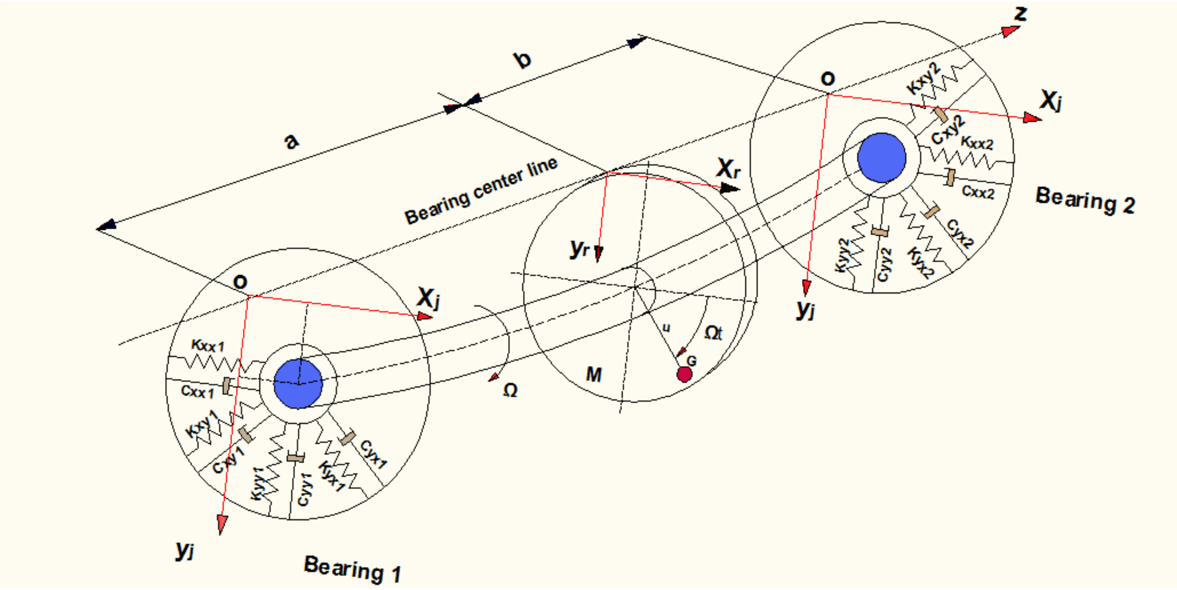


Figure 1. Rotor supported on the fluid film journal bearings.

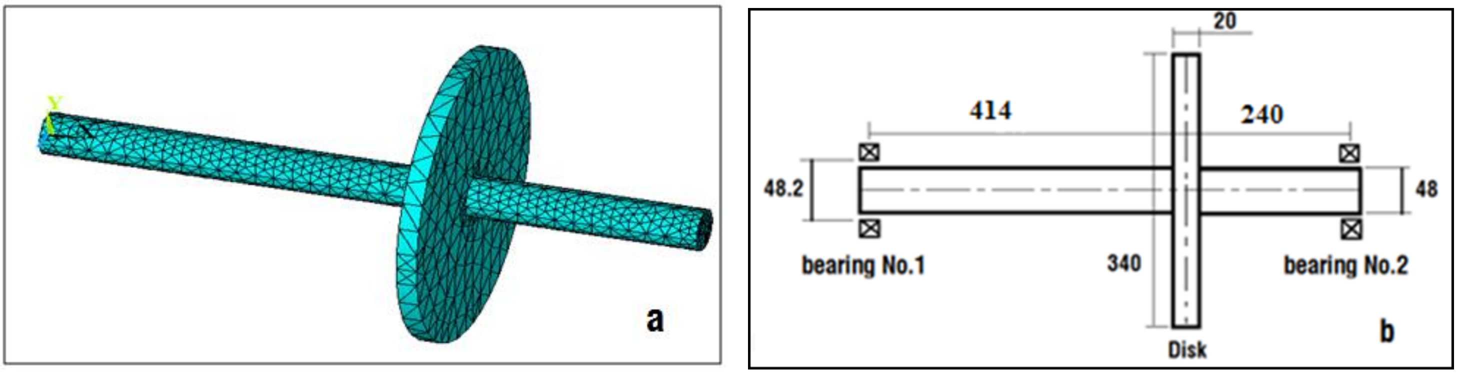


Figure 2. (a) Ansys rotor model (b) Mechanical drawing of present rotor bearing system.

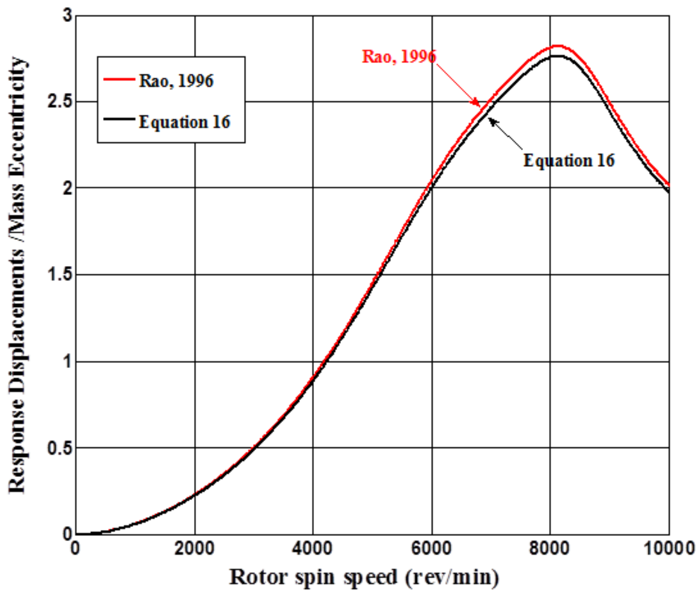


Figure 3. Steady state harmonic unbalance response.

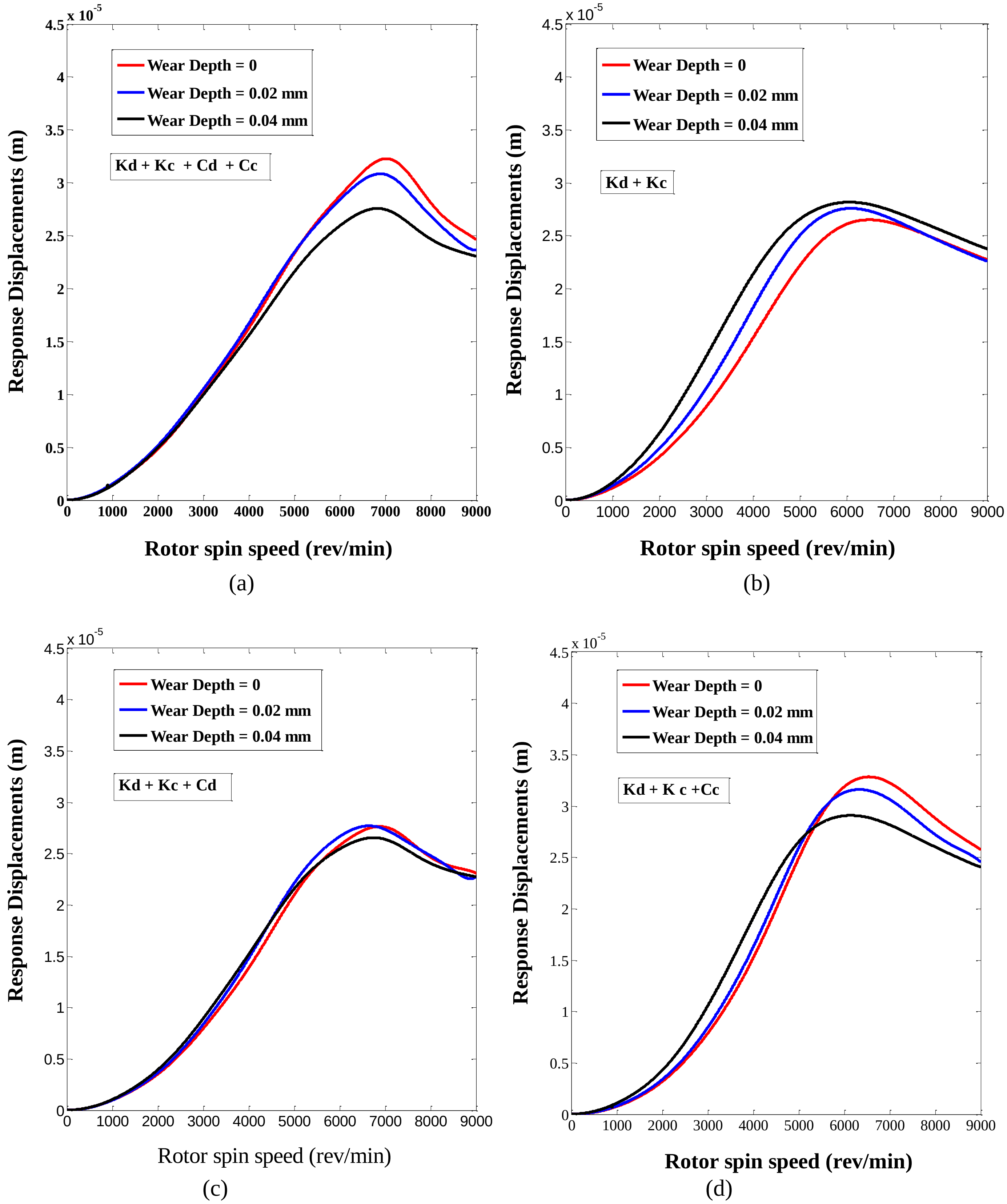
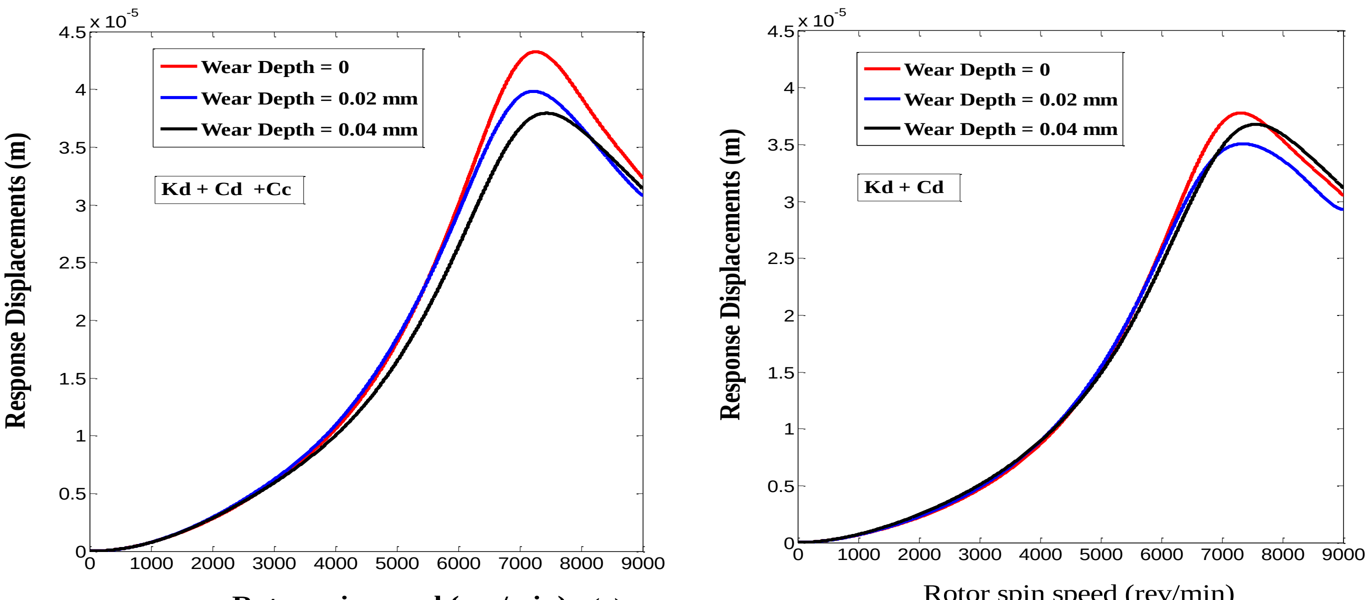


Figure 4. Effect of wear in journal bearing on the harmonic response of rotor bearing system (a): Using all coefficients, (b): Ignoring damping, (c): Ignoring cross coupled damping, (d): Ignoring direct damping.



(a)

(b)

Figure 5. Effect of wear in journal bearing on the harmonic response of rotor bearing system (a): Ignoring cross coupled stiffness, (b): Ignoring cross coupled stiffness and damping.

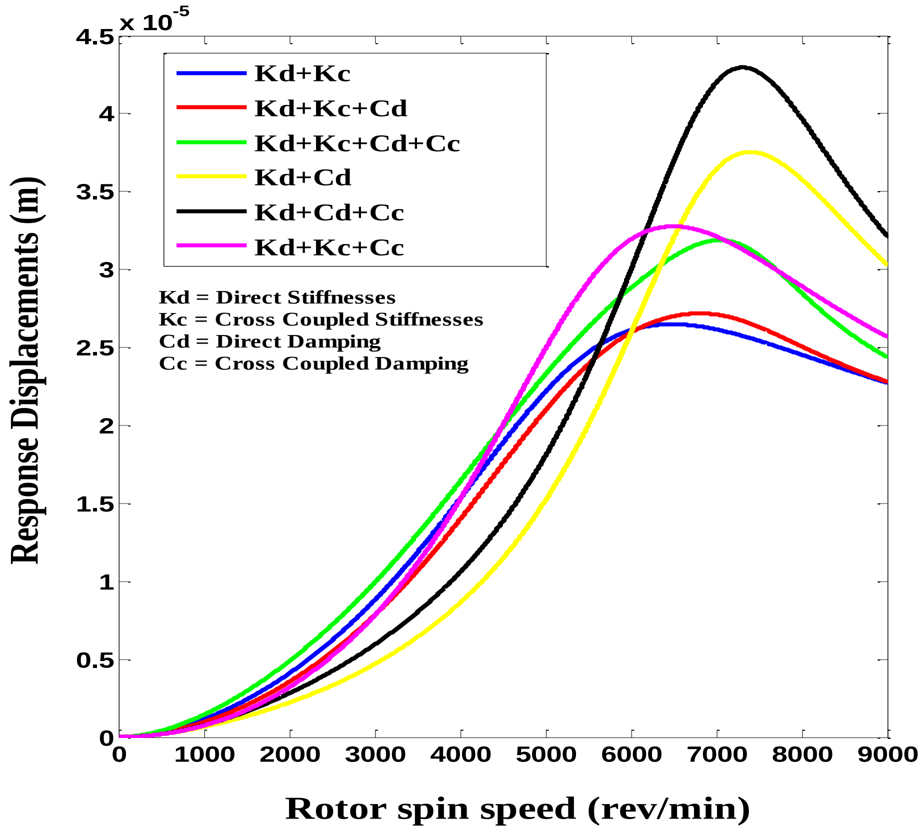


Figure 6. Harmonic unbalance response of rotor bearing system with different combinations of stiffnesses and damping of non-worn journal bearing.

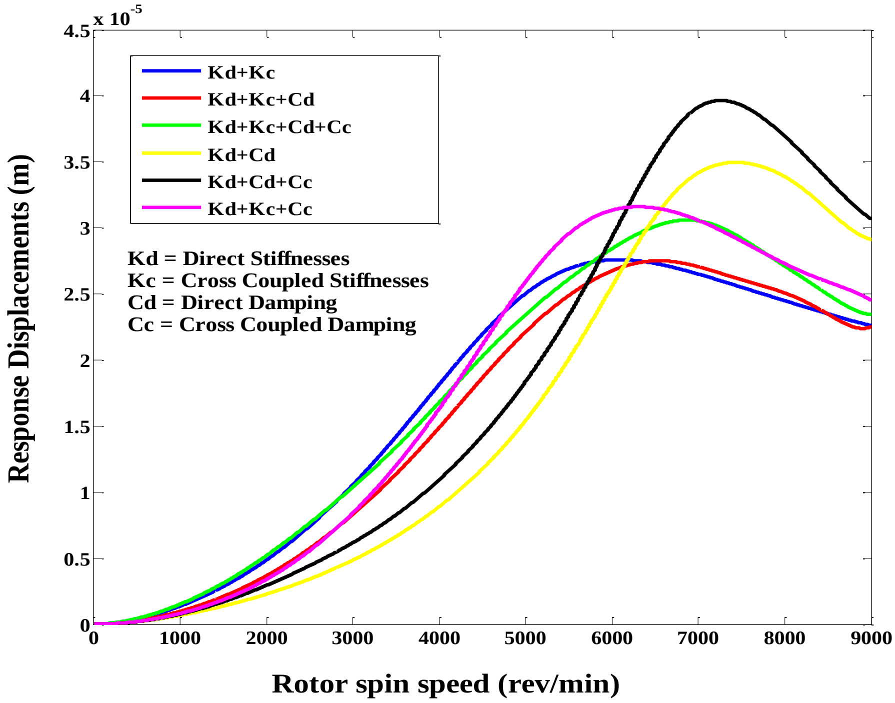


Figure 7. Harmonic unbalance response of rotor bearing system with different combinations of stiffnesses and damping of worn journal bearing ,wear depth = 0.02 mm.

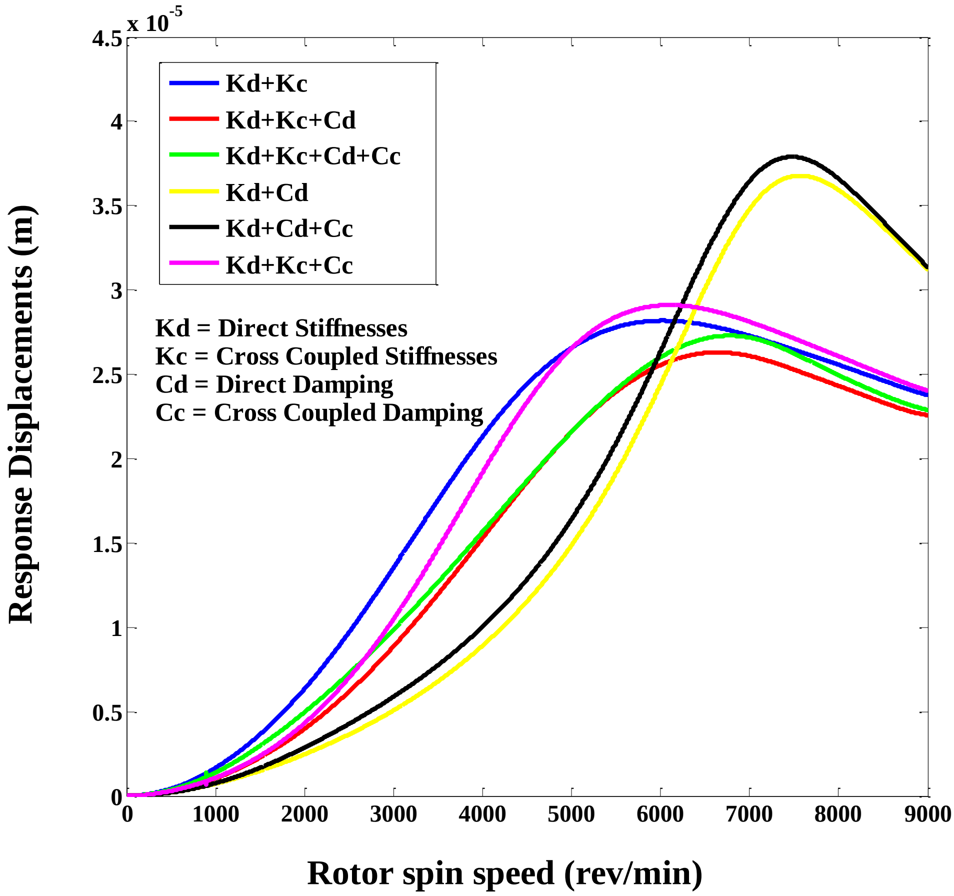


Figure 8. Harmonic unbalance response of rotor bearing system with different combinations of stiffnesses and damping of worn journal bearing ,wear depth = 0.04 mm.

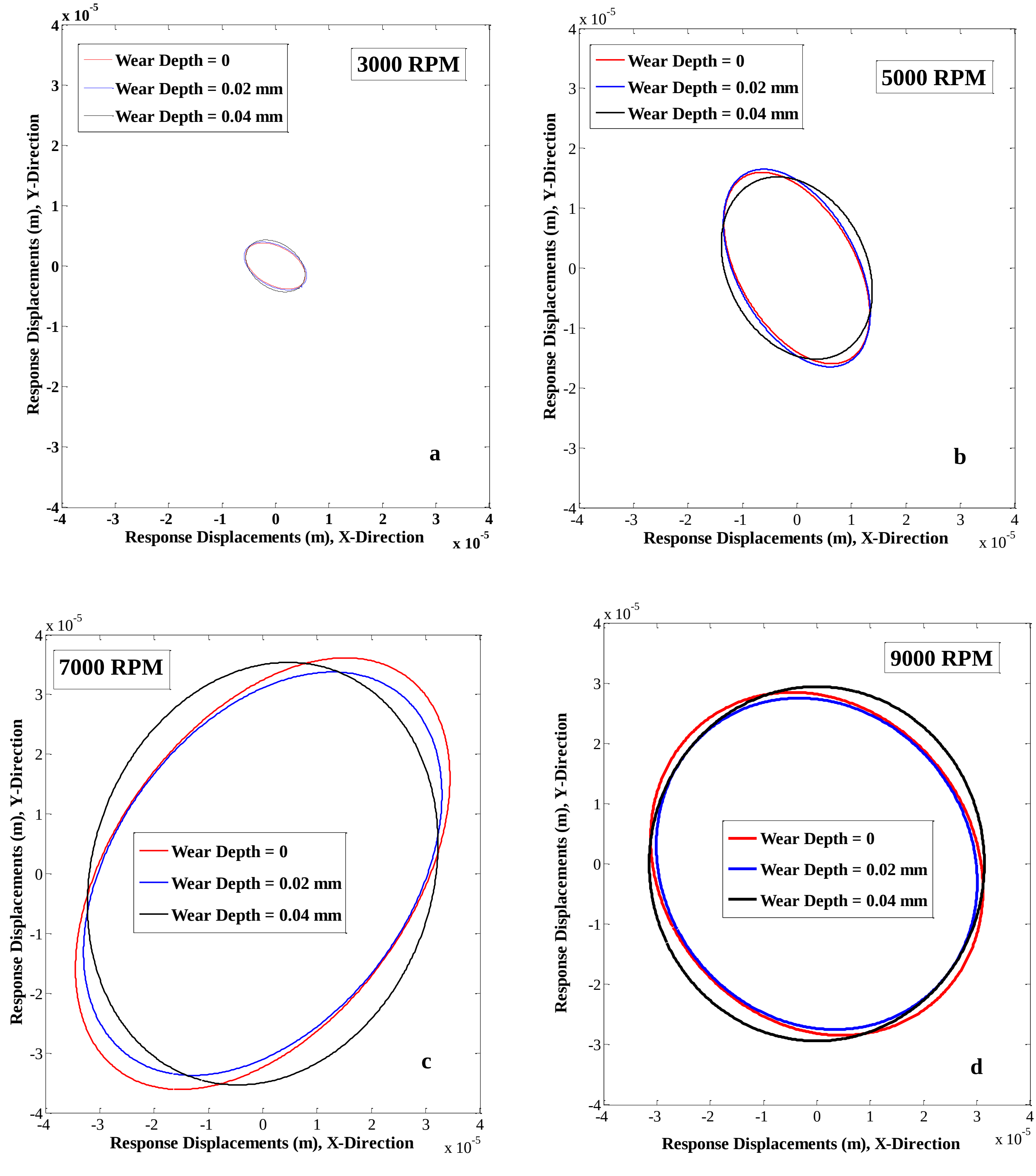


Figure 9. Effect of wear depth in journal bearing on the rotor whirl orbit at disk center, (a) At 3000 rpm, (b) At 5000 rpm, (c) At 7000 rpm and (d) At 9000 rpm.

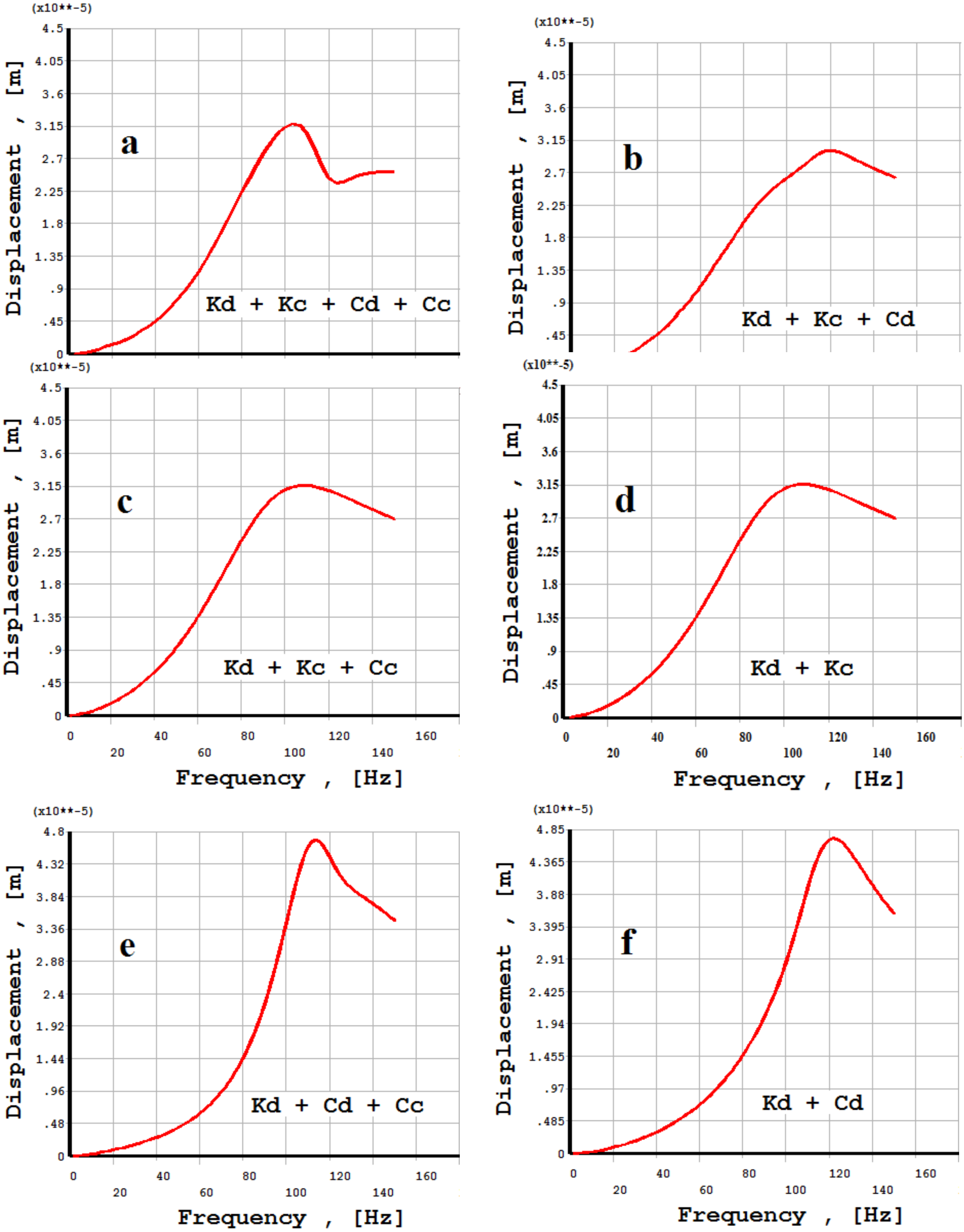


Figure 10. Steady state harmonic response of rotor mounted on non-worn fluid film journal bearing with different combinations of dynamic coefficients (ANSYS).

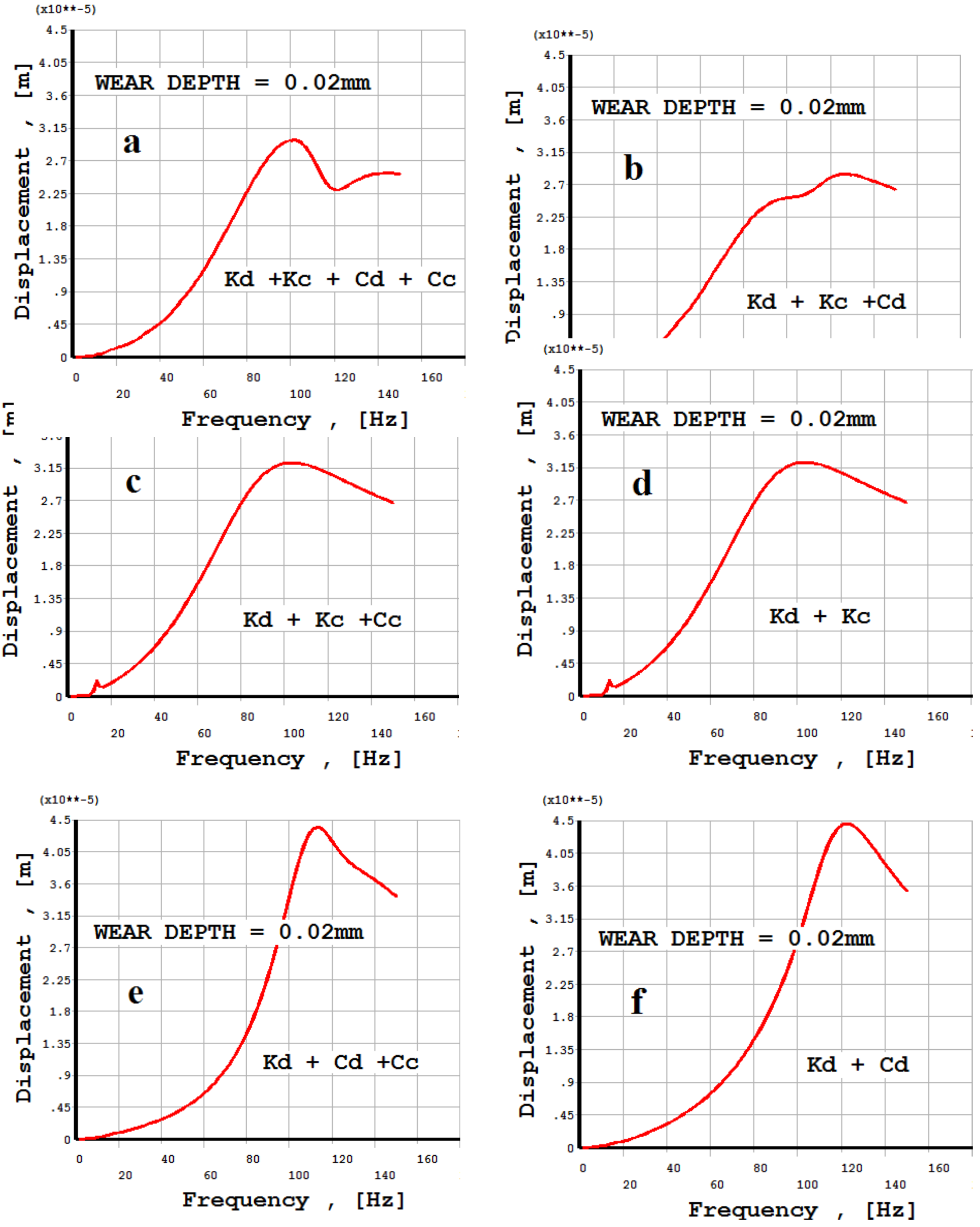


Figure 11. Steady state harmonic response of rotor mounted on worn fluid film journal bearing (wear depth = 0.02 mm) with different combinations of dynamic coefficients (ANSYS).

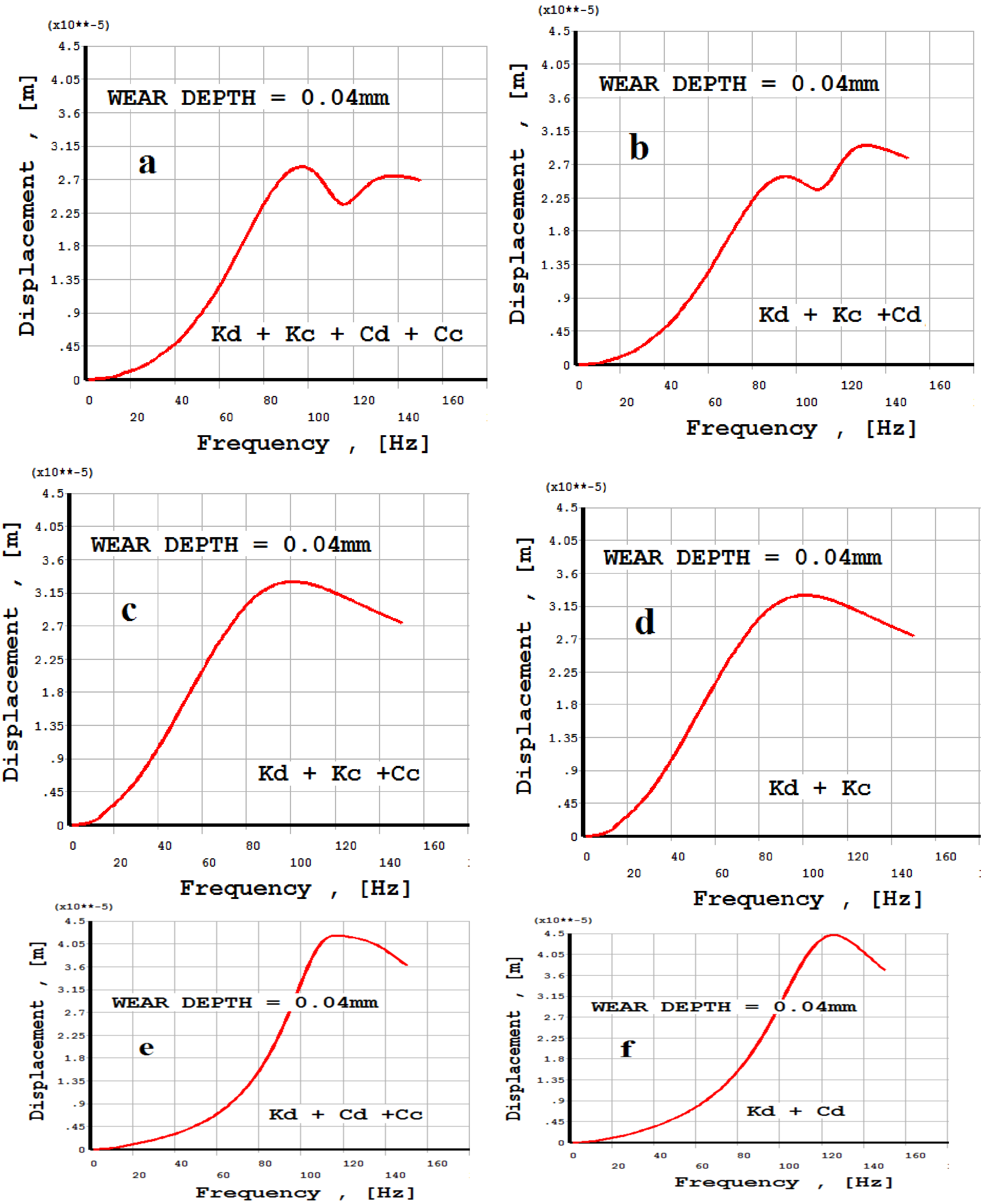


Figure 12. Steady state harmonic response of rotor mounted on worn fluid film journal bearing (wear depth = 0.04 mm) with different combinations of dynamic coefficients (ANSYS).

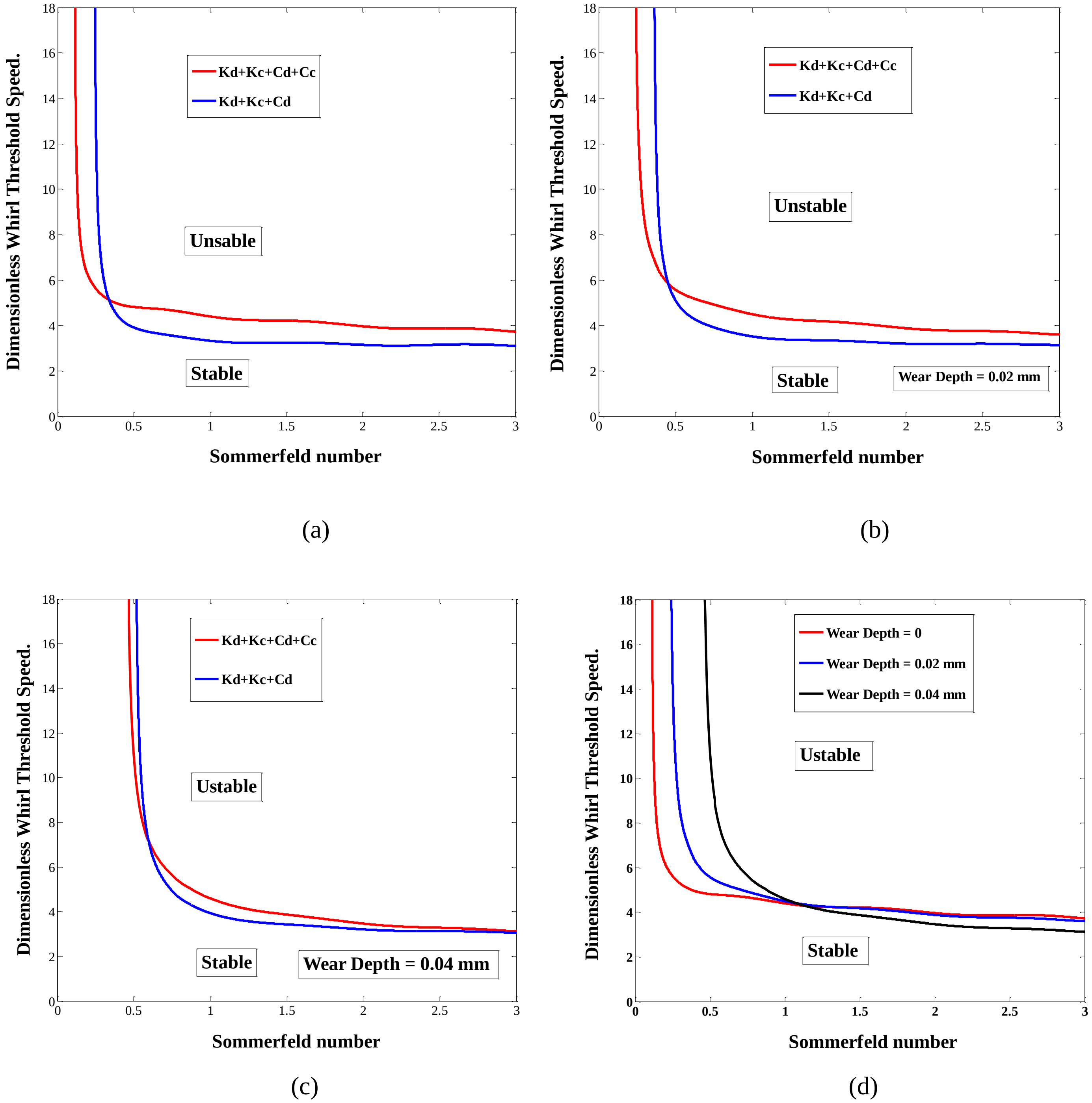


Figure 13. Instability threshold speed (a) Non-worn journal bearing, (b) Wear depth = 0.02 mm, (c) Wear depth = 0.04mm, (d) Comparison between three cases.

Table 1. Shows rotor dimensions and material properties and lubricant oil specifications.

Shaft length m	Shaft diam. m	Disc diam. m	Disc thickness m	Modulus of elasticity pa	Shaft and disk density Kg/m^3	Lubricant oil viscosity Pa s	Unbalance force Kg - m
o.654	0.048	0.34	0.02	$2.1x10^{11}$	7850	0.032	$0.323x10^{-3}$

Table 2. Effect of wear depth in the fluid film journal bearing on the steady state harmonic response amplitude and critical speed of rotor mounted on worn fluid film journal bearing.

Wear Depth mm	Analytical		ANSYS		% Error	
	Max Response Amplitude m	Critical Speed rpm	Max Response Amplitude m	Critical Speed rpm	Critical Speed	Response Amplitude
0	3.22×10^{-5}	7000	3.182×10^{-5}	6400	8.5	1.18
0.02	3.08×10^{-5}	6700	2.983×10^{-5}	6100	8.9	3.14
0.04	2.77×10^{-5}	6500	2.873×10^{-5}	5950	8.4	3.58