NEW METHOD FOR DERIVATION OF LORENTZ TRANSFORMATION

O. M. Al-Kazaleh
Electrical Department /College of
Engineering /University of Baghdad

ABSTRACT
The free solution of the Dirac equation is so important in the modern field of heavy ion atomic physics, determined through Lorentz transformation [W. Greiner 1997]. At high energies one must investigate the relative wave equations. This means equations, which are invariant under Lorentz transformation. The Lorentz transformation is so important, that it needs, to be derived on a more logical basis. Many textbooks and research papers have derived Lorentz transformation by different postulates [Anderson, J.L. 1967][G. Nadean 1962][H.A. Atwater 1974], which are not treated electro-dynamically this paper exhibits a method of finding the Lorentz transformation by exploiting the Maxwell's famous equations of the electromagnetic field equations.

الخلاصة
أن حل معادلة ديراك ذات أهمية عظمى في الحقل الحديث للأيون الثقيل في الفيزياء الذرية، ويعتمد الحل على تحويل لورانز. حيث يجب أن يبحث المرء معادلات الموجة النسبية عند وجود الطاقات العالية، وهذه تعني أن تلك المعادلات لازمة تحت تحويل لورانز وذلك بموجب فرضيات متعددة. أن هذا البحث يبين طريقة معتمدة على معادلات ماكسويل الكهرومغناطيسية المشهورة وذلك بفرض أن هذه المعادلات لازمة.

KEY WORDS
Relativity theory, Lorenz Transformations.

INTRODUCTION

The Galilean transformation has left the phenomena of classical mechanics invariant (unaltered) relative to inertial reference frames, that is, independent in form of the reference frames which are moving uniformly with respect to each other. Yet the equations of electrodynamics will change under the Galilean transformation.

Thus Galilean transformation is totally inadequate for electrodynamics phenomena. Several persons discovered independently that the invariance of all physical laws could be achieved by using Lorentz transformations, which would leave both the equations of mechanics and the equations of electrodynamics invariant. Equations of Maxwell's as have been verified experimentally in every inertial frame of reference. Einstein in 1905 put forward the theory of relativity based on the assumption that all physical laws are invariant, this could be achieved by means of Lorentz transformation. The transformation determined by many authors. Here the derivation of Lorentz transformations are based on a new assumption, that the equations of Maxwell are invariant.

INVARIANCE OF MAXWELL EQUATIONS:

Assuming that some body ignorant of the work of Copernicus, Galileo, and Newton but otherwise gifted with highest experimental abilities and mathematical skill (a quite imaginary assumption) and familiar with Maxwell equations, they appeared only after a prolonged analysis of precise experiments, with power of thought. Just as Newton equations where an inspired postulate so Maxwell equations can be regarded an inspired postulate. He declares as postulate:

Maxwell equations are invariant with respect to inertial frames of reference, which give at once the invariance of the wave equation:

$$\Omega\phi = (\frac{\partial^2}{c^2\partial t^2} - \nabla^2)\phi = 0 \qquad (*)$$

Where Φ= E or Φ= H, the electric and the magnetic fields respectively.

Now, let S be a coordinate system moving in the X- direction with uniform velocity V relative to another system S'. let x, y, z, t of any event in S, and those x', y',z', t' of the same event in S'. In order that the relation between these variables is to be as simple as possible, let us choose our axes so that the x and x'- axes are both parallel to the velocity V, and in fact, sliding along each other; and let the y'- and z'- axes be parallel to y and z respectively.

The times t and t' measured from the instant at which the two origins of coordinates, o and o' momentarily coincide.

Working with the differential operates Ω = Ω', and y=y', z=z', then;

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial' y^2} \quad ; \quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial' z^2} \quad \text{hence;}$$

$$\Omega = \frac{\partial^2}{c^2\partial t^2} - \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{c^2\partial t'^2} - \frac{\partial^2}{\partial' x^2} = \Omega'$$

Where c is a universal constant.

Put:

$\alpha = x - ct$, $\beta = x + ct$, in S, and similarly

$\alpha' = x' - ct'$, $\beta' = x' + ct'$ in S'

Now, we have:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial\alpha}\frac{\partial\alpha}{\partial x} + \frac{\partial}{\partial\beta}\frac{\partial\beta}{\partial x} = \frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta},$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial\alpha}\frac{\partial\alpha}{\partial t} + \frac{\partial}{\partial\beta}\frac{\partial\beta}{\partial t} = -c\frac{\partial}{\partial\alpha} + c\frac{\partial}{\partial\beta}$$

and

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) = \frac{\partial}{\partial\alpha}(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) + \frac{\partial}{\partial\beta}(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) = \frac{\partial^2}{\partial\alpha^2} + 2\frac{\partial^2}{\partial\beta\partial\alpha} + \frac{\partial^2}{\partial\beta^2} \qquad (1)$$

Similarly,

$$\frac{\partial^2}{\partial t^2}=-c^2(\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\beta\partial\alpha}+\frac{\partial^2}{\partial\beta^2}) \qquad (2)$$

Thus from (1) and (2) we have:

$$\Omega=4\frac{\partial^2}{\partial\beta\partial\alpha} \qquad (3)$$

And likewise,

$$\Omega=4\frac{\partial^2}{\partial\beta'\partial\alpha} \qquad (4)$$

The invariance of the wave equation (*) will assume the form

$$\frac{\partial^2\phi}{\partial\beta\partial\alpha}=\frac{\partial^2\phi}{\partial\beta'\partial\alpha'}$$

$$\frac{\partial^2}{\partial\beta\partial\alpha}=\frac{\partial^2}{\partial\beta'\partial\alpha'} \qquad (5)$$

Let α',β' as functions of α,β,

$\alpha'=\alpha'(\alpha,\beta)$: $\beta'=\beta'(\alpha,\beta)$

Now,

$$\frac{\partial}{\partial\alpha}=\frac{\partial}{\partial\alpha'}\frac{\partial\alpha'}{\partial\alpha}+\frac{\partial}{\partial\beta'}\frac{\partial\beta'}{\partial\alpha}$$

$$\frac{\partial^2}{\partial\beta\,\partial\alpha}=\frac{\partial\alpha'}{\partial\alpha}\frac{\partial\alpha'}{\partial\beta}\frac{\partial^2}{\partial\alpha'^2}+\frac{\partial\beta'}{\partial\alpha}\frac{\partial\beta'}{\partial\beta}\frac{\partial^2}{\partial\beta'^2}+(\frac{\partial\alpha'}{\partial\beta}\frac{\partial\beta'}{\partial\alpha}+\frac{\partial\alpha'}{\partial\alpha}\frac{\partial\beta'}{\partial\beta})\frac{\partial^2}{\partial\beta'\partial\alpha'}$$

From (5), then:

$$\frac{\partial^2\alpha'}{\partial\beta\partial\alpha}=0\ ,\quad \frac{\partial^2\beta'}{\partial\beta\partial\alpha}=0 \qquad (6)$$

$$\frac{\partial\alpha'}{\partial\alpha}\frac{\partial\alpha'}{\partial\beta}=0\ ,\quad \frac{\partial\beta'}{\partial\alpha}\frac{\partial\beta'}{\partial\beta}=0 \qquad (7)$$

$$\frac{\partial\alpha'}{\partial\beta}\frac{\partial\beta'}{\partial\alpha}+\frac{\partial\alpha'}{\partial\alpha}\frac{\partial\beta'}{\partial\beta}=1 \qquad (8)$$

To satisfy the first equation of (7), put:

$\frac{\partial\alpha'}{\partial\beta}=0,$ then $\frac{\partial\alpha'}{\partial\alpha}\frac{\partial\beta'}{\partial\beta}=1$ from (8)

Then for the second equation of (7) we must have $\frac{\partial\beta'}{\partial\alpha}=0$, therefore;

$\alpha' = \alpha'(\alpha)$ and $\beta' = \beta'(\beta)$

The first and second of the above conditions are identically satisfied:

then $\frac{\partial \alpha'}{\partial \alpha}\frac{\partial \beta'}{\partial \beta} = 1$ becomes $\frac{d\alpha'}{d\alpha}\frac{d\beta'}{d\beta} = 1$

But $\alpha' + \beta' = 2x'$

Let P be any fixed point on the x'-axis. Then,

x'= constant in S', but p with velocity $v = \frac{dx}{dt}$ relative to S.

Therefore $\frac{d\alpha'}{dt} + \frac{d\beta'}{dt} = 0$

$$\frac{d\alpha'}{dt} = \frac{d\alpha'}{d\alpha}\frac{d\alpha}{dt} \quad , \quad \frac{d\beta'}{dt} = \frac{d\beta'}{d\beta}\frac{d\beta}{dt}$$

And

$$\frac{d\alpha}{dt} = \frac{dx}{dt} - C = \mathrm{V} - \mathrm{C} \ , \quad \frac{\mathrm{d}\beta}{\mathrm{dt}} = \mathrm{V} + \mathrm{C}$$

So that;

$$(V\text{-}C)\frac{d\alpha'}{d\alpha} + (v + C)\frac{d\beta'}{d\beta} = 0$$

$$(V\text{-}C)\ ((\frac{d\alpha'}{d\alpha})^2 + (V + C)\frac{d\alpha'}{d\alpha}\frac{d\beta'}{d\beta} = 0'$$

But,

$$\frac{d\alpha'}{d\alpha}\frac{d\beta'}{d\beta} = 1$$

Then; $\frac{d\alpha'}{d\alpha} = \sqrt{\frac{C + V}{C - V}}$, $\frac{d\beta'}{d\beta} = \sqrt{\frac{C - V}{C + V}}$

Integrating, then

$$\alpha' = \sqrt{\frac{C + V}{C - V}}\alpha + \mathrm{C1} \quad , \quad \beta' = \sqrt{\frac{\mathrm{C - V}}{\mathrm{C + V}}}\beta + C2$$

But the choice of S and S', implies that x'=0, t'=0, when x=0, t =0 , or $\alpha',\beta'=0$ where $\alpha,\beta=0$, thus C1=C2=0.

$$2x' = \alpha' + \beta' = \sqrt{\frac{C + V}{C - V}}\alpha + \sqrt{\frac{C - V}{C + V}}\beta$$

$$\beta = \sqrt{\frac{C+V}{C-V}}(x-ct) + \sqrt{\frac{C-V}{C+V}}(x+ct)$$

Hence, $x' = \gamma(x - vt)$. Similarly $\beta' - \alpha' = 2ct$

Gives; $t' = \gamma(t - \frac{vx}{c2})$

The last two equations are the well-known Lorentz transformation.

Where,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

REFERENCES:-

W. Greiner: (1997), Relativistic Quantum Mechanics, Wave Equations , Springer.

Anderson, J.L. (1967), Principles of Relativity Physics", New York, London: Academic press .

G. Nadean (1962), the Lorentz- Einstein Transformation obtained by a vector method , Amer. J. Phys30,pp.602-3,.

H.A. Atwater (1974), Introduction to General Relativity , Pergamon press.