RELIABILITY-BASED DESIGN PROCEDURE OF AXIALLY LOADED PILES

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ABSTRACT:
Geotechnical engineering involves many different and complex materials and many different mechanisms of behaviour. The direct use of experience as a guide to the prediction and design is effective, provided these are understood. Geotechnical engineering is a relatively new science. Its successful application to prediction requires realistic assumptions to be made, and predictions must be tested against reality. Methods of prediction need then to be refined. An example of the importance of making realistic assumptions is examined in this paper. It includes the effects of soil properties on the ultimate capacity of axially loaded piles.

Better analysis offers better prediction and better understanding. Both are only possible when reality is modelled. There are occasions when mechanisms are too complex for predictive analysis. Prediction must then be based directly on experience, applied with an understanding of the mechanisms involved. Moreover, methods of analysis may become too sophisticated for everyday use. However, pseudo-analysis, involving standardized methods based on oversimplified and unrealistic mechanisms of behaviour and material properties, is dangerous. The use of engineering experience as a guide to prediction and design may offer a more effective alternative, provided it is based on a realistic understanding of mechanisms and materials.

In this paper, a procedure is recommended to estimate the bearing capacity of axially loaded piles based on reliability calculations. The procedure is an extension of the point estimate method in which the expected values of the standard deviation of the capacity and demand functions are calculated. The probability of failure, the reliability, central factor of safety and reliability index are calculated as appropriate. The procedure is then applied to two cases where the pile in the first case is driven in sand while in the second, it is driven in clay.

طريقة تصميم للركائز المحملة محوريا مبنية على أساس مبدأ الاعتماد

الخلاصة:
تتطلب الهندسة الجيوتقنية العديد من المواد المعقدة و العديد من آليات السلوك. يعتبر الاستعمال المباشر للخبرة كدليل في التخمين و التصميم فعالا عند إدراك هذه المتطلبات. تعتبر الهندسة الجيوتقنية علما حديثا نسبيا و يتطلب تطبيقها بنجاح في التخمين و صنع فرضيات مقبولة, و هذه التخمينات يجب أن تختبر من

ناحية قبولها و اعتمادها. و عليه فان طرق التخمين في حاجة إلى تهذيب. في هذا البحث أختبر مثال على أهمية وضع فرضيات معقولة. و هذا يتضمن تأثير خواص التربة على قابلية التحمل القصوى للركائز المحملة محوريا.

إن التحليل الجيد يوفر تخمينا جيدا و فهما جيدا, و كلاهما يصبحان ممكنين عند التمثيل بشكل معقول. هناك حالات تصبح عندها الآليات معقدة لوضع تحليل تخميني, و عندها يجب أن يستند التخمين مباشرة على الخبرة مقترنا مع فهم الآليات المطلوبة, فضلا عن ذلك قد تصبح طرق التحليل معقدة بالشكل الذي يجعلها صعبة الاستعمال بشكل يومي. و على كل حال, إن التحليل الساكن الذي يتطلب طرقا قياسية مبنية على آليات غير معقولة و مبسطة بشكل مفرط لتصرف المواد و خواصها يعتبر خطرا. ان استعمال الخبرة الهندسية كدليل للتخمين و التصميم قد يوفر بديلا فعالا عندما يكون مستندا إلى فهم معقول للآليات و للمواد.

في هذا البحث أقترح أسلوب لتخمين قابلية تحمل الركائز المحملة محوريا بناءا على حسابات مبدأ الاعتماد و المعقول. إن هذا الأسلوب يعتبر امتدادا لطريقة تخمين النقطة التي من خلالها تحسب القيم المتوقعة للانحراف المعياري لقابلية التحمل و دوال الحاجة. و تحسب احتمالية الفشل و الاعتماد و معامل الأمان المركزي و مؤشر الاعتماد و المعقول حسب الحاجة. ثم طبق الأسلوب على حالتين حيث في الأولى الركيزة مساقة في رمل و في الثانية الركيزة مساقة في طين.

INTRODUCTION

The trend in civil engineering today, more than ever before, is toward providing economical designs at specified levels of safety. Often these objectives necessitate a prediction of the performance of a system for which there exists little or no previous experience. Current design procedures, which are generally learned only after many trial-and-error iterations, lacking precedence, often fall short of expectations in new or non conventional situations. In addition, there is an increasing awareness that the raw data, on which problem solutions are based, themselves exhibit significant variability. It is the aim of reliability methods of design to demonstrate how concepts of probability analysis may be used to supplement the geotechnical engineer's judgment in such matters.

Quite often, deterministic approaches are employed in the analysis and design of engineering structures. These approaches are characterized by the use of specified minimum factors of safety or specified minimum material properties. Deterministic approaches do not rigorously account for uncertainties in engineering analysis and design. In order to address uncertainty, probability theory has been widely accepted and used in engineering design in which some statistical knowledge of random variables such as their mean values and standard deviations is used to introduce them into applications (Kaymaz et al., 1998). Probabilistic methods, especially, reliability analysis, have frequently been used in structural engineering (e.g. Grigoriu, 1983; Afolayan, 1998) as well as in geotechnical and geoenvironmental engineering (e.g. Christian et al., 1994; Rowe and Fraser, 1995; Gui et al., 2000). Reliability calculations provide a means of evaluating the combined effects of uncertainties, and a means of distinguishing between conditions where uncertainties are particularly high or low (Duncan, 2000). Moreover, reliability analysis provides a framework for establishing appropriate factors of safety and other design targets and leads to a better appreciation of the relative importance of uncertainties in different parameters (Christian and Baecher, 2001).

The interaction between theory and practice is complex. To predict real behaviour, realistic assumptions for material properties must be made, based on the geological origins of materials, on their intrinsic nature and on their behaviour observed in the laboratory and in the field. Realistic assumptions for boundary conditions must also be made. Theory must be examined regularly to see whether or not it meets these requirements, and it must be refined when it does not. Terzaghi ,Peck and Mesri (1996) recognized the need to link theory and practice, and to test one against the other, nearly 60 years ago. Theory is much more coherent now. Perhaps old habits should be recovered. Often, if problems are understood, simple methods of analysis will provide adequate solutions.

The primary value of realistic analysis is to aid understanding of problems. If they are understood they can usually be solved. Understanding provides a framework within which uncertainties which cannot be avoided can be defined and managed. The ability to make exact deterministic theoretical predictions, even by the most advanced methods, is uncertain. There are many areas where analysis may help to explain a problem, but not solve it in a predictive way. Then the direct use of field experience is necessary, and this is proper and safe provided the mechanisms governing behaviour are understood. Recognition of when this approach is appropriate needs to be improved. A theoretical framework provides the language with which experience can be digested, learned from and made generally available, but reality must be incorporated if success is to be achieved. Such an approach often makes things more complicated than they need be, and so introduces the risk of unnecessary errors. Doubtless these can be eliminated by quality assurance, but quality assurance applied to unrealistic and therefore irrelevant calculations merely eliminates the possibility of being right by accident.

There is an alternative: practical guidance carefully linked to both geology and type of construction, with warnings as to where uncertainties lie. This offers a much more effective approach to everyday problems. Probably the most important question facing the profession, and the one of greatest economic significance, is in what form should effective guidance in geotechnical engineering be given to non-specialist engineers? It controls how effectively the considerable expertise of geotechnical engineers is used, (Vaughan, 1994).

Phoon (2004) presented an overview of the evolution in structural and geotechnical design practice over the past half a decade or so in relation to how uncertainties are dealt with. The key elements of reliability-based design (RBD) were briefly discussed and the availability of statistics to provide empirical support for the development of simplified RBD equations is highlighted. Several important implementation issues were presented with reference to an EPRI study for reliability-based design of transmission line structure foundations (Phoon et al., 1995).

PROBABILISTIC PRELIMINARIES

The **probability** of the success of a structure is called its **reliability**, R, Symbolizing the probability of failure as p(f), we have the important expression:

$$\mathbf{R + p(f) = 1} \qquad (1)$$

Moments

Consider a system of discrete parallel (vertical) forces, P(1), P(2), …, P(N), acting on a rigid beam at the respective distances x(1), x(2), …, x(N), as in Fig. 1(a). From statics, we have that the magnitude of the *equilibrant, M,* is:

$$M = \sum_{i=1}^{N} P(i) \tag{2}$$

and its point of application, $\bar{x}$, is

$$\bar{x} = \frac{\sum_{i=1}^{N} x(i).P(i)}{\sum_{i=1}^{N} P(i)} \tag{3a}$$

Suppose now that the discrete forces $P(i)$ in Fig. (1a) represent the frequencies of the occurrence of the N outcomes $x(1)$, $x(2)$, …, $x(N)$. As the distribution is exhaustive, the magnitude of the equilibrant must be unity, $M = 1$. Hence, Eq. (3b) becomes:

$$E[x] = \bar{x} = \sum_{i=1}^{N} x(i)P(i) \tag{3b}$$

The expected value (mean) provides the locus of the central tendency of the distribution of a random variable. To characterize other attributes of the distribution, recourse is had to higher moments. Again, returning to statics, a measure of the dispersion of the distribution of the force system about the centroidal axis, at $x = E[x]$ in Fig.1(b), is given by the *moment of inertia* (the *second central moment*),

$$I(y) = \int_{x(a)}^{x(b)} (x - \bar{x})^2 \, p(x)dx \tag{4}$$

The equivalent measure of the scatter (variability) of the distribution of a random variable is called its **variance**, denoted as $v[x]$ and defined as:

$$\langle Discrete \rangle \quad v[x] = \sum_{all\ x(i)} [x(i) - \bar{x}]^2 . p(i) \tag{5a}$$

$$\langle Continuous \rangle \quad v[x] = \int_{x(a)}^{x(b)} (x - \bar{x})^2 \, p(x)dx \tag{5b}$$

In terms of the expectation, these can be written as:

$$v[x] = E[(x - \bar{x})^2] \tag{6}$$

which, after expansion, leads to a form more amenable to computations:

$$v[x] = E[x^2] - (E[x])^2 \tag{7}$$

This expression is the equivalent of the parallel-axis theorem for the moment of inertia.

As an example, let us find the expected value and the variance of the exponential distribution:

p(x) = a.exp(-a x), x > 0 a is a constant.

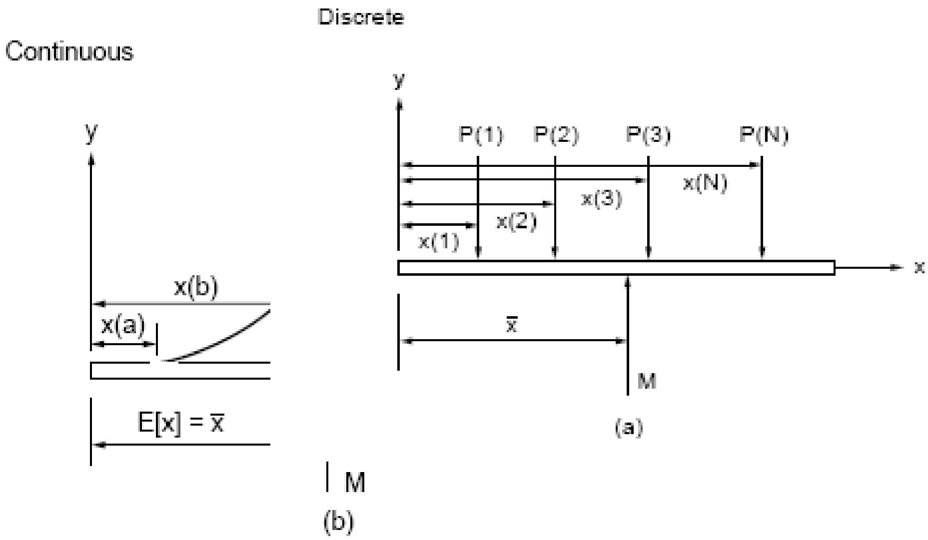

Figure (1): Equilibrant for discrete and continuous distributions, (Harr, 2002).

It is first required to show that $p(x)$ is a valid probability distribution:

$$\int_0^\infty p(x)dx = a\int_0^\infty e^{-ax}dx = 1 \quad ,$$

The expected value is:

$$E[x] = a\int_0^\infty xe^{-ax}dx = \frac{1}{a}$$

Continuing,

$$E[x^2] = a\int_0^\infty x^2e^{-ax}dx = \frac{2}{a^2}$$

whence, using Eq. (7),

$$v[x] = \frac{2}{a^2} - \left(\frac{1}{a}\right)^2 = \frac{1}{a^2}$$

It is seen that the variance has the units of the square of those of the random variable. A more meaningful measure of dispersion of a random variable (x) is the positive square root of its variance (compare with radius of gyration of mechanics) called the standard deviation, $\sigma[x]$,

$$\sigma[x] = \sqrt{v[x]} \tag{8}$$

From the results of the previous example, it is seen that the standard deviation of the exponential distribution is $\sigma[x] = 1/a$.

An extremely useful relative measure of the scatter of a random variable (x) is its coefficient of variation V(x), usually expressed as a percentage:

$$V[x] = \frac{\sigma[x]}{E[x]} x100(\%) \tag{9}$$

It should be emphasized that a straight line fit can be assumed. The reasonableness of this assumption is provided by the correlation coefficient, ρ defined as:

$$\rho = \frac{\mathrm{cov}[x, y]}{\sigma[x].\sigma[y]} \tag{10}$$

where σ [x], and σ [y] are the respective standard deviations and cov[x, y] is their covariance which is defined as:

$$\mathrm{cov}[x, y] = \frac{1}{N}\sum_{i=1}^{N}\left[x(i) - \bar{x}\right]\left[y(i) - \bar{y}\right] \tag{11}$$

With analogy to statics, the covariance corresponds to the product of inertia, (Harr, 2002).

Point Estimate Method — Several Random Variables

Rosenblueth (1975) generalized the methodology for any number of correlated variables. For example, for a function of three random variables say, $y = y[x(1), x(2), x(3)]$, where $\rho(i, j)$ is the correlation coefficient between variables $x(i)$ and $x(j)$,

$$E[y^N] = p(+++)y^N(+++) + p(++-)y^N(++-) + ... + p(---)y^N(---) \tag{12a}$$

where:

$$y(\pm\pm\pm) = y\left[\bar{x}(1) \pm \sigma[x1], \bar{x}(2) \pm \sigma[x2], \bar{x}(3) \pm \sigma[x3]\right] \tag{12b}$$

$$p(+++) = p(---) = \frac{1}{2^3}\left[1 + \rho(1,2) + \rho(2,3) + \rho(3,1)\right]$$

$$p(++-) = p(--+) = \frac{1}{2^3}\left[1 + \rho(1,2) - \rho(2,3) - \rho(3,1)\right]$$

$$p(+-+) = p(-+-) = \frac{1}{2^3}\left[1 - \rho(1,2) - \rho(2,3) + \rho(3,1)\right]$$

$$p(+--) = p(-++) = \frac{1}{2^3}\left[1 - \rho(1,2) + \rho(2,3) - \rho(3,1)\right] \tag{12c}$$

where σ[*xi*] is the standard deviation of *x* (*i*).

The sign of ρ(*i*, *j*) is determined by the multiplication rule of *i* and *j*; that is, if the sign of *i* = (–), and of *j* = (+), then (*i*)(*j*) = (–)(+) = (–).

Equation (12a) has 2^3 = 8 terms, all permutations of the three + s and – s. In general, for *M* variables there are 2^M terms and *M*(*M* – 1)/2 correlation coefficients, the number of combinations of *M* objects taken two at a time. The coefficient on the right-hand side of Eqs. (12c), in general, is $(1/2)^M$, (Harr, 2002).

RELIABILITY ANALYSIS

Capacity–Demand

The adequacy of a proposed design in geotechnical engineering is generally determined by comparing the estimated resistance of the system to that of the imposed loading. The resistance is the **capacity** *C* (or strength) and the loading is the induced **demand** *D* imposed on the structure. In the present procedure, because of its greater generality, we shall use a *capacity– demand* concept. Some common examples are the bearing capacity of a soil and the column loads, allowable and computed maximum stresses, traffic capacity and anticipated traffic flow on a highway, culvert sizes and the quantity of water to be accommodated, and structural capacity and earthquake loads.

Conventionally, the designer forms the well-known factor of safety as the ratio of the single-valued nominal values of capacity *C* and demand *D (*Ellingwood et al., 1980), depicted in Fig. 2(a),

$$FS = \frac{\overline{C}}{\overline{D}} \tag{13}$$

For example, if the allowable load is 400 tons per square meter and the maximum calculated load is 250 tons per square meter, the conventional factor of safety would be 1.6. The design is considered satisfactory if the calculated factor of safety is greater than a prescribed minimum value learned from experience with such designs. Thus, in concept, in the above example, if a factor of safety of 1.6 was considered intolerable, the system would be redesigned to decrease the maximum induced load.

In general, the demand function will be the resultant of the many uncertain components of the system under consideration (vehicle loadings, wind loadings, earthquake accelerations, location of the water table, temperatures, quantities of flow, runoff, and stress history, to name only a few). Similarly, the capacity function will depend on the variability of material parameters, testing errors, construction procedures and inspection supervision, ambient conditions, and so on.

A schematic representation of the capacity and demand functions as probability distributions is shown in Fig. 2(b). If the maximum demand (D_{max}) exceeds the minimum capacity (C_{min}), the distributions overlap (shown shaded), and there is a nonzero probability of failure.

The difference between the capacity and demand functions is called the safety margin (*S*); that is,

$$\mathbf{S = C - D} \tag{14}$$

Obviously, the safety margin is itself a random variable, as shown in Fig. 2(c). Failure is associated with that portion of its probability distribution wherein it becomes negative (shown shaded); that is, that portion wherein $S = C - D \leq 0$. As the shaded area is the probability of failure $p(f)$, we have:

$$p(f) = P[(C - D) \leq 0] = P[S \leq 0] \tag{15}$$

Reliability Index

The number of standard deviations that the mean value of the safety margin is beyond $S = 0$, Fig. 2(c), is called the *reliability index*, β; that is,

$$\beta = \frac{\overline{S}}{\sigma[S]} \tag{16a}$$

The reliability index is seen to be the reciprocal of the coefficient of variation of the safety margin, or

$$\beta = \frac{1}{V(S)} \tag{16b}$$

Application to their definitions, produces the following identities (a, b, and c are constants), (Ditlevesen, 1981):

$$E[a + bx + cy] = a + bE[x] + cE[y] \tag{17a}$$

$$v[a + bx + cy] = b^2 v[x] + c^2 v[y] + 2bc.\mathrm{cov}[x, y] \tag{17b}$$

$$\mathrm{cov}[x, y] \leq \sigma[x].\sigma[y] \tag{17c}$$

$$v[a + bx + cy] = b^2 v[x] + c^2 v[y] + 2b.c.\sigma[x].\sigma[y].\rho \tag{17d}$$

From Eq. (17a), we have:

$$\mathbf{E[S] = E[C] - E[D] = \overline{C} - \overline{D}} \tag{18}$$

Equation (17c) produces:

$$\sigma^2[S] = \sigma^2[C] + \sigma^2[D] - 2\rho.\sigma[C].\sigma[D]$$

Hence,

$$\beta = \frac{\bar{C} - \bar{D}}{\sqrt{\sigma^2[C] + \sigma^2[D] - 2\rho.\sigma[C].\sigma[D]}} \qquad (19)$$

It is seen that β is a maximum for a perfect positive correlation and a minimum for a perfect negative correlation. It can be shown that the sum of difference of two normal variates is also a normal variate (Haugen, 1968). Hence, if it is assumed that the capacity and demand functions are normal variates, it follows that:

$$p(f) = \frac{1}{2} - \psi[\beta] \qquad (20)$$

where ψ[β] is standard normal probability as given in standard normal probability tables.

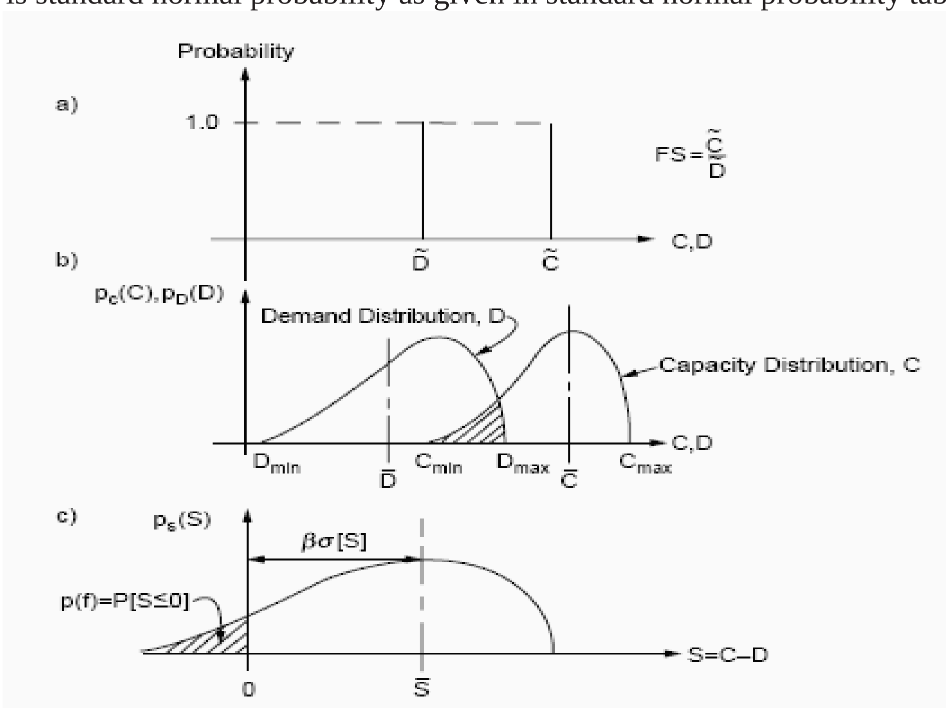

Figure (2): (a) Conventional factor of safety, (b) capacity–demand model, (c) safety margin, (Harr, 2002).

RECOMMENDED PROCEDURE

The following points represent the desirable attributes of a reliability-based design procedure (Harr, 2002):

- It should account for the pertinent capacity and demand factors, their components, and their interactions.
- It should produce outputs that can be related to the expected performance during the design life
- of the system under consideration.

- It should employ as input into formulations quantities, parameters, or material characterizations that can be ascertained within the present state of the art.
- It should not disregard indices currently considered to be pertinent, such as factor of safety or reliability index. It should serve to supplement this knowledge and reduce uncertainty.
- Ideally, mathematical computations should be reduced to a minimum.

All of the above can be accommodated by an extension of the point estimate method. The recommended procedure is as follows, where it is here applicable to the problem of bearing capacity of axially loaded piles:

- Using the point estimate method, or an equally valid probabilistic formulation, the expected values and standard deviations of the capacity and demand functions: $E[C]$, $E[D]$, $\sigma[C]$, $\sigma[D]$ are obtained.
- Calculating the expected value and standard deviation of the safety margin, $E[S]$, σ [S].
- Fitting a beta distribution (and normal distribution, as a check) to the safety margin, using appropriate upper and lower bounds. If unknown, take them as $E[S] \pm 3\sigma[S]$.
- Obtaining the probability of failure, $p(f) = P[S \leq 0]$, the reliability, central factor of safety, and reliability index, as appropriate.

Application to Axially Loaded Piles

I. Piles in Sand

The ultimate bearing capacity Q of a pile in a dry sand (cohesion, c = 0) is given by the following equation (Tomlinson, 1993):

$$\boldsymbol{Q = Q_b + Q_s}$$

where: Q_b = base resistance.

Q_s = shaft resistance

$$\boldsymbol{Q_b = A_b . \sigma'_{vb} . Nq}$$

$$\boldsymbol{Q_s = Ks . \tan(\delta) . As . \sigma'_v}$$

where: A_b = area of the pile base,

As = the surface area of the pile,

σ_v' = effective overburden pressure along the pile,

σ'_{vb} = effective overburden pressure at the pile base,

Ks = a factor depending on the pile type and the relative density of the soil,

δ = the coefficient of friction between the pile and the soil (= 3/4 ϕ for concrete piles),

N_q = bearing capacity factor.

Example: If a 12 m length concrete pile having a square cross-section (0.285 x 0.285 m) is to be driven in a sandy soil with the following parameters:

Parameter, x	Expected Value	Standard Deviation	x (+)	x (-)
Unit weight, γ (kN/m^3)	18	2	20	16
Angle of friction, ϕ (^{o})	35	5	40	30
Cohesion, c (kN/m^2)	0	-	-	-

The correlation coefficient $\rho(\phi, c) = -0.5$:

a. Estimate the expected value and the standard deviation of the bearing capacity.
b. If a central factor of safety (CFS) of 4 is required, and it is assumed that the coefficient of variation V(D) = 50%, estimate the probability of failure.

Solution:

(a) Forming the required values of Nq as the bearing capacity factors are functions of ϕ only (Terzaghi, Peck and Mesri, 1996):.

Bearing capacity factors

ϕ	N_γ	N_q	N_c
0	0.00	1.00	5.14
5	0.45	1.57	6.49
10	1.22	2.47	8.34
15	2.65	3.94	10.98
20	5.39	6.40	14.83
25	10.88	10.66	20.72
30	22.40	18.40	30.14
35	48.83	33.30	46.12
40	109.41	64.20	75.31
42.5	170.25	91.90	99.20
45	271.75	133.87	134.87
50	762.86	319.06	266.88

Nq (+) = 64.20 and Nq (-) = 18.40

Since c = 0, the bearing capacity Q becomes Q(ϕ, γ).

The respective values of Q(ϕ, c, γ) are calculated in (kN):

Q (i, j) (kN)	**Q^2 (i, j)**
Q (+ +) = 2087.94	4359493.44
Q (+ –) = 964.30	929874.49
Q (– +) = 1670.36	2790102.53
Q (– –) = 771.44	595119.67

and

$$\text{p}\ (+\,+) = \text{p}\ (-\,-) = \frac{1}{2^2}(1+\rho) = \frac{1}{8}$$
$$\text{p}\ (+\,-) = \text{p}\ (-\,+) = \frac{1}{2^2}(1-\rho) = \frac{3}{8}$$

From equation (3b):

$$E[Q] = \overline{Q} = \sum Q(ij)\,p(ij) \qquad = 1345.42 \text{ kN}$$

$$E[Q]^2 = \sum Q^2(ij)\,p(ij) \qquad = 2014318.02$$

and from equation (7), we have:

$$V[Q] = E[Q^2] - (E[Q])^2 \qquad = 204163.04$$

and equation (8) gives:

$$\sigma[Q] = \sqrt{V(x)} = \sqrt{204163.04} \quad = 451.84$$

Equation (9) requires:

$$V[Q] = \frac{\sigma[x]}{E[x]}x100 \qquad = \frac{451.84}{1345.42}x100 \quad = 33.6\%$$

(b) For a CFS = 4:

$$\overline{D} = \frac{\overline{Q}}{4} = \frac{1345.42}{4} \quad = 30.57$$

As V (D) = 50%

$$\sigma[D] = E(D)*V(D) \quad = 30.5 * 0.5 = 15.28$$

Forming the characteristics of the safety margin with ρ (Q, D) = $+\frac{3}{4}$, we have:

$$E[S] = \overline{Q} - \overline{D} \quad = 1345.42 - 30.57 \ = 1314.85 \text{ kN}$$

From equation (19), we have:

$$\beta = \frac{1314.85}{\sqrt{(451.84)^2 + (15.28)^2 - 2(0.75)(451.84)(15.28)}} = \frac{1314.85}{40.49} \quad = 2.98$$

If S is to be taken as normal:
From equation (20):

$$p(f) = \frac{1}{2} - \psi(\beta)$$

since β > 2.2:

then: $\psi(\beta) = \frac{1}{2} - \frac{1}{\beta}(2\pi)^{-\frac{1}{2}}.\exp\left[-\frac{\beta^2}{2}\right]$ = 0.49

The probability of failure $p(f)$ = 0.01

II. Piles in Clay

The ultimate bearing capacity Q of a pile in a saturated clay (ϕ = 0) is given by the following equation (Tomlinson, 1993):

$$Q = Q_b + Q_s$$

where: Q_b = base resistance.

Q_s = shaft resistance

Since ϕ = 0, the bearing capacity factors, Nq = 1 and Nγ = 0

$$Q_b = Cu.Nc.A_b$$

$$Q_s = \alpha.P.L.Cu$$

where: P = perimeter of the pile,

L = length of the pile,

Cu = undrained cohesion,

Nc = bearing capacity factor,

α = a factor depending on the undrained shear strength of the pile.

Example: If the same concrete pile of the previous example is to be driven in saturated clay having the following parameters:

Parameter, x	Expected Value	Standard Deviation	x (+)	x (-)
Unit weight, γ (kN/m^3)	20	2	22	18
Angle of friction, ϕ (^{o})	0	-	-	-
Cohesion, c (kN/m^2)	60	20	80	40

The correlation coefficient $\rho(\phi$, c) = -0.5:

c. Estimate the expected value and the standard deviation of the bearing capacity.

d. If a central factor of safety (CFS) of 4 is required, and it is assumed that the coefficient of variation (VD) = 50%, estimate the probability of failure.

Solution:

(a) The required values of the bearing capacity factor Nc are constant since ($\phi = 0$).

α is calculated following Tomlinson method.
Since $\phi = 0$, the bearing capacity Q becomes Q(c, γ).

The respective values of Q(ϕ, c, γ) are calculated in (kN):

Q (i, j) (kN)	**Q^2 (i, j)**
Q (+ +) = 631.43	376296.36
Q (+ −) = 454.76	206806.65
Q (− +) = 613.43	376296.36
Q (− −) = 454.76	206806.65

and

$$p\,(+\,+) = p\,(-\,-) = \frac{1}{2^2}(1+\rho) = \frac{1}{8}$$

$$p\,(+\,-) = p\,(-\,+) = \frac{1}{2^2}(1-\rho) = \frac{3}{8}$$

From equation (3b):

$$\boldsymbol{E[Q] = \bar{Q} = \sum Q(ij)P(ij)} \qquad = 379.06 \text{ kN}$$

$$\boldsymbol{E[Q]^2 = \sum Q^2(ij)P(ij)} \qquad = 291551.50$$

and from equation (7), we have:

$$\boldsymbol{V[Q] = E[Q^2] - (E[Q])^2} \qquad = 147865.0$$

and equation (8) gives:

$$\boldsymbol{\sigma[Q] = \sqrt{V(x)} = \sqrt{147865.0}} \qquad = 384.53$$

Equation (9) requires:

$$\boldsymbol{V[Q] = \frac{\sigma[x]}{E[x]}x100 \quad = \frac{384.53}{379.06}x100} \qquad = 101.44\%$$

(b) For a CFS = 4:

$$\boldsymbol{\bar{D} = \frac{\bar{Q}}{4} = \frac{379.06}{4}} \qquad = 94.76$$

As V (D) = 50%

$$\boldsymbol{\sigma[D] = E(D)*V(D)} \qquad = 94.76 * 0.5 = 47.38$$

Forming the characteristics of the safety margin with ρ (Q, D) = $+\frac{3}{4}$, we have:

$$E[S] = \overline{Q} - \overline{D} \quad = 1379.06 - 94.76 \ = 284.3 \text{ kN}$$

From equation (19), we have:

$$\beta = \frac{284.3}{\sqrt{(384.53)^2 + (47.38)^2 - 2(0.75)(384.53)(47.38)}} = \frac{284.3}{250.39} \quad = 1.13$$

If S is to be taken as normal:

From probability tables, $\psi(\beta)$ = 0.370762

From equation (20):

$$p(f) = \frac{1}{2} - \psi(\beta)$$

The probability of failure $p(f)$ = 0.129

CONCLUSIONS

A procedure is recommended to estimate the bearing capacity of axially loaded piles based on reliability calculations. The procedure is an extension of the point estimate method in which the expected values of the standard deviation of the capacity and demand functions are calculated. The probability of failure, the reliability, central factor of safety and reliability index are calculated as appropriate. The procedure is then applied to two cases where the pile in the first case is driven in sand while in the second, it is driven in clay.

It was found that the proposed procedure is simple and can be extended to other applications in geotechnical engineering.

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