CALCULATION OF SOIL WATER DIFFUSIVITY USING A MODEL FOR SOIL MOISTURE PROFILE UNDER DIFFERENT SALINITY CONDITIONS

Dr. Mahdi I. Aoda Taghreed F. Younan
College of Agriculture, University of Baghdad

ABSTRACT

A closed formed model modified from that of van Genuchten (1980) was used to fit the data of soil moisture profile (θ vs. λ) from which the slope dλ/dθ can be evaluated and then the soil water diffusivity [D(θ)] can be calculated. Diffusivity was calculated for three soil textures under different salinity conditions.

The results showed that the model fitted the data very well with 1% confidence level (R2 > 0.93). D(θ) increased sharply with soil moisture for all soils but its values were lower when the texture got finer. At a certain sodium adsorption ratio (SAR), D(θ) increase with the increase of salt concentration (C) of the water infiltrating. Increasing SAR of water caused a decrease in D(θ) at any level of C. Same trends were obtained for the values of weighted mean diffusivity $\overline{D}$.

حساب الانتشارية المائية للتربة باستخدام انموذج لمنحنى المقد الرطوبي تحت ظروف ملحية مختلفة

الخلاصة

استخدم انموذج مغلق محور عن انموذج van Genuchten (1980) لمطابقة بيانات منحنى المقد الرطوبي (θ vs. λ) لتقييم ميل منحنى المقد الرطوبي (dλ/dθ) والذي باستخدامه تم حساب الانتشارية المائية للتربة. حسبت الانتشارية لثلاث ترب مختلفة النسجة وتحت ظروف ملحية مختلفة لماء الغيض.

اظهرت النتائج بأن الانموذج طابق البيانات التجريبية لمنحنى مقد رطوبة التربة بدرجة كبيرة عند مستوى قناعة 1% ($R^2 > 0.93$). ازدادت الانتشارية بشدة بزيادة رطوة التربة لكافة الترب ولكن قيمها انخفضت عندما تصبح التربة اكثر نعومة . عند مستوى واحد من SAR ازدادت الانتشارية بزيادة التركيزالملحي للماء الغائض. زيادة SAR في التربة ادت الى انخفاض قيم الانتشارية بثبوت التركيز. تم الحصول على نفس التوجه في سلوك قيم معدل الانتشارية الموزون ($\overline{D}$) .

INTRODUCTION

The rate of water movement through the soil is of considerable importance in many aspects of agricultural and urban life. The entry of water into soil, the movement to plant roots, the flow of water to drains and wells, and the evaporation of water from soil surface are but a few of the various situations, in which the rate of movement plays an important role (Klute and Dirksen, 1986). The soil properties that determine the behavior of soil water flow system are the hydraulic conductivity, water- retention characteristics, and soil water diffusivity. These properties are often called soil hydraulic properties. Soil water diffusivity [D((θ)] as a function of volumetric water content (θ), is the ratio of hydraulic conductivity K(θ) to the differential water capacity C(θ)=dθ/dψ, may be used to analyze the behavior of soil water system. Because of the importance of soil water diffusivity, a lot of work has been done to evaluate, calculate, or predict this transport function. Hence, several empirical and physically-based methods were proposed for this purpose (Gardner and Mayhugh,1958; Gardner,1962; Dirksen,1975; Hillel, 1980).

Soil water diffusivity is calculated directly with some known methods, but can be derived easily from measured conductivity data and water content-suction head relationship (Childs and Collis-George, 1950 and Gardner and Mayhugh, 1958) as the follows:

$$D(\theta)= - K(\theta)/C(\theta) = - K(\theta)\, d\psi/d\theta \quad \text{…………….(1)}$$

The soil-water diffusivity appears in the diffusion form of the one-dimensional (x-direction) horizontal flow equation with time t (Philip, 1957). This equation can be written in the following form:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta) \frac{\partial \theta}{\partial x} \right] \quad \text{…………………(2)}$$

with the following initial and boundary conditions:

$$\theta(0, t) = \theta_1 \quad ; \quad \theta(x, 0) = \theta_0 \quad \text{…………………(3)}$$

The Boltzmann variable, ($\lambda = xt^{-1/2}$), was used to transform Eq.(1) to an ordinary differential equation. Integrating, using the given boundary conditions in Eq.(2), yield the following equation in terms of soil–water diffusivity:

$$D(\theta) = -\frac{1}{2}\left(\frac{d\lambda}{d\theta}\right)\int_{\theta_0}^{\theta} \lambda\, d\theta \quad \text{……………….. (4)}$$

where D(θ) and dλ/dθ are evaluated at any value of water content θ,as it will be explained later in this introduction .

Bruce and Clute (1956) described a method in which the spatial distribution of water content, determined by destructive gravimetric sampling at a fixed time (t) in a horizontal infiltration flow system, was used to calculate the diffusivity function [D(θ)] .

Whisler et al.(1968) described a method in which the water content as a function of time at a fixed position (soil- water transient) was measured. This procedure requires a nondestructive method of determining the water content in the soil column, such as gamma attenuation.

The water content versus position (θ vs. x) at a series of fixed times, or the water content vs. time (θ vs. t) at a series of fixed positions can be used to construct a plot of λ vs. θ. If the flow is described by the nonlinear diffusion equation and the boundary and initial water contents are constant, the transformed water content – distant –time data should give a unique λ(θ) function. The derivative and integral in Eq.(4) can then be evaluated from the plot of λ vs. θ .

If λ in Eq.(4) is substituted by $xt^{-1/2}$ (Boltzmann variable) in the derivative and integral and arranging the terms result in:

$$D(\theta) = -\frac{1}{2t}\left(\frac{dx}{d\theta}\right)\int_{\theta_0}^{\theta} x\,d\theta \quad \text{.................}(5)$$

In this case dx/dθ needs to be evaluated for the calculation of D(θ).

The variable λ can be calculated from the data of x and t data ($\lambda = xt^{-1/2}$) and plotted vs. θ to obtain a curve of λ vs. θ. Fitting these data resulted in an equation from which the derivative can be obtain and evaluated and so can be the integral term.

This procedure may be done graphically, numerically, or by analytical means if an equation for the fitted curve is available . So , the diffusivity as a function of water content D(θ) can then be calculated using Eq.(4) or Eq.(5).

A closed form model similar to that used by van Genuchten (1980) for the soil moisture characteristic curve was used to describe the data of θ vs. λ (can also be used for the data of θ vs. x). This model can be written in the following form:

$$\theta = a + b[1+(c\,\lambda)^{n}]^{-m} \quad \text{………………} (6)$$

Where a, b, c, n, and m are fitted parameters, and m has no relation with n . Taking the derivative of Eq.(6) with respect to λ (dθ/dλ) resulted in:

$$d\theta/d\lambda = -bcnm\,(c\,\lambda)^{n-1}\,[1+(c\,\lambda)^{n}]^{-(m+1)} \quad \text{…………..}(7)$$

Also, Solving [Eq. (6)] for λ results in:

$$\lambda = (1/c)\,\{[(\theta-a)/b]^{-1/m} - 1\}^{1/n} \quad \text{..………………} (8)$$

Equations (6) and (8) were used by Aoda et al. (2005) and Younan (2008) and resulted in very successful results in terms of fitting the soil moisture profile data [either θ(λ), or θ(x)].

Taking the derivative of λ with respect to θ of Eq.(8), ($d\lambda/d\theta$) can be obtained and calculated for each value of θ as follows:

$$d\lambda/d\theta = (-1/bcnm)\ \{[(\theta-a)/b]^{-1/m}-1\}^{(1-n/n)}\,[(\theta-a)/b]^{-(1+m/m)} \quad \ldots\ldots\ldots(9)$$

This derivative can also be obtained by taking the reciprocal of Eq.(7).

Weighted mean diffusivity ($\overline{D}$) was calculated using the following equation (Crank, 1956):

$$\overline{D}(\theta) = 5/3\left[1/\left(\theta_s-\theta_0\right)\right]^{5/3}\int_{\theta_0}^{\theta_s}\left(\theta-\theta_0\right)^{2/3}D\left(\theta\right)d\theta \qquad \ldots\ldots\ldots(10)$$

Where θ_s is the volumetric water content at the inlet end of soil column.

The objective of the work reported here is to use an empirical model [Eq.(6)] to describe the relationship between λ and θ and then to find the slope $d\lambda/d\theta$ [Eq.(7)] which is required in Eq.(4) for calculating soil-water diffusivity as a function of volumetric water content [$D(\theta)$]. Values of soil water diffusivity was used as a hydraulic property to study the effect of the salinity and sodicity of irrigation water on soil water diffusion.

MATERIALS AND METHODS

Three different soil samples of different texture (namely, sandy loam, loam , and clay loam) were collected from the surface layer (0 – 30 cm) from Jadriya location in Baghdad. These soils were classified under the subgroup of Typic torrifluvents. Samples were air-dried and sieved through 2-mm sieve openings. Chemical and physical properties of these samples were determined using the procedures given by Page et al. (1982) and Klute et al. (1986), respectively (see table 1).

Water of different salinity[C=50,100 and 200 meq/l] and sodicity [SAR = 0, 10, and 20 $(\text{mmole/l})^{1/2}$] were used for performing the flow experiments. Nine combinations of C and SAR plus the control treatment (C=0, SAR=0) were used to run horizontal infiltration experiments in uniform soil columns packed homogeneously (see Aoda, 1982), with the three different soil textures. Plexiglas columns of 40–cm long and 3.17-cm inside diameter were constructed by combining tightly the rings of 2-cm length using transparent tape. One end of the column was closed by a perforated Perspex glass plate, while the other end was connected to the water applicator (see Al- Douri, 2002).

Horizontal water flow experiments were performed to obtain the required data for calculating soil water diffusivity following the procedure of Bruce and Klute (1956). Volume of water infiltrated into the soil and wet front advance with time were recorded until the wet front reached a distance of 30 cm from the inlet end of the column. At this instance, water entry was cut off and the column was sectioned by sharp blade to determine the water content and bulk density for each ring. Bulk density was also determined for each ring to test the uniformity of the whole column. The column was rejected if the coefficient of variability (C.V) exceeded 2% (Nofziger and Swartzendruber, 1976).

Soil water diffusivity [D(θ)] was calculated using Eq.(4) where the derivative $d\lambda/d\theta$ was calculated from the proposed equation [Eq.(7)]. The procedure and calculation were done for ten different water qualities for each soil texture.

Table 1: Some physical and chemical properties of the soils used.

Soil property	Soil texture		
	Sandy loam	Loam	Clay loam
Sand (g/Kg)	663.8	490.7	364.2
Silt (g/Kg)	252.3	326.1	330.9
Clay (g/Kg)	83.9	183.2	304.9
Bulk density (Mg/m^3)	1.500	1.447	1.410
ECe* (dS/m)	0.82	1.60	2.40
pH	7.82	7.78	7.60
Carbonates (g/Kg)	195.6	204.8	232.8
Gypsum (g/Kg)	Nill	3.8	4.00
Organic matter (g/Kg)	11.17	13.07	13.67
CEC**(cmole charge/Kg)	15.85	19.20	21.80
Total porosity (m^3/m^3)	0.434	0.454	0.468
Mean weight diameter (mm)	0.252	0.309	0.482

* ECe is the electrical conductivity of the extract soil saturated paste .

** CEC is the cation exchangeable capacity.

RESULTS AND DISCUSSION

Table 2 shows the results of fitting Eq.(6), θ vs. λ.The parameters a,b,c,m, and n are listen in the table along with the values of residual mean squares of θ (RMSθ)and the coefficient of determination(R^2).The fitting was done by using the Statistical Analysis System (Statistica) . The model fitted the data very well (0.01 level) indicated by the high values of R^2 and the small values of RMS θ. This finding is the same for the three soils and for all experiment combination of salinity (salt concentration ,C) and sodicity (sodium adsorption ratio, SAR).

Values of parameter a in table 2 represent the values of the initial volumetric water content θ_0, and the values of b in the table represent the values of θ behind the wet front θ_m minus the values of θ_0 (i.e., b= θ_m- θ_0).If parameter a is added to parameter b, the result is θ_m. If this θ_m is divided by the total porosity (f) for each soil (Table 1), the result is the saturation ratio(S). When this is done using tables 2 and 1, the percentage of saturation was about 83% ,85%,and 87% for sandy loam, loam, and clay loam soils , respectively.Values similar to these were found by van Duin (1955) , Aoda (1982) and Aoda et al.(1993). Some good consistence for the c and n values

Table 2: Results of fitting Eq.(6) for all experiments of salinity combinations.

Texture	SAR	Conc.	a	b	*c*	*n*	m	RMSθ	R^2
Sandy Loam	o	0	0.0260	0.3400	0.4679	26.854	73.396	$6.052*10^{-4}$	0.93657**
		50	0.0270	0.3390	0.4605	26.845	96.005	$6.134*10^{-4}$	0.93545**
		100	0.0270	0.3390	0.4552	27.069	106.990	$6.125*10^{-4}$	0.93548**
		200	0.0270	0.3390	0.4427	27.031	143.901	$5.998*10^{-4}$	0.93669**
	10	50	0.0260	0.3402	0.4701	26.475	89.430	$6.159*10^{-4}$	0.93567**
		100	0.0270	0.3396	0.4622	26.741	89.939	$6.026*10^{-4}$	0.93668**
		200	0.0270	0.3390	0.4527	27.038	81.807	$6.235*10^{-4}$	0.93435**
	20	50	0.0256	0.3414	0.4798	26.991	103.912	$6.227*10^{-4}$	0.93492**
		100	0.0260	0.3401	0.4683	26.720	90.979	$6.267*10^{-4}$	0.93432**
		200	0.0270	0.3390	0.4527	26.775	84.806	$6.208*10^{-4}$	0.93462**
Loam	0	0	0.0282	0.3588	0.5740	27.174	77.428	$6.783*10^{-4}$	0.93572**
		50	0.0282	0.3588	0.5703	27.366	77.020	$6.709*10^{-4}$	0.93645**
		100	0.0286	0.3584	0.5637	27.144	78.755	$6.656*10^{-4}$	0.93677**
		200	0.0283	0.3587	0.5514	27.658	61.967	$6.902*10^{-4}$	0.93445**
	10	50	0.0280	0.3598	0.6103	27.794	90.493	$6.730*10^{-4}$	0.93632**
		100	0.0280	0.3595	0.5903	27.983	81.623	$6.995*10^{-4}$	0.93377**
		200	0.0280	0.3590	0.5702	27.359	76.058	$6.852*10^{-4}$	0.93502**
	20	50	0.0281	0.3589	0.6429	27.473	88.964	$6.809*10^{-4}$	0.93557**
		100	0.0280	0.3593	0.6169	27.355	91.416	$6.821*10^{-4}$	0.93531**
		200	0.0280	0.3590	0.5971	27.134	83.897	$6.673*10^{-4}$	0.93683**
Clay Loam	0	0	0.0300	0.3760	0.6967	27.466	98.834	$7.947*10^{-4}$	0.93080**
		50	0.0320	0.3750	0.6924	27.173	92.915	$7.468*10^{-4}$	0.93437**
		100	0.0300	0.3780	0.6811	27.174	102.943	$7.663*10^{-4}$	0.93369**
		200	0.0323	0.3757	0.6471	27.058	75.557	$7.391*10^{-4}$	0.93528**
	10	50	0.0300	0.3780	0.7888	27.634	114.939	$7.686*10^{-4}$	0.93336**
		100	0.0300	0.3790	0.7468	27.135	88.897	$7.492*10^{-4}$	0.93513**
		200	0.0300	0.3780	0.7084	26.720	98.729	$7.434*10^{-4}$	0.93548**
	20	50	0.0300	0.3790	0.8743	27.851	104.316	$7.311*10^{-4}$	0.93674**

		100	0.0300	0.3800	0.8258	27.079	96.382	$7.253*10^{-4}$	0.93620**
		200	0.0290	0.3810	0.7951	26.549	106.695	$7.561*10^{-4}$	0.93537**

** Significant at 0.01 level

were found for each soil. Wide variations in values of parameter m were found even in the same soil texture.

Results of table 2 suggest that the model is quite capable of describing the data of θ vs. λ with more than 93% (values of R^2). This finding indicates the capability of using this model to describe the slope $d\theta/d\lambda$ and hence $d\lambda/d\theta$ of Eq.(4) ,therefore, the parameters listed in table 2 can be used in Eq.(8) to find $d\theta/d\lambda$ and hence $d\psi/d\theta$ or in Eq.(9) to find $d\lambda/d\theta$ directly. The slope $d\lambda/d\theta$ can then be used in Eq.(4) along with the integral term to calculate the soil water diffusivity $[D(\theta)]$.

Soil water diffusivity as a function of water content was calculated for all treatment combinations listed in table 2 (30 experiments). Diffusivity was then plotted against the relative water content (θ/θ_m). Plotting the three curves representing the three soils on one figure resulted in total of ten figures. Three typical examples of these figures are shown in Figures 1, 2, and 3.

It is obvious from all figure that $D(\theta)$ increases sharply with water content and the increase becomes sharper near saturation .This finding has also been found by Aoda et al.(1993). This increase was explained by Bruce and Klute (1956) on the basis of the gradual increase in the radii of water conducting pores and consequently a decrease in tortousity of the actual flow path when the water content increases towards saturation .

For each concentration, an increase in SAR caused a decrease in D for all levels of θ especially near saturation. This decrease is more pronounced for low concentration and high SAR levels. This can be explained by the theory of diffuse double layer when applied to mixed electrolyte systems in the following manner: at a given water content, the distance between clay particles increase with the increase in SAR and decrease in C. For a constant total soil volume, changes in spacing between clay platelets result in changes in pore size geometry(Aoda et al.,1993).An increase in spacing would, therefore, cause a decrease in ratio of macro to micropores. Then, at a given θ, D decrease with an increase in SAR and a decrease in C.

Values of weighted mean diffusivity ($\bar{D}$) calculated by Eq.(10) for all treatment combinations are listed in table 3. Increasing salt concentration(C) caused a significant increase (at 0.01 level) in $\bar{D}$ for a certain SAR level and the increase was higher for higher C levels. At each level of C, $\bar{D}$ was higher with lower level of SAR. For example, for sandy loam soil, at C=100 meq/l , $\bar{D}$ was decreased in the following order : 0.3303 , 0.3251 , 0.3162 cm^2/mim when SAR increase from 0, 10, and 20 respectively. Similar trend is more clear for the other two soils. The results of $\bar{D}$ decreased due to the increase in SAR indicates again the deterioration of pores and this

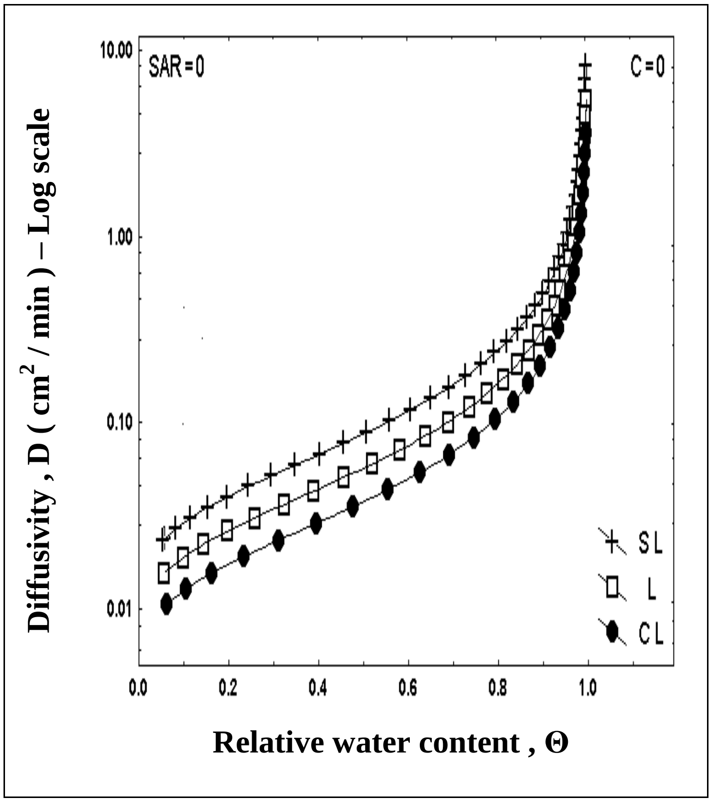

Figure 1: Soil water diffusivity vs. relative water content for the three soils (C=0,SAR=0)

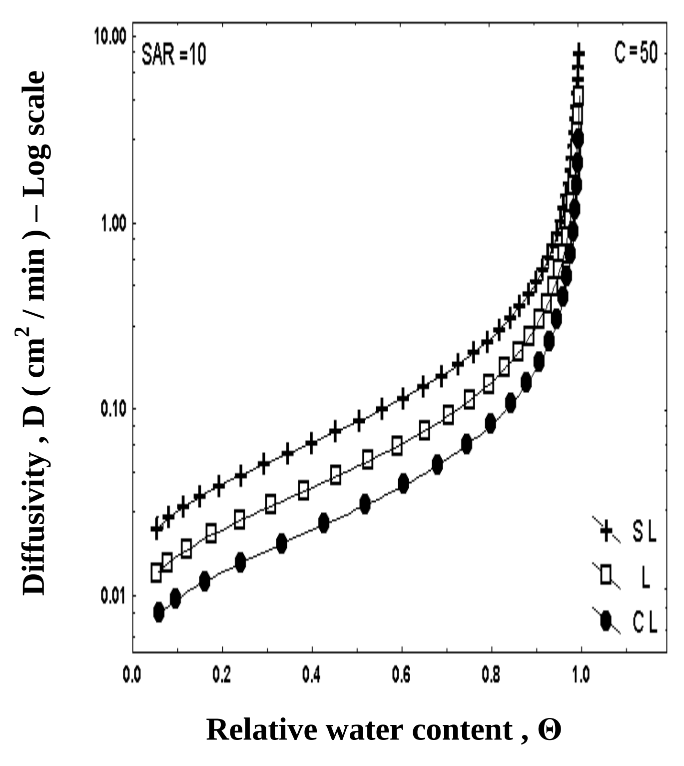

Figure 2: Soil water diffusivity vs. relative water content for the three soils(C=50,SAR=10)

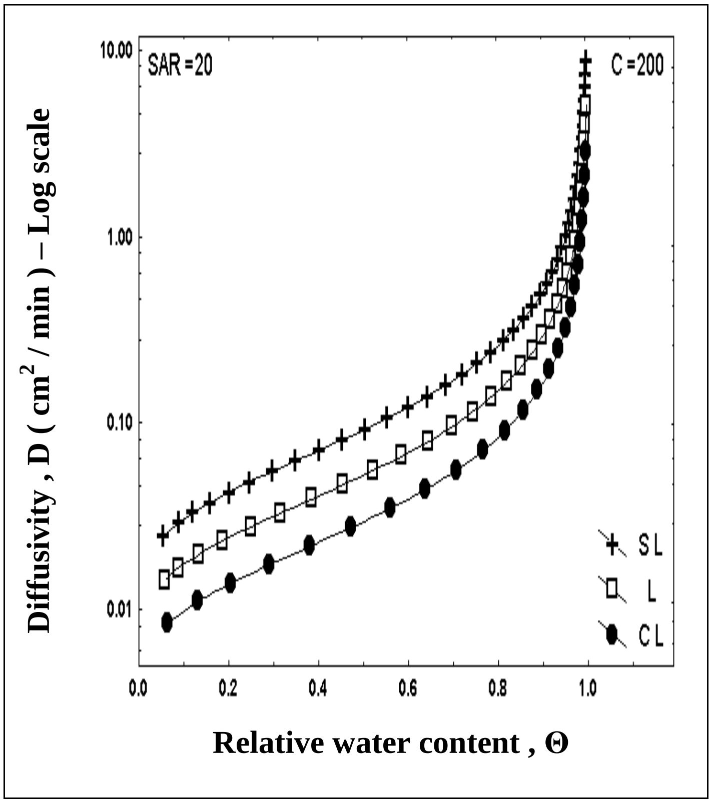

Figure 3: Soil water diffusivity vs. relative water content for the three soils (C=200,SAR=20)

deterioration is reduced by increasing salt concentration (C) . For example, when C=50 meq/l, $\overline{D}$=0.2095cm^2/mim for SAR=0 while $\overline{D}$= 0.1602 cm^2/mim for SAR=20 in the loam soil. Same conclusion is obtained for the other two soils. It is clear from the table that at the same level of SAR, increasing C would increase the values of $\overline{D}$. This finding is very important in leaching alkali soils using saline water which would protect soil pores from being deteriorated.

Table 3: Mean weighted diffusivity for the three soils and all salinity combinations.

SAR $(mmol/l)^{1/2}$	C (meq/l)	Weighted Mean Diffusivity, $\overline{D}(\theta)$ $cm^2 . min^{-1}$		
		Texture		
		Sandy Loam	Loam	Clay Loam
0	0	0.3213	0.2073	0.1342
	50	0.3257	0.2095	0.1380
	100	0.3303	0.2156	0.1417
	200	0.3409	0.2272	0.1616
10	50	0.3155	0.1777	0.1014
	100	0.3251	0.1914	0.1175
	200	0.3408	0.2097	0.1315
20	50	0.2959	0.1602	0.0815
	100	0.3162	0.1713	0.0943
	200	0.3400	0.1897	0.1028
LSD		*0.001*	*0.0007*	*0.0004*

CONCLUSIONS

From this study it can be concluded :

- The suggested model fitted the data of soil moisture profile very well and so it can be used to evaluate the slope dλ/dθ which is required for the determination of soil water diffusivity [D(θ)] .
- Increasing the salt concentration (C) of the water infiltration at constant level of sodium adsorption ratio (SAR) increased the values of D(θ) .
- Increasing SAR at constant C of the water infiltration decreased the values of D(θ) .

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