NUMERICAL ANALYSIS OF THIN BEAMS RESTING ON NONLINEAR ELASTIC FOUNDATIONS

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ABSTRACT

The finite difference method is used for solving the basic differential equation for the elastic deformation of a thin beam supported on a nonlinear elastic foundation. A tangent approach is used to determine the modulus of subgrade reaction after constructing a second degree equation for load-deflection diagram. Results of plate loading test of soil obtained in Iraq were used in the analysis. An iterative approach is used for solving the nonlinear problem until the convergence of the solution. The method of analysis, as programmed for a computer solution, considers the continuous elastic, nonlinear foundation to be active only when the beam is pressing against the foundation. Two examples of with simply supported beams are presented to illustrate the application of the method of analysis.

التحليل العددي للعتبات النحيفة المسندة على اساس غير خطي مرن

الخلاصة

إنّ طريقةَ الفروقِ المحدودةِ تمستعملُ لحَلّ المعادلة التفاضليةِ الأساسيةِ للتشوه المرن للعتباتِ النحيفة المسندة على اسسِ مرنةِ لاخطّيةِ. المعامل المماسي يُستَعملُ لتحديد معاملِ ردٍّ فعل التربة باستعمال معادلة من الدرجةِ الثانيةِ من مخطط الحمل مع الازاحة. تم الحصول على نَتائِج إختبارِ تحميل الصفيحة لتربةِ في العراق والتي تم استخدامها في التحليلِ.تم اعتماد طريقة الحل المتكرر لحَلّ مسألة الاسس اللاخطّيةِ حتى يتقاربِ الحَلّ. ان طريقة التحليلِ، تم برمجتها باستخدام الحاسوب، حيث يَعتبرُ الاسسَ اللاخطّيةَ المرنة المستمرةَ الّتي سَتَكُونُ فعالةَ فقط عندما الحمل يَضْغطُ الاسس. تم انتخاب إثنان مِنْ أمثلةِ العتبات المسند اسناد بسيط لكي تُقدّمُ تَصور لكيفية تطبيق طريقةِ التحليلِ.

KEWORDS

Beams, finite differences, Winkler foundation, nonlinear analysis, tangent modulus.

INTRODUCTION

The analysis of a linear elastic thin beam supported on a linear elastic foundation and subjected to lateral loads has been accomplished by many different techniques. The beam may have different support types such as fixed, simply supported and a load bearing media. Elastic support provided for beams is referred to foundation. The basic assumption is that the reaction forces of the foundation are proportional at every point to the deflection of the beam at that point. Winkler first introduces that assumption in 1871. Many researches were carried out this works such as Heteny (1946); Vlasov and Leontiev (1960); Selvadurai (1979); Chen (1999); Yin (2000) and Guo and Y. Jack Weitsman (2002).

Modulus of subgrade reaction is a conceptual relationship between soil pressure and deflection. It can be measured by using plate-loading test. Using this test, a load-deflection curve is adapted. The modulus of subgrade reaction K_s can be calculated using:

$$K_z = \frac{p}{w} \tag{1}$$

where :

K_z is the modulus of subgrade reaction,
p is the applied pressure and
w is the deflection.

The value of K_z is obtained from the concept of secant or tangent approach as shown in **Fig. (1)**.

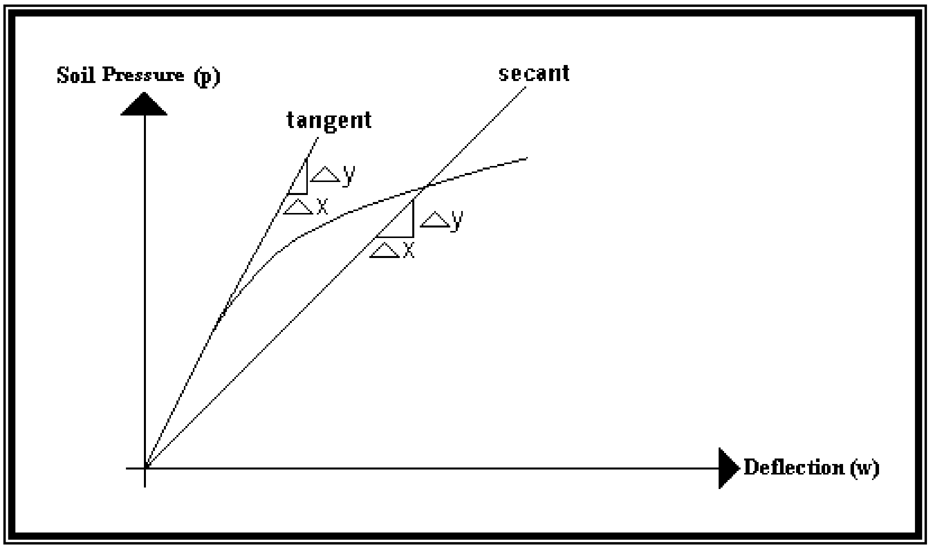

Fig. (1): Typical load-deflection curve.

There are a wide range of K_z values for different types of soils. In the present study a quasi linearization method or iteration procedure to get the value of K_z was used. This lineariazation by method iteration was developed using tangent method as a basic approach.

THEORY

The basic or differential equation for beams resting on Winkler foundation is [**Fig. (2)**]:

$$\frac{d^4w}{dx^4} + \frac{K_z}{EI} w = \frac{q}{EI} \qquad (2)$$

where:

w is the deflection of the beam,
E is the modulus of elasticity of the beam,
I is the moment of inertia of beam section and
q is the distributed load per unit length.

There are many methods used to solve this equation depending on type of support and applied loading:

- Exact solutions.
- Finite difference method.
- Finite element method.
- Fourier series method.

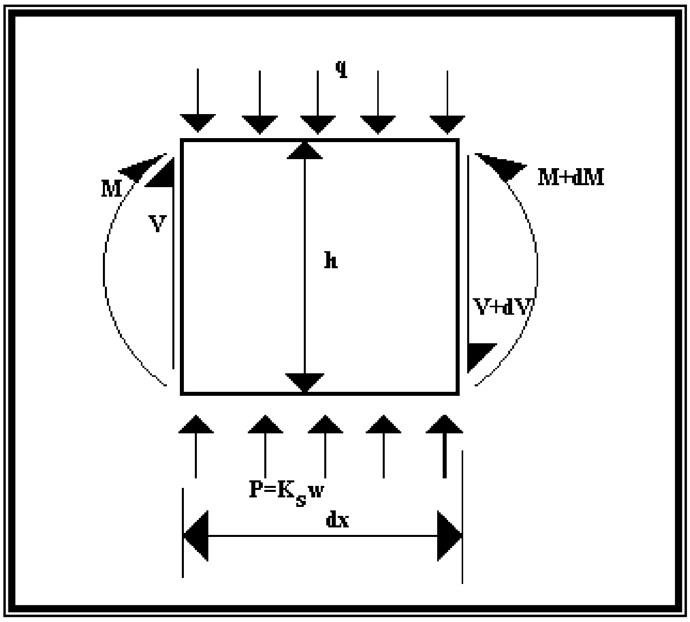

Fig. (2): Beam element supported by load bearing media.

FINITE DIFFERENCE METHOD

The method of finite difference was used in the present study to solve the differential eq. (2). The basic concept of this method is to replace the derivatives by ratio of infinite small quantities or difference at selected points [**Fig. (3)**].

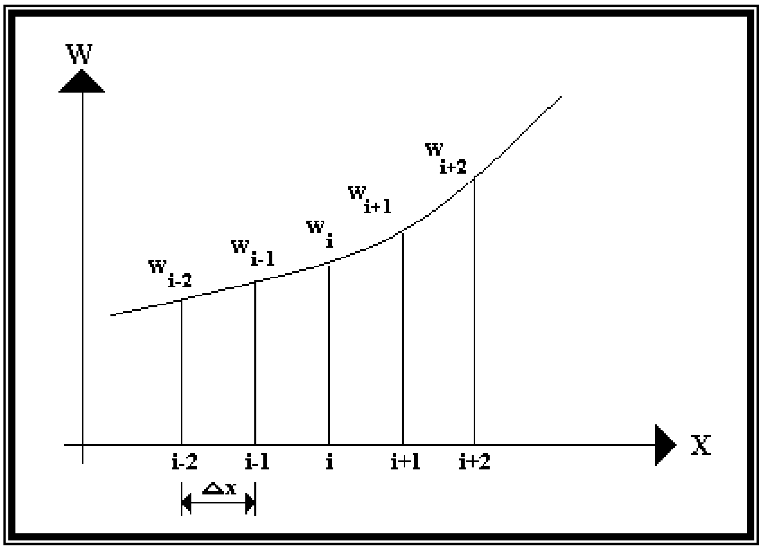

Fig. (3): Finite difference at selected points.

In this method the derivatives $\frac{dw}{dx}, \frac{d^2w}{dx^2}, \frac{d^3w}{dx^3}, \frac{d^4w}{dx^4}$ are converted into equivalent finite difference representations as follows:

$$\frac{dw}{dx} = \frac{w_{i+1} - w_{i-1}}{2\Delta x} \tag{3}$$

$$\frac{d^2w}{dx^2} = \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \tag{4}$$

$$\frac{d^3w}{dx^3} = \frac{w_{i+2} - 2w_{i+1} + 2w_{i-1} - w_{i-2}}{2\Delta x^3} \tag{5}$$

$$\frac{d^4w}{dx^4} = \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} \tag{6}$$

The beam is divided into small elements of length Δx. Depending on boundary condition (support conditions) the differential equation can be solved as shown in **Fig. (4)**.

$$\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}+\frac{K_z}{EI}w_i=\frac{q_i}{EI}$$
(7)

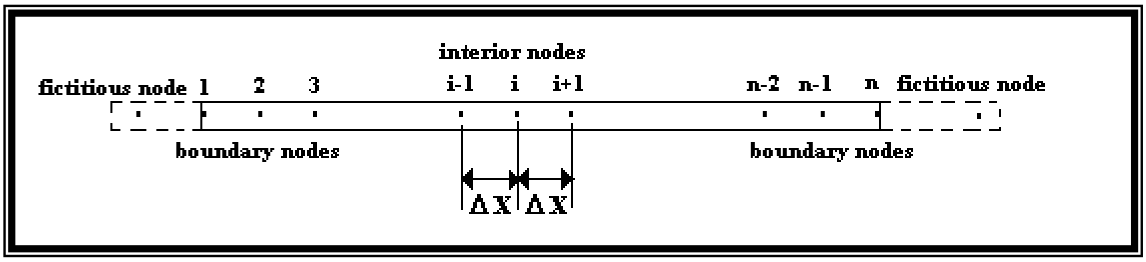

Fig. (4): Finite difference mesh.

In this study, the nonlinear behavior is adapted using iterative values of K_z. A typical p-w diagram was taken from a plate loading test was carried out on a soil in Baghdad. The reading of this test is shown in **Fig. (5)**. The consultant Bureau in the University of Baghdad carried out this test.

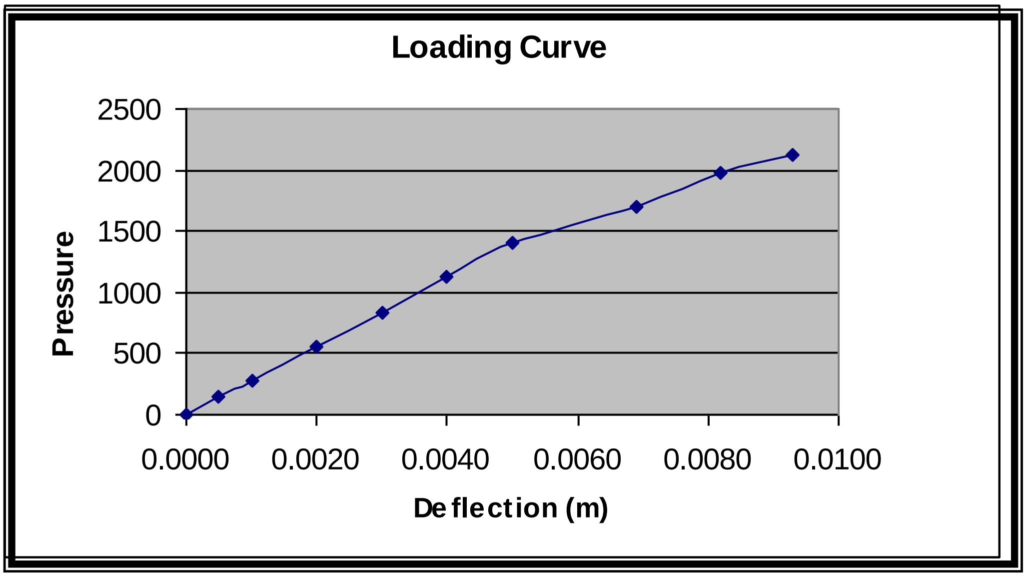

Fig. (5): Plate test data.

The data shown in the load-deflection curve is used to obtain the following second degree polynomial equation:

$$K_z(w)=280000+1.962*10^7 w-8.021*10^9 w^2 \quad (8)$$

which gives, the initial modulus of subgrade reaction= 280000 kN/m^3 and the final modulus of subgrade reaction= 171429 kN/m^3 for $w \geq 0.0051$

APPLICATIONS

Two cases were considered in this study for a simply supported beam (concentrated load and uniform distributed load) as shown in **Fig. (6)** and **Fig. (7)**.

Case 1

Beam depth = 0.25m
Beam height =0.25m
Concentrated load P = 500 kN
Length = 5m
E = $25*10^5$ kN/m^2

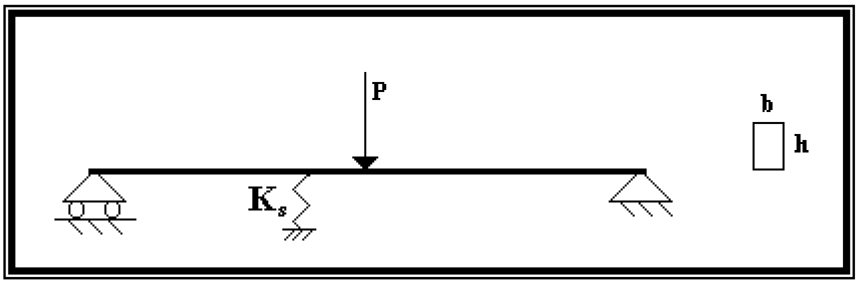

Fig. (6): Beam characteristics in case 1.

Case 2

Beam depth = 0.25m
Beam height =0.25m
Uniform load q = 250 kN/m^2
Length = 5m
E = $25*10^5$ kN/m^2

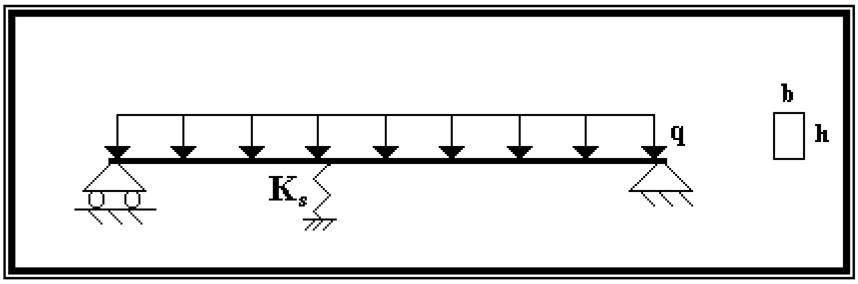

Fig. (7): Beam characteristics in case 2.

Here, the boundary conditions used to solve case 1 and 2 are:

- At x = 0 , deflection = bending moment = 0.
- At x = L, deflection = bending moment = 0.

RESULTS

For the simply supported beam, figure (8) shows the deflection profile along x-direction for the linear elastic and non-linear elastic Winkler foundation while figures (9) and (10) show the bending moment and shearing force along x-direction. The results show nonlinear effect of K_z in the two solutions. For the beam under a concentrated load, figures (11), (12) and (13) show the deflection profile, bending moment diagram and shearing force diagram. The results show nonlinear effect of K_z in the two solutions. Figure (14) shows that the mid-span deflection for the linear and nonlinear modulus decreases as the depth of the beam increases because the section flexural rigidity EI of the beam increases. Figures (15) and (16) show that the mid-span moment and maximum shear force increase as the depth of the beam increases because also the section flexural rigidity EI of the beam increases for the two approaches (linear and nonlinear).

CONCLUSIONS

The obtained results show different values for both deflection and bending moment but rather close values for shearing force for high values of applied loads on the beam, which is resting on linear or nonlinear elastic Winkler foundation. The nonlinear behavior of soil was obtained by using high-applied loads (to make the difference in results much obvious). This study shows that the elastic method for analyzing beam resting on Winkler foundation is still valid for ordinary applied loading on beams. The effect of beam depth on maximum beam deflection and bending moment is found to be significant but not much on shearing force.

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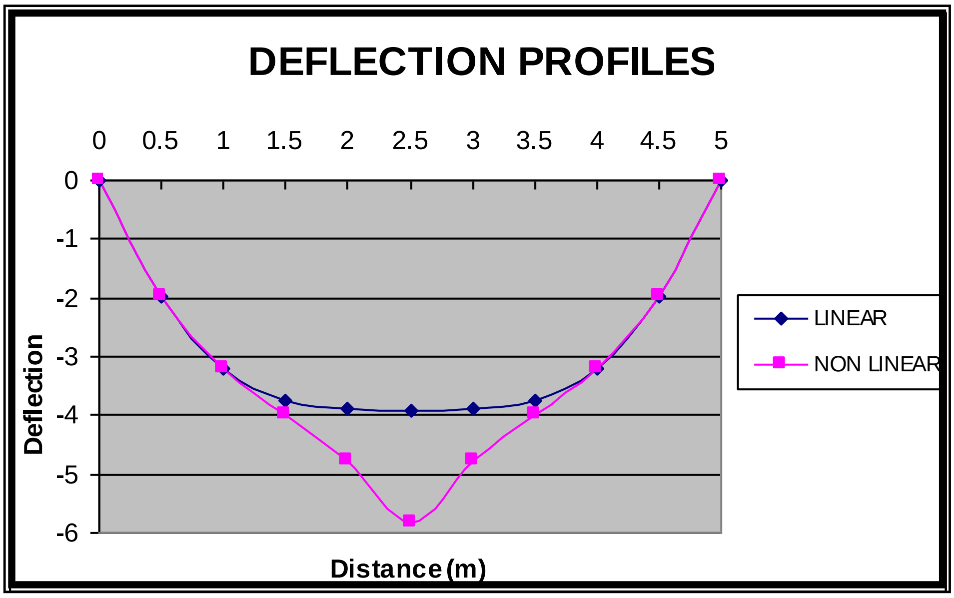

Fig. (8): Deflection profile of beam in case 1.

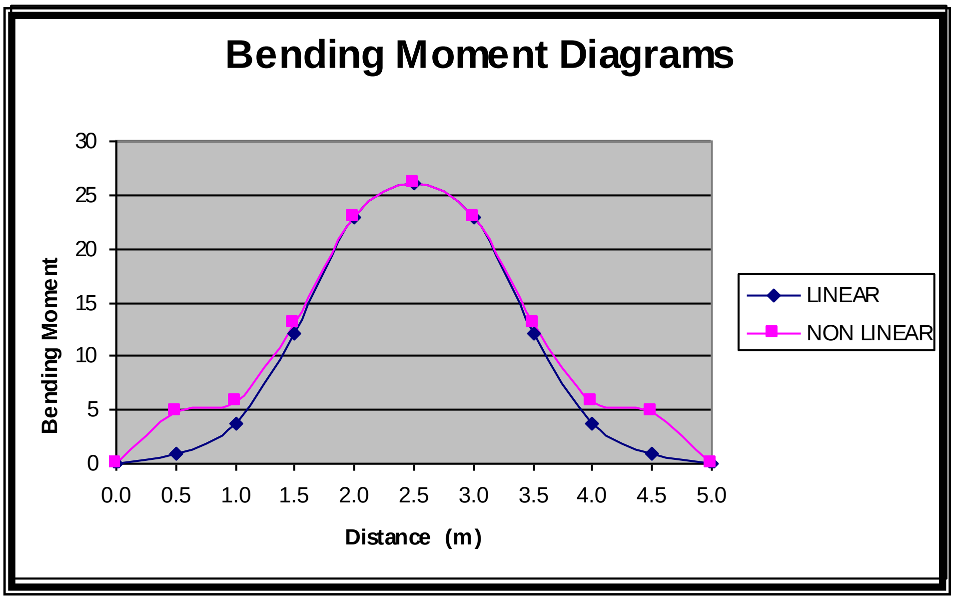

Fig. (9): Bending moment diagram of beam in case 1.

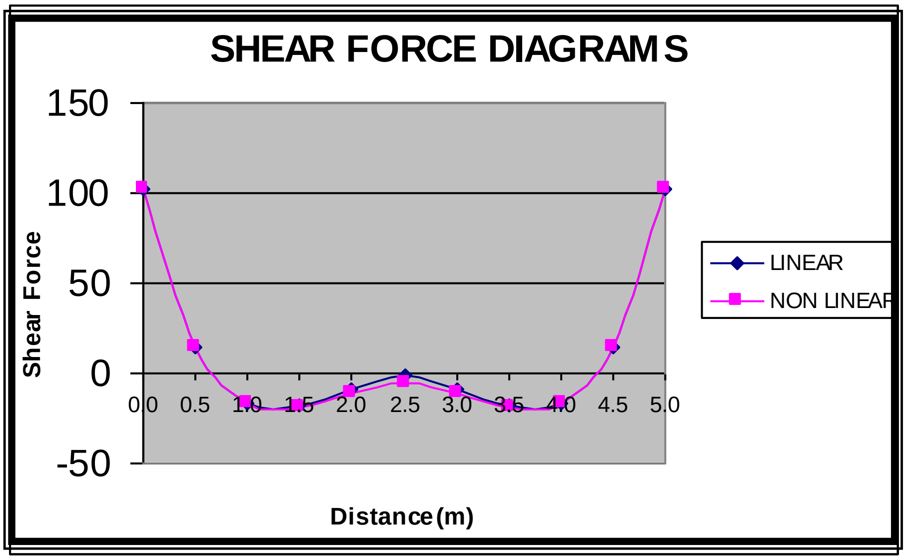

Fig. (10): Shearing force diagram of beam in case 1.

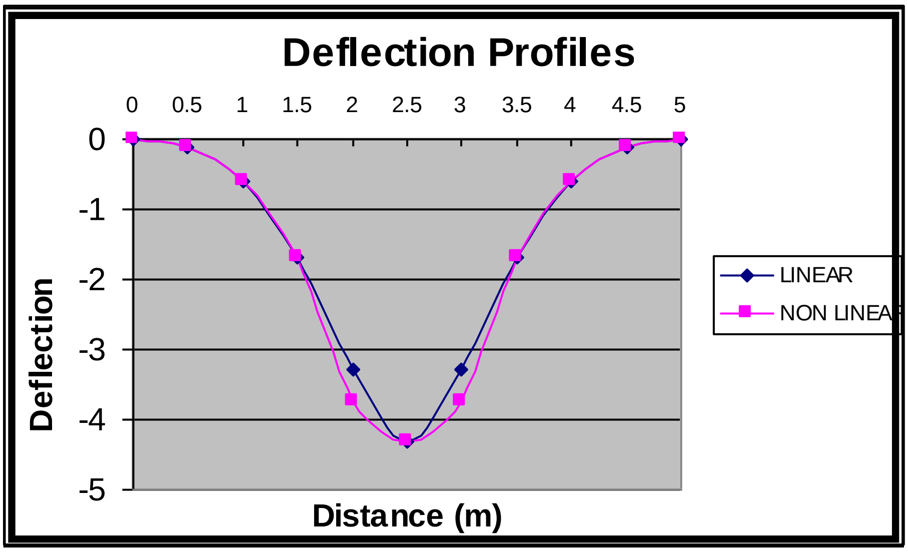

Fig. (11): Deflection profile of beam in case 2.

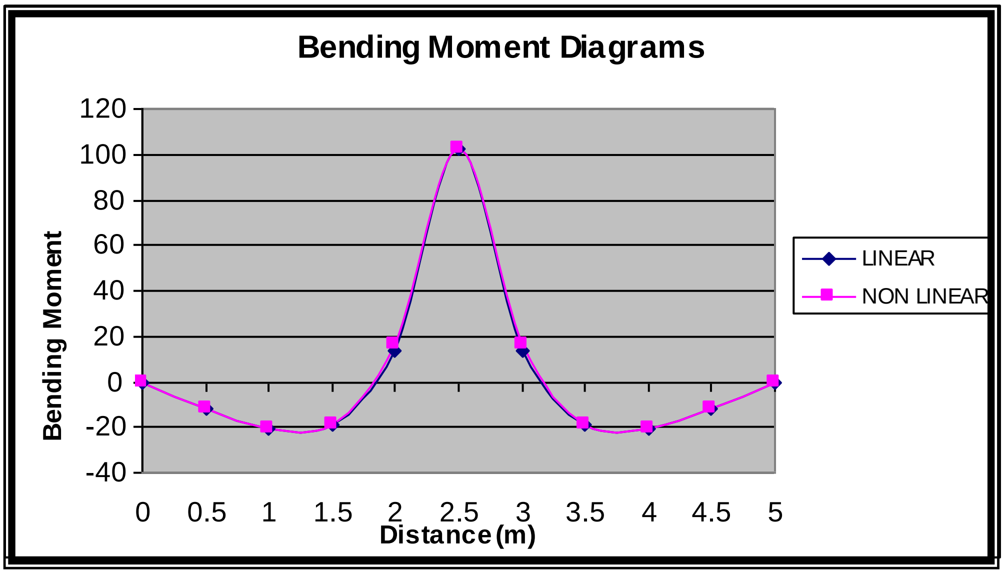

Fig. (12): Bending moment diagram of beam in case 2.

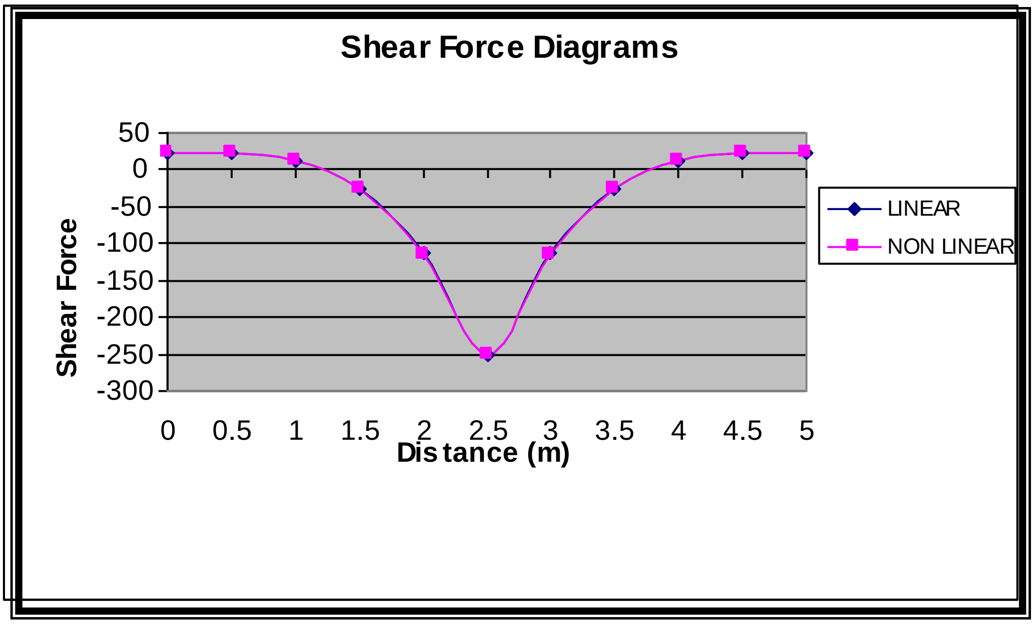

Fig. (13): Shearing force diagram of beam in case 2.

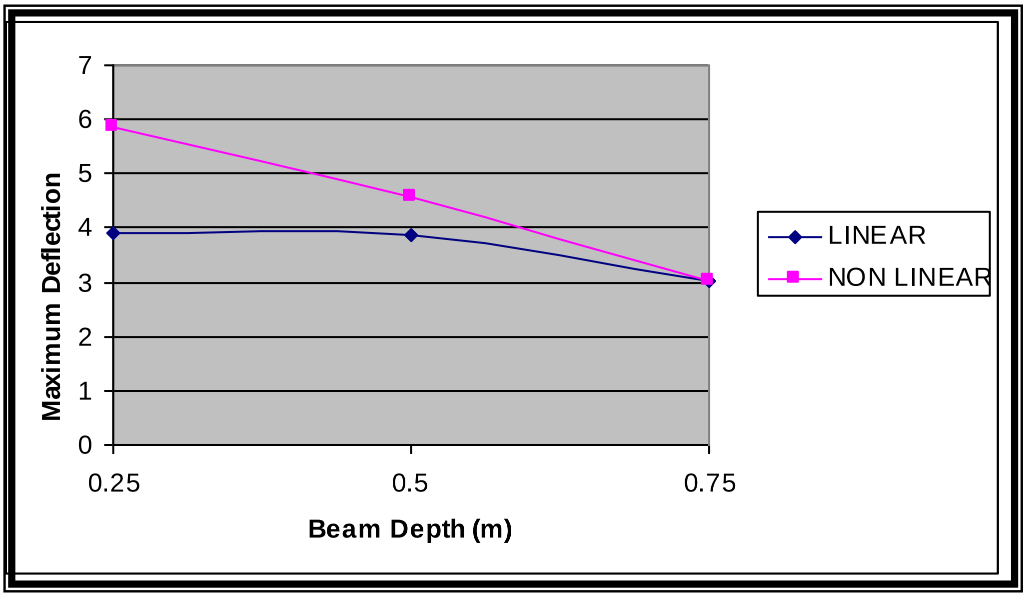

Fig. (14): Effect of beam depth on maximum deflection (case1).

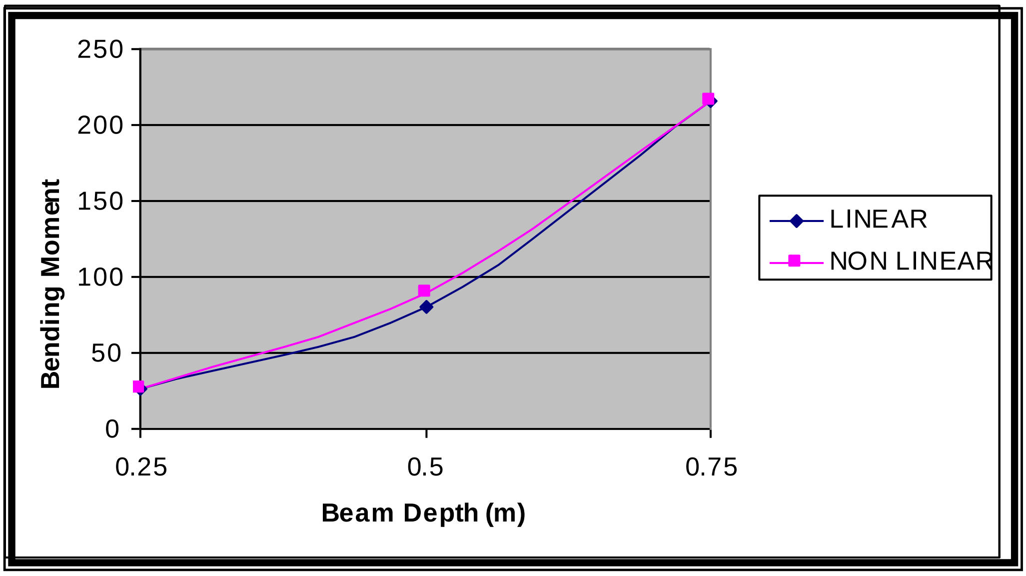

Fig. (15): Effect of beam depth on maximum bending moment (case1).

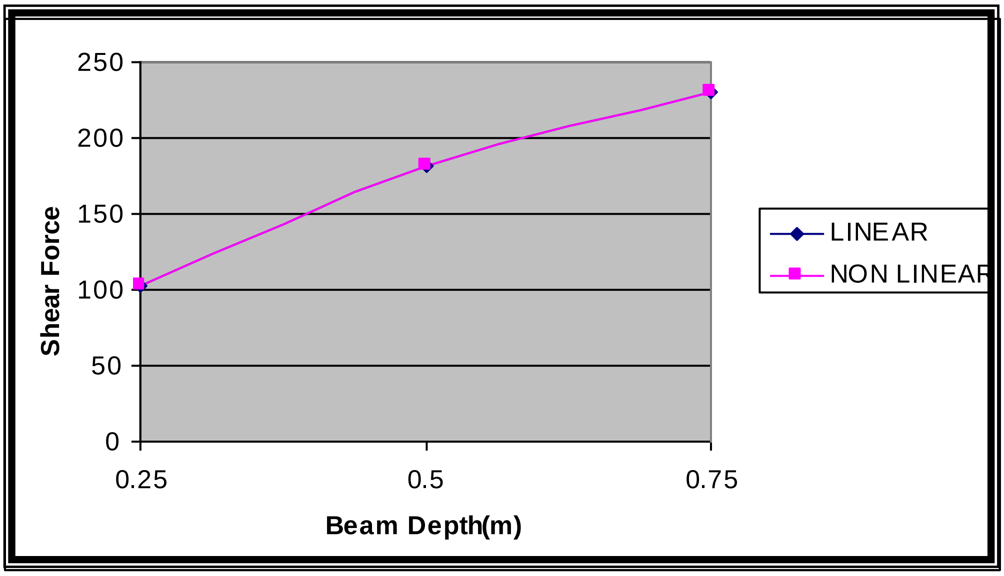

Fig. (16): Effect of beam depth on maximum shearing force (case1).