NUMERICAL INVESTIGATION OF LAMINAR NATURAL CONVECTION IN RECTANGULAR ENCLOSURES OF POROUS MEDIA

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ABSTRACT

In this investigation, steady two-dimensional natural convective heat transfer in a rectangular porous cavity, (heated from below) with horizontal walls heated to uniform but different temperatures and adiabatic sides has been studied numerically. The numerical results of heat transfer rates are presented for porous Rayleigh numbers (Ra^*), based on width of cavity, in the range ($Ra^* \leq 500$), with layer aspect ratios (Ar), (height/width) ranging between ($0.5 \leq Ar \leq 5$). Plots of streamlines and isotherms to show the behavior of the flow and temperature distribution are presented. The current study shows that the Nusselt number is a strong function of the porous Rayleigh number, and the geometry of the cavity is represented by aspect ratios. Porous Rayleigh number has a large effect on the flow field, whereas any increase in (Ra^*) results in changing the flow pattern from unicellular to multicellular flow. Correlation equation has been obtained to show the dependence of Nusselt number on the porous Rayleigh number, and aspect ratio (Ar), as this correlation will be beneficial in design of systems of thermal insulators in the energy storage engineering applications.

الخلاصـــــــــة

في هذا البحث ، تم إجراء دراسة عددية لانتقال الحرارة بالحمل الطبيعي المستقر ثنائي البعد في تجويف مسامي مستطيل الشكل، (مسخن من الأسفل) له جدران أفقية مسخنة إلى درجات حرارة منتظمة ولكن مختلفة والجوانب معزولة. تم تمثيل النتائج العددية لمعدلات انتقال الحرارة بعدد رالي المسامي (Ra^*) المحسوب على أساس عرض التجويف، في المدى ($Ra^* \leq 500$) ولنسب أبعاد (Ar) ، (الارتفاع / العرض) تمتد من ($0.5 \leq Ar \leq 5$). لقد تم عرض رسومات لخطوط الانسياب ولخطوط درجات الحرارة لوصف سلوك الجريان وتوزيع درجات الحرارة. بالإضافة ، بينت إن عدد نسلت هو دالة قوية من عدد رالي المسامي ونسبة الأبعاد. أن عدد رالي المسامي يمتلك تأثير كبير على مجال الجريان، حيث أن أي زيادة بعدد رالي المسامي (Ra^*) يسبب تغيير لشكل الجريان من جريان أحادي الخلايا الى جريان متعدد الخلايا. تم إيجاد معادلة تقريبية تبين اعتمادية عدد نسلت على عدد رالي المسامي ونسبة الأبعاد، وان هذه العلاقة يمكن الاستفادة منها في تصميم العوازل الحرارية من اجل حفظ الطاقة في التطبيقات الهندسية.

KEY WORDS: Heat Transfer, Numerical Solution, Free Convection, Rectangular Porous Medium

INTRODUCTION

Natural convection heat transfer in enclosures involves different aspects of problems. This variety of problems comes from possibly geometry characteristic of enclosures, type of fluid, nature of fluid flow, orientation of the enclosure etc. The most studies of natural convection in enclosures, based on tow-dimensional or three- dimensional parallelogram enclosure investigation, annuli and cylinders with different aspect ratio or diameters, or caliber. It's very interesting because of sensibility of natural convection phenomena from geometry. Important thing like aspect ratios of enclosures according to acceleration gravity vector, produce variety of physical situation. Also the type of fluid with influence on natural convection phenomena. If new phenomena are added like radiation, change of fluid phase, porous media, and chemical reaction and so on, have very difficult physical models often unsolvable, **(Miomir, 2001)**.

In general natural convection is one of the important modes of heat transfer. This phenomenon has been observed in numerous environmental circumstances. It occurs frequently as a result of density inversion caused by either the thermal expansion of a fluid, or the concentration gradients within a fluid system. Natural convection can also happen in a porous medium saturated with a fluid. Generally, the porous medium is a solid with voids in it. These voids are interconnected with each other so that it is possible for a fluid to penetrate the medium. There are many fields of application of flow through porous media ranging from industrial processes in factories to the movement of oil or gas in an oil field, **(Bouwer, 1978)**, and **(Raudkivi, and Callauder, 1976)**. Natural convection heat transfer in porous enclosures commonly takes place in nature, and engineering and technological applications. This phenomenon plays an important role in diverse applications including thermal insulators, storage of solar energy in underground containers, underground cable systems, heat exchangers, food industry, biomedical applications and heat transfer from nuclear fuel rod bundles in nuclear reactors, **(El Kady, 1999)**. Over the past years, more emphasis is put on natural convection in porous media due to its growing importance in engineering and geophysical areas. The analysis of the fluid flow and heat transfer for natural convection is difficult. Only a few problems have been solved analytically, many more have been solved numerically, **(Dawood, 1991)**. In particular, when air is trapped in the void space of fibrous porous media, the overall thermal conductivity of the medium is very low, consequently these emphasis is put on natural convection in porous media due to its growing importance in engineering and geophysical areas.

The present work involves a numerical study of the effect of porous Rayleigh number on laminar natural convection heat transfer in a rectangular enclosure filled with a porous medium heated from below. Also, the object of this investigation is to study the influence of geometry of enclosure represented by aspect ratios (*Ar*) on the behavior of fluid flow and heat transfer by free convection through a porous medium.

MATHEMATICAL FORMULATION

The problem under investigation, consists of a two-dimensional porous cavity has opposite isothermal hot and cold walls, at temperatures (T_{ho} & T_{co}), respectively, and adiabatic vertical walls. Physical model of the enclosure is represented on **Fig. (1)**. The cavity is fully filled with a porous media saturated with fluid, and all the surfaces are impermeable.

In the porous medium, Darcy's law is assumed to hold, and the fluid is assumed to be a normal Boussinesq fluid. The viscous drag and inertia terms in the governing equations are neglected, which are valid assumptions for low Darcy and particle Reynolds numbers. With these assumptions, the continuity, momentum and energy equations for steady, two-dimensional flow in an isotropic and homogeneous porous medium are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{-K}{\mu}\left(\frac{\partial P}{\partial x}\right) \tag{2}$$

$$v=\frac{-K}{\mu}\left(\frac{\partial P}{\partial y}-\rho_f g\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_e\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{4}$$

In the above equations, (u, v, μ, P, T) are the fluid velocity components **Fig. (1)**, the viscosity, the pressure and the temperature. The two momentum equations (2), and (3) reflect the Darcy flow model, where (K) stands for the permeability of the porous material. It is assumed that the fluid and the porous solid matrix are in local thermal equilibrium, at temperature T (x,y). The thermal diffusivity (α_e)is defined as ($\alpha_e = k_e/\rho_f cp_f$) where, (k_e) is the effective thermal conductivity of fluid-saturated porous matrix composite and ($\rho_f cp_f$) is the thermal capacity of the fluid alone.

The governing equations (1)-(4) reflect also the Boussinesq approximation, where by the fluid density (ρ_f) is regarded as a constant except in the body force term of the vertical momentum Eq. (3) where it is replaced by

$$\rho_f \cong \rho_o[1-\beta(T-T_o)] \tag{5}$$

Using this approximation, and eliminating the pressure terms between eqs. (2), and (3), yields a unique momentum conservation statement,

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{\rho_o g\beta k}{\mu}\frac{\partial T}{\partial x} \tag{6}$$

The above equations are subjected to the following boundary conditions:

Impermeable walls $\quad u=0$ at $x=0,L$, $\quad v=0$ at $y=0,H$, (7)

Isothermal horizontal walls $\quad T=T_{ho}$ at $y=0$, $\quad T=T_{co}$ at $y=H$, (8) adiabatic vertical walls $\frac{\partial T}{\partial x}=0 \quad at \quad x=0, L,$ (9)

NUMERICAL PROCEDURE

The mathematical problem formulated above was first placed in dimensionless from by defining the new dimensionless variables

$$x_* = \frac{x}{L},\ y_* = \frac{y}{L}$$
$$\psi_* = \frac{\psi}{\alpha_e},\ \theta = \frac{T - T_{co}}{T_{ho} - T_{co}} \tag{10}$$

where, (ψ) is stream function ($u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$) and ($\Delta T = T_{ho} - T_{co}$). The dimensionless forms of the momentum and energy equations are then

$$\frac{\partial^2 \psi_*}{\partial x_*^2} + \frac{\partial^2 \psi_*}{\partial y_*^2} = -Ra^* \left[\frac{\partial \theta}{\partial x_*}\right] \tag{11}$$

$$\frac{\partial \psi_*}{\partial y_*}\frac{\partial \theta}{\partial x_*} - \frac{\partial \psi_*}{\partial x_*}\frac{\partial \theta}{\partial y_*} = \frac{\partial^2 \theta}{\partial x_*^2} + \frac{\partial^2 \theta}{\partial y_*^2} \tag{12}$$

Where, (Ra^*) is porous Rayliegh number, based on width of the enclousre, and defined as:-

$$Ra^* = Da * Ra = \frac{Kg\beta L \Delta T}{\nu \alpha_e} \tag{13}$$

The corresponding dimensionless from of boundary conditions (7) – (9) is:-

$$\theta = 1,\ \psi_* = 0 \quad at \quad y_* = 0 \quad , for \quad 0 \le x_* \le 1$$

$$\theta = 0,\ \psi_* = 0 \quad at \quad y_* = Ar \quad , for \quad 0 \le x_* \le 1$$

$$\frac{\partial \theta}{\partial x_*} = 0, \psi_* = 0 \quad at \quad x_* = 0 \quad and \quad 1 \quad , for \quad 0 \le y_* \le Ar$$

The physical quantity of interest in this problem is the average Nusselt number along the hot wall, defined by

$$Nu = \frac{Q_{conv.}}{Q_{cond.0}} \tag{14}$$

The ratio of the convective to conductive heat transfer rates and (0) indicates porous Rayleigh number ($Ra^* = 0.0$). The convective heat transfer is calculated from:-

$$Q_{conv.} = -k_e \int_0^L \left(\frac{\partial T}{\partial y}\right)_{y=0 \quad or \quad H} dx \tag{15}$$

Where as the conductive heat transfer is calculated from:

$$Q_{cond.0} = k_e L \frac{\Delta T}{H} \tag{16}$$

Thus, eq.(14) becomes

$$Nu = -Ar \int_0^1 \left(\frac{\partial \theta}{\partial y_*}\right)_{y_*=0} dx_* \tag{17}$$

Numerical method is used to compute the stream function and temperature distribution for the porous cavity. A finite-difference technique is applied to solve the governing equations. The upwind differences method is used for the transport terms in the energy equation. All other terms in the energy and momentum equations are discretized by central differencing. The successive substitution formulas derived in this way satisfy the convergence criterion and are quite stable for many circumstances, **(Najdat, 1987)**. The choice of an already-used numerical scheme was intentional, in order to be able to check the validity of the present results against published results for the no-obstruction case (l/L=0); this test is presented later in this section. The finite difference approximation of the governing equations was based on dividing the ($0 \le x_* \le 1$) interval into (m) equal segments separated by ($m+1$) nodes. Likewise, the (y_*) interval was divided into (n) segments. The numerical work starts with postulating a certain distribution of flow and temperature in the ($x_* - y_*$) space ; in the present solution of distributions were taken as ($\Psi_*=0$) and ($\theta = y_*$), i.e. no flow and pure conduction. Based on these old fields, the momentum equation (12) is used to determine point-by-point the new (Ψ_*) field, while the energy equation (13) is used to determine the new (θ) field. The iteration process is terminated under the following condition

$$\sum_{i,j} \left|\tau^{r+1}{}_{i,j} - \tau^{r}{}_{i,j}\right| \Big/ \sum_{i,j} \tau^{r+1}{}_{i,j} \le 10^{-5} \tag{18}$$

where,(τ) stands for either (ψ_* or θ); (r) denotes the iteration step.

Before starting the computational solution, the grid independence of the results must be tested. Thus, numerical experiments have been carried out to solve a two-dimensional convection problem. The porous Rayleigh number in this test is set to be (300), while the grid size varies from (10×10) to (70×70) for different values of aspect ratio as shown **Fig.(2)**. It is found that the change in the heat flow rate for grid size of (40×40), and (50×50) is less than (0.45) percent for the whole range of aspect ratio ($0.5 \le Ar \le 5$). Therefore , the number of grid that is adopted in the present study is (40×40). The number of grid was selected as a compromise between accuracy and speed of computation.

RESULTS AND DISCUSSION

Effect of porous Rayliegh number on the temperature distribution and flow fields can be clearly seen in **Figs. (3)** to **(6)**. For ($Ra^* \to 0$) the case, heating from below at constant surface temperature, the energy is transported from hot wall to cold wall by pure conduction (i.e. Nu= 1.0) for saturated porous media at (Ra^*) less than its critical value. In the conduction regime, the isotherms are almost parallel to isothermal walls. The conduction mode of heat transfer continues until a critical value of Rayleigh number is reached.

For the case of (Ar=1), the value of (Ra_c^*) has been found to be equal to (39.5). This is in a good agreement with the value predicted from the linear theory ($Ra_c^* = 4\pi^2$). At this value the onset of convection begins because of the buoyancy effects. Thus, the flow field comprises a primary cell circulating around the entire enclosure with clockwise (this is an arbitrary direction). It may be counterclockwise, **Fig. (3)** and has a maximum value for the stream function (ψ_{*max}= 1.95). The small value of (ψ_{*max}) characterizes a very weak convective flow. The isotherms deviate only slightly from those of the pure conduction state. The extremum value of the stream function becomes larger as (Ra^*) increases, indicating a more effective motion, a circulatory motion is established because of the buoyancy influences. In addition, further increase in (Ra^*) results in changing the direction of the isotherms and change the flow pattern from unicellular to multicellular flow. **Fig. (4)** shows the streamlines at (Ra^*=100), (Ar=1). This flow exhibits two counter- rotating cells, each covering half of the cavity. It also indicates the flow rising slightly in the middle, turning at the top of the cavity, moving adjacent the cold wall, turning, and falling down the insulated wall. The number of cells are increased to three at (Ra^*=300) and then reduced to two at (Ra^*=500), see **Figs. (5)** , and **(6)**. The same phenomenon has been noticed by (Prasad and Kulacki, 1985).

Effect of aspect ratio on the flow pattern can be inferred with reference to **Figs. (7)** to **(10)**. It is worthwhile to note that any increase in aspect ratio delays the appearance of convective mode. The reasoning for this is as follows. As the aspect ratio increase, the isothermal walls become smaller than the insulated walls. Thus, there is a small area for convective contribution, compared to the path for flow. Also it is seen the flow change to multicellular flow at (Ra^*=100) for (Ar=1). Now, different values of aspect ratio will be taken to examine the appearance of the multicellular flow. The results of the numerical computations for streamlines and isotherms at (Ra^*=100) with (Ar=0.5,1,1.5, and 2) are plotted in **Figs. (7)** to **(10)** which show that the number of cells depends strongly on the value of aspect ratio. As depicted in this Figures, two cells appeared at (Ar = 1) while the number of cells reduced to one at (Ar=1.5). This is expected because the distance between isothermal wall at (Ar=1) is smaller than that of (Ar=1.5). Thus, the resistance to flow is lower. Also, it is interesting to note that for (Ar>1) and (Ra^*=100) the flow is still unicellular. While, the onset of convection starts at (Ra^*=39.5) for (Ar=1). It is worthwhile to note that any increase in aspect ratio delays the appearance of convective mode. **Fig.(11)** represents the relation between the maximum value of stream function (ψ_{*max}) and porous Rayleigh number compared for different values of aspect ratio (Ar). At low porous Rayleigh number ($Ra^* \leq 50$), (ψ_{*max}) seems to be invariable with aspect ratio this is due to dominance of conduction as mentioned before. At higher porous Rayleigh number or when convective becomes dominant, (ψ_{*max}) increases with increasing (Ar), since for a higher aspect ratio, the path along which the ascending flow is heated is longer, the velocity as well as the circulation (ψ_{*max}) becomes higher. It is also show that the peak value of (ψ_{*max}) depends on (Ra^*).

Figure (12) show the variation of Nusselt number versus porous Rayleigh number for aspect ratio (Ar=1). Porous Rayleigh numbers take values in the range (Ra^*=50 to 500), because of the stability of problem and program into computer. It is clear that (Nu) equal to one in the conduction regime (i.e. at $Ra^* \leq Ra_c^*$). The reason is that the viscous force is greater than the buoyancy force therefore the heat is transported by conduction as discussed previously. In general form, the value of (Nu) increases with increase (Ra^*), as shown in **Fig. (12)**. Then, (Nu) increase rapidly as (Ra^*) increases expressing the existence and increase of convective heat transfer.

The effect of aspect ratio on the Nusselt number, and porous Rayleigh numbers in range $(Ra^*=100,150,\ 300,\ \text{and}\ 500)$ is depicted in **Fig. (13)**. It is noted that the aspect ratio has a great effect on the heat transfer results. For $(Ra^*=150)$, it appears that the value of (Nu) increases with an increase in aspect ratio beyond (0.5). It reaches a maximum value of (Nu) at about $(Ar=1.5)$. Then, the value of (Nu) decreases with an increase in aspect ratio in the case $(Ar>1.5)$. This can be explained as follows. For large aspect ratio, the fluid encounters much more flow resistance in the y-direction due to the increased path length. The location of the maximum Nusselt number change with changing porous Rayleigh number. The above Figure also indicated that the Nusselt number is a strong function of porous Rayleigh number. For the range of (Ra^*) which is used in this investigation, the maximum (Nu) is found to be lay between aspect ratio from (0.5 to 1.5). The present result of the rate of heat is in good agreement with those reported by **(Caltagirone, 1975)**, for a porous layer heated from below and **(Chan et.al., 1970)**, for a porous layer heated from side.

Finally, correlation equation has been predicted depending on variation of porous Rayleigh number, and aspect ratio, by using least square method.

$$Nu = 0.2174\left(Ra^*\right)^{0.5256}\left(Ar\right)^{-0.4966} \qquad (19)$$

The above correlation is acceptable in the range of porous Rayleigh number (0 to 500), and aspect ratio (0.5 to 5).

To ensure that this approximation correlation is usable, the correlation coefficient (R) had been obtained for each equation. The minimum value of (R) was (0.96), that means this approximate equation are good for predicting the value of Nusselt number. **Fig. (14)** shows the comparison between predicted and numerical results. Agreement between numerical and predicted is close, although most the predicted points lie near the theoretical line.

The problem is modeled in a rectangular domain subjected to different temperature on it's horizontal sides with the left and right sides are insulated. All the analytical and numerical solutions, and experimental study show that the onset of natural convection in a porous layer starts at $(Ra^*=4\pi^2)$. The numerical solution agrees with those solutions, this is shown **Table (1)**.

Further, values of the average Nusselt number along the hot wall of the cavity at the steady-state flow for $(Ra^*=50,\ 100,\ \text{and}\ 200)$, are given in **Table (2)**. It is seen again that the present values of (Nu) are in very good agreement with that obtained by different authors, such as (Chan et al, 1970). (Burns et al, 1974), have analyzed a similar problem for different values of aspect ratio. The comparison with their results for $(Ar=0.5$, and 1) show agreements within (± 8 %) except the case for $(Ar=1$ and $Ra^*=200)$ where the agreements is (± 4 %). (Chan et al, 1970), and experimental investigation presented by (Close, Symons, and White, 1985), presented their results in a graph and some errors might have been introduced in reading the graph. Also, as shown in the table, there are some difference between the present work and those of **(Burns et al, 1974)**, and **(Bejan and Tien, 1978)**. These differences are attributed to the finite difference approximation.

CONCLUSIONS

The problem of steady laminar natural convection in a two-dimensional, porous cavity under uniform temperature on two opposite walls while the other walls are insulated has been studied numerically. The main conclusions of the present study are:

1. For the porous enclousres that have been solved, it has been demonstrated that the Nusselt number (Nu) is a strong function of porous Rayleigh number, the value of (Nu) increases with increase (Ra^*) for same aspect ratio.

2. The heat transfer is represented by Nusselt number (Nu) as a function of the geometry represented by the aspect ratio (Ar). As the aspect ratio increase, the isothermal walls become smaller than the insulated walls. Then the value of (Nu) decreases with an increase in aspect ratio beyond (0.5) for the same porous Rayleigh number. Later, any increase in aspect ratio delays the appearance of convective mode should have been noted.

3. The effect of porous Rayleigh number has a large effect on the flow field. At low value of (Ra^*), the flow field comprises a primary cell circulating around the entire enclosure. The extremum value of the stream function becomes larger as (Ra^*) increases, indicating a more effective motion, a circulatory motion is established because of the buoyancy influences. Further, increase in (Ra^*) results in changing the flow pattern from unicellular to multicellular flow. The number of cells are increased to three at $(Ra^*=300)$ and then reduced to two at $(Ra^*=500)$.

4. Correlation equation (19), can be used to calculate the rate of heat transfer as a function of (Ra^*), and (Ar). As this correlation will have been benefiting in a design for systems of thermal insulators so that storage of energy in the engineering applications.

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NOMENCLATURE

List of Symbols

Ar = Aspect ratio = (H/L)

C_P = Specific heat at constant pressure, $(J/kg.K)$

Da = Darcy number = (K/L^2)

g = Acceleration due to gravity, (m/s^2)

H = Height of cavity, (m)

L = Width of porous cavity, (m)

K = Permeability of porous medium, (m^2)

k_e = Effective thermal conductivity of fluid-saturated porous medium, $(W/m.K)$

Nu = Average of Nusselt number

P = Pressure, (Pa)

Q_{conv} = Convection heat transfer rate, (W)

$Q_{cond.}$ = Conduction heat transfer rate, (W)

Ra^* = Porous Rayleigh number based on width of cavity

T = Temperature, (K)

T_{co} = Temperature of cold horizontal wall, (K).

T_{ho} = Temperature of hot horizontal wall, (K).

u = Fluid velocity in x-direction, (m/s)

v = Fluid velocity in y-direction, (m/s)

x, y = Cartesian coordinates

Greek Symbols

α_e = Thermal diffusivity of porous medium, (m^2/s)

β = Thermal coefficient of volumetric expansion, (K^{-1})

ΔT = Temperature difference between isothermal surfaces = $(T_{ho} - T_{co})$, (K)

θ = Dimensionless temperature = $(T-T_{co})/(T_{ho}-T_{co})$

μ = Dynamic viscosity, $(kg/m.s)$

υ = Kinematic viscosity of fluid, (m^2/s)

ρ = Density, (kg/m^3)

ψ = Stream function, (m^2/s)

$(\ )_*$ = Dimensionless variables

Table (1) Comparison of the Onset of Convection in two-dimensional porous layer.

	Dawood 1991 Numerical	Caltagirone 1975 Numerical	Present work Numerical
Ra^*	39	44.41	39.5

Table (2) Nusselt number comparison for the present work with the past studies at the same boundary condition.

Ar	Ra^*	*Nu*					
		Numerical Study Raed 2003	Numerical Study Chan1970	Analytical and Numerical Study Burns1970	Analytical Study Bejan and Tien 1978	Experimental Study Close, Symons, and White 1985	Present work Numerical
0.5	50	—	1.480	1.430	1.770	1.234	1.363
	100	—	2.500	2.854	2.800	2.244	2.689
1	100	1.897	2.100	2.200	2.120	—	2.289
	200	3.815	3.560	3.600	3.250	—	3.413

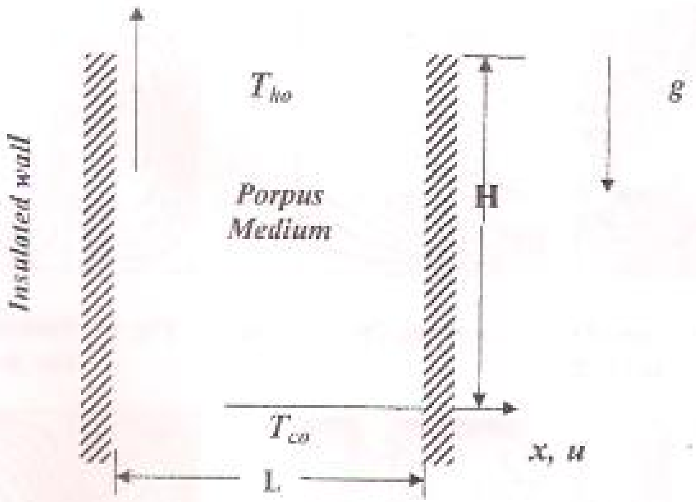


Fig.(1) Schematic diagram of the physical model and coordinate system.

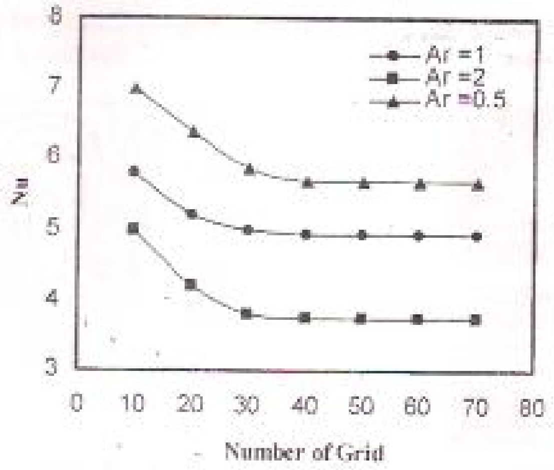
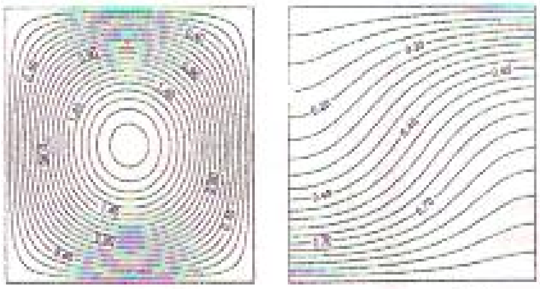
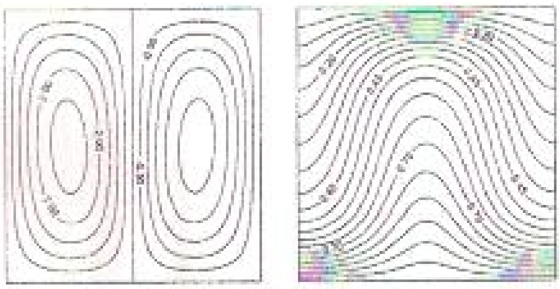


Fig.(2) Variation of Nusselt number with number of grid for different aspect ratio



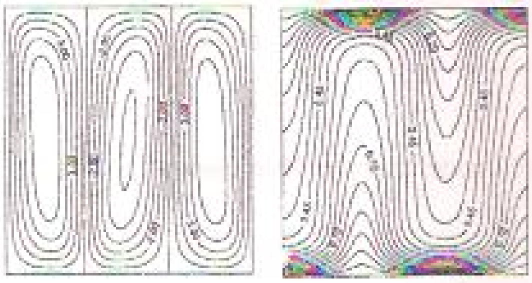
(a) (b)

Fig.(3) Pattern of (a) streamlines, (b) isotherms For $Ar=1$, $Ra^*=50$



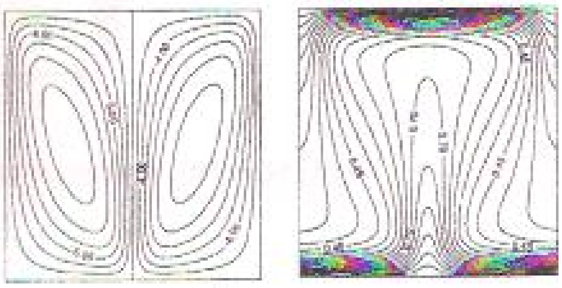
(a) (b)

Fig. (4) Pattern of (a) streamlines, (b) isotherms For $Ar=1$, $Ra^*=100$



(a) (b)

Fig.(5) Pattern of (a) streamlines, (b) isotherms For $Ar=1$, $Ra^*=300$



(a) (b)

Fig.(6) Pattern of (a) streamlines, (b) isotherms For $Ar=1$, $Ra^*=500$

(a)

(b)

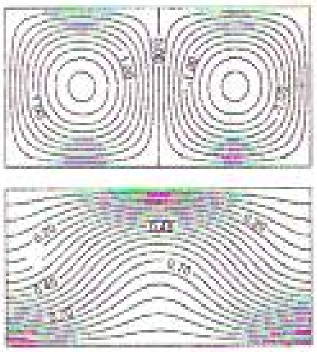
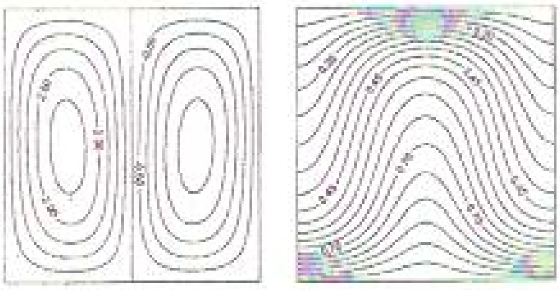


Fig. (7) Pattern of (a)streamlines, (b)isotherms For $Ar=0.5$, $Ra^*=100$

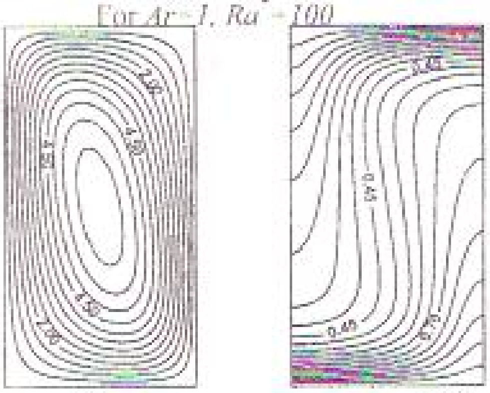


(a) (b)

Fig. (8) Pattern of (a) streamlines, (b) isotherms For $Ar=1$, $Ra^*=100$

(a) (b)

Fig. (9) Pattern of (a) streamlines, (b) isotherms For $Ar=1.5$, $Ra^*=100$



(a) (b)

Fig.(10)Pattern of (a)streamlines,(b)isotherms For $Ar=2$, $Ra^*=100$

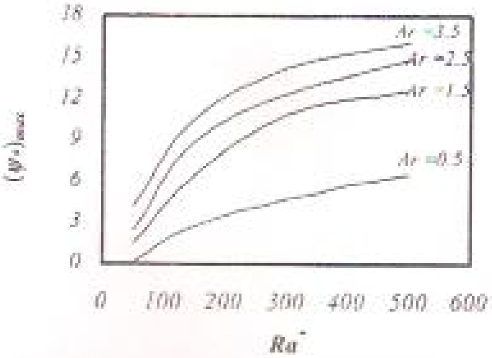


Fig.(11) Variation of maximum stream function with a porous Rayleigh number for different values of aspect ratios (*Ar*)

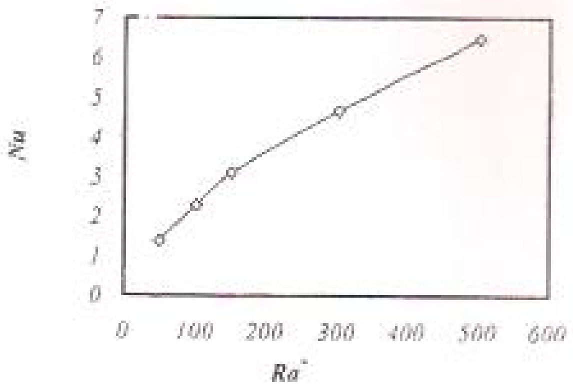


Fig.(12) Nusselt number vs. a porous Rayleigh number for *Ar*=1

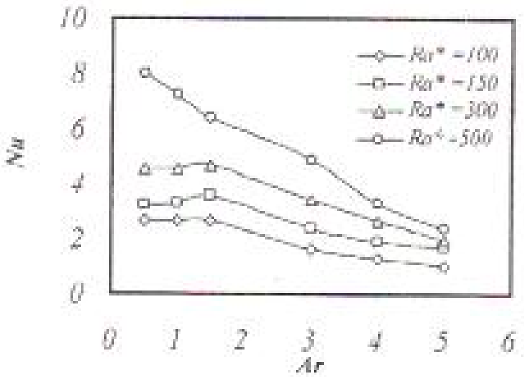


Fig.(13) Variation of Nusselt number vs.aspect ratio for different values of (Ra^*).

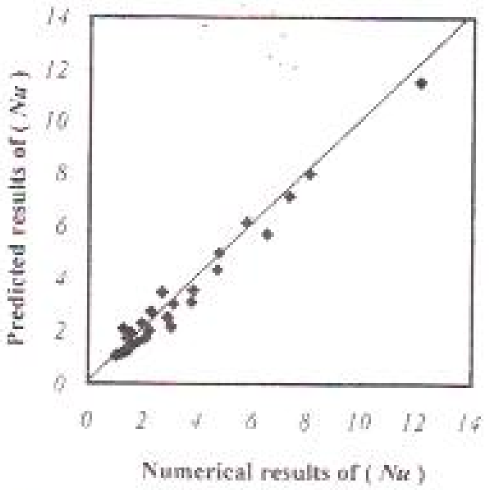


Fig. (14) Predicted vs. numerical results of heat transfer rate.