



PAPR Reduction of OFDM Signals Using Clipping and Coding

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ABSTRACT

The problem of the high peak to average ratio (PAPR) in OFDM signals is investigated with a brief presentation of the various methods used to reduce the PAPR with special attention to the clipping method. An alternative approach of clipping is presented, where the clipping is performed right after the IFFT stage unlike the conventional clipping that is performed in the power amplifier stage, which causes undesirable out of signal band spectral growth. In the proposed method, there is clipping of samples not clipping of wave, therefore, the spectral distortion is avoided. Coding is required to correct the errors introduced by the clipping and the overall system is tested for two types of modulations, the QPSK as a constant amplitude modulation and 16QAM as a varying amplitude modulation.

Key words: OFDM (orthogonal frequency division multiplexing), PAPR (peak to average power ratio), clipping, QPSK (quadrature phase shift keying) modulation, QAM (quadrature amplitude), channel coding.

تخفيض قيمة PAPR لاشارات OFDM باستعمال القطع والترميز

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مدرس

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الخلاصة

في هذا البحث دُرست مشكلة ارتفاع نسبة القمة الى المعدل القدرة PAPR في اشارات OFDM مع استعراض مختصر للطرق المختلفة المستعملة للتقليل الـ PAPR والتركيز على طريقة القطع. تم تقديم طريقة بديلة للقطع حيث يُجرى القطع بعد مرحلة الـ IFFT بعكس الطريقة التقليدية للقطع حيث يُجرى القطع في مرحلة مكبر القدرة والتي تولد نمو الطيف خارج الحزمة الخاصة بالاشارة. في الطريقة المقترحة تم قطع العينات وليس الموجة لذلك يتم تجنب تشويه الطيف. تكون الحاجة الى الترميز لاجل تصحيح الاخطاء المتولدة بسبب القطع وقد تم اختبار المنظومة ككل على تحميل QPSK الذي هو ذو سعة ثابتة وتحميل 16QAM الذي هو ذو سعة متغيرة.

الكلمات الرئيسية: التقسيم الترددي المتعامد، نسبة القمة الى معدل القدرة، القطع، تضمين الطور الرباعي، تضمين السعة الرباعي



1- INTRODUCTION

In modern communications the Orthogonal Frequency Division Multiplexing (OFDM) is becoming increasingly a widely used technique to transmit data due to its attractive advantages especially its immunity to multipath fading and impulsive noise, bandwidth efficiency and the fact that recent technology advancement made its complexity no longer an obstacle against implementation. This made it possible to have the OFDM to be adopted in many wireless communication systems like 802.11a, 802.11n, 802.16 and 4G mobile systems. Despite the wide spread applications of OFDM but this is mostly in systems where the high cost is affordable and non-battery operated devices, on the other hand OFDM still has limited application in mobile cell phones application because of the high peak to average power ratio (PAPR) of the OFDM signal. Such problem requires the power amplifier (PA) stage to have a relatively wide dynamic range to accommodate for the high peaks of the OFDM signal. This can lead to a low efficiency of the PA, **Reynaert, 2006** and **Hu, 2010**, and will cause a deleterious effect of the battery life which is a very undesirable to the user. There are many techniques that try to solve the high PAPR problem but each has its disadvantage; these techniques will be discussed later. Clipping is the direct and simplest technique to deal with the high PAPR problem, where the waveform is clipped whenever this waveform exceeds a specified threshold. This has the disadvantage of increasing the out of band spectral power due to the nonlinear effect on the waveform, and as it is so obvious to anyone who is involved in wireless communications, the bandwidth is becoming a very scarce resource and the OFDM is basically a solution to this problem and by using direct brutal clipping to the signal waveform, we are eliminating the advantage of OFDM that wireless communication is relying on. On the other hand regulatory authorities have strict requirements regarding radiation outside the subscriber's accommodated bandwidth. Filtering can be used to eliminate the out of band spectral growth due to clipping but this may reproduce the high peaks again, so, multistage of clipping and filtering may be required which is very demanding in terms of amount of calculation and/or cost. The traditional way of clipping where the OFDM signal is allowed to grow at a certain instance of time and then this signal is applied to a power amplifier with insufficient backoff input power which leads to clipping of the wave. In this work the clipping is made by software to the samples produced by the IFFT stage, therefore, this clipping will be referred to as the **sample clipping** as opposed to the wave clipping produced by the traditional clipping, see **Fig. 1**. Although sample clipping affects the certain sample values (that are higher than a certain threshold) and hence affects the transmitted data carrying symbols but it does not cause out of band spectral growth and does not distorts the in band spectrum. The modification of the sample value can be considered as an impulsive noise because from Eq. (4) there are N values of the data carrying symbol c_k that depend on the value of a specific OFDM symbol $s(n)$.

2- OFDM AND THE PAPR

The idea of the OFDM transmission is that instead of sending a high rate data with rate $R_s=1/T_s$, where T_s is the symbol duration, on a single carrier, the data are converted into a bank of N parallel subchannels each is transmitted by a different carrier with a fixed frequency separation given by **,Rohling, 2011**.

$$f_s = 1/(NT_s) \quad (1)$$

where T is the symbol duration, in this way the new symbol duration will be NT and the new rate $R_m=1/(NT_s)$. Although the situation in terms of bandwidth seems the same because in the single carrier we are sending at rate $R_s=1/T_s$ while in the multicarrier N carriers are sent with rate $R_m=1/(NT_s)$ so theoretically it is the same bandwidth, but actually wireless channels suffer from the multipath situation, where the signal arrives at different time delays at the receiver which makes different symbols to be added up at the same time at the receiver which causes what is commonly known as Inter Symbol Interference (ISI). Such channel is referred to as time dispersive channel and the amount of the ISI is considered negligible if the symbol time is much greater than the channel maximum delay spread T_m . So, in the single carrier situation it should be such that $T_s \gg T_m$ while in multi carrier it should be $NT_s \gg T_m$. From this it can be seen that the maximum delay spread T_m is a limiting factor against increasing the symbol rate while in multicarrier transmission the condition can be met by increasing the number of carriers N .

The complex OFDM signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} c_k \exp(2\pi(f_o + kf_s)t) \quad 0 < t < NT \quad (2)$$

where c_k is the data carrying symbol, it is a complex quantity and its values depend on the modulation and constellation shape, f_o is the OFDM carrier frequency and f_s is the separation frequency given by Eq. (1) which is a requirement to maintain orthogonality. In the receiver the received signal is sampled N times within each OFDM symbol, that is within time equals NT_s which according to Eq. (2) contains N data symbols (c_k) this gives N equations with N unknowns that can be solved for the c_k 's. In practice this is not how it is done, returning to Eq. (2) if the carrier term is excluded and making the sampling at $t=nT_s$, the OFDM sampled signal will be

$$s(n) = \sum_{k=0}^{N-1} c_k \exp(2\pi mk / N) \quad (n=0,1,\dots,N-1) \quad (3)$$

Eq. (3) looks so much like the Inverse Fourier Transform (IFFT) of the data symbols c_k . This is an important advantage of the OFDM because the IFFT can be implemented by efficient algorithms. In the receiver side the values of data carrying symbols c_k are obtained by Fast Fourier Transform (FFT) as in Eq. (4) below

$$c_k = \sum_{n=0}^{N-1} s(n) \exp(-2\pi mk / N) \quad (k=0,1,\dots,N-1) \quad (4)$$

A simple block diagram featuring the main components of the OFDM transmitter-receiver is shown in **Fig. 2**. There are other component in the OFDM system which are not discussed here like the windowing and the cyclic prefix because they deal with the spectral shaping and equalization and these issues are not within the scope of this work and do not affect our treatment of the PAPR.

After this brief overview of the OFDM we turn our attention to the PAPR problem. The PAPR of a signal $s(t)$ can be expressed as **Bhad et al., 2012**.

$$PAPR = \frac{\max[s(t)]^2}{E\{[s(t)]^2\}} \quad (5)$$

where $\max[s(t)]^2$ is the maximum power, $E\{.\}$ is the expected value operator and $E\{[s(t)]^2\}$ is the average power of $s(t)$. From Eq. (2) it is clear that the OFDM signal is the sum of multiple sinusoids and it can happen that at certain instances these sinusoids add up constructively and lead to high

peak as shown in **Fig. 1**. The peak and average power of the OFDM symbol can be found from Eq. (3) as

$$P(n) = s(n)s^*(n)$$

$$P(n) = \left(\sum_{k=0}^{N-1} c_k \exp(j2\pi nk / N) \right) \left(\sum_{m=0}^{N-1} c_m^* \exp(-j2\pi mm / N) \right)$$

$$P(n) = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} c_k c_m^* \exp(j2\pi m(k - m) / N) \tag{6}$$

By making the change of variables $r=m-k$ Eq. (6) becomes:

$$P(n) = \sum_r \sum_{m=0}^{N-1-r} c_{m-r} c_m^* \exp(-j2\pi r / N) \tag{7}$$

In the above equation the range of values of r in the outer summation is from $-(N-1)$ to $(N-1)$. The difference between the summations in Eqs. (6) and Eq. (7) can be viewed like summing elements of an $N \times N$ matrix; in Eq. (6) the summation is in row-column direction, while in Eq. (7) is in diagonals direction. The inner summation is called the aperiodic autocorrelation of c at displacement r . To find the average power the expected value of Eq. (7) is taken over the random variable c and it is assumed that the data symbols are independent and each has zero mean (because the constellations are usually symmetrically distributed) the terms in Eq. (7) vanish except when $r=0$, so the average power will be reduced to:

$$P_{av} = E[P(n)] = \sum_{m=0}^{N-1} E[|c_m|^2] \tag{8}$$

And since it is usually assumed without loss of generality that the constellation of any modulation scheme (QPSK or QAM) has normalized average power, then Eq. (8) reduces to simply

$$P_{av} = N \tag{9}$$

To find an upper bound of the expression in Eq. (7) it is known that the complex exponential term has the upper bound of 1 ($|e^{jX}| \leq 1$) and the correlation term has an upper bound when all coefficients are equal to the maximum coefficient which we denote c_M , so from Eq. (7)

$$P(n) \leq \sum_r \sum_{m=0}^{N-1-r} |c_M|^2 \tag{10}$$

Now the term inside the summation is constant (independent of m and r) and remember that the outer summation is over the values $-(N-1) \leq r \leq (N-1)$ so we will separate the case when $r=0$ and combine the two cases when r is positive and negative because the inner summation is the same, so, Eq. (10) will be:

$$P(n) \leq |c_M|^2 N + 2 \sum_{r=1}^{N-1} \sum_{m=0}^{N-1-r} |c_M|^2 = |c_M|^2 N + 2 |c_M|^2 \sum_{r=1}^{N-1} (N-r)$$

$$P(n) \leq |c_M|^2 N + 2 |c_M|^2 (N(N-1) - N(N-1)/2)$$

$$P(n) \leq |c_M|^2 N^2 \quad (11)$$

In other words the peak power is proportional to N^2 with constant of proportionality equals the magnitude square of the data symbol of maximum amplitude. It should be noted that this constant equals unity for the MPSK modulation and depends on the constellation shape for the amplitude varying constellations like the QAM case. Hence we get from Eqs. (9) and (11) the peak to average power ratio

$$PAPR = N|c_M| \quad (12)$$

Sometimes this ratio is given in dB and its square root is called the crest factor. It can be seen that PAPR increases with the number of carriers and this makes the reduction of this ratio an important job.

3- TECHNIQUES FOR PAPR REDUCTION IN OFDM

In this section, an overview is presented for the different techniques used to reduce PAPR in OFDM. In general there is no single technique that is best in every aspect for PAPR reduction. Every technique has its own drawback in one or more of the following aspects:

- 1- Cost which is either in hardware or amount of calculations
- 2- Reduction in coding rate
- 3- Distortion
- 4- Increasing of transmission power
- 5- Transmission of side information

It is up to the designer to determine which of the above can be tolerated by the specific application. A brief description is presented here for the different techniques used for PAPR reduction.

3.1- Clipping and Filtering

Clipping is the simplest and the most straightforward technique for reducing PAPR, actually clipping alone can hardly be considered a technique because in this case the system is doing nothing and the high peaks of the OFDM signal are passed to the power amplifier that does not have enough input backoff which causes clipping of the high peaks. The clipped signal can be represented by **Joshi and Saini, 2011**.

$$\tilde{s}(t) = \begin{cases} s(t) & |s(t)| \leq T \\ T e^{j\phi(t)} & |s(t)| > T \end{cases} \quad (13)$$

where $s(t)$ is the OFDM given by eq. (1), T is the threshold and $\phi(t)$ is the phase of $s(t)$.

As a consequence the out of band spectrum will grow due to the signal distortion as shown in **Fig. 3**. Therefore, clipping is usually combined with filtering as in the work of **Bhad et al., 2012**, but the filtering will cause the clipped peaks to grow again so further clipping-filtering stages are usually required which in turn causes increase in cost.

3.2- Coding

The method of coding is based on the idea of choosing a forward error coding (FEC) scheme where the codewords have low PAPR. In this technique two goals are achieved; reducing the PAPR and providing more reliable transmission via coding. Many researches have been developed in this field



and most adopt Golay sequence to achieve low PAPR. Golay pairs are defined as the two sequences whose aperiodic autocorrelation add to zero i.e. if $a(n)$ and $b(n)$ are two sequences of length N then

$$C_a(k) + C_b(k) = 0 \text{ for } k=1, 2, \dots, N \quad (14)$$

Where

$$C_x(k) = \sum_{n=0}^{N-k-1} x(n)x(n+k) \quad (15)$$

Is called the aperiodic autocorrelation of the sequence $x(n)$. Any two sequences that satisfy Eq. (14) are called *Golay pairs* and either sequence is called a *Golay sequence*. It can be shown that Golay sequences when chosen as OFDM symbols will have PAPR of at most of 2 ($PAPR \leq 2$) irrespective of the sequence length N , **Davis and Jedwab, 1999**. In 1999 Davis and Jedwab devised an analytical scheme to produce Golay sequences and found a relationship between Golay sequences and Reed-Muller codes. Although their work was limited to MPSK signals up to 32 carriers it was considered a landmark that many other researchers like **Röbbling et al., 2001**, who expanded Davis and Jedwab work to 16QAM and archived $PAPR \leq 3.6$ or **Huang et al., 2010**, who expanded to 64QAM achieving $PAPR \leq 2.85$, **Lee and Golomb, 2006** also expanded to 64QAM achieving $PAPR \leq 4.66$.

The problem with the coding method for reducing the PAPR is that the hamming distance between sequences that have low PAPR is not always high. Therefore, many sequences with low PAPR are excluded in order to satisfy the high hamming distance criteria of the coding scheme. Satisfying both conditions (low PMPR and high hamming distance) results in reduction of the coding rate and hence the information rate which strips the OFDM technique from its main advantage.

3.3- Selected Mapping

In selected mapping (SLM), each information sequence D is mapped to U different sequence blocks X^u (where $u=0,1,\dots,U-1$) in many cases the mapping is performed by multiplying the information sequence by random vector B^u i.e.

$$X^u = B^u D \quad u = 0,1,\dots,U-1 \quad (16)$$

The effect of this mapping is supposed to introduce randomness in the new sequence X^u such that the PAPR of X^u will be lower than the PAPR of D for some u . In other words selected mapping selects the vector B^u such that $PAPR(X^u)$ is minimum over u and sends the value of u as a side information so that the receiver can extract the information sequence D . The advantage of the selected mapping is that it works for any type of modulation, while the disadvantage is that in order to find the minimum PAPR, the transmitter requires to perform the IFFT stage U times and selects the output that with minimum PAPR, and this increase the cost requirement of the system. On the other hand, the requirement of sending side information is also a cost in terms of bandwidth requirement. There is another technique that is closely related to SLM which instead of the mapping used by Eq. (16), a set of interleavers is used to find one output whose PAPR is lowest to be selected for transmission. In literature, selected mapping received great deal of attention to improve performance especially in terms of computational cost, for example, **Ding and Lin-Bo, 2011** used a

method called Random Screening (RS-SLM) to find the mapping vector B^u of Eq. (16) before the IFFT stage, so, only one IFFT stage is required. RS-SLM is based on calculating coefficients for each X^u sequence, these coefficients are measures of randomness, periodicity, and period of the sequence and by using a special criterion, the sequence with the lowest PAPR can be found and sent to the IFFT stage. Although this method requires only one IFFT stage but it still requires some calculations and the authors only presented the results for limited parameters where the number of carriers $N=64$, the number of vectors $U=8$ and BPSK modulation type. **Breiling et al., 2001**, proposed a method that does not require explicit transmission of the side information (the mapping vector index u), in the traditional methods where the mapping vector index u is explicitly transmitted, the whole information sequence can be lost if transmission error occurs in the value of u . Breiling proposed to concatenate the index u at the beginning of the information sequence and feeding the resulting sequence to a scrambler, according to the value of u different scrambled vectors are produced and fed to U IFFT stages to choose the output with lowest PAPR. At the receiver, the received sequence is descrambled and if no transmission errors occurred, the information sequence is retrieved with the index u .

3.4- Partial Transmit Sequence

In Partial Transmit Sequence (PTS) method the signal block $X(k)$, of length N , is partitioned into M blocks, where M is a divisor of N . New M vectors $X^m(k)$ ($m=1,2,\dots, M$) are generated such that

$$X^m(k) = \begin{cases} X(k) & (m-1)L < k \leq mL \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

where $L=N/M$ is the length of the partition vector $X^m(k)$. The signal block $X(k)$ can be expressed in terms of the partition vectors $X^m(k)$ by

$$X(k) = \sum_{m=1}^M X^m(k) \quad (18)$$

Now the vectors $X^m(k)$ are fed to M IFFT stages to generate M time-domain vectors $x^m(n)$ which are in turn each multiplied by a fixed phase shift number b^m . The sum is optimized by choosing the set of values of b^m for the lowest PAPR, i.e.

$$x(n) = \sum_{m=1}^M b^m x^m(n) \quad (19)$$

where the values of b^m are chosen from a set of phase shifts $b^m = e^{j2\pi h/W}$, where $h=0,1,\dots,W-1$ and W is a given integer whose value affects the performance. Eq. (19) says that the transmitted sequence is a linear combination of the outputs of the IFFT stages with each output phase-shifted by b^m . Now the system must make an exhaustive search for the values of b^m that minimize the PAPR. In practice, without loss of performance, $b^1=1$ so the exhaustive search for the $M-1$ vectors from a set of W values gives a value of $W^{(M-1)}$ iterations and the size of the side information is $\lfloor \log_2(W^{(M-1)}) \rfloor$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x . In the receiver, performing the FFT to $x(n)$ results in



$$\text{FFT}[x(n)] = \sum_{m=1}^{m=M} b_m \text{FFT}[x_m(n)] = \sum_{m=1}^{m=M} b_m X_m(k) \quad (20)$$

And because of the special arrangement of $X^m(k)$ as in Eq. (17), the output of the FFT stage will be the signal vector $X(k)$ with each partition multiplied by the corresponding phase shift b^m that are obtained from the transmitted side information. The main disadvantage of the PTS method is the large amount of iterations that depend of the values of M and W , where increasing their values enhances the performance in terms of lowering the value of the PAPR while degrades the performance in terms of amount of computations **Han et al., 2005**. The partitioning expressed by Eq. (17) is called adjacent partitioning and it is not the only way of partitioning, there is the interleaved and the pseudo random partitioning but the adjacent partitioning is the simplest to implement. **Lee et al., 2012**, presented a tree-based method for reducing the number of iterations required to find the best phase vector, the performance is better than the exhaustive search in terms of number of iterations but it is suboptimal in terms of achieved PAPR values.

3.5-Tone Reservation

In tone reservation (TR) the frequency domain data sequence D (of length N) is not composed entirely of data symbols, rather it has zero values at certain frequency indices on the other hand a sequence A , same length as sequence D , is composed of nonzero entries where X has zero entry and a zero entry where D has a nonzero entry. The overall sequence $X=D+A$ is fed to the IFFT stage such that

$$x(n)=\text{IFFT}[X]=\text{IFFT}[D]+\text{IFFT}[A] \quad (21)$$

The problem now become to find A such that the time domain added signal $\text{IFFT}[A]$ results in reduction of PAPR of the overall time domain signal $x(n)$. **Hu et al., 2010**, used TR and used iterative procedure in which the vector of the reserved tones is recursively updated by a special formula. The iterations stop if the number of iterations reaches a predefined number or the PAPR becomes lower than a desired threshold. In TR technique no side information is transmitted but part of the bandwidth is allocated for the reserved tones.

3.6-Active Constellation Expansion

Active Constellation Expansion (ACE) or Active Point Modification (APM) is to dynamically expanding the outer points of the constellation (used in mapping the data symbols) further outside or equivalently increasing the amplitude of these points in a way that does not affect the decision thresholds used to detect the received symbols and at the same time allows manipulating the values of some of the transmitted symbols in order to reduce the peaks in the time domain signal. The problem is to find a vector C whose components represent the magnification factor such that $\text{IFFT}[CX]$ has a minimized PAPR. **Zhou and Jian, 2013**, presented an ACE method based on minimizing quadratic form of $M<N$ variables, the choice of the number of variables M affects performance and complexity. The ACE method does not require sending side information but its drawback beside complexity is the increase of transmitted power.

3.7-Tone Injection

In tone injection (TI) Technique, each point of the M constellation points is mapped to another set of K constellation points and hence the overall constellation is mapped to a new larger KM -point constellation that is divided into M sub-constellations each represents one symbol. The modulator can use either one of the K points to represent each symbol and the choice criteria will be to reduce the PAPR of the transmitted OFDM symbol. **Zhou and Jian, 2009**, proposed a multipoint mapping based scheme to eliminate the need for transmitting side information by using a QPSK constellation as information symbols and mapping this constellation to a 16 QAM constellation. The authors presented the work as a PTS scheme, but it actually falls under the TI category. Although this technique does not require transmission of side information, but the constellation expansion of the QPSK to 16 QAM means increase of average transmission power. **Goel et al., 2013**, introduced a similar concept except that the mapping is performed from the QPSK constellation to a concentric circles constellation. Goel et al., made a performance evaluation of these to schemes and results show the symbol error rate (SER) of the concentric circles base mapping has a 1 dB advantage over the QPSK to 16QAM mapping.

4- PROPOSED SYSTEM

It has been mentioned before, that clipping the OFDM waveform causes undesirable out of band spectral growth and ICI. In this work the clipping is performed in an earlier stage at the output of the IFFT stage where the signal is still in terms of samples, therefore it is referred to as sample clipping. This has the advantage of having no spectral distortion, but it still has the disadvantage of modifying samples in the time domain and this reflects on all the frequency domain samples which are actually the transmitted symbols.

$$e(n) = s(n) - \hat{s}(n) \quad (22)$$

Where, $e(n)$ is the error (noise) term, $s(n)$ is the OFDM symbol given by Eq. (3) and $\hat{s}(n)$ is the clipped OFDM symbol. This means that $e(n)$ is nonzero whenever there is a clipping event and zero when there is no clipping. So, when clipping occurs and $s(n)$ is modified, and it will be reflected on all data symbols c_k (see Eq. (3)). This can be viewed as introducing a sort of impulsive noise on a block of symbols that are the input of the IFFT stage, and this situation can be managed by introducing an error correcting code to the system. Before going into more details, it should be noted that the use of coding in this work is in the general sense and it is not related to the type of coding mentioned earlier that is used to reduced the PAPR. In other words the use of coding here alone does not reduce the PAPR by itself, therefore clipping is still required. The role of the coding is to correct the errors caused by the sample clipping. Another point that is worth mentioning is that introducing a coding scheme into the system does not necessarily mean that we are adding more complexity to the system because almost in every communication system, channel coding is used, so we are only considering the system as a whole instead of focusing on specific part of the system. On the other hand the designer has more freedom to choose the appropriate coding scheme unlike the systems that rely on coding schemes alone to reduce PAPR which are limited and hence may impose certain limitation to the system like the type of modulation, code rate and coding/decoding complexity.

In this work, the convolutional coding is proposed to correct the errors introduced by the sample clipping and since it is expected that the errors due to clipping to be in the form of burst errors,

interleaving is also required and the simplest interleaving technique is used which is the block interleaving. Where the symbols are stored in a rectangular matrix with N row and L columns with each row containing L convolutionally encoded symbols and each column with N symbols to be fed to the IFFT stage, so when clipping is performed at the output of the IFFT stage in case of having a high peak sample, the error is distributed over N different symbols each separated by L symbols, so, the convolutional decoder will not have a burst of clipping errors in its input. The block diagram shown in **Fig. 4** illustrates the components of the proposed system.

5- PERFORMANCE EVALUATION

The system was tested by simulation using Matlab R2010a for two types of modulation; the QPSK and 16QAM modulations. These two modulation types were chosen to study the effect of clipping on the constant amplitude modulation (QPSK in this case) and the varying amplitude modulation (16QAM). Additive White Gaussian Noise (AWGN) was introduced as a perturbing factor to the OFDM signal in addition to the clipping error given by Eq. (22). As has been mentioned earlier, the error correcting code used is the convolutional code, the parameters used where coding rate R equals $1/2$ and constraint length K (which represents the error correcting capability of the convolutional code) taking the values 3, 4 and 5, with generator polynomials having octal representation [3 5], [13 15] and [23 35] respectively. The circuit diagrams of these encoders are shown in **Fig. 5**. Viterbi algorithm is used as decoding algorithm where the complexity is proportional to 2^K . The interleaver size is 64×128 and the number of OFDM carriers is $N=64$. The bit error rate (BER) was evaluated by simulation for the different values of K . since the clipping will make the OFDM have maximum amplitude equals the value of the sample clipping threshold T , then, the PAPR can be evaluated from eq. (5) as

$$PAPR = \frac{T^2}{N} \quad (23)$$

Looking at **Fig. 6** and **Fig. 7**, it can be seen that there is a significant difference in the performance of the system for the QPSK modulation for sample clipping thresholds $T=2$ and $T=4$, which are already very small. Referring to Eq. (12) the PAPR of such signal equals to $N=64$ and from Eq. (23) the PAPR is reduced to 0.0625 and 0.25 respectively. On the other hand, **Fig. 8** and **Fig. 9** show the performance of the system for the 16QAM modulation with square constellation, it has been found that in order to achieve a comparable performance to the QPSK, the threshold of the sample clipping was chosen $T=10$ and $T=11$ which gives PAPR values of 1.56 and 1.89, this is expected because QPSK is a constant amplitude modulation and the demodulation criteria depends on the phase of the received signal not the amplitude, on the other hand, the QAM modulation is an amplitude varying modulation which makes it more sensitive to amplitude clipping. It is clear from these results that there is a significant decrease of the PAPR and if the PAPR is further slightly increased the BER will be reduced much more, so, it is up to the designer to make the proper compromise between the acceptable BER and the desired PAPR.

Further investigation of the graphs in **Fig. 6** to **Fig. 9**, it can be seen that the curves show asymptotic behavior at high values of SNR, remember that the SNR in these graphs are for the AWGN and there is already the clipping-generated noise of Eq. (22), so, when the power of the AWGN is

reduced, the clipping noise will be dominant and cause the graph to reach a certain value of BER because the clipping noise is directly related to the clipping threshold value. The graph shown in **Fig. 10** shows the amount of the SNR when a noise-free OFDM signal is clipped by a given threshold and it clearly shows that the clipping noise decrease (increase of SNR) when the clipping threshold is increased since less clipping events occurs. There are no theoretical formulas for BER performance against clipping-generated noise, but in order to assess the simulation results of **Fig. 6** to **Fig. 9**, the BER performance against AWGN is used as in **Fig. 11**, where the upper bound theoretical values of BER for 1/2 rate convolutionally encoded QPSK and 16QAM signals are plotted against AWGN energy per bit to noise spectral power (E_b/N_o), these curves are Matlab generated using BERTool. And the quantity (E_b/N_o) is related to SNR by

$$SNR(dB) = E_b / N_o (dB) + 10 \log(B) \quad (24)$$

Where B is the amount of information in bits per symbol and it equals to the coding rate times the number of bits per modulation symbol, so for the QPSK $B=1/2 \times 2=1$ so $(E_b/N_o)=SNR$ and for the 16QAM $B=1/2 \times 4=2$ so there is a 3 dB difference in this case. The vertical lines shown in the figure reflect the corresponding E_b/N_o for a given clipping threshold and this is obtained from **Fig. 10** and Eq. (24). BER values are obtained from **Fig. 6** to **Fig. 9** where the curves saturate, i.e. where the clipping noise is dominant. It is seen that the QPSK case have BER values below the theoretical curves and there is 3-4 dB advantage, this means that the system has better performance handling clipping noise than AWGN. In the QAM case the BER values are over the curves and there is about 1 dB performance degradation when compared to the AWGN performance and this is because the clipping has direct effect on the symbol amplitude and demodulating QAM signal depends on phase as well as amplitude, therefore the performance is reduced unless higher value of PAPR (or clipping threshold) is allowed.

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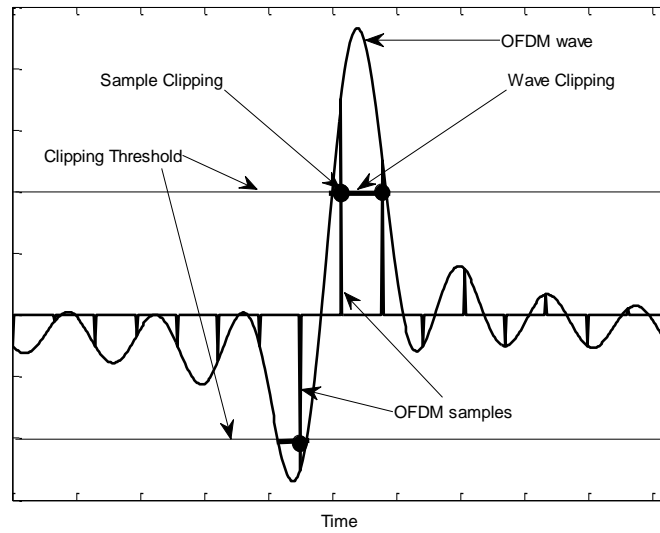


Figure 1. An OFDM signal before and after the LPF showing high peaks over the clipping threshold.

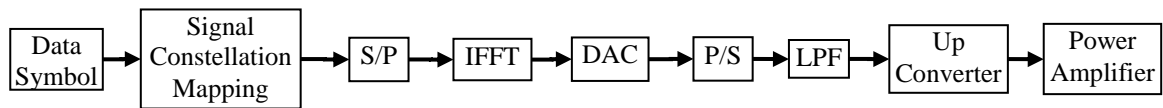


Figure 2-a. OFDM transmitter.

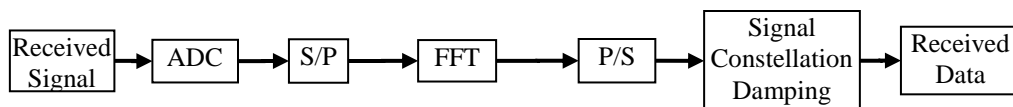


Figure 2-b. OFDM receiver.

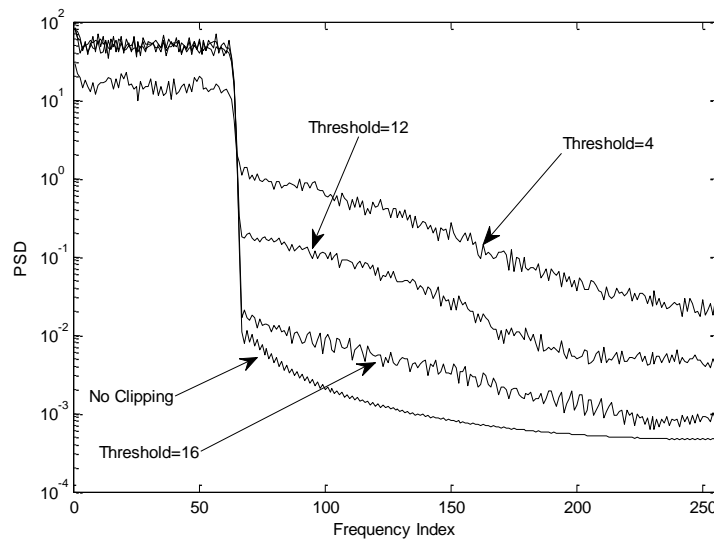


Figure 3. The spectrum of OFDM signals with $N=64$ and different clipping thresholds.

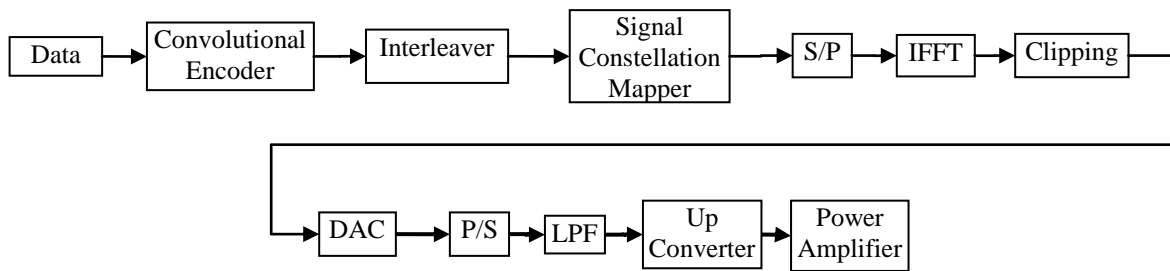


Figure 4-a. Proposed OFDM transmitter with clipping and coding.

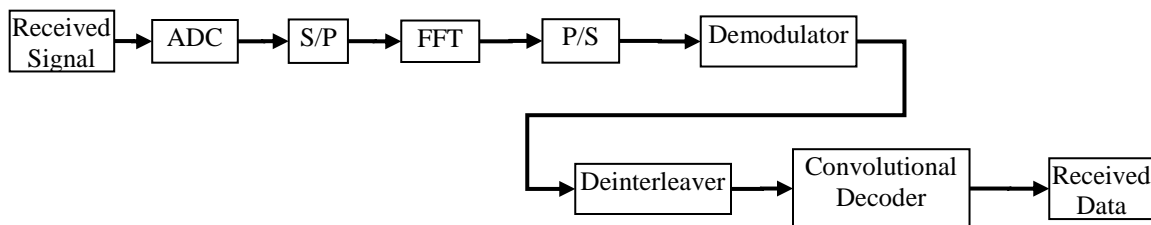


Figure 4-b. Proposed OFDM receiver with clipping and coding.

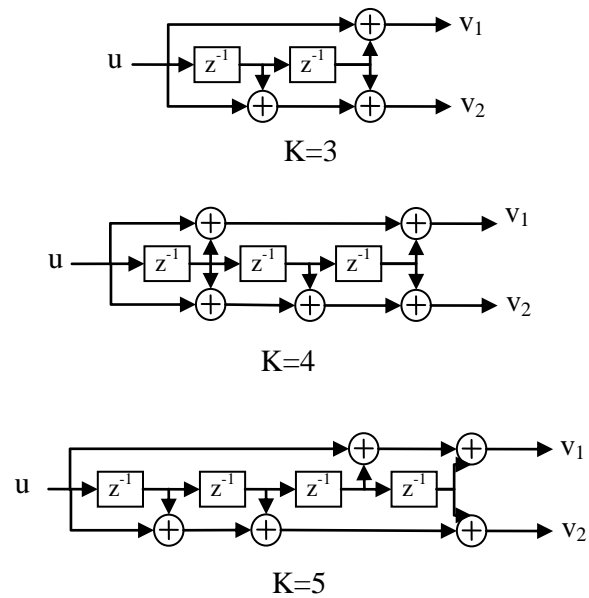


Figure 5. The three convolutional encoders used in the proposed transmitter.

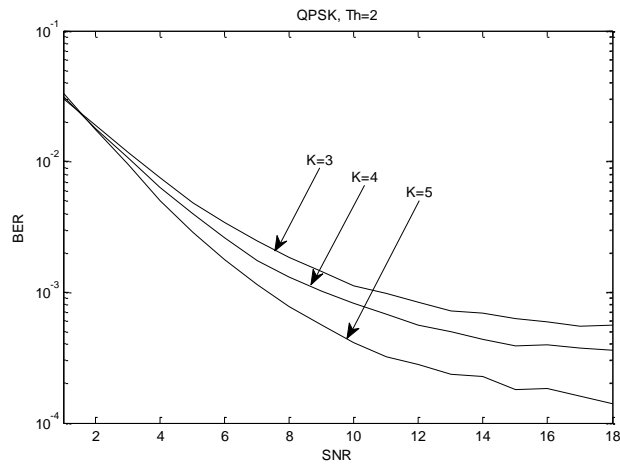


Figure 6. BER vs. SNR for QPSK signal and clipping threshold $T=2$ (PAPR=0.0625).

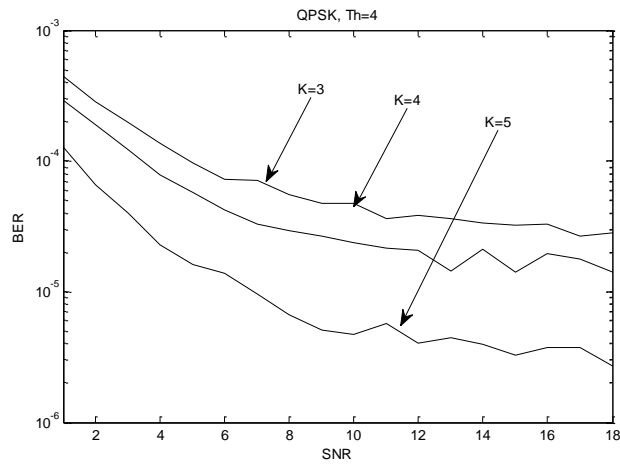


Figure 7. BER vs. SNR for QPSK signal and clipping threshold $T=4$ (PAPR=0.25).

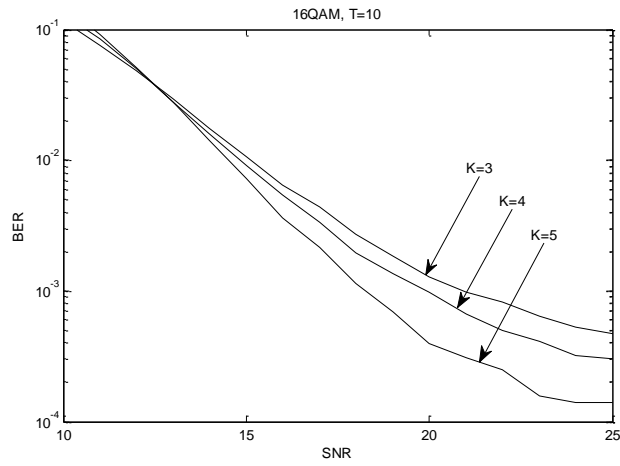


Figure 8. BER vs. SNR for 16QAM signal and clipping threshold $T=10$ (PAPR=1.56).

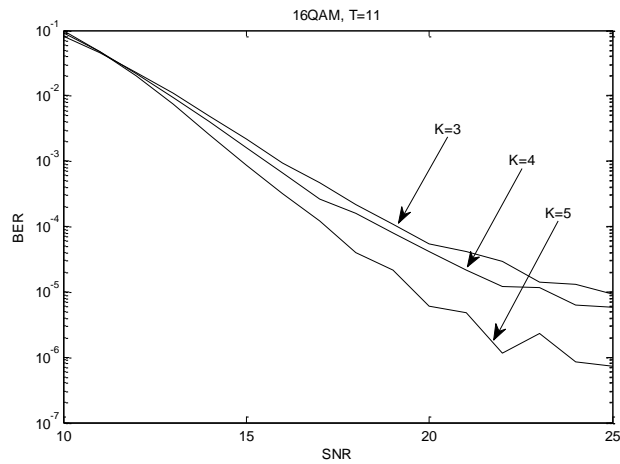


Figure 9. BER vs. SNR for 16QAM signal and clipping threshold $T=11$ (PAPR=1.89).

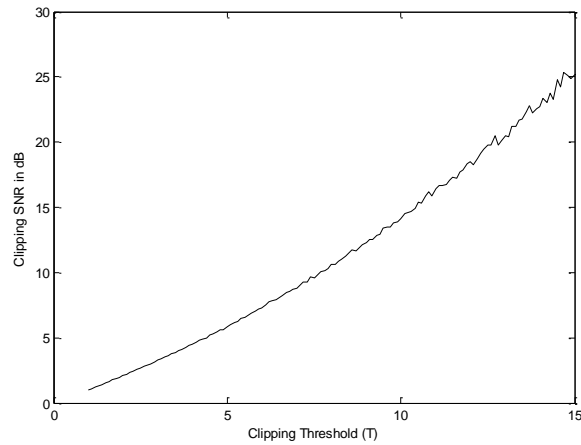


Figure 10. Clipping signal-to-noise ratio versus clipping threshold.

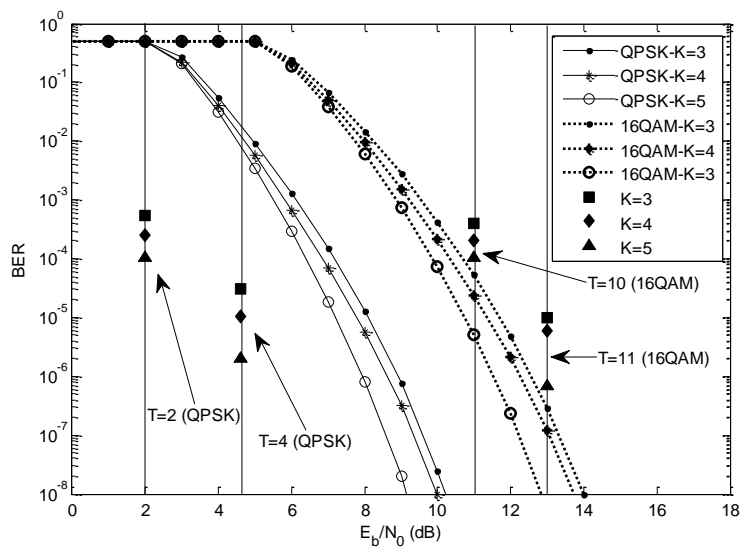


Figure 11. Comparison of simulated BER results with theoretical upper bound performance against AWGN.