Solving Time-Cost Tradeoff Problem with Resource Constraint Using Fuzzy Mathematical Model

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ABSTRACT

Scheduling considered being one of the most fundamental and essential bases of the project management. Several methods are used for project scheduling such as CPM, PERT and GERT. Since too many uncertainties are involved in methods for estimating the duration and cost of activities, these methods lack the capability of modeling practical projects. Although schedules can be developed for construction projects at early stage, there is always a possibility for unexpected material or technical shortages during construction stage.

The objective of this research is to build a fuzzy mathematical model including time cost tradeoff and resource constraints analysis to be applied concurrently. The proposed model has been formulated using fuzzy theory combining CPM computations, time-cost trade off analysis and resource constraint. MATLAB software has been adopted to perform ranking process, for each case, that facilitates obtaining the optimum solution. This research infers that it is possible to perform time-cost trade off analysis with resource restriction simultaneously, which ensures achieving scheduling optimum solution reducing the effort and the time when performing these techniques in succession using traditional methods.

Key words: fuzzy mathematical model, CPM, resource constraint, MATLAB, linear programming.

حل توافق الزمن- الكلفة مع محددات المصادر باستخدام نموذج التعرج الرياضي

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الخلاصة

تعتبر الجدولة واحدة من اهم المبادىء والقاعدة الاساسية في ادارة المشاريع. توجد عدة طرق لجدولة المشاريع مثل (CPM, PERT, GERT). ومع وجود الكثير من عدم التيقن المتضمنة في طرق تخمين مدد الفعاليات وكلف الفعاليات, فأن هذه الطرق تفتقد القدرة لنمذجة المشاريع العملية (الواقعية). بالرغم من أن الجداول ممكن ان تعد للمشاريع الانشائية في مراحل مبكرة من المشروع، توجد أحتمالية ظهورعجزتقني او نقص في الموارد غير متوقع أثناء مرحلة الانشاء.

أن الهدف من هذا البحث هو بناء أنموذج التعرج الرياضي متضمناً تطبيق توافق الكلفة مع الزمن و محددات المصادربالتوازي. لقد صيغ الأنموذج المقترح باستخدام نظرية التعرج مدموجاً مع تطبيق طريقة المسار الحرج، توافق الوقت مع الكلفة ومحددات الموارد. تم تبني برنامج (MATLAB) لانجاز عملية الترتيب لكل حالة والتي تسهل أيجاد الحل الامثل. استنتج البحث الى امكانية اجراء تحليل توافق الوقت مع الكلفة ومحددات الموارد في آن واحد مما يضمن الوصول الى الحل الامثل للتخطيط وبشكل يقلل من الجهد والوقت عند اجراء هذه العمليات بالتعاقب وبالطرق التقليدية المعمول بها والتي من المطلوب فيها غالباً تكراراعادة الجدولة للوصول الى الحل الأمثل.

1-INTRODUCTION

Decisions in construction management are made based on schedules that are developed during the early planning stage of projects, while many possible scenarios need to be considered during actual construction stage which may cause many changes in schedule. Many restrictions appear during construction stage, therefore, taking these restrictions into account helps project managers to evaluate situations and make better decisions. In order to adopt more integrated construction project plans including the requirements for implementing the project plans in possible least costly manner, time-cost trade off analysis with resource constraints techniques were developed to apply in succession. In real projects, the trade-off between the project cost and the completion time, and the uncertainty are both considerable aspects for managers. Resources are the main factor that affect implementing project schedule, providing the accurate resources at the right time means that the schedule will probably run smoothly. But when there are insufficient resources available for activities (especially concurrent activities), which use the same type of resource, some of these activities are delayed to relieve the resource constraints. Usually, the solutions for the optimum time cost trade off may not be suitable for resource allocation. Although optimization programming processes the capability of producing accurate solutions, it requires elaborate formulation and extensive computation.

Fuzzy Logic has emerged as a nontraditional tool in construction management applications and as such has been employed in resource scheduling and time cost trade off analysis individually. To obtain optimum solution for time schedule, it is necessary to make time cost trade off, resource allocation applied simultaneously within fuzzy environment to produce optimum time schedule considering cost and resource constraints.

Many researcher performed studies about using fuzzy theory in project scheduling. **Zhang et al, 2005** Incorporate fuzzy set theory and a fuzzy ranking measure with discrete-event simulation in order to model uncertain activity duration in simulating a real-world system, especially when insufficient or no sample data are available. **LIANG, 2006** presents an interactive Fuzzy Linear Programming (FLP) approach for solving project management (PM) decision problems in a fuzzy environment. **Soltani** and **Haji, 2007** have developed a new method based on fuzzy theory to solve the project scheduling problem under fuzzy environment. Assuming that the duration of activities are trapezoidal fuzzy numbers (TFN), in this method they compute several project characteristics such as earliest times, latest times, and, slack times in term of TFN.

Lin, 2008 introduces a fuzzy time-cost tradeoff problem based on statistical confidence-interval estimates and a distance ranking for fuzzy numbers to derive the level $(1 - \alpha)$ of fuzzy numbers from $(1 - \alpha) \times 100\%$ statistical data confidence-interval estimates. **Shankar, et al. 2010** presents a method for finding critical path in the fuzzy project network. Trapezoidal fuzzy numbers are used to represent activity times in the project network. **Liang, et al. 2011** presents a fuzzy mathematical programming approach to solve imprecise project management decision problems with fuzzy goal and fuzzy cost coefficients.

The research has many difficulties when applying such models due to unavailability of the required information or emphasizing it on the logic part of fuzzy theory rather than presents a new method to solve scheduling problem. Thus this research will focus on making integration between fuzzy logic and the management theories to provide an improved method used in project scheduling.

2- FUZZY SETS

Fuzzy sets can be considered as an extension of classical or 'crisp' set theory. In classical set theory, an element x is either a member or non-member of set A. Thus, the membership μA (x) of x into A is given by:

$$\mu A\,(x) = \begin{cases} 1, \text{ if } x \in A \\ 0, \text{ if } x \notin A \end{cases}$$

In contrast to classical set theory, the fuzzy set methodology introduces the concept of degree to the notion of membership. More formally, a fuzzy set A of a universe of discourse X (the range over which the variable spans) is characterized by a membership function μA(x): X → [0, 1] which associates with each element x of X a number μA(x) in the interval [0, 1], with μA(x) representing the grade of membership of x in A. So, Membership Function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. **Sivanandam et al, 2007.**

Various membership functions can be established depending on how we can represent the context of the practical problem, the most familiar membership function presented in **Fig. 1 Lorterapong and Moselhi, 1996.** The mathematical representation of the membership function presented in **Fig. 1** is as follows:

2-1 Triangular Membership Function **Fig. 1 a,** The membership function for this type is

$$\mu\,(x) = \begin{cases} 1- (|x-b|/a-b) & \text{if } a < x < c \\ 0, & \text{otherwise} \end{cases}$$

2-2 Trapezoidal Membership Function **Fig.1 a,** The membership function for this type is

$$\mu\,(x) = \begin{cases} 0 & \text{if } x \le a \\ x\text{-}a\ /b\text{-}a & \text{if } a < x \le b \\ 1 & \text{if } b < x \le c \\ x\text{-}d\ /c\text{-}d & \text{if } c < x \le d \\ 0 & \text{if } x > d \end{cases}$$

2-3 Open Right Membership Function **Fig. 1 c,** The membership function for this type is

$$\mu\,(x) = \begin{cases} 1 & \text{if } x \ge b \\ x\text{-}a\ /b\text{-}a & \text{if } a < x < b \\ 0 & \text{if } x < a \end{cases}$$

2-4 Open Left Membership Function **Fig. 1 d,** The membership function for this type is

$$\mu\,(x) = \begin{cases} 1 & \text{if } x \le a \\ b\text{-}x\ /b\text{-}a & \text{if } a < x < b \\ 0 & \text{if } x > b \end{cases}$$

3- FUZZY NUMBERS ARTHIMATIC

Let A and B be two trapezoidal fuzzy numbers parameterized by the quadruple A= $[a_1, b_1, c_1, d_1]$ and B= $[a_2, b_2, c_2, d_2]$ respectively. The simplified fuzzy number arithmetic operations between the trapezoidal fuzzy numbers A and B are as follows **Shankar et al (2010)**.

Addition ⊕

$A \oplus B = [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]$

Subtraction ⊖

$A \ominus B = [a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2]$

Multiplication ⊗

$A \otimes B = [a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2]$

Division ⊘

$A \oslash B = [a_1 / d_2, b_1 / c_2, c_1 / b_2, d_1 / a_2]$

4-FUZZIFICATION AND DEFUZZIFICATION

According to **Ross, 2004**, Fuzzification is the process of making a crisp quantity fuzzy. This is done by simply recognizing that many of the quantities that are considered to be crisp and deterministic are actually not deterministic at all: They carry considerable uncertainty.

While defuzzification is a mathematical process used to extract crisp output from fuzzy output set(s). This process is necessary because all fuzzy sets inferred by fuzzy inference in the fuzzy rules must be aggregated to produce one single number as the output of the fuzzy model, **Asmuni, 2008.**

Many methods that have been proposed in the literature in recent years, seven are described here for defuzzifying fuzzy output functions (membership functions), **Ross, 2004,** Max membership principle, Centroid method, Weighted average method, Mean max membership, Center of sums, Center of largest area and First (or last) of maxima.

5- TIME- COST TRADEOFF ANALYSIS

Time-Cost Trade off (TCT) analysis represents the process of optimally reducing the project duration in a least costly manner. The objective of TCT analysis is to search for the optimum set of activities methods of construction that minimizes the total project cost (direct and indirect) while not increasing project completion time. The TCT analysis involve estimating, if possible, the cost of crashing normal time for the project activities so as total project completion time will be decreased.

6- RESOURCE RESTRICTION

Two problems arise in developing a resource constrained project schedule. **Hendrickson and Au (2003)** First, it is not necessarily the case that a critical path schedule is feasible. Because one or more resources might be needed by numerous activities, it can easily be the case that the shortest project duration identified by the critical path scheduling calculation is impossible. The difficulty arises because critical path scheduling assumes that no resource availability problems or bottlenecks will arise. As a second problem, it is also desirable to determine schedules which have low costs or, ideally, the lowest cost. To overcome these problems, all the possible scenarios of resource allocation with associated time schedule get developed, considering restricted availability of recourses, and the schedule that satisfies both the time and cost criteria is identified. This will be done by changing different activities start time, depending on the availability of the resources, and the most optimum schedule is selected.

7- FUZZY MATHEMATICAL MODEL

The time and cost of the project activities, as well as the project itself, may be expressed using a range of values rather than exact numbers. This fact makes the theory of Fuzzy logic applicable in such cases to represent the uncertainty in time and cost of construction project. Since the time and cost of the activities are considered fuzzy numbers, the project total time and cost will be expressed by fuzzy numbers. To prepare a mathematical model for project scheduling using Fuzzy theory the following proposed algorithm will be applied

1- Define the project activities by answering the question " what must be done"
2- Define the logical relationship between activities
3- Estimate the activities cost and time
4- Specify, if possible, the crash time and cost for crashing the activity
5- Convert the activity time to fuzzy time for the project activities; this is done by finding the max number of crash time in all activities considering it as the fuzzy membership function. For example if the max number of crash time in all project activities is 3 days and the normal time for this activity is 8 days, then we will use four point fuzzy membership function (trapezoidal Function) and fuzzy numbers will be (5, 6, 7 and 8).
6- For the activities having no or less crash time, consider the following: -
A- The activities with less than max crash time, repeat one of the numbers. For example if an activity have 2 days crash time and 3 days normal time then fuzzy time may be written as minimum (1, 1, 2 and 3) , middle (1, 2, 2 and 3) and maximum fuzzy time (1, 2, 3 and 3).
B- For the activities with no crash time, repeat the same activity time. For example if an activity has 2 days of time then the fuzzy time will be (2, 2, 2 and 2).
7- For the activities with crashing cost per unit of time greater than indirect cost set the fuzzy time equal to normal time.
8- Specify: -
A- The required resources for each activity.
B- The available resources and the time of availability.
9- Examine the possible scenarios for the project scheduling (consider availability of the resources and the time of availability).
10- For each scenario, develop a number of networks by considering different fuzzy time.
11- Develop a mathematical model for each scenario and solve it by using fuzzy logic toolbox presented in the commercial program (MATLAB).

8- CASE STUDY

For applying the proposed algorithm a case study project from **Mohammed, 2004** will be adopted, normal, crash time and cost presented in **Table 1.** While the **Fig. 2** presents the AOA network for this project.

While the first four steps in the proposed algorithm are satisfied in the case study, other steps will be implemented as follows: -

1- Converting the activity duration to fuzzy duration, according to step 5 & 6 in the proposed algorithm, the conversion process was done depending on the maximum amount of crashed time available in the project activities, while fuzzy cost depending on crashing rate for each activity, that's mean increasing in unit of time will be associated with decreasing in cost using crashing rate, **Table 2** present the fuzzy duration and cost for each activity, the above conversion actually determines the linguistic variables. As mentioned in paragraph four "Fuzzification is the process of making a crisp quantity fuzzy" that means the activity time, and cost, are converted from crisp (single value) value to fuzzy value (membership value) by using linguistic variables .

2- For step number 8, the required resources for each activity are presented in **Table 1.** While the available resources and the time of availability are stated by only 12 units of material (m) available and the rest are going to be delivered after days 12.

3- Considering step number 9 in the proposed algorithm, in addition to the case of normal resources availability, there are two possible scenarios for the project scheduling consider availability of the resources and the time of availability.

4- In step number 10, each scenario developed in step number 9 will be tested with the possible combination of activity fuzzy time (min, middle and max fuzzy time) using the scenario network, the result represents project total fuzzy times (the project membership functions). This step will be done by using the planned case (Normal resource availability) and resource restriction cases as follows

A- Normal resources availability

Three networks are developed considering normal availability of the resources. For each network the project completion time represents project membership function, that means three trapezoidal membership functions are developed, **Fig. 3** presents the networks of normal recourse availability scenario with min fuzzy time. It is clear that crashing some activity in the above network will not reduce the project total time while increasing the total cost (activity D, F and H) because the following activities have greater start time than their finish time, so set the activity time equal to normal time as in **Fig. 4**. The same procedure will be implemented in the other network (middle and maximum fuzzy time) presented in **Fig. 5** and **Fig. 6**.

B- Resource restriction first scenario

In this scenario, the activities (A, B, C, D and E) will be performed during the first 12 days and the rest of activities will be implemented after that. This action requires inserting dummy activity (40-45) with early start time equal to 13 days. **Fig. 7** shows the network developed for this scenario and the resulting project completion time. The network in **Fig. 7** shows that crashing some activities will not reduce the project total time while increasing the total cost because the following activities have greater start time than their finish time, so the backward adjustment involves activities (A, D, E and G) by setting activity time equal to normal time as in Figure **Fig. 8**. The same procedure will be implemented in the other network (middle and maximum fuzzy time) presented in **Fig. 9** and **Fig. 10**.

C- Resource restriction second scenario

The second scenario shows that the activities (A, B, C, E and F) will be performed during the first 12 day and the rest of activities will be implemented after that. To perform this action, the dummy activity (30-35) will be inserted in the project network with early starting time of 13 days. **Fig. 11** illustrates the network developed for this scenario with the project fuzzy completion time. The same procedure of backward adjustment is implemented in this scenario involving activities (E, F, G and H) by setting the normal activity time as a fuzzy time. **Fig. 12, Fig. 13** and **Fig. 14** show the network for second scenario.

The associated cost for each time will be calculated using the following equation

$$TC= \sum C+[IC * Xn] \qquad (1)$$

Where:
TC= total Cost
C= activity direct cost for specified time (**Table 2**).
Xn= project completion time
IC= indirect cost / unit of time (1500$/day)
Table 3 summarizes the project fuzzy time for each scenario with the associated fuzzy total cost. The information in this table is the basis for creating membership functions for using in fuzzy mathematical models.

9- THE MATHEMATICAL MODEL

Developing the mathematical model for each scenario as follows
A- For normal resource availability, the proposed model will be developed using the information resulting from planned case (**Table 3**). The cost model can be expressed as: -

$$\mu (t) = \begin{cases} 1 & \text{if } t \le 21 \\ (27-t) / 5 & \text{if } 22<t\le 27 \\ 0 & \text{otherwise} \end{cases}$$

Where the variable (t) represents completion time for the project, 22 is the preferred completion time and 27 is the normal completion time. The graphical representation for this model is shown in **Fig. 15**. While the following model represents the cost model with graphical illustration in **Fig. 16**.

$$\mu(c) = \begin{cases} 1 & \text{if } c\le 68200 \\ (70000-c)/1800 & \text{if} 68200<c\le 70000 \\ 0 & \text{otherwise} \end{cases}$$

Where the variable (c) is the completion time for the project, (68200) represents the preferred completion cost and (70000) is the normal completion cost.

B- For resource restriction first scenario the following model represents time and cost model using the information in **Table 3** with same procedure used in developing planned case models. The models graphical illustration is presented in **Fig. 17** and **Fig. 18**.

$$\mu (t) = \begin{cases} 1 & \text{if } t\le 27 \\ (29-t) / 2 & \text{if } 27<t\le 29 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu (c) = \begin{cases} 1 & \text{if } c\le 72100 \\ (73000-c) / 900 & \text{if } 72100<c\le 73000 \\ 0 & \text{otherwise} \end{cases}$$

C- The models for case two is developed using information in **Table 3** (case two) as shown below with graphical representation in **Fig. 19** and **Fig. 20**.

$$\mu\ (t) = \begin{cases} 1 & \text{if } t \leq 25 \\ (27-t) / 2 & \text{if } 25 < t \leq 27 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu\ (c) = \begin{cases} 1 & \text{if } c \leq 70000 \\ (72000-c) / 2000 & \text{if } 70000 < c \leq 72000 \\ 0 & \text{otherwise} \end{cases}$$

10- SOLVING THE MATHEMATICAL MODEL USING MATLAB PROGRAM

Now the proposed mathematical model developed for each case will be solved by using MATLAB fuzzy logic toolbox (Graphical User Interface GUI) as follows

1- Construct two inputs (time and cost) one output (rank) system using FIS Editor. While the inputs represent the fuzzy time and cost for each case, which are defined in the mathematical model, the output will represent the scale to measure optimum time and cost as shown in Figure **Fig. 21**.

2- Define the membership function for system. One trapezoidal membership functions will be used for each input and one triangular membership function for the output, while the defuzzification method will be smallest of maximum (som). State the range for the time input (0-30), while the cost input will be entered in thousands and the range will be (0-80). The output range will be (0-1) which represents the rank for each time and associated cost.

3- Write down the rules using Rule Editor. The rule will be added as presented in Figure **Fig. 22**.

4- Finally the time value can be fed with associated cost by using rule viewer to get their rank. The value of each input variable can be classified by sliding the lines in the input column and generating the output value or by writing those in the input field as shown in figure **Fig. 23**. The output of each input is presented in **Table 4** which summarizes the final rank for each case.

11- DEVELOPING GENERAL MODEL

Now if it is required to choose between the times generated from the restriction cases, case one and two, a general model will be developed which represents the min and max time and cost of the restriction cases, the resulting model is as follows

$$\mu\ (t) = \begin{cases} 1 & \text{if } t \leq 25 \\ (29-t) / 4 & \text{if } 25 < t \leq 29 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu\ (c) = \begin{cases} 1 & \text{if } c \leq 70000 \\ (73000-c) / 3000 & \text{if } 70000 < c \leq 73000 \\ 0 & \text{otherwise} \end{cases}$$

The graphical representation is shown in Figures **Fig. 24** and **Fig. 25**.

The final rank for each time and associated cost is obtained using MATLAB (GUI) and the final result is summarized in **Table 5.**

12- MODEL VERIFICATION

The verification processes involve converting normal and resource restriction cases to Linear optimization models and solving them using commercial computer program called (WinQSB). The results generated from solving these models summarized in **Table 6.**

13- RESULTS DISSCUSION

The results generated from solving fuzzy mathematical models reflect the required purposes of the models developed for each case individually which can be summarized in finding the optimum reduction time and the associated cost. For the planned case, the result shows that the optimum time is 22 days with associated cost of $68200, which has the highest rank, and this result matches the result of the optimization model which satisfy the model verification requirement, but another time and cost has the same rank which is 21 days with cost of $68500, and that have reflect the enhancement of this method which gives the decision maker flexibility to choose what he favorite, min time or min cost. For case one the results shows that the optimum time is 27 days with associated cost of $72100 which is exactly the same result of the optimization model. For the case two, the result shows that the optimum time is 26 days with cost of $71000 and this result differs from the result of the optimization model, but again this result gives the decision maker option to choose between minimum time and cost.

The general model result shows that the optimum time is 26 days with cost of $71000, but still the decision maker has the option to choose what he favors min time or min cost according to project situation. The above models provide decision makers with a range of time that is between the normal time and the maximum crash time.

14- CONCLUSIONS

1- Fuzzy mathematical model has the capability to determine the optimum solution for time-cost trade off analysis with inclusion of resource restriction simultaneously. The presented solution is identical to manual solution in which time-cost trade off analysis and resource allocation are performed in succession, and requires no effort of network rescheduling as it is performed manually.

2- Fuzzy mathematical model provides accurate results and that the optimization model is performed correctly. In addition optimization model finds the minimum completion time for projects while fuzzy model provides a range of time that is between the normal time and the maximum crash time.

3- The model allows the decision maker to examine different scenarios for project execution, and their impact on total time and cost, done by changing the order of performing activities which causes automatic change in project duration and cost.

4- This model could be used for examining the possibility of material or technical shortages. The analysis could be done by comparing other alternatives such as using a more costly material that could be delivered at the right time.

5- This model can be used for the project in Iraqi construction sector which have the right required information for project scheduling such as normal and crash time and cost, the expected resource shortage and the cost of the available alternatives.

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Table 1. Case study project information.

Activity	Followed by	Time (Days)		Cost ($)		The required no. of resources
		Normal	Crash	Normal	Crash	
A	B C	3	1	1000	3000	1
B	D E	4	3	4000	6000	3
C	E F	2	2	2000	2000	2
D	I	6	4	3000	6000	3
E	G H	5	4	2500	3800	3
F	H	3	2	1500	3000	2
G	I	7	4	4500	8100	4
H	I	5	4	3000	3600	3
I		8	5	8000	12800	5

Table 2. Fuzzy duration and cost for case study project activities.

Activity	Activity fuzzy duration				Activity fuzzy cost			
A	1	1	2	3	3000	3000	2000	1000
	1	2	2	3	3000	2000	2000	1000
	1	2	3	3	3000	2000	1000	1000
B	3	3	3	4	6000	6000	6000	4000
	3	3	4	4	6000	6000	4000	4000
	3	4	4	4	6000	4000	4000	4000
C	2	2	2	2	2000	2000	2000	2000
D	4	4	5	6	6000	6000	4500	3000
	4	5	5	6	6000	4500	4500	3000
	4	5	6	6	6000	4500	3000	3000
E	4	4	4	5	3800	3800	3800	2500
	4	4	5	5	3800	3800	2500	2500
	4	5	5	5	3800	2500	2500	2500
F	2	2	2	3	3000	3000	3000	1500
	2	2	3	3	3000	3000	1500	1500
	2	3	3	3	3000	1500	1500	1500
G	4	5	6	7	8100	6900	5700	4500
H	4	4	4	5	3600	3600	3600	3000
	4	4	5	5	3600	3600	3000	3000
	4	5	5	5	3600	3000	3000	3000
I	5	6	7	8	12800	11200	9600	8000

Table 3. Case study project fuzzy total time and total cost.

Case		Fuzzy total time				Fuzzy total cost			
Planned case(Normal resource availability)		21	22	24	27	68500	68200	69000	70000
		21	23	25	27	68500	68700	69200	70000
		21	24	26	27	68500	68900	69700	70000
Resource restriction	Case one	27	27	27	29	72100	72100	72100	73000
		27	27	29	29	72100	72100	73000	73000
		27	29	29	29	72100	73000	73000	73000
	Case two	25	25	26	27	72000	72000	71000	70000
		25	26	26	27	72000	71000	71000	70000
		25	26	27	27	72000	71000	70000	70000

Table 4. Final rank for the three cases.

Planned case	Time	21	22	23	24	24	25	26	27
	Cost	68500	68200	68700	68900	69000	69200	69700	70000
	Rank	0.42	0.42	0.33	0.25	0.24	0.17	0.09	0.01
Case one	Time	27	29						
	Cost	72100	73000						
	Rank	0.5	0.02						
Case two	Time	25	26	27					
	Cost	72000	71000	70000					
	Rank	0.02	0.26	0.02					

Table 5. Final rank of the general case.

Two cases	Time	25	26	27	27	29
	Cost	72000	71000	70000	72100	73000
	Rank	0.18	0.34	0.26	0.16	0.01

Table 6. Model verification results.

Case		Total time	Total cost
Planned case (Normal resource availability)		22	68200
Resource restrictions	Case one	27	72100
	Case two	27	70000

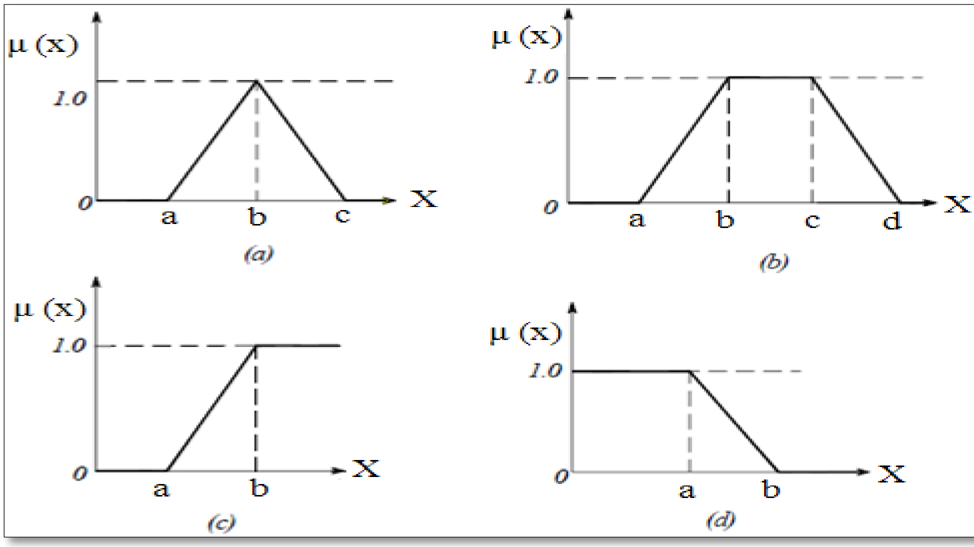

Figure 1. Common fuzzy membership functions.

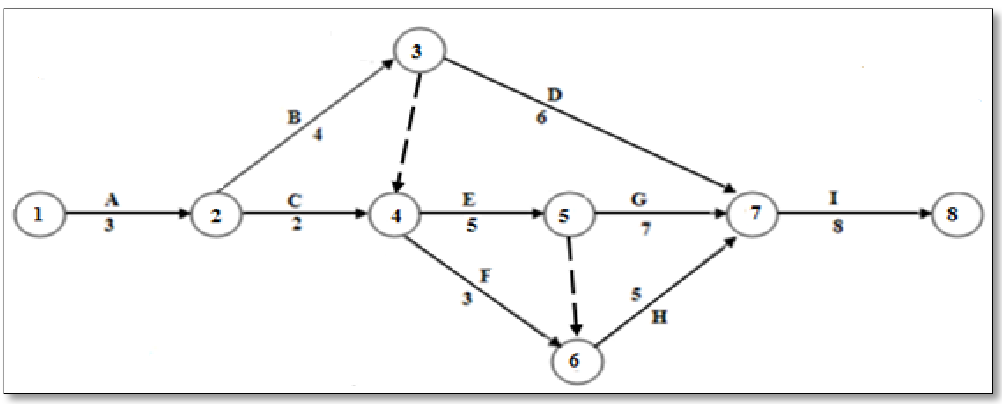

Figure 2. AOA network for case study project.

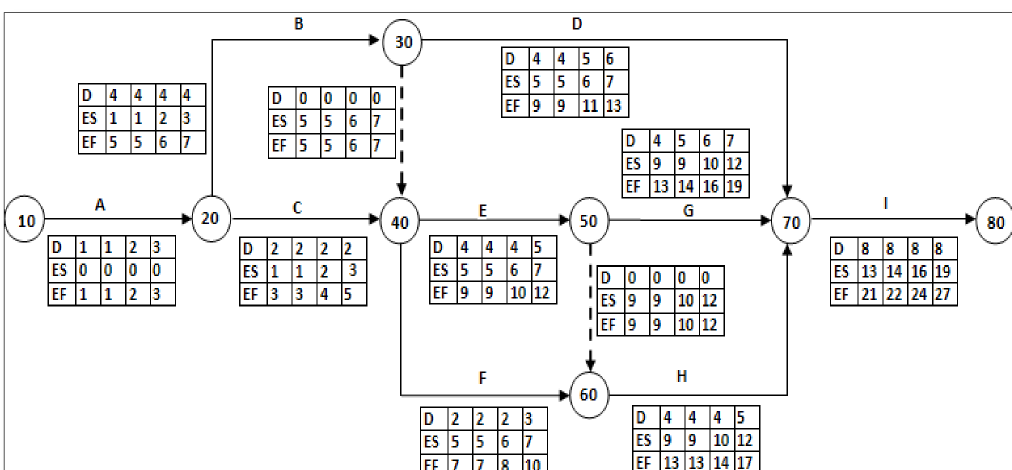

Figure 3. AOA network for normal recourse availability with min fuzzy time.

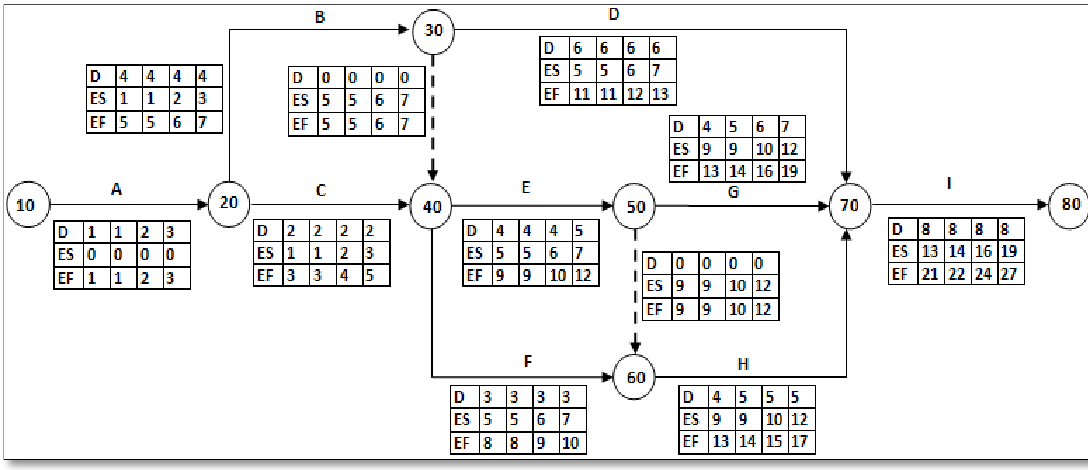

Figure 4. Adjusted AOA network for normal recourse availability with min fuzzy time.

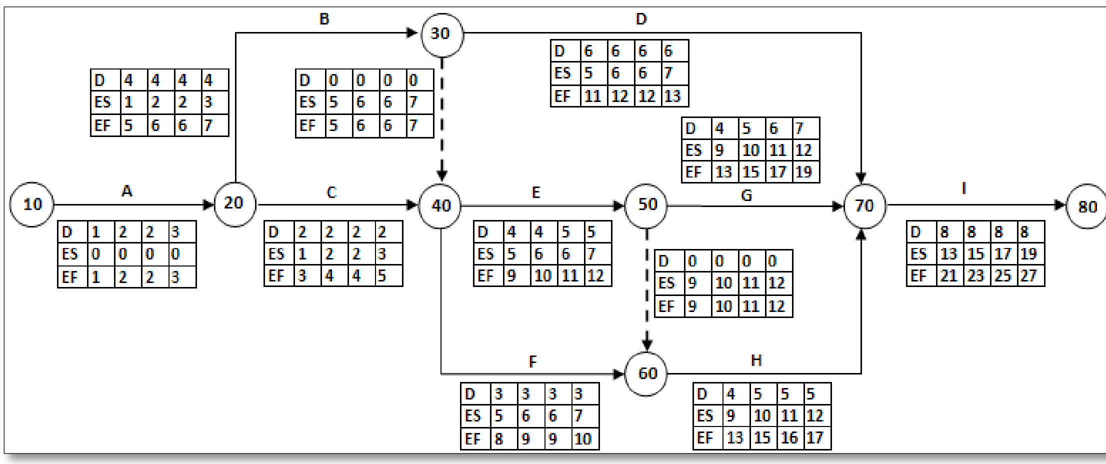

Figure 5. Adjusted AOA network for normal recourse availability with middle fuzzy time.

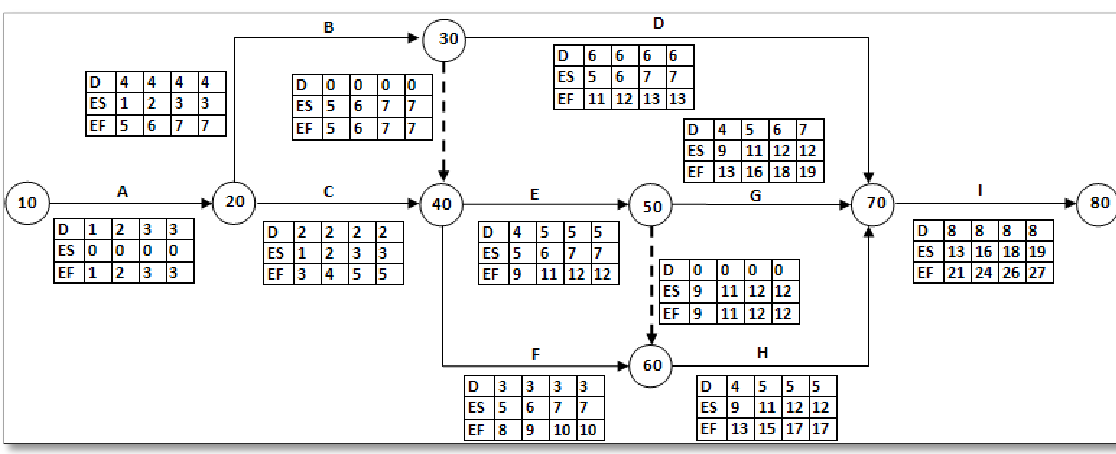

Figure 6. Adjusted AOA network for normal recourse availability with max fuzzy time.

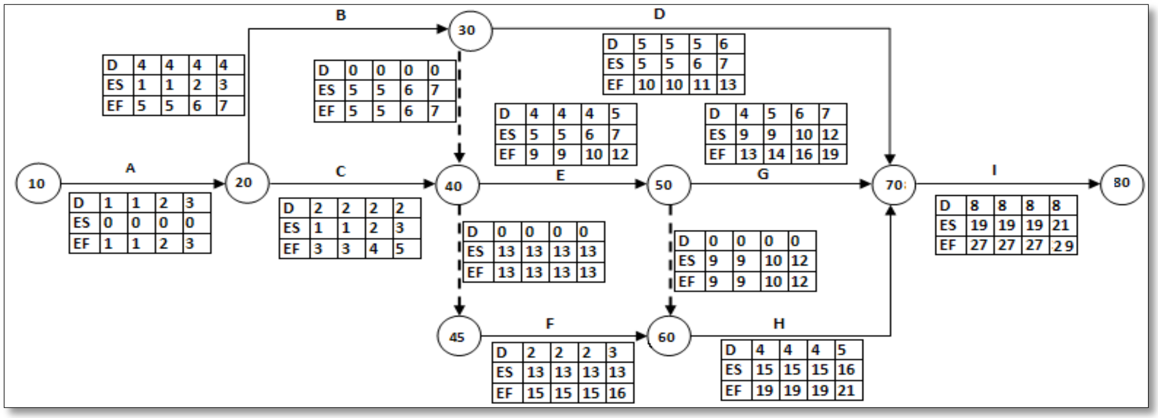

Figure 7. AOA network for the first scenario with min fuzzy time.

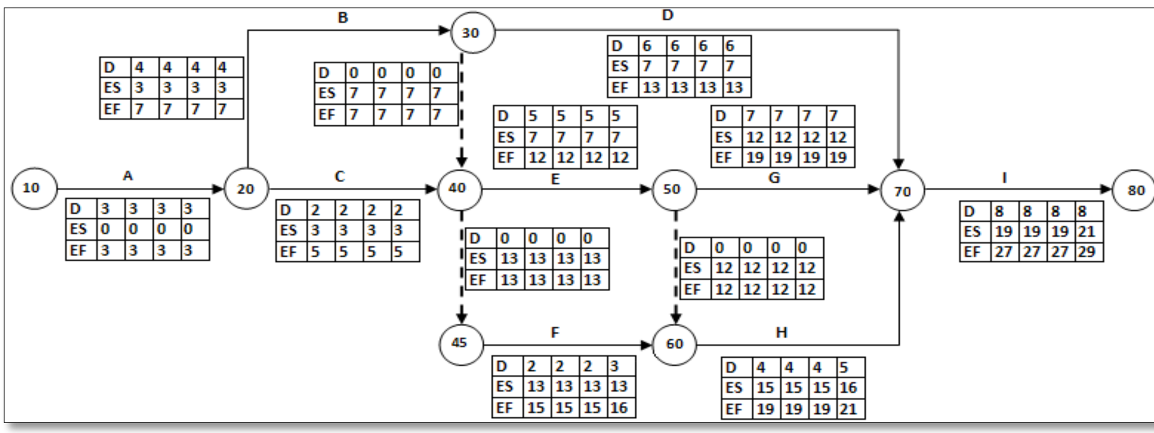

Figure 8. Adjusted AOA network for the first scenario with min fuzzy time.

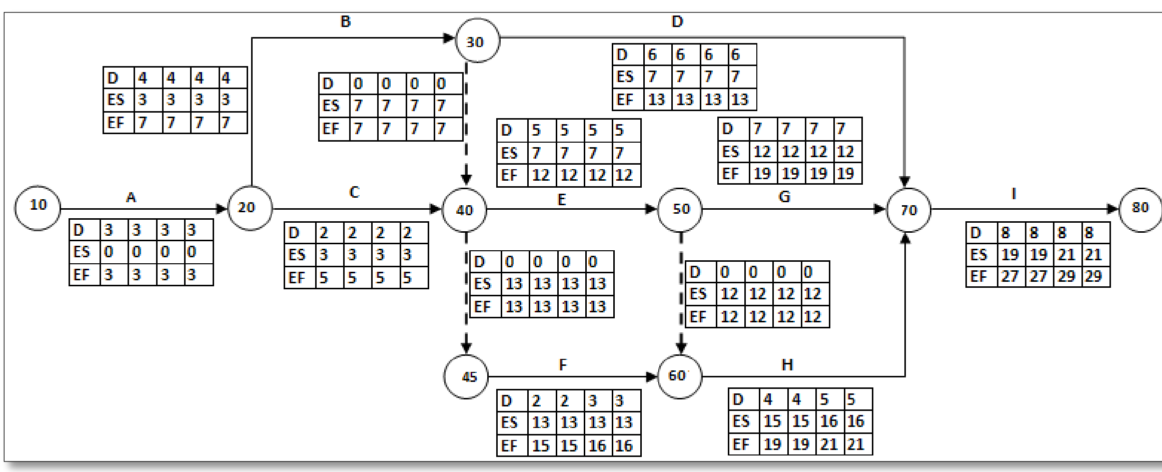

Figure 9. Adjusted AOA network for the first scenario with middle fuzzy time.

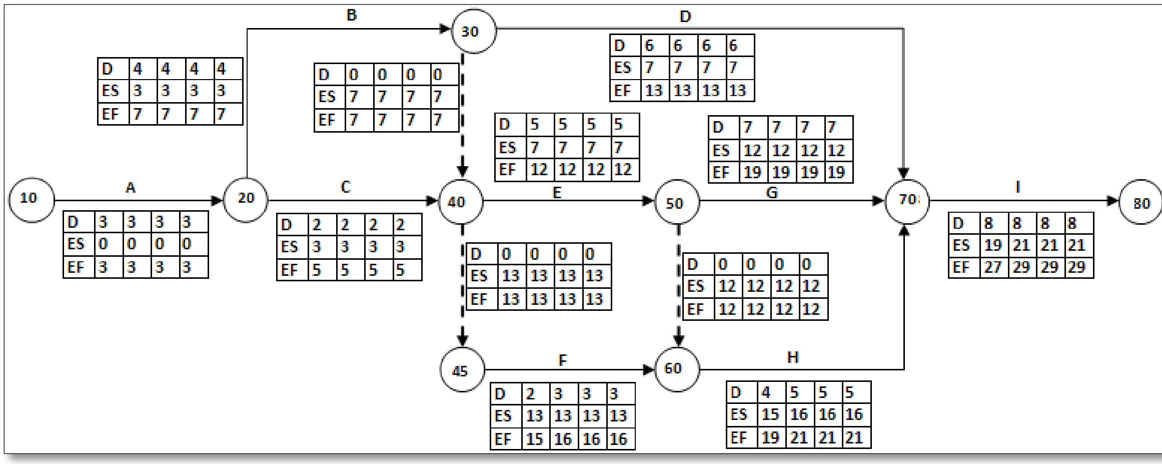

Figure 10. Adjusted AOA network for the first scenario with max fuzzy time.

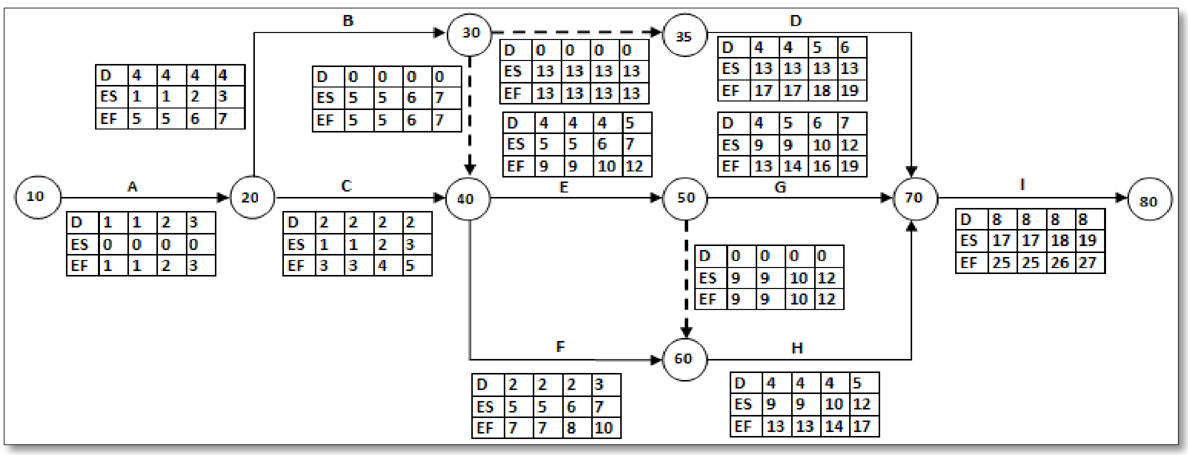

Figure 11. AOA network for the second scenario with min fuzzy time.

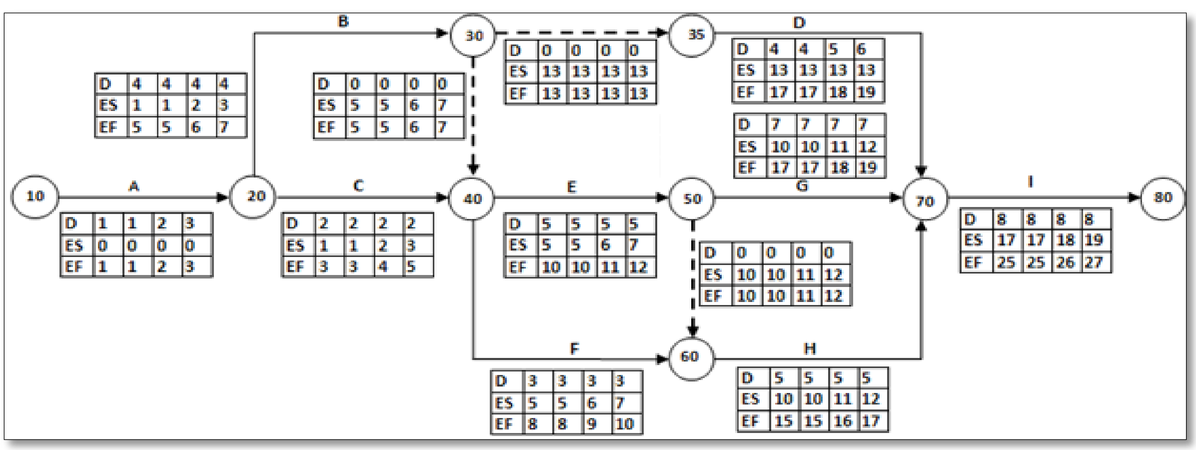

Figure 12. Adjusted AOA network for the second scenario with min fuzzy time.

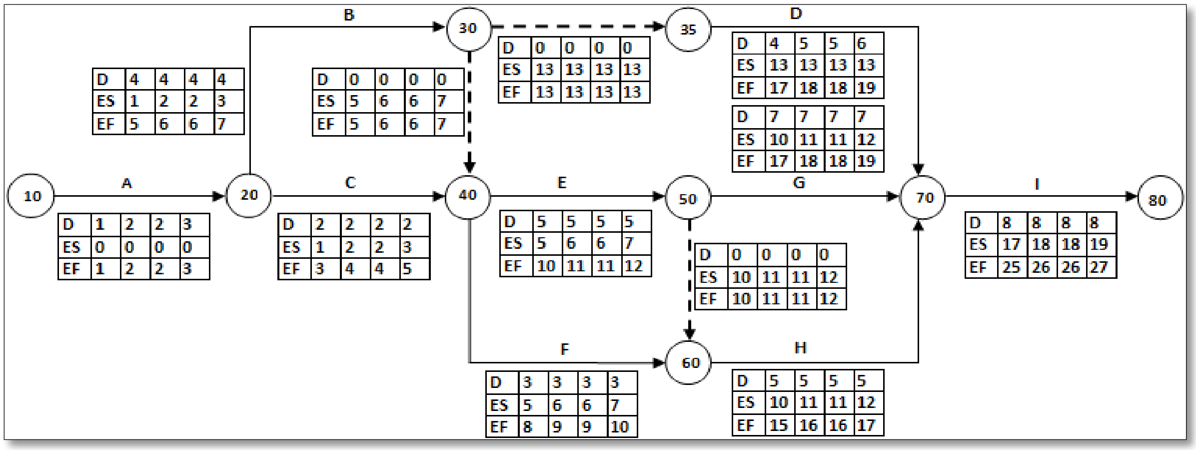

Figure 13. Adjusted AOA network for the second scenario with middle fuzzy time.

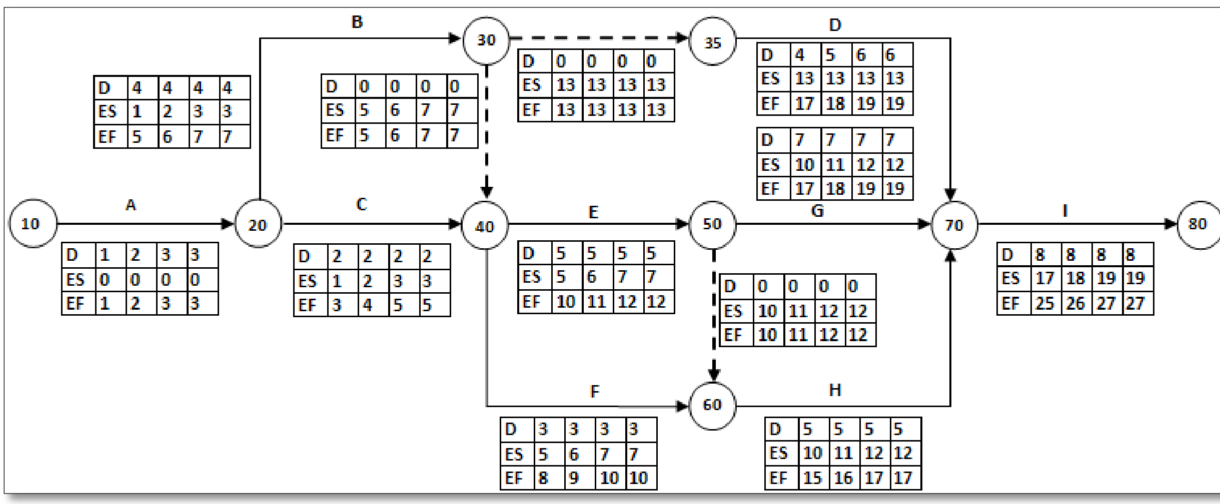

Figure 14. Adjusted AOA network for the second scenario with max fuzzy time.

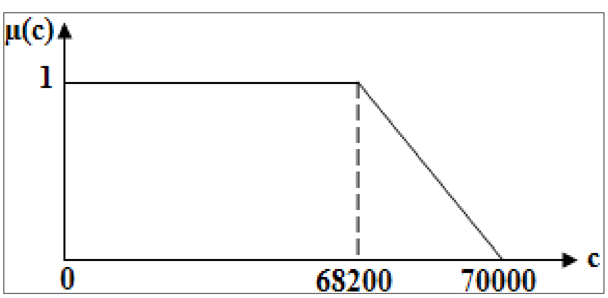

Figure 15. Graphical representation of cost model (planned case).

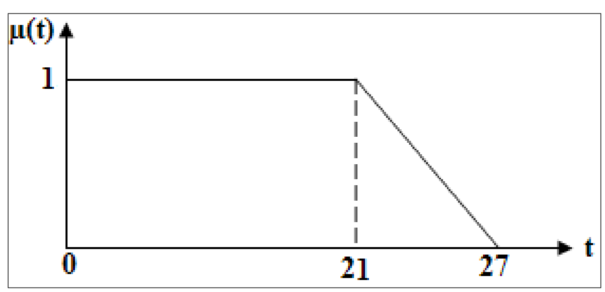

Figure 16. Graphical representation for time model (planned case).

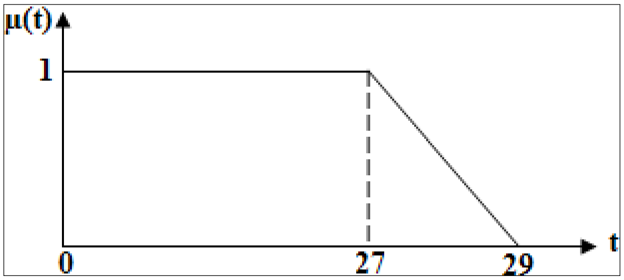

Figure 17. Graphical representation of time model (first scenario).

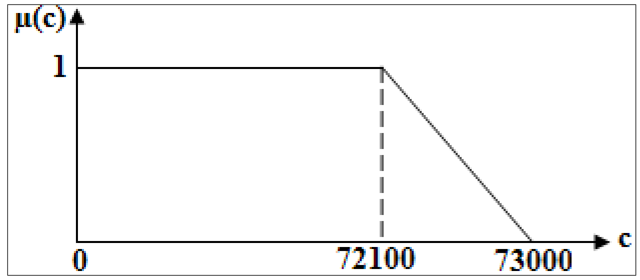

Figure 18. Graphical representation for cost model (first scenario).

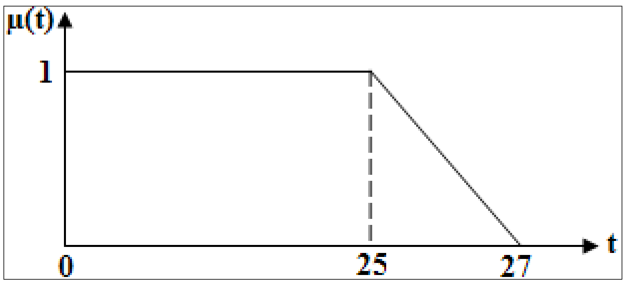

Figure 19 . Graphical representation of time model (second scenario).

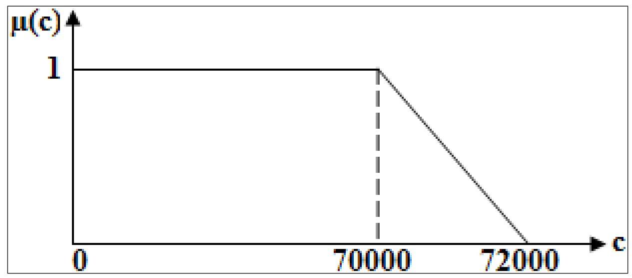

Figure 20. Graphical representation for cost model (second scenario).

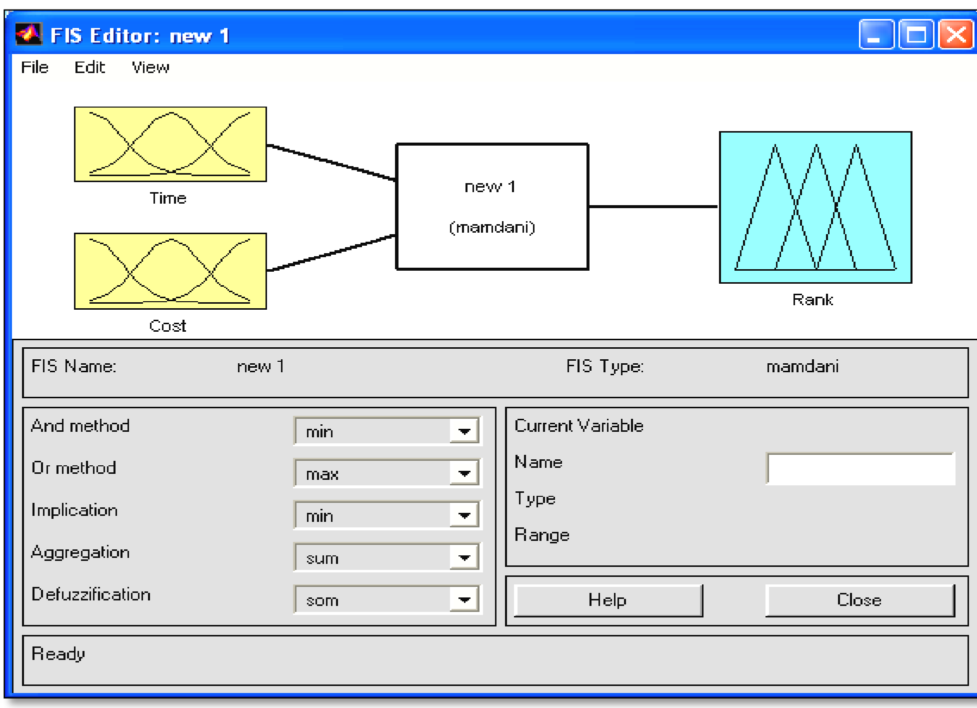

Figure 21. Fuzzy inference system for the mathematical model.

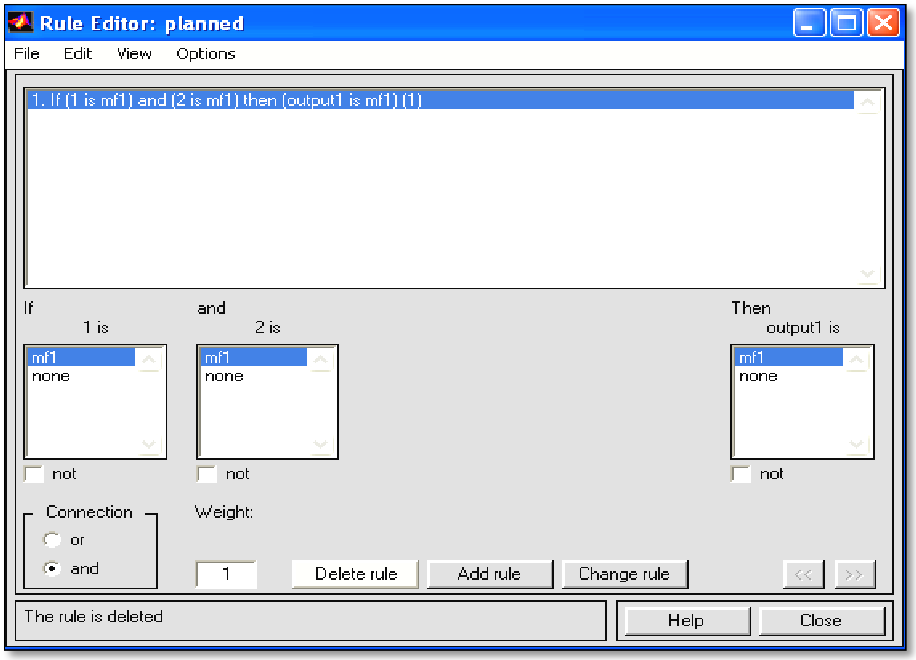

Figure 22. Rule editor for the mathematical model.

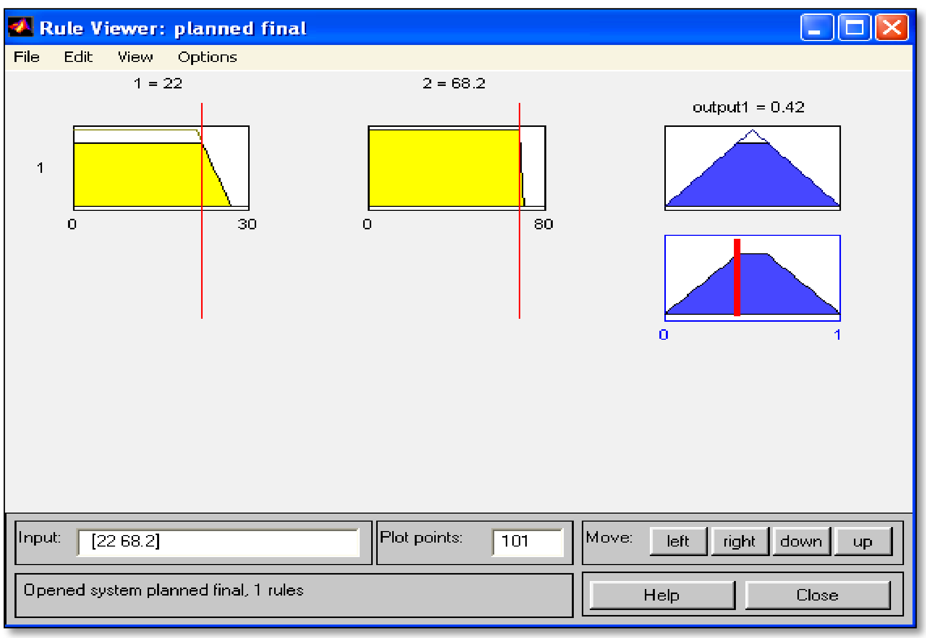

Figure 23. Rule viewer for the mathematical model.

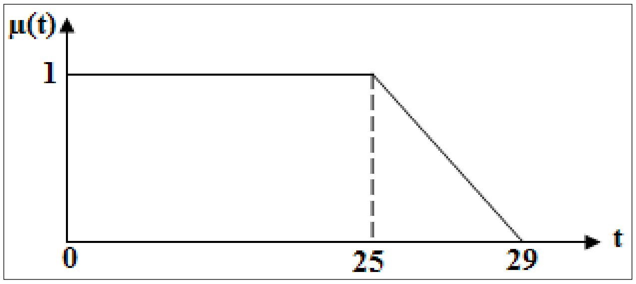

Figure 24. Graphical representation of time model (General case).

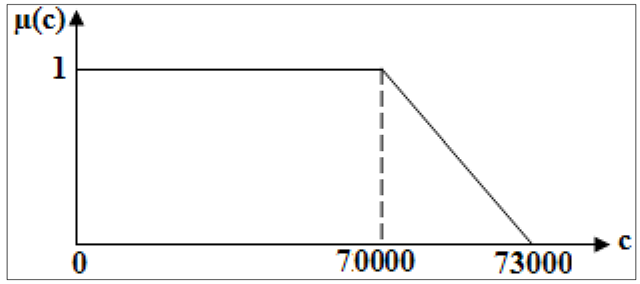

Figure 25. Graphical representation for cost model (General case).