1-DOF Model for Fluid-Structure-Interaction Vibration Analysis

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ABSTRACT

In this paper an attempt to provide a single degree of freedom lumped model for fluid structure interaction (FSI) dynamical analysis will be presented. The model can be used to clarify some important concept in the FSI dynamics such as the added mass, added stiffness, added damping, wave coupling ,influence mass coefficient and critical fluid depth . The numerical results of the model show that the natural frequency decrease with the increasing of many parameters related to the structure and the fluid .It is found that the interaction phenomena can become weak or strong depending on the depth of the containing fluid .The damped and un damped free response are plotted in time domain and phase plane for different model parameters It is found that the vibration free response is still sinusoidal for weak FSI coupling ,however for strong coupling it behaves as modulated periodic response .To justify some of the theoretical aspects such as; the effects of the fluid density and the interact shape on the natural frequency an experiment was conducted .The results of the experiment shows a good agreement with the theory where the error is not exceeded 7%.

Key words: FSI, mass influence coefficient , interact shape ,added mass ,added damping

نموذج ذو درجة حرية واحدة لتحليل الاهتزاز للهيكل المتفاعل مع المائع

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الخلاصة

في هذا البحث محاولة لتوفير موديل ذو درجه حريه واحدة للتحليل الديناميكي للهياكل المتفاعله مع الموائع .يمكن استخدام هذا النموذج لتوضيح مفاهيم اساسيه في الديناميك التفاعلي مثل الكتله المضافة والجساءة المضافة والتخميد المضاف والتداخل الموجي ومعامل الكتلة المؤثر وعمق المائع الحرج .بينت النتائج العددية ان التردد الطبيعي يتناقص بتاثير زيادة بعض المعاملات التي تخص الهيكل والمائع .كما بينت النتائج بان ظاهرة التفاعل يمكن ان تكون ضعيفه او قوية وذلك اعتمادا على عمق المائع الحاوي .تم رسم الاستجابة الحره المخمدة والغير مخمدة باستخدام احداثيات الزمن وطريقة مستوي الطور لعدة متغيرات فبينت المخططات ان استجابة الاهتزاز تبقى موجيه في حالة التفاعل الضعيف بينما تكون من النوع الدوري المتضمن في حالة التفاعل القوي .للتحقق من بعض النتائج النظريه مثل تاثير كثافة السائل والشكل المتفاعل فقد تم اجراء تجارب عملية .بينت النتائج العمليه تطابق جيد مع النظري حيث لم تتعدى نسبة الخطأ 7%.

الكلمات الرئيسية: الهياكل المتفاعلة,معامل تأثير الكتلة, الشكل المتفاعل, الكتله المضافه, التخميد المضاف.

1. INTRODUCTION

The dynamics of fluid structure interaction (FSI) has wide applications in many branches of engineering such as aerospace ,aerodynamics, ship motion, medical applications and other flow induced vibration problems.

In the FSI systems the couplings between, fluid and structure include many kinds. It is well known now as frictional coupling, Poisson coupling, junction coupling, Bourdon coupling, wave-.flow coupling and wave–wave coupling, etc ,**Amabili,2000.**

In general the dynamical behavior of structures interacts with fluid is very complicated and they are normally evaluated by numerical technique like finite element or finite difference methods .Many models are treated by using simulation methods based on experiments under wind tunnel .An exact analytical models are seldom available in the literature, however there existed many approximate models for analyzing some special problems under some assumptions . For example an idealized case of elastic structure in free lateral vibration and interacting with an enclosure fluid cavity where the fluid medium has been infinite was investigated by **Aitkinson, et al.,2007** ,in their analysis, the wall reflecting of the fluid pressure waves was neglected. **Daniel et al., 2007** constructed a model for analyzing vibration of cantilever beam interact with finite volume air cavity for using in vibration of health monitoring .The analysis of the coupled free vibration of distributed structure such as beams, plates and shells in interaction with a fluid-filled was performed by **Gorman et al., 2001**, which produced natural frequencies of the coupled system .The results were agreed well with the finite element analysis and experiment . **Sarkar and Paidoussis, 2004,** treated the dynamical behavior of pipes conveying fluid as another type of FSI syatems .It was found that, the fluid adds additional forces on the structure such as axial and coriolis forces . An extended literature survey about the FSI can be found in a book published by **Paidoussis, 1998.**

In the present work an attempt to provide a lumped one degree of freedom model for treating the FSI dynamic will be presented. The present model treats the various fluid effects such as the added mass, added stiffness, added damping and the fluid pressure wave for compressible and incompressible fluids. Such a model may has the benefit of its simplicity for the approximate analysis of FSI as a first insight in this vital field.

2. THEORITICAL CONSIDERATIONS

Consider a coupled fluid-structure 1-DOF mass-damping–stiffness (*m-c-k*) model shown in **Fig.1,** is interacting with a enclosed fluid space of depth *H*. The coordinate of the mass is y and that of the fluid container is *Y* as shown in the figure .The containing fluid is considered inviscid (non viscous) which may be compressible or incompressible. For small vibration the effect of vortex is neglected.

The equation of motion of 1-DOF mass spring damper, taking into account the effect of the induced pressure force is;

$$(m_s + m_d)\frac{d^2 y}{dt^2} + (C_s + C_d)\frac{dy}{\partial t} + (k_s + k_d)y = AP\big|_{y=H} \tag{1}$$

Where *s* and *d* refer to the structure and the added parameters due the fluid effect, respectively. Eq.(1) can be written in the following form ;

$$m\frac{d^2 y}{dt^2} + C\frac{dy}{\partial t} + ky = AP\big|_{y=H} \tag{2}$$

Where,
$m = m_s + m_d$
$C = C_s + C_d$
$k = k_s + k_d$, (3)

are the total mass ,damping and stiffness, respectively .*P* is the pressure and *A* is the interacted area of the mass block .

Now dividing Eq.(2) by *m* giving;

$$\frac{d^2y}{dt^2} + \frac{C}{m}\frac{dy}{\partial t} + \frac{k}{m}y = \frac{A}{m}P\Big|_{y=H} \quad (4)$$

As shown in **Fig. 1**, the fluid boundary lies only under the mass block . The fluid domain is bounded above by the block mass and by perfectly absorbent walls on all other sides. The fluid is assumed to be inviscid and may be compressible or incompressible and can well described by Laplace's equation over the domain. The boundary condition of the fluid-structure interface is described as a Neumann boundary condition by coupling the velocities across the interface **Blevins, 2010**. The sides and bottom containing the fluid represent a Dirichlet boundary at which the fluid potential is zero. The fluid system can be characterized by the following Laplace's equations **Paidoussis,1998**.

$$\nabla^2\phi = \left(\frac{1}{S}\right)^2 \frac{\partial^2\phi}{\partial t^2} \quad (5a)$$

Where ϕ is potential flow function and *S* is the speed of sound .For one dimensional motion (in *Y* direction only) the potential function takes the following form;

$$\frac{\partial^2\phi}{\partial Y^2} = \left(\frac{1}{S}\right)^2 \frac{\partial^2\phi}{\partial t^2} \quad (5b)$$

Eq.(5b) can be solved by trying the following harmonic solution;

$$\phi(Y,t) = \eta(Y)e^{i\omega t} \quad (6)$$

Separating the variable leads to the following general solution;

$$\phi(Y,t) = [C_1 \sin\frac{\omega}{S}Y + C_2 \cos\frac{\omega}{S}Y]e^{i\omega t} \quad (7)$$

Where C_1 and C_2 are arbitrary constant depending on the boundary conditions and ω is the natural frequency.
The velocity at the bottom is zero; hence the first boundary condition is;

$$V_y = \frac{\partial\phi}{\partial Y}\Big|_{Y=0} = 0 \quad (8)$$

Substituting Eq.(8) into Eq.(7) ,Gives;C_1=0
Hence Eq.(7) is reduced to;

$$\phi = C_2 \cos(\frac{\omega}{S} Y) e^{i\omega t} \tag{9}$$

Now the fluid pressure, P excreted at the bottom surface of the mass block can be evaluated as follows, **Atikinson and Marique, 2007**,

$$P\big|_{Y=H} = -\rho S \frac{\partial \phi}{\partial t} \big|_{Y=H} \tag{10}$$

Combining of Eqs. (4,9) and (10) giving;

$$\frac{d^2 y}{dt^2} + \frac{C}{m}\frac{dy}{\partial t} + \frac{k}{m} y = -C_2 \frac{i\omega A \rho S}{m} \cos\frac{\omega H}{S} e^{i\omega t} \tag{11}$$

The solution of Eq.(11) can be written as;

$$y = \frac{-iC_2 \omega A \rho S \cos(\omega H / S)}{k - m\omega^2 + i(Ck\omega / m)} e^{i\omega t} \tag{12}$$

Eq.(12) can be used to plot the free vibration response due to a given initial conditions in the time domain.
At Y= H, the fluid velocity V_y and the lateral velocity of the mass are equal, hence the second boundary condition is ;

$$\frac{\partial \phi}{\partial Y}\bigg|_{Y=H} = \frac{\partial y}{\partial t} \tag{13}$$

Substituting Eq.(13) into Eq.(12), gives the frequency equation of free vibration of the FSI as follows;

$$\frac{\omega A \rho S^2}{k - m\omega^2 + i(Ck\omega / m)} + \tan\frac{\omega H}{S} = 0 \tag{14}$$

As stated above Eq.(3) the parameters m, C and k combine the structure parameters and the added ones due to the fluid interaction .The added parameters may be evaluated as the following

A-The added mass;

The added mass represents the fluid mass displayed by the block as it vibrates (Archimedes principle).It can be calculated from the following equation **Paidoussis, 1998** ;

$$m_d = \rho A C_I \tag{15}$$

Where C_I denoted the *added mass influence coefficient* which depends on the geometry of the body interacts with the fluid and can be found from tables presented by **Blevins, 2010** .
For example CI=1 and 1. 86 for circular and square cross sections , respectively.

B-The Added Damping

The damping mechanism is complex phenomenon which depends on many factor associated to the fluid and structure parameters such as viscosity, types of fluid ,the lift and drag forces , boundary conditions ,structure mass and stiffness ,fluid hammering, etc. However for still and non-viscous fluid with rigid bounded structure as it is the presented case, it is assumed that the predominate damping effect is due to the fluid hammering .This phenomena can be resulted from the influence of the shock resulting from the pressure wave as it strike the block due to vibration motion .The additional pressure induced by the surface area of the block can be evaluated according to Joukowsky equation as;

$$P_h = \rho S \frac{dy}{dt} \qquad (16)$$

Due to this pressure the force excreted on the block is ;

$$F = P_h A \qquad (17)$$

This force will create an additional damping force given by;

$$F = C_d \frac{dy}{dt} \qquad (18)$$

Combining, Eqs. (16) ,(17)and (18) ,the added damping , C_d can be evaluated as;

$$C_d = \rho SA \qquad (19)$$

C-The Added Stiffness

The added stiffness depend on the compressibility of the fluid and the geometry of its container In other words on the bulk modulus of the enclosed fluid .However for the present model since the fluid is not perfectly enclosed and can be easily escaped ,the effect of the added stiffness is so small and can be neglected .

Finally , considering the above effects the free time response Eq.(12) and the frequency equation Eq.(14) will take the following forms ;

$$y = C_2 \frac{-i\omega A\rho S\cos(\omega H/S)}{k + [k(C+\rho SA)/(m_s + C_I\rho A)]\omega - (m_s + C_I\rho A)\omega^2} e^{i\omega t} \qquad (20)$$

$$\frac{S^2\omega A\rho}{k + [k(C+\rho SA)/(m_s + C_I\rho A)]\omega - (m_s + C_I\rho A)\omega^2} + \tan\frac{\omega H}{S} = 0 \qquad (21)$$

Eq. (21) has two terms; the first is characterized by the structure stiffness ,damping and mass and the second term is characterized by the fluid container .The FSI natural frequency is a contributing of the two effects. An inspection of the second term it can be seen that this term can take two extreme values , which are; zero and infinity .When this term becomes zero this means that the FSI coupling effect due to the fluid is weak ,however, when it becomes infinity the FSI effect becomes strong . Hence one gets;

For weak FSI ; $\tan\frac{\omega H}{S}$ =0, which gives , ω=Sπ/H

For strong FSI; $\tan\frac{\omega H}{S}$ =∞, which gives ,ω=Sπ/2H (22)

Keeping in mind that only one root of the tangent function is taken since the model is 1-DOF.
For a certain fluid, the speed of sound *S* is constant, so that Eq. (22) indicates that the depth *H* is the only parameter which define the weakness or strengthens effects of the fluid on the natural frequency .In other words there are certain critical values of *H* at which the FSI coupling becomes maximum or minimum. This conditions are also observed by other FSI models for example the model given by **Daniel *et al*., 2007** .

3. EXPERIMENTAL INVESTIGATION

To justify some of the theoretical concepts an experiment was carried out .The main aim of the experiment is to investigate the effects of fluid and the interact shape on the natural frequency To maintain a same mass weight with different interact shapes. three plastic shapes was prepared and attached by using the arrangement shown in **Fig.2**. The dimensions and the corresponding mass influence coefficient of these samples are given in **Table.1**.
The test rig consists of the apparatus and the measuring instruments as shown in **Fig.4**.The apparatus consists of mass-spring system (k=2500N/m ,m=0.28 kg), as well as the fluid container (D=0.1 m, *H*=0.23 m) .These parameters are chosen to insure that the system is at the strong coupling regions. The measuring instruments are the accelerometer, charge amplifier and oscilloscope .In this test, water and kerosene (ρ =780 kg/m^3) are used as working fluids .For each fluid the three samples were used .The vibration signal was picked up by the accelerometer and amplified by the charge amplifier and fed to the scope .The natural frequency was recorded from the scope as shown in **Fig.4** .

4. RESULTS AND DISCUSSIONS

Exanimating of Eqs .(19) and (20) indicate that, the dynamical behavior of the present FSI model is affected by two main groups of parameters .The first are associated to the structure which are m_s , *k A*, C_I and *C*. The seconds are associated to the containing fluid which are *ρ, S,* and *H* .The effects of m_s , *k* and *C* can be clearly understood from considering the elementary analysis of a damped single degree of freedom oscillator The goal of the present investigation is to focus on the additional FSI parameters .

Figs.5,6, and 7, display the effects of the interacted area of the block mass ,fluid density and the influence mass coefficient on the natural frequency . In plotting these figures the following model parameters are chosen; m_s=0.25 kg (round cross section) ,C=20 Ns/m , k=500 N/m and *H*=0.25 m. With these data the vacuum natural frequency (i.e. without fluid) can simply be calculated as ω= 44.72 r/s .

In **Fig.5,** the area of the block mass is varied from 0.05x10^{-5} to 2.05x10^{-5} m^2 .As it is clear from this figure that ; increasing the area tends to reduce the natural frequencies .This behavior is logical since increasing the area produces resistance against the mass motion .

Fig.6 shows the effect of the influence coefficient which depends on the geometry of the block mass area .The minimum value of this coefficient is associated to the circular shape .The coefficient value increases as the shape become more complicated **Blevins.2010**.Examining of **Fig.6** indicates that increasing of the influence coefficient reduce the natural frequency, also This can be attributed to the increasing of the total mass which decreases the natural frequency .
In **Fig.7**, the effect of the fluid density is investigated for rang values from 1 to 1000 kg/m^3 Again the natural frequencies decrease with the increasing of the fluid density.
The un damped (C=0) natural response of the same model with the additional parameter data ρ=1000,H=0.25 , $A=0.4x10^{-4}$ with rectangular shape , is plotted in **Figs. 8, a and b** in time domain and phase plane ,respectively .As it can be seen from these figures that the response is clean sinusoidal . To identify the response of the model near and at the strong zone of FSI coupling , the depth is assigned a new value which are (H=0.7 and 0.8 m) .The results are plotted in **Figs.9 and 10** for ($2x10^{-7}$m,0) initial displacement and velocity .In general these figures tell that the response is still sinusoidal but with variable amplitude this means that the response is a result of the modulation of two wave frequencies .These two frequencies are the acoustic wave frequency and the system natural frequency. The effect of this modulation becomes more clearly visible when the depth reach critical value (0.8 m) as it is shown in **Fig. 10,a and b** in both time and phase plane response .
The effect of damping is investigated in **Figs.11** .The damping ratio of the model is assigned ζ=0.1 value ($\zeta=C/2m\omega$) .The under damping response at the strong zone is plotted in time domain and phase plane. The spiral trajectory of the phase plane indicates that the system is stable and the oscillation will be diminished after about ten periods.

The experimental and the corresponding theoretical results of the tested samples are collected in **Table 2**.In general the results show the decreasing in the natural frequencies due to the effects of the fluid and the interacting shape as it compared with the vacuum (without fluid) natural frequency .This confirm the theoretical conclusion that ;as the interact area becomes more sharply edge (from circular to squarer shape) the natural frequency decrease .Such a behavior can be attributed to the effect of drag force which increases as the profile sharpness increases .Moreover ,that; the natural frequency takes lower values as the fluid density increases ($\rho_{water}=1000\ kg/m^3$, $\rho_{kerosene}=780kg/m^3$) . From comparing the experimental and theoretical the results show good agreements where the maximum error is not exceed the 7% .Also that the experimental results in general is lower than the corresponding theoretical .This may be due to the effect of the several sources of damping (friction ,drag ,internal , etc.) which are very difficult to be taking into account .

5. CONCLUTIONS

A lumped single degree of freedom analysis for FSI dynamics is treated in this work .It is found that; despite of its simplicity , it can serve a good model for investigating several main aspects of the FSI .The natural frequency was investigated .It is found that increasing the interacting area and the fluid density will reduce the natural frequency .Also, that the natural frequency can be increased as the shape of the interact area become irregular or complex .The free damped and un damped response are plotted in time domain and phase plain , It is concluded that the response in general is sinusoidal but it highly affected by the depth of the containing fluid .At certain critical fluid depths the FSI coupling become weak so that the response takes a pure sinusoidal form .However at other depth values the FSI coupling become strong leading to a modulated sinusoidal form .

Some aspects of the FSI was investigated experimentally such as the effects of fluid density and the shape of the interacting area .Comparing the results show good agreements where the maximum error is not more than 7% .
The model can be regarded as a useful tool for engineers for the crude estimating of the fundamental natural frequency and response of FSI systems .

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7. NOMENCLATURE

Ω= natural frequency, rad/s
ζ = damping ratio
A= interact area , m^2
C:=total damping , Ns/m
C_s=structure damping , Ns/m
C_d= added damping ,Ns/m
C_I =mass influence coefficient
K=total stiffness ,N/m
k_s= structure stiffness,N/m
k_d= added stiffness , N/m
m = total mass ,kg

m_d= added mass ,kg
m_s= structure mass ,kg
H= fluid depth ,m
S =speed of sound ,m/s

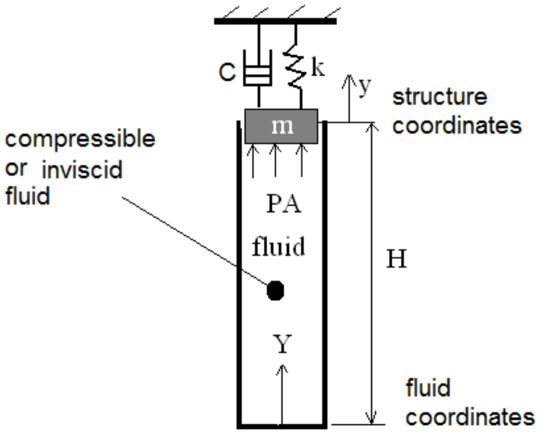


Figure 1. Schematic diagram of the idealized coupled FSI system.

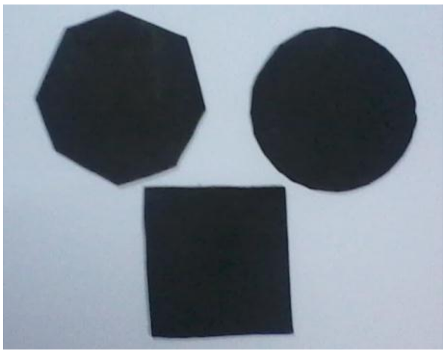


Figure 2. The tested samples.

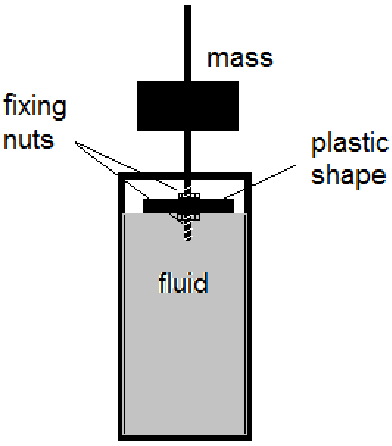


Figure 3. The mass-interacting shape arrangement.

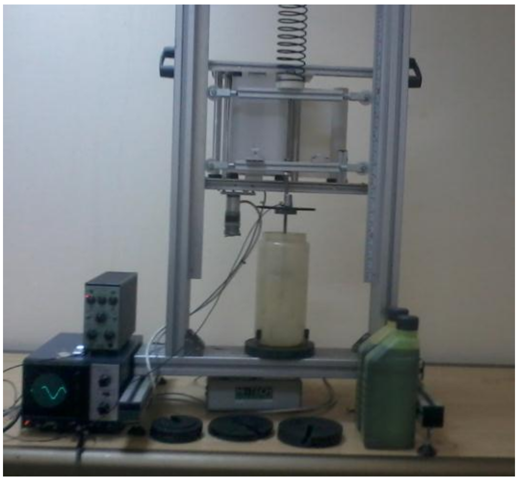


Figure 4. The test rig.

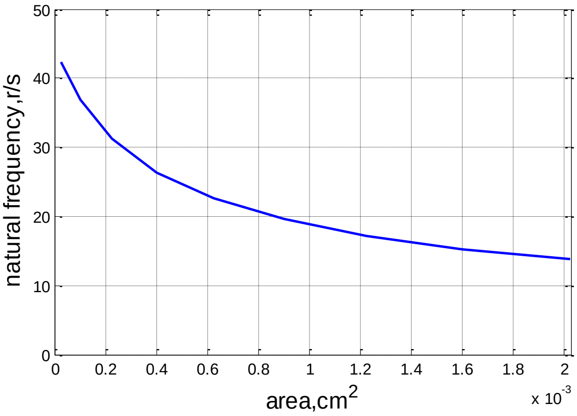


Figure 5. Effect of interacting area on the natural frequency.

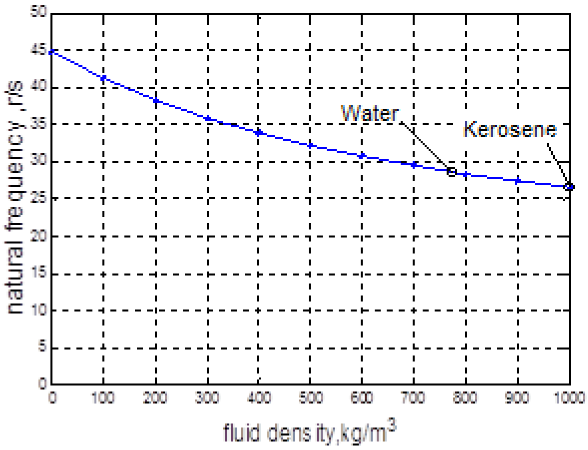


Figure 6. Effect of fluid density on the natural frequency.

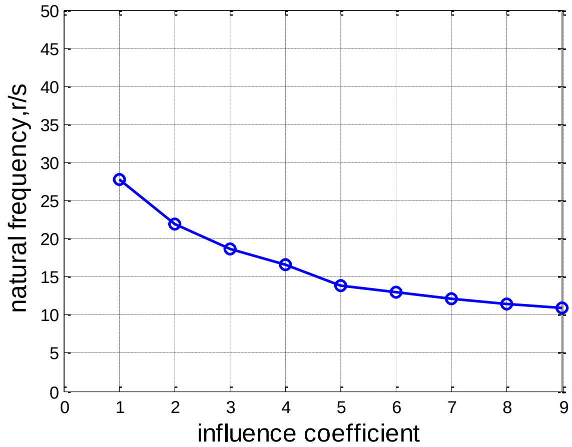
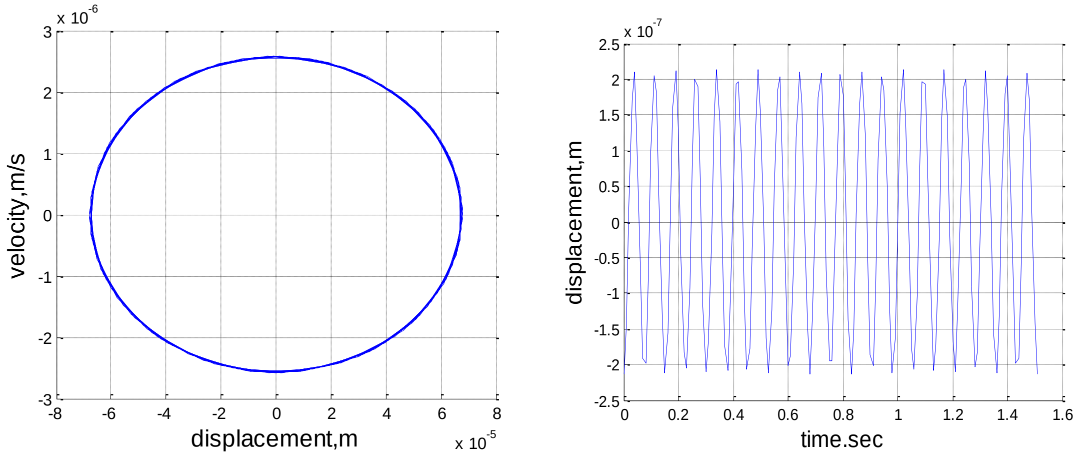
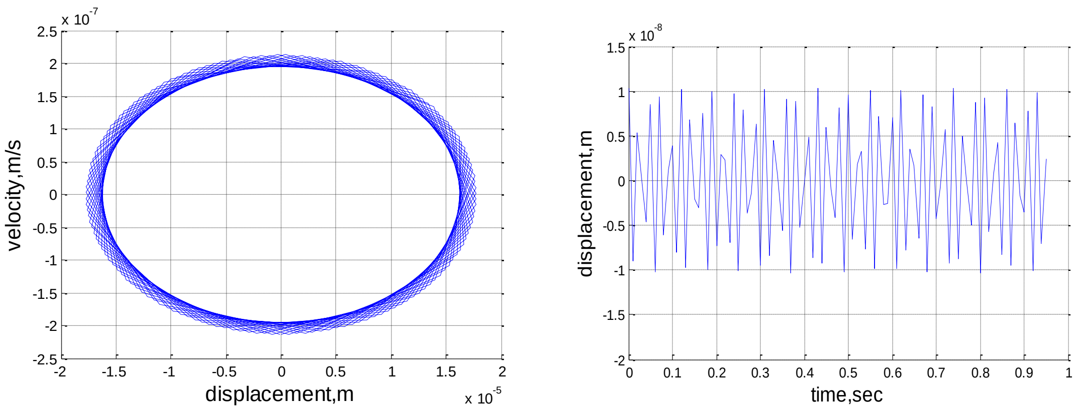


Figure7. Effect of mass influence coefficient on the natural frequency.



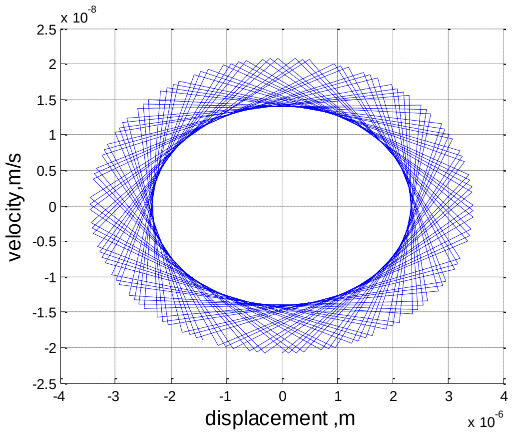
(a) **(b)**

Figure 8. Natural response at weak interaction zone,
(a) in phase plane (b) in time domain .

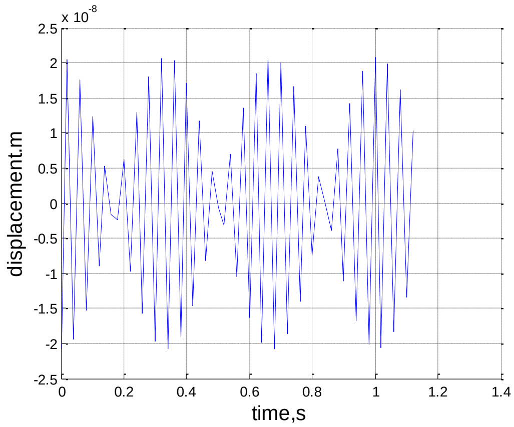


(a) **(b)**

Figure 9. Natural response close to strong interaction zone,
(a) in phase plane (b) in time domain .

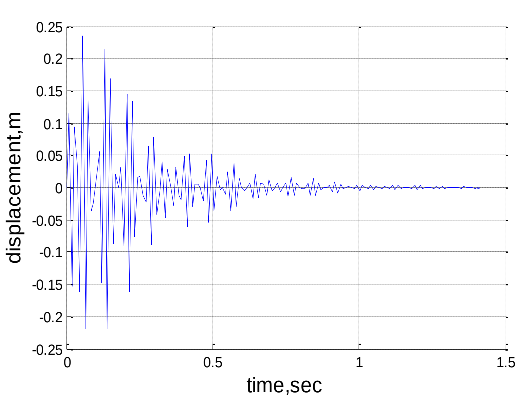


(a)

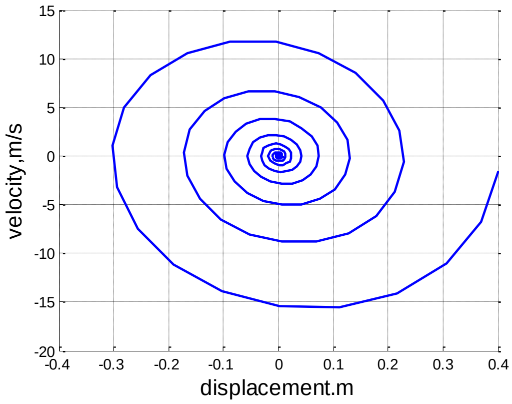


(b)

Figure 10. Natural response at strong interaction zone,
(a) in phase plane (b) in time domain.



(a)



(b)

Figure 11. Damping response at ζ=0.1,
(a) in phase plane (b) in time domain.

Table 1. The tested samples dimensions and influence coefficients.

Sample no.	Shape	Radius or side length(m)	Area	C_I
1	Circle	0.082	πa2	1
2	Polygon	0.032	2(1+√2)a2	1.23
3	Squarer	0.073	a2	1.86

Table 2. Experimental and theoretical natural frequencies (Hz**).**

Interacting Shape	Vacuum (without fluid)		Kerosene		Water	
	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.
Circle	11.2540	10.5	7.0337	6.5	6.4975	6
Polygon			6.5523	6	6.0155	5.5
Square			5.6975	5.2	5.1801	4.8