DRAG FORCE OF TWO SPHERES IN POWER LAW FLUIDS

Prof. Dr. Abbas H. Sulaymon Abeer Ibrahim Alwared
Environmental Engineering Department - Collage of Engineering-University of Baghdad

ABSTRACT

The present research is concerned with studying of the drag force on two spheres moving side by side and in line in non-Newtonian liquid. Polyacryamide (PAA) solution with different concentrations (0.01, 0.03, 0.05 and 0.07)% by weight and water is used for comparison for obtaining the effect of fluid properties on the drag force.

Different types of spheres stainless steel, glass and plastic with different sizes and densities were used.

Within the considered range of power law index (0.6 - 1), and generated Reynolds number (1.1 -75) in power law fluid and Reynolds number (100 - 1000) for water it was found the drag coefficient increases with increasing the power law index for constant generated Reynolds number and the drag force increases with fluid density increases but it decreases with the sphere density increases.

الخلاصة

يهتم البحث بدراسة قوة الاحتكاك بين كرتين تتحركان جنباً إلى جنب أوعلى طول الخطّ الواصل بين مركزيهما داخل اسطوانة مملوءة بمادة البولي اكريمايد (PAA) وبتراكيز مختلفة (0.01, 0.03, 0.05 و0.07%) وزناً كسائل غير نيوتيني ولغرض المقارنة مع السوائل النيوتينية تم استخدم الماء، وباستخدام انواع مختلفة من الكرات (حديدية مقاومة للصدأ، زجاجية وبلاستيكية) ذات اقطار وكثافات مختلفة. تبين من هذه التجارب ان ضمن مدى الـ power law index (مؤشر سلوك الجريان) المحدد بين (0.6 − 1) كان generated Reynolds number بين(1.1 −75) في سائل البولي اكرلامايد (PAA) في حين كانت قيمة Reynolds number في الماء تتراوح ما بين (100 − 1000).

تبين من هذه التجارب ان معامل الاحتكاك يزداد بزيادة مؤشر سلوك الجريان عند ثبات قيمة الـ generated Reynolds number كما تم التوصل الى ان قوة الاحتكاك تزداد بزيادة كثافة السائل وتقل مع زيادة كثافة الكرة.

KEY WORDS: Two spheres, power law fluids, drag coefficient, interaction correlation, drag force,

INTRODUCTION

The fluid dynamic drag on a sphere in an infinite fluid are important in numerous fields including chemical, mechanical and environmental engineering. As such, they have been subject to many experimental and theoretical investigations; despite such interest existing knowledge is less than perfect, and improvements in predictions could be useful in mathematical modeling of particle behaviour. In environmental engineering such predictions are used for modeling all particle process in water (flocculation, sedimentation, boiling, flotation and filtration including back wash of filter media, liquid–liquid extraction) and particle capture and deposition in air (Brown and Lawler, 2003).

Considerable progress has been made towards the understanding of the accelerating motion of a sphere in Newtonian fluids, Clift et al., (1978) have presented a concise summary of the theoretical developments on this subject in their classic treatise. In contrast to this, the acceleration motion of a sphere in non-Newtonian fluids received less attention, some investigators have described the unsteady motion of a sphere in viscoelastic fluids that have constant viscosity (i.e. no shear thinning behavior) and exhibit quadratic normal stress differences.

However, most materials of practical interest(e.g. polymer solution , polymer melts ,… etc.) display what is called shear thinning characteristics ,which its apparent viscosity decreases with increasing shear rate .This research concerned with the acceleration motion of two spheres in shear thinning type power law fluid.

The most common approach by previous investigators is by using the standard Newtonian relationships ($C_D = f(\mathrm{Re})$) but using a modified (non-Newtonian or generalized) Reynolds number Re_{gn} (Kelessidis, 2003).

DRAG COEFFICIENT CORRELATION DUE TO GENERATED REYNOLDS NUMBER

In Newtonian fluids there are many relationships for the standard drag $C_D = f(\mathrm{Re})$ are available, the more widely used empirical form of drag correlation with the Reynolds number employed from 10 to 10^3 is Schiller and Naumann's correlation (1933).

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + \frac{1}{6}\mathrm{Re}^{2/3}\right) \qquad (1)$$

Lali et al. (1989) used five different carboxymethycellulose (CMC) solutions covering the range of power law fluid (n) between (0.555 – 0.85), with different diameters of glass beads and steel balls, thus covering a range of Re_{gn} $(0.1 - 200)$,the data correlated very well with the Newtonian curve defined by:

$$C_D = \frac{24}{R_e}\left[1 + 0.15 R_e^{0.687}\right] \qquad 0.1 < \mathrm{Re} < 1000 \qquad (2)$$

Darby (1996) predicted the following expression which give an accurate representation for the drag in non-Newtonian fluids on condition that A and B are made depend on the power law index(n).

$$C_D = \left(\frac{A}{\sqrt{\mathrm{Re}}} + B\right)^2 \qquad (3)$$

Where :

$$A = 4.8\left(\frac{1.33 + 0.37n}{1 + 0.7n^{3.7}}\right)^{1/2} \qquad (4)$$

and

$$B = \left[\left(\frac{1.82}{n} \right)^8 + 34 \right]^{-1/8} \tag{5}$$

Equation (3) is based on the numerical results of Tripathi et al., (1994) and Tripathi and Chhabra (1995), it shows fair agreement with experimental results in the range $0.4 \le n \le 1$ (as cited in Chhabra et al.,1998).
Kelessidis (2004) predicted the following drag coefficient correlation for power law fluid.

$$C_D = \frac{24}{\mathrm{Re}_{gn}} \left[1 + 0.1466\, \mathrm{Re}_{gn}^{0,378} \right] + \frac{0.44}{1 + 0.2635 / \mathrm{Re}_{gn}} \qquad 0.1 < \mathrm{Re}_{gn} < 1000 \tag{6}$$

A comparison of non-Newtonian data with Newtonian data on a $C_D - \mathrm{Re}$ plot reveals that the non - Newtonian data fall slightly below the underlying curve of the Newtonian data.
Dhole et al. (2006) correlated the following equation over the ranges of $5 < \mathrm{Re}_{gn} < 500$ and power law index values $0.5 \le n \le 2$.

$$C_D = \frac{24}{\mathrm{Re}_{gn}} \left[1 + a\, \mathrm{Re}_{gn}^{\, bn/(cn+d)} \right] \tag{7}$$

Where: a = 0.148, b = 2.346, c= 2.423, and d = 0.918.
They also concluded that the drag is seen to be enhanced above the Newtonian value in shear thickening fluids and reduces in shear thinning fluids at constant Reynolds number.
For the settling of sphere in shear thinning fluids (n<1), as noted by Graham and Jones(1994); Koziol and Glowacki (1988) ; Tripathi et al., (1995); Renaud et al., (2004);and Zhu et al., (2003) there exists a crossover Reynolds number of $\mathrm{Re}_{gn} \approx 4$, up to which the total drag decreases with the power law index increases . From the above discussions, it is appeared that the majority of the investigators concluded that the use of Newtonian correlations for non-Newtonian fluids is justified by using the apparent viscosity. However, a single equation yielding accurate results is not available for non-Newtonian fluids (Kelessidis, 2003).

DYNAMIC DRAG CORELATION IN NEWTONIAN FLUID

It is worthwhile to mention that, a large number of investigators worked theoretically and experimentally on the dynamic interaction coefficient of drag force of two spheres in Newtonian fluid. There are many correlations for particle interaction effects on drag coefficient $\lambda = C_D/C_{Do}$ for two particles fall side by side with the same velocity and for two particles motion in line in Newtonian fluid. For two bubbles rising side by side Kok (1993) obtained for the drag coefficient the following correlation:

$$C_D = \frac{48}{\mathrm{Re}} \left(1 + g(S) \right) + O\left(\mathrm{Re}^{-3/2} \right) \tag{8}$$

With:

$$g(S) = S^{-3} + \frac{3}{4}S^{-6} + \frac{11}{3}S^{-8} + ... \qquad (9)$$

Where $S = l/a$

The presence of the second bubble tend to increase the drag force due to the interaction results in a higher strain rate in the fluid located between the two bubbles. Legendre and Magnaudate (2003) found an improved expression for the drag force in Reynolds number range, $0.02 \le \text{Re} \le 500$.

$$C_D = \frac{48}{\text{Re}}(1 + g(S)) + (1 + S^{-3})\frac{2.211}{\text{Re}^{1/2}} \qquad (10)$$

They use a correlation factor for the drag force as:

$$\lambda = (1 + S^{-3}) = 1 + \frac{1}{8}\left(\frac{l}{d}\right)^{-3} \qquad (11)$$

For two horizontally side by side bubbles at high Reynolds number ,Kendoush (2005) derived the following equation:

$$\lambda = \left[1 + \frac{1}{2}\left(\frac{a}{l}\right)^3 + \frac{1}{4}\left(\frac{a}{l}\right)^6\right]^2 \qquad (12)$$

For two spheres falling one above the other at very low Reynolds numbers Smoluchowski (1911) showed that:

$$\lambda = 1 - \frac{3}{4}\left(\frac{d}{l}\right) + \left(\frac{3d}{4l}\right)^2 \qquad (13)$$

Equation (13) is an approximate expression and can only be applied for two equal-sized spheres falling with center-to-center distance greater than three diameters.

Oseen (1927) extended Smoluchawski's solution, equation (13) for the motion of two spheres so that it might be applied to higher Reynolds numbers. Oseen's equation for one sphere falling above another can be approximated when the spheres are not too far apart by the following equation: (Happel and Pfeffer, 1960)

$$\lambda = 1 - \frac{3}{4}\left(\frac{d}{l}\right) + \frac{3}{8}\text{Re} \qquad (14)$$

The correlation proposed by Rowe and Henwood (1961) for two spheres in line was in a hyperbolic form. This is invalid for small interparticle distances since the drag ratio approaches either positive or negative infinity at contact (Zhu et al., 1994).

$$\lambda = 1 - \frac{0.85}{\delta} \qquad (15)$$

Where $\delta = x/d$

The above correlation is invalid for small inter-particle distance since the drag ratio approaches infinity at contact (Zhu et al., 1994).
Tsuji et al. (1982) measured the drag force on two spheres in the longitudinal direction with Reynolds number less than 10^3.

$$\frac{C_D}{C_{Do}} = 1 - A\exp\left(-B\frac{l}{d}\right) \tag{16}$$

Where A =1.0083 and B = 0.4995 with correlation coefficient R=0.9980.
Zhu et al. (1994) proposed an empirical equation for the drag force variation in the intermediate range of Reynolds number in a form similar to Tsuji et al.(1982)equation.

$$\frac{C_D}{C_{Do}} = 1 - (1-A)\exp\left(-B\frac{l}{d}\right) \qquad 20 \le \mathrm{Re} \le 150 \tag{17}$$

The coefficients A and B are both function of Re and are empirically correlated as:

$$A = 1 - \exp\left(-0.483 + 3.45\times10^{-3}\,\mathrm{Re} - 1.07\times10^{-5}\times\mathrm{Re}^2\right) \tag{18}$$

$$B = -0.115 - 8.75\times10^{-4}\,\mathrm{Re} + 5.61\times10^{-7}\,\mathrm{Re}^2 \tag{19}$$

DYNAMIC DRAG CORRELATIONS IN NON-NEWTONIAN FLUIDS

Analogous to Newtonian multiphase flows groups of multi-particles do not behave like those of an isolated particle unless the flow is extremely dilute, hence the study of the multi-particle group behavior and hydrodynamic interactions among particles is of great importance. Kawase and Ulbrecht (1981) used a free surface model and the boundary layer theory to estimate the motion of an assemblage of spheres moving in a power law non -Newtonian fluid at high Reynolds numbers and concluded that the drag coefficient for an assemblage of solid spheres decrease with the increase of the shear thinning anomaly at high and low Reynolds number. Staish and Zhu (1992) and Jaiswal et al., (1993) numerically investigated unbounded slow flows of a power law non-Newtonian fluid through an assemblage of spheres. Considering the effect of wall and the flow disturbance induced by upstream and downstream particle groups.
Subramaniam and Zuritz (1994) determined the drag force on multiple assemblies of spheres suspended in CMC solutions, from which an averaged drag force on each particle was deduced. However, this averaged drag force should not be used in the calculation of dynamic motions of particles with strong wake interactions. (Zhu et al.,2003).
Zhu et al. (2003) concluded experimentally and theoretically the drag coefficient ratio of an interacting spheres is independent from the power law index but strongly depends on the separation distance and the particle Reynolds number in a range of $0.6 \le n \le 1$ and , $0.7 \le \mathrm{Re}_{gn} \le 23$.

Up to date, there is little published information about the interactions between two spheres in non-Newtonian fluids most of them are dealing with viscoelastic, viscoplastic and yeildstress fluid.
It is worth to mentioning that due to lack of any definitive dynamic correlation for two spheres (aligned side by side or in line) motion in power law fluid and due to the concluded of Zhu et al. (2003) that the drag coefficient ratio of an interacting sphere is independent from the power law index but strongly depends on the separation distance and the particle Reynolds number at power

law index range between 0.6 to 1, the corresponding correlation for two spheres in Newtonian fluid will be used in this study as a first approximation.

EXPERIMENTAL APPARATUS AND MEASUREMENT SYSTEM

The experimental apparatus is shown in figure 1. It consisted of a borosilicate glass cylindrical column of length 2.0 m and diameter of 0.3 m. Two types of liquid were used in this study polyacryamaide (PAA) solution as a non -Newtonian liquid with different concentrations of (0.01, 0.03 , 0.05 and 0.07 %) by weight and water as a Newtonian liquid .

The spheres used in the experiments were made of stainless steel, glass and plastic of different size and densities table (1). Each sphere pair was connected by a thin steel rod of approximately 1/50 of the sphere diameter. The effect of the connecting rod on the motion of the spheres is considered to be negligible. The distance between spheres was varied from 2 to 10 radii. A fishing string of 0.18 mm diameter, passed over an aluminum pulley to a drive weight that provided the driving force, suspended the spheres.

The external friction was reduced to a minimum with ball bearings on the pulley's shaft. The spheres were submerged in the liquid of the column at an initial position of approximately 0.5 m from the bottom. Upon release of the string the spheres rose under the action of falling weights. Measurements of the velocity of the pairs of spheres were carried out for different sphere diameters and sphere separation distances. On the top of the column there was a system of light source and a photo-cell. A small pieces of eight light blocks were fixed on the part of string that was un-submerged. As the spheres moved in the liquid, the light blocks also moved up through the collimator, which made the light intensity seen by the photo-cell varied and hence its resistance. This causes a variable voltage drop across the photo-cell. An electronic circuit was constructed to measure the time elapsed between two successive light blocks. The electronic circuit components consist of light source, photo-cell detector and an interface unit connected to a personal computer.

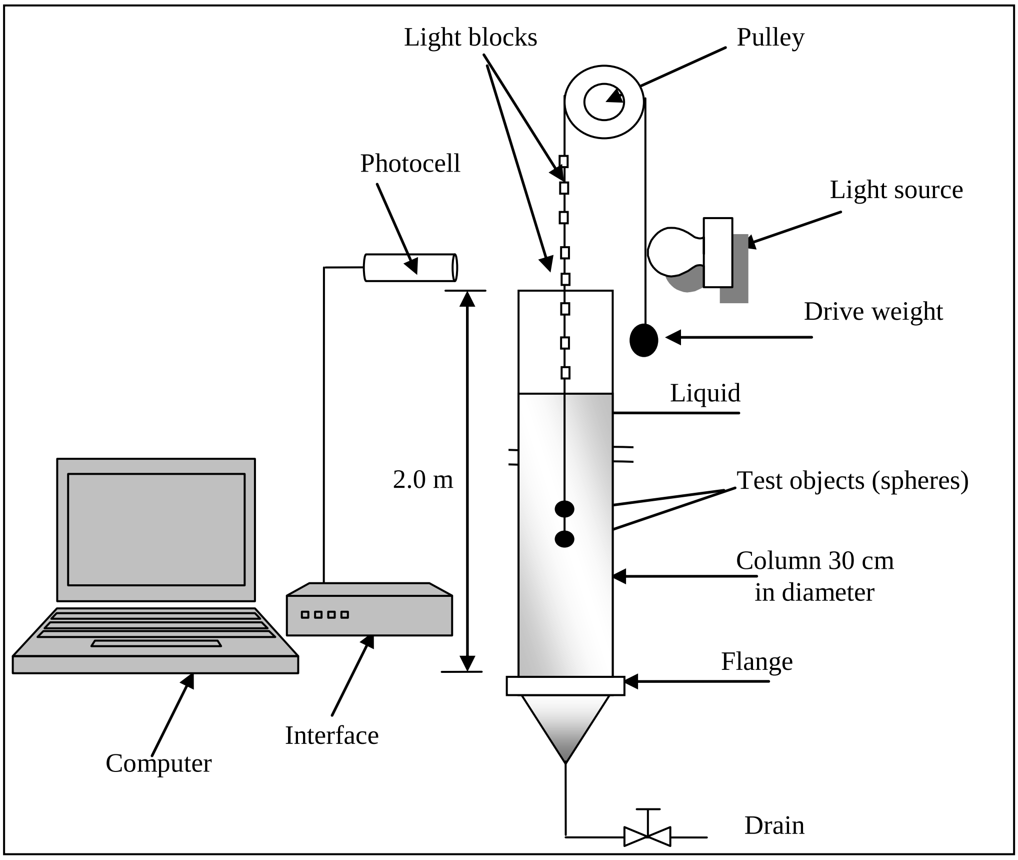

Fig. 1 Schematic diagram of experimental apparatus

Table1 Properties of spheres

Type	*Diameter(m)*	*Density (kg/m³)*
stainless steel	0.01	7660.22
	0.012	
	0.134	
	0.016	
glass	0.072	2520
	0.093	
	0.0204	
	0.0256	
Plastic	0.0114	1355
	0.0138	

EXPERIMENTAL PROCEDURE

For calibrating the system, a single sphere submerged in the liquid column was accelerated under the action of a falling weight that was slightly heavier than the sphere. As the sphere moved up, the light blocks also moved up and passed between the light and the photo-cell. The interface unit fed the response to the computer until all light blocks were passed. The online computer displayed the velocity of the spheres versus time on the screen.

The experimental procedure for two spheres is similar to that with single sphere. The motion of two identical solid spheres rising along their line of centers and side by side was carried out for different sphere diameters and separation distances.

RESULTS AND DISCUSSIONS

- **Modification of Stokes Drag Coefficient due to Reynolds Number**

Consistent with previous studies (Clift et al, 1978; Bagchi and Chhabra, 1991; and Chhabra et al.1998), it is assumed here that the drag coefficient of the accelerated sphere is similar to that under constant velocity conditions. Equation (1) was used to evaluate the drag coefficient in water while equation (10) was used to evaluate the drag coefficient in the power law fluid to take into account the effect of generated Reynolds number $1.1 \leq \mathrm{Re}_{gn} \leq 76$ and power law index on the drag coefficient of the spheres.

The drag coefficient against Re_{gn} for non-Newtonian with different concentrations for two steel spheres side by side (d=10mm, l/d=2) is plotted in figure 2 .This figure shows good agreement with published data in the literature.

It can be seen that drag coefficient decreases for the generated Reynolds number increases at the same liquid , but when the shear thinning is increasing (power law index decrease) the generated Reynolds number will be decreased with increasing the drag coefficient

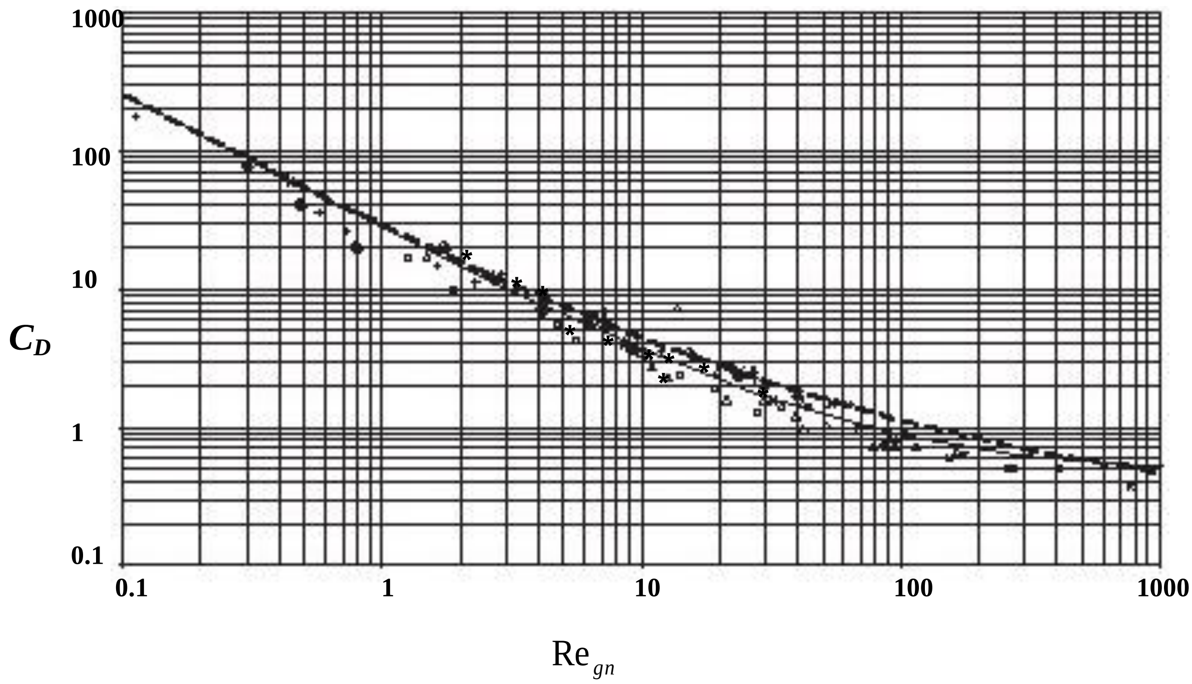

Fig.2 Comparison of drag coefficients with generated Reynolds number for the present results and many other predicted results in non -Newtonian fluids

- **Drag Force Correction due to Interaction**

For the motion of two spheres in power law fluids, no reliable correlations to describe this interaction were available and according to the result of Zhu et al. in 2003 , the ratio of drag coefficient in power law fluids is exponentially dependent on the separation distance and is strongly related to the particle Reynolds number but nearly independent of the power law index $0.6 \le n \le 1$.Therefore, Newtonian correlations are used in this study to predict an expression that relates the drag coefficient with the inter-particle distance in power law fluid.

- **Drag Force Correction for Two Spheres Moving Side by Side**

The interaction parameter ($\lambda = C_D/C_{Do}$) was calculated using equation (11) for two spheres moving side by side in polyacryamaide solution for $0.02 \le \mathrm{Re}_{gn} \le 500$.

Figure 3 shows a comparison between the drag force calculated by equation (11) and Tsuji et al. (1982) result. This figure shows that when the spheres are close to each other, the drag increases, but the effect disappears at a distance larger than l/d = 2~3, and the drag ratio reaches that of a single sphere (i.e. C_D/C_{Do} =1). According to Kim et al., (1998), the drag increases when the two spheres get closer from each other probably due to the increase of the shear stress on the sphere surface and the change in the pressure distribution owing to the flow acceleration in the gap between them.

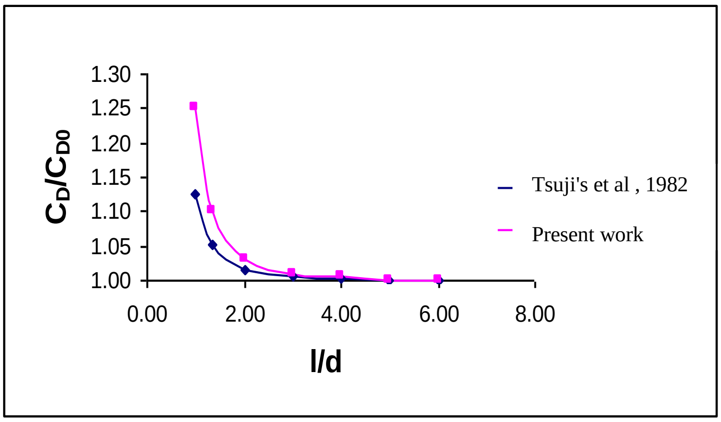

Fig. 3 Drag ratio versus inter-particle distance for two steel spheres side by side

• **Drag Force Correction for Two Spheres Moving Along their Line of Centers**

Within the considered range of Reynolds number, $0.1 < \mathrm{Re}_{gn} < 10^3$, no dependable correlation available to describe the interaction parameter($\lambda = C_D/C_{Do}$) for two spheres moving in the direction parallel to their line of centers. So the interaction parameter was calculated from different correlations depending on the Reynolds number, Oseen's equation (14) was used for low generated Reynolds number(less than 20).

Zhu (1994) equation (17) was adopted for intermediate range of Reynolds number. This equation was examined at two limits of the separation distances between the two particles. As $l \to \infty$, the second term in equation (17) vanishes and the drag ratio becomes unity as expected. At contact, the drag ratio equals the coefficient A, which gives the minimum value of the drag ratio. Zhu (1994) found that the particle Reynolds number affects not only the magnitude of the drag force of an interacting particle but also its variation with the separation distance.

Zhu et al., (1994) results agree with the measurements of Rowe and Henwood (1961) and Tsuji et al. (1982).

For Newtonian fluids, equation (13) was used. The coefficients of the fitted equation are determined as *A*=1.0083 and *B*= 0.4995 with correlation coefficient R=0.9980, so equation (13) will be :

$$\frac{C_D}{C_{Do}} = 1 - \exp\left(-0.5\frac{l}{d}\right) \quad (20)$$

The drag ratio is decreasing during the decreasing of distance between the spheres, but this effect disappears at a distance larger than l/d = 5 ~10 and approaches the single sphere value.

The reason for this reduction according to Zhu et al., (1994) was the particle interaction renders the wake vortex of the leading particle longer than that of a single non-interacting particle. In addition, Liang et al. (1996) found that the reduction in the drag ratio results from the wake effect.

After that the interaction parameter ($\lambda = C_D/C_{Do}$) was introduced into equation (6) in order to evaluate the effect of interaction on the drag coefficient. The drag force (F_D) is then determined using:

$$F_D = \frac{1}{2} C_D \pi \rho_f a^2 u^2 \quad (21)$$

For the motion of spheres in water, the same method of evaluating the drag force will be used but instead of equation (6), equation (1) was used.

- **Effect of Fluid Rheology on the Drag Force**

Figure (4) shows the relationship between the drag coefficient and power law index at constant generated Reynolds number for one steel sphere (d=10mm). It can be seen from this figure that the drag coefficient increases with increases the power law index at constant generated Reynolds number and shows fair agreement with the result of Dohle et al., (2006).

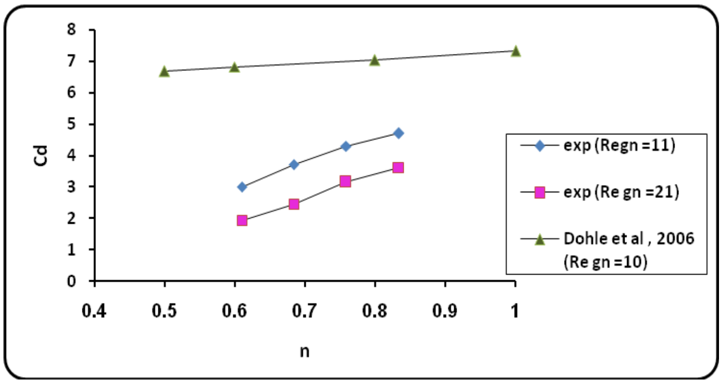

Fig. 4 Relationship between drag coefficient and power law index at constant generated Reynolds number for one steel sphere, d=10mm

- **Density Effect on the Drag Force**

Examining the effect of fluid density on the drag force, the drag force was plotted as a function of velocity for different PAA concentration compared with water. Figure 5, shows the effect of fluid density on the drag force, i.e. the drag force increased as the fluid density increased.

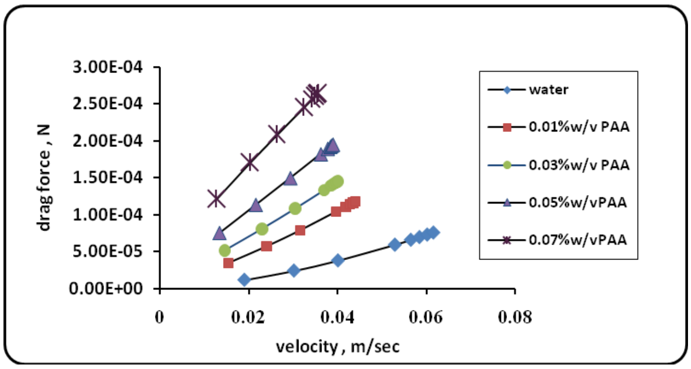

Fig. 5 Change of drag force with velocity at different fluid densities, for two steel spheres side by side (d=10mm, l/d=3)

To evaluate the effect of sphere density on the drag force, the drag forces are plotted as a function of sphere density at constant velocity. Figure 6, shows the effect of sphere density on the drag force for two spheres, side by side with 0.07% w/v PAA solution (l/d=1), which indicates that the drag force decreases as the sphere density increases.

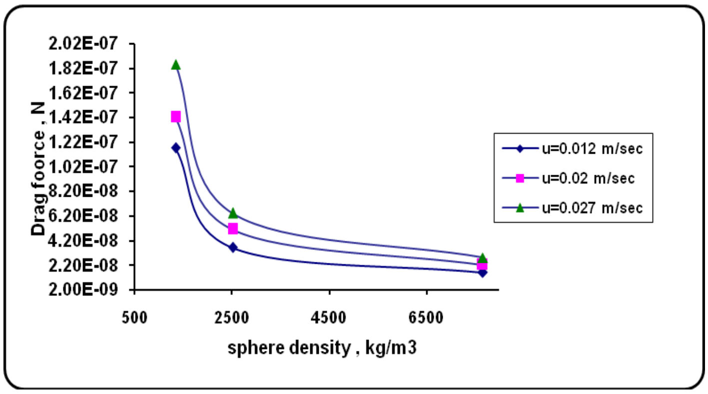

Fig. 6 Effect of sphere density on the drag force, for two spheres, side by side (l/d=1) in 0.07% w/v PAA solution

CONCLUSIONS

- Newtonian correlations were used to predict an expression that relates the drag coefficient with the inter-particle distance in power law fluid in the rage of generated Reynolds number between 1.1 and 76 and power law index is $0.6 \le n \le 1$,the following conclusion were obtained :
- For two spheres moving side by side, the drag increases when the spheres are close to each other, but the effect disappears at a distance larger than *l/d* about 3 and the drag ratio asymptotically reaches that of a single sphere similar to that in Newtonian fluid.
- For two spheres moving in line, according to this correlation, the drag decreases with decreasing distance between the spheres, but the effect of interaction disappears at a distance larger than *l/d* about 5 to 10 and asymptotically approaches the single sphere value.
- Drag coefficient increases with increases the power law index at constant generated Reynolds number.
- The drag force is related to the fluid and sphere density; it increases as the fluid density increased while it decreases as the sphere density increase, within the considered range of generated Reynolds number.

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Symbols

a	*Sphere radius, m*
n	*power law index (flow behavior index),dimension less*
C_D	*Drag coefficient,-*
C_{Do}	*Drag coefficient of an isolated sphere,-*
d	*Sphere diameter, m*
F_D	*Drag force, N*
k	*consistency index, (Pa.s^n)*
l	*Distance between the centers of spheres, m*
u	*Sphere velocity, m/s*
Re	*Reynolds number based on the sphere diameter ($\rho f .u.d/ \mu$)*
R_{egn}	*Generated Reynolds number for power law fluid ,* $\rho V^{2-n} d^n / k$
S	*Dimensionless separation (l/a)*
V	*Fluid velocity, m/s*
x	*Distance between particle surfaces, m*
μ	*Viscosity, kg/m.s*
δ	*Dimensionless separation (x/d), -*
λ	*Drag interaction parameter (C_D / C_{Do}), -*
ρf	*Density of fluid, kg/m3*