A STUDY OF FREE VIBRATION AND FATIGUE FOR CROSS-PLY CLOSED CYLINDRICAL SHELLS USING GENERAL THIRD SHELL THEORY (GTT)

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ABSTRACT:

Free vibration solution will be developed for laminated simply supported closed cylindrical shells. This solution is obtained using General Third Shell Theory (G.T.T.). Also the critical in-plane fatigue load is studied and the required equilibrium equations are developed, the effects of tension or compression in-plane load on the natural frequencies are discussed also. The natural frequencies and in-plane fatigue load results are very close to those obtained by other researchers.

الخلاصة:

الاهتزاز الحر لقشريات طباقية مسندة بصورة بسيطة تم تطوير حل له باستخدام نظرية القشريات العامة الرتبة الثالثة. كذلك تم دراسة حمل الكلال الحرج في المستوي كما تم تطوير معادلات التوازن الضرورية, كذلك تم مناقشة تاثير حمل الانضغاط او التمدد في المستوي على التردد الطبيعي. نتائج التردد الطبيعي وحمل الكلال في المستوي قريبة جدا لتلك المحصل عليها من قبل باحثين اخرين.

KEYWORDS:

Free vibration, fatigue, cross-ply, cylindrical shells, general third shell theory.

INTRODUCTION:

The last decay is marked by a transition in use of composites has expanded from aerospace and defense applications into a wider commercial arena. Today composites are used in the power generation industry, the automotive industry, biomedical engineering and various consumer goods. In service similarly to other materials, advanced composites age, suffer physical or chemical degradation and accumulate micro mechanical damage. As cured laminated composites cool to room temperature, stresses develop at the laminae and fiber-matrix interfaces due to the different consistent expansion characteristics. Hence, initially stable composites could become unstable in an actual space application. In all above applications, it is important to achieve low weight, high strength, stiffness and safety which can be achieved by good fatigue performance. Most of structural theories used till now to characterize the behavior of composite laminates fall into the category of equivalent single layer (ESL) theories. In these theories the material properties of constituent layers are "smeared "to form a hypothetical single layer whose properties are equivalent to through-the-thickness integrated sum of its constituents. This category of theories have been found to be adequate in predicting global response characteristics of laminates, like maximum deflection, maximum stresses, fundamental frequencies, forced response, or critical buckling load. Continuum based theories give an analytical (3-D) elasticity solutions for beam, plate and shells but it cumbersome. High order shell theories are those in which the transverse strains are accounted. The (in-plane) fatigue loading (presented as buckling load in present work) of shell of revolution such as cylinder or conical has been studied by many researchers.

(Altan Kayran & Vinson 1990) presented an analysis for the free vibration characteristics of isotropic and laminated composite truncated circular shells including transverse shear deformation. All components of translatory and rotatory inertia are included. The applicability of linear shell theory due to Reissner is assumed, and governing equations are solved for the natural frequencies and mode shapes by using a combination of modal iteration and transfer matrix approach for different boundary conditions.

(Narita et al 1992) developed a theoretical method for solving the free vibration angle-ply laminated cylindrical shells. The angle-ply laminated shell is macroscopically modeled as a thin shell of General anisotropy by using the classical lamination theory. Shell theory is minimized by following the Ritz procedure, and arbitrary combinations of boundary conditions at both ends are accommodated by introducing newly developed admissible functions.

(Liyong Tong 1993) used a particularly convenient coordinate system, a simple and exact solution is obtained directly for the Donnell-type governing equations of the free vibration of composite laminated conical shells, with orthotropic stretching-bending coupling. The solution is in the form of a power series, and its convergence condition is investigated.

(Tong 1996) obtained an analytical solution in the form of a power series for the three governing equations of free vibrations of axially loaded orthotropic conical shells. Numerical results are presented for the frequency parameters and associated circumferential wave numbers of axially loaded shells with different geometric and material parameters and under two types of boundary conditions.

(Korjakin et al (1998) investigated the damping of free vibrations of laminated composite conical shells. Finite element analysis of conical shells is performed by using first-order shear deformation theory (FOSDT). Based on proposals of other researchers a damping model is developed in connection with energy method (EM) and applied in order to calculate the modal loss factors of laminated composite conical shells.

(Xi et al 1999) investigated the effects of shear non-linearity on free vibration of laminated composite shell of revolution using a semi-analytical method based on Reissner-Mindlin shell theory. The coupling between symmetric and anti-symmetric vibration modes of the shell is considered in the shear deformable shell element.

(Pinto Correia et al 2001) presented a numerical method for the structural analysis of laminated conical shell panels using a quadrilateral isoperimetric finite element based on the higher order shear deformation theory.

(Werner Hufenbach et al 2002) developed analytical solution for lightweight design using dynamically loaded fiber-reinforced composite shells. The analytic results were fully corroborated by accompanying FE calculations for special lay-ups.

(Lee et al 2002) used the finite element method based on Hellinger-Reissner principle with independent strain to analyze the vibration problem of cantilevered twisted plates, cylindrical and conical laminated shells.

(Kabir et al 2003) presented a hitherto unavailable analytical solution to the boundary value problem of the free vibration response of shear flexible antisymmetric cross-ply laminated cylindrical shell, using (FSDT) theory.

(Young-Shin Lee et al 2003) investigated the free vibration analysis of a laminated composite cylindrical shell with an interior rectangular plate by analytical and experimental methods. The frequency equations of vibration of the shell including the plate are formulated by using the reacceptance method.

(Darvizeh et al 2005) presented a calculation of overall dynamic response of thin orthotropic cylindrical shells. Due to the obvious importance of the effects of transverse shear deformation and rotary inertia, these terms are included in the analysis. The exact method is modified to predict the dynamic behavior of an orthotropic circular cylindrical shell.

The (in-plane) fatigue loading of shell of revolution such as cylinder or conical has been studied by many researchers. **(George J. Simitses & John S. Anastasiadis 1992)** developed a higher order theory, which includes initial geometric imperfections and transverse shear effects for a laminated cylindrical configuration under the actions of lateral pressure, axial compression, and eccentrically applied torsion.

(**Seishi Yamada & Croll 1993**) used nonlinear Ritz analysis to investigate the elastic (in-plane) loading behavior of pressure loaded cylinders. Careful analysis of the energy changes during the loading process allows definition of a reduced stiffness theoretical model.

(**NG et al 1998**) studied the dynamic stability of then, laminated cylindrical shells under combined static and periodic axial forces using Love's classical theory of thin shells. A normal-mode expansion of the equations of motion yields a system of Mathien-Hill equations. Bolton's method is then employed to obtain the dynamic instability regions. The present study examines the dynamic stability of antisymmetric cross-ply circular, cylindrical shells of different lamination schemes. The effect of the magnitude of the axial load on the instability regions is also examined.

(**NG et al 1998**) investigated the parametric resonance of rotating cylindrical shells periodic axial loading. The formulation is based on the dynamic version of Connell's equations for thin rotating cylindrical shells. A modified assumed-mode method is used to reduce the partial differential equation of motion to a system of coupled second order differential equations with periodic collisions of the Mathieu Hill type. The instability regions are determined based on the principle of Bolton's method. Or special interests here are the effects of the centrifugal and Carioles forces on the instability regions which were examined in detail.

(**Romil Tanov et al 1999**) presented the results obtained while investigating the behavior of cylindrical laminated shells under suddenly applied lateral pressure. The investigations were based on a finite-element approach using an explicit time integration scheme. The Budiansk- Roth and phase- plane criteria were used to assess (in-plane) loading.

(**Meyers & Hyer 1999**) used results from semi analytical predictions and experiments to study the response of composite cylinders with elliptical cross sections loaded axially to a significant percentage of their (in-plane) loading load. The semi analytical approach is based on the methods of Marguerre, Rayleigh-Ritz, and Kantorovich.

(Youngjin Chung (2001) aimed in this study, is to improve the strength of conical shells and reduce the weight of the structure. Buckling of composite conical shells subjected to combine axial loading, external pressure, and bending is investigated using energy and finite element methods.

(Andrea Spagnoli 2001) studied the local shell and stringer buckling modes and global buckling mode in conical shells under axial compression through a linear eigenvalue finite element analysis. In order to examine buckling modes in isolation as well as competing modes together, use is made of different finite element model, including discrete and smeared models.

(**Geier et al 2002**) studied the (in-plane) loading load of laminated cylinders which strongly depend on the position of the differently oriented layers, his work deals with two different laminated orthotropic cylinders with opposite stacking sequence of the laminate layers. Analytical and semi analytical method have been used to predict the (in-plane) loading loads, the results are compared with those tested.

(**Hunt et al 2003**) presented a hypothesis for prediction of the circumferential wave number of (in-plane) loading of thin axially-compressed cylindrical shell, based on the addition of a length effect to classical (Koiter circle) critical load result.

(**Sofiyev 2005**) provided an analytical solution for stability behavior of cylindrical shell made of compositionally graded ceramic-metal materials under axial compressive loads varying as a power function of time. The material properties of compositionally graded shells are assumed to vary continuously through the thickness of the shell according to arbitrary distribution of the volume fraction of the constituents. The fundamental equations for thin shells of compositionally graded ceramic-metal material are obtained Loves shell theory.

(**Azam Tafreshi 2005**) carried out a series of finite element analyses on the delaminated composite cylindrical shell subjected to combined axial compression and pressure by which the delamination thickness and length, material properties and stacking sequence are varied. The characteristics of (in-plane) loading and post (in-plane) loading behavior of delaminated composite cylindrical shell are investigated.

In present work, a unified third order theory (G.T.T.) to evaluate the performance of some displacement based (ESL) theories in natural frequencies and fatigue characteristics in laminated composite cylindrical shell.

EQUATIONS OF MOTION:

In present study high-order theory displacement field is:

$$\begin{aligned} u(x,\theta,z,t) &= u_0(x,\theta,t) + z\times\phi_1(x,\theta,t) + z^2\times\psi_1(x,\theta,t) + z^3\times\theta_1(x,\theta,t) \\ v(x,\theta,z,t) &= v_0(x,\theta,t) + z\times\phi_2(x,\theta,t) + z^2\times\psi_2(x,\theta,t) + z^3\times\theta_2(x,\theta,t) \\ w(x,\theta,z,t) &= w_0(x,\theta,t) + z\times\psi_3(x,\theta,t) + z^2\times\theta_3(x,\theta,t) \end{aligned} \quad \ldots\ldots\ldots(1)$$

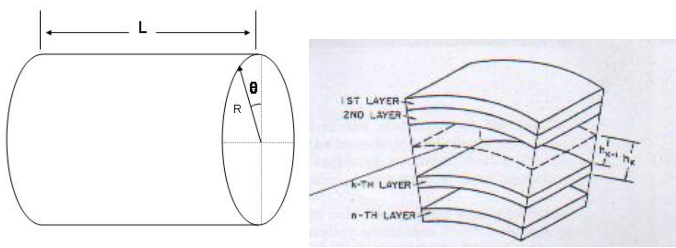

Figure (1): Coordinate system and structure of laminated cylinder.

$$
\begin{aligned}
\varepsilon_1 &= \frac{\partial u}{\partial x} \\
\varepsilon_2 &= \frac{1}{R} \times \left(\frac{\partial v}{\partial \theta} \right) + \frac{w}{R} \\
\varepsilon_3 &= \frac{\partial w}{\partial z} \\
\varepsilon_6 &= \frac{1}{R} \times \left(\frac{\partial u}{\partial \theta} \right) + \frac{\partial v}{\partial x} \\
\varepsilon_5 &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
\varepsilon_4 &= \frac{\partial v}{\partial z} - \frac{v}{R} + \frac{1}{R} \times \left(\frac{\partial w}{\partial \theta} \right)
\end{aligned}
\qquad \ldots\ldots (2)
$$

Assuming vanishing transverse shear stress at top and bottom of laminated composite layers, and hence transverse strain also vanishes, so:

$$
\begin{aligned}
\varepsilon_5 (x, \theta, \pm h/2, t) &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \\
\varepsilon_4 (x, \theta, \pm h/2, t) &= \frac{\partial v}{\partial z} - \frac{v}{R} + \frac{1}{R} \times \left(\frac{\partial w}{\partial \theta} \right) = 0
\end{aligned}
\qquad \ldots\ldots (3)
$$

According to Hamilton's′ Principles:

$$
\int_{t2}^{t1} (\delta U - \delta K) \partial t = 0 \qquad \ldots\ldots (4)
$$

Where:

$$
\delta U = \iint_{A\, z} (\sigma_1 \delta\varepsilon_1 + \sigma_2 \delta\varepsilon_2 + \sigma_3 \delta\varepsilon_3 + \sigma_6 \delta\varepsilon_6 + \sigma_5 \delta\varepsilon_5 + \sigma_4 \delta\varepsilon_4) \times R d\theta dz dx
$$

$$
\iint_{A\, z} \sigma_1 \delta\varepsilon_1 = R \times \left\{ \begin{array}{l} \iint_{x\, \theta} - \frac{\partial N_1}{\partial x} \delta u_0 - \frac{\partial M_1}{\partial x} \delta\phi_1 - \frac{1}{2} \times \frac{\partial^2 P_1}{\partial x^2} \delta\psi_3 + \left(\frac{4}{3h^2} \right) \times \frac{\partial S_1}{\partial x} \delta\phi_1 - \left(\frac{4}{3h^2} \right) \times \frac{\partial^2 S_1}{\partial x^2} \delta w_0 \\ - \left(\frac{1}{3} \right) \times \frac{\partial^2 S_1}{\partial x^2} \delta\theta_3 \end{array} \right\} d\theta dx
$$

$$
\iint_{A\, z} \sigma_2 \delta\varepsilon_2 = R \times \left\{ \begin{array}{l} \iint_{x\, \theta} - \frac{\partial N_2}{\partial \theta} \delta v_0 + N_2 \delta w_0 - \frac{\partial M_2}{\partial \theta} \delta\phi_2 + M_2 \delta\psi_3 - \frac{1}{2R} \times \frac{\partial^2 P_1}{\partial \theta^2} \delta\psi_3 + P_2 \delta\theta_3 + \left(\frac{4}{3h^2} \right) \times \frac{\partial S_2}{\partial \theta} \delta\phi_2 - \\ \left(\frac{4}{3h^2 R} \right) \times \frac{\partial S_2}{\partial \theta} \delta v_0 - \left(\frac{4}{3h^2 R} \right) \times \frac{\partial^2 S_2}{\partial \theta^2} \delta w_0 - \left(\frac{1}{3R} \right) \times \frac{\partial^2 S_2}{\partial \theta^2} \delta\theta_3 \end{array} \right\} d\theta dx
$$

$$
\iint_{A\, z} \sigma_3 \delta\varepsilon_3 = R \times \left\{ \iint_{x\, \theta} N_3 \delta\psi_3 + 2 M_3 \delta\theta_3 \right\} d\theta dx
$$

$$
\iint_{A\, z} \sigma_6 \delta\varepsilon_6 = R \times \left\{ \begin{array}{l} \iint_{x\, \theta} -(\frac{1}{R}) \frac{\partial N_6}{\partial \theta} \delta u_0 - \frac{\partial N_6}{\partial x} \delta v_0 - (\frac{1}{R}) \frac{\partial M_6}{\partial \theta} \delta\phi_1 - \frac{\partial M_6}{\partial x} \delta\phi_2 - \frac{1}{R} \times \frac{\partial^2 P_6}{\partial \theta \partial x} \delta\psi_3 + \left(\frac{4}{3h^2} \right) \times \\ \left[\frac{\partial S_6}{\partial x} \delta\phi_2 - \left(\frac{2}{R} \right) \times \frac{\partial^2 S_6}{\partial x \partial \theta} \delta w_0 - \left(\frac{h^2}{2R} \right) \times \frac{\partial^2 S_6}{\partial x \partial \theta} \delta\theta_3 - (\frac{1}{R}) \frac{\partial S_6}{\partial x} \delta v_0 + (\frac{1}{R}) \frac{\partial S_6}{\partial \theta} \delta\phi_1 \right] \end{array} \right\} d\theta dx
$$

$$
\iint_{A\, z} \sigma_5 \delta\varepsilon_5 = R \times \left\{ \iint_{x\, \theta} Q_5 \delta\phi_1 - \frac{\partial Q_5}{\partial x} \delta w_0 - (\frac{4}{h^2}) K_5 \delta\phi_1 + (\frac{4}{h^2}) \frac{\partial K_5}{\partial x} \delta w_0 \right\} d\theta dx
$$

$$
\iint_{A\, z} \sigma_4 \delta\varepsilon_4 = R \times \left\{ \iint_{x\, \theta} Q_4 \delta\phi_2 - \frac{Q_4}{R} \delta v_0 - (\frac{1}{R}) \frac{\partial Q_4}{\partial \theta} \delta w_0 - (\frac{4}{h^2}) K_4 \delta\phi_2 + (\frac{4}{h^2}) K_4 \delta v_0 + (\frac{4}{h^2 R}) \frac{\partial K_4}{\partial \theta} \delta w_0 \right\} d\theta dx
$$

Where:

$$\left(Ni, Mi, Pi, Si\right) = \int \sigma_i \left(1, z, z^2, z^3\right) \times dz \longrightarrow i = 1,2,3,6 \qquad \ldots\ldots (5)$$

$$\left(Qi, Ki\right) = \int \sigma_i \left(1, z^2\right) \times dz \longrightarrow i = 4,5$$

$$\delta K = -\int_t \iiint_V R\rho\left(\ddot{u}\,\delta u + \ddot{v}\,\delta v + \ddot{w}\,\delta w\right) dz\,dx\,d\theta\,dt$$

$$\frac{\partial N_1}{\partial x} + \left(\frac{1}{R}\right)\frac{\partial N_6}{\partial \theta} = I_1 \ddot{u} + I_2 \ddot{\phi}_2 - \left(\frac{I_3}{2}\right)\frac{\partial \ddot{\psi}_3}{\partial x} - \left(\frac{4I_4}{3H^2}\right)\left(\ddot{\phi}_1 + \frac{\partial \ddot{w}}{\partial x}\right) - \left(\frac{I_4}{3}\right)\frac{\partial \ddot{\theta}_3}{\partial x} \qquad \ldots\ldots (6)$$

$$\frac{\partial N_2}{\partial \theta} + R\frac{\partial N_6}{\partial x} + Q_4 - \left(\frac{4}{H^2}\right)K_4 + \left(\frac{4}{3H^2}\right)\left(\left(\frac{1}{R}\right)\frac{\partial S_2}{\partial \theta} + \frac{\partial S_6}{\partial x}\right) = \left(RI_2 - \left(\frac{4RI_4}{3H^2}\right) - \left(\frac{16I_7}{9H^4R}\right) + \left(\frac{4I_5}{3H^2}\right)\right)\ddot{\phi}_2 \qquad \ldots\ldots (7)$$

$$+\left(RI_1 + \left(\frac{8I_4}{3H^2}\right) + \left(\frac{16I_7}{9H^4R}\right)\right)\ddot{v} + -\left(\left(\frac{I_3}{2}\right) + \left(\frac{4I_6}{6RH^2}\right)\right)\frac{\partial \ddot{\psi}_3}{\partial \theta} + \left(\left(\frac{4I_4}{3H^2}\right) - \left(\frac{16I_7}{9H^4R}\right)\right)\frac{\partial \ddot{w}}{\partial \theta} - \left(\frac{I_4}{3} + \left(\frac{4I_7}{9H^2R}\right)\right)\frac{\partial \ddot{\theta}_3}{\partial x}$$

$$\left(\frac{4}{3H^2}\right)\left(\frac{1}{R}\frac{\partial^2 S_2}{\partial \theta^2} + R\frac{\partial^2 S_1}{\partial x^2} + 2\frac{\partial^2 S_6}{\partial x \partial \theta}\right) - N_2 - \left(\frac{4}{H^2}\right)\left(\frac{\partial K_4}{\partial \theta} + R\frac{\partial K_5}{\partial x}\right) + \left(\frac{\partial Q_4}{\partial \theta} + R\frac{\partial Q_5}{\partial x}\right) + N_x\frac{\partial^2 w}{\partial x^2} = \left(\left(\frac{4I_4}{3H^2}\right) + \left(\frac{16I_7}{9H^4R}\right)\right)\frac{\partial \ddot{v}}{\partial \theta} \qquad \ldots\ldots (8)$$

$$+\left(\frac{4I_4}{3H^2}\right)\frac{\partial \ddot{u}}{\partial x} + \left(-\left(\frac{16I_7}{9H^4R}\right) + \left(\frac{4I_5}{3H^2}\right)\right)\frac{\partial \ddot{\phi}_2}{\partial \theta} + RI_2\ddot{\psi}_3 - \left(\frac{4I_6}{6RH^2}\right)\frac{\partial^2 \ddot{\psi}_3}{\partial \theta^2} + \left(\frac{4I_4}{3H^2}\right) - \left(\frac{16I_7}{9H^4}\right)\left(\left(\frac{1}{R}\right)\frac{\partial^2 \ddot{w}}{\partial \theta^2} + R\frac{\partial^2 \ddot{w}}{\partial x^2}\right)$$

$$-\left(\frac{4I_7}{9H^2R}\right)\left(R\frac{\partial^2 \ddot{\theta}_3}{\partial x^2} + \frac{1}{R}\frac{\partial^2 \ddot{\theta}_3}{\partial x^2}\right) + R\left(-\left(\frac{16I_7}{9H^4R}\right) + \left(\frac{4I_5}{3H^2}\right)\right)\frac{\partial \ddot{\phi}_1}{\partial x} + RI_1\ddot{w} + RI_3\ddot{\theta}_3$$

$$R\frac{\partial M_1}{\partial x} - \left(\frac{4R}{3H^2}\right)\frac{\partial S_1}{\partial x} + \frac{\partial M_6}{\partial \theta} - \left(\frac{4R}{3H^2}\right)\frac{\partial S_6}{\partial \theta} - RQ_5 + \left(\frac{4R}{H^2}\right)K_5 = \left(RI_2 - \frac{4RI_4}{3H^2}\right)\ddot{u} + \left(RI_3 - \frac{8RI_5}{3H^2} + \frac{16RI_7}{9H^4}\right)\ddot{\phi}_1 + \qquad \ldots\ldots (9)$$

$$\left(\frac{4RI_6}{6H^2} - \frac{RI_4}{2}\right)\frac{\partial \ddot{\psi}_3}{\partial x} + \left(-\frac{8RI_5}{3H^2} + \frac{16RI_7}{9H^4}\right)\frac{\partial \ddot{w}}{\partial x} + \left(-\frac{RI_5}{3} + \frac{4RI_7}{9H^2}\right)\frac{\partial \ddot{\theta}_3}{\partial x}$$

$$\frac{\partial M_2}{\partial \theta} - \left(\frac{4R}{3H^2}\right)\frac{\partial S_2}{\partial \theta} + R\frac{\partial M_6}{\partial x} - \left(\frac{4R}{3H^2}\right)\frac{\partial S_6}{\partial x} - RQ_4 + \left(\frac{4R}{H^2}\right)K_4 = \left(RI_2 - \frac{16RI_7}{9H^4}\right)\ddot{v} + \left(RI_3 - \frac{8RI_5}{3H^2} + \frac{16RI_7}{9H^4}\right)\ddot{\phi}_2 + \qquad \ldots\ldots (10)$$

$$\left(\frac{4I_6}{6H^2} - \frac{I_4}{2}\right)\frac{\partial \ddot{\psi}_3}{\partial \theta} + \left(-\frac{4I_5}{3H^2} + \frac{16I_7}{9H^4}\right)\frac{\partial \ddot{w}}{\partial \theta} + \left(-\frac{I_5}{3} + \frac{4I_7}{9H^2}\right)\frac{\partial \ddot{\theta}_3}{\partial \theta}$$

$$\left(\frac{R}{2}\right)\frac{\partial^2 P_1}{\partial x^2} + \left(\frac{1}{2R}\right)\frac{\partial^2 P_2}{\partial \theta^2} + \frac{\partial^2 P_6}{\partial x \partial \theta} - RN_3 - M_2 = \left(\frac{RI_3}{2}\right)\frac{\partial \ddot{u}}{\partial x} + \left(\frac{RI_4}{2} - \frac{4RI_6}{6H^2}\right)\frac{\partial \ddot{\phi}_1}{\partial x} + \left(\frac{I_3}{2} + \frac{4I_6}{6RH^2}\right)\frac{\partial \ddot{v}}{\partial \theta} \qquad \ldots\ldots (11)$$

$$-\left(\frac{RI_5}{4}\right)\frac{\partial^2 \ddot{\psi}_3}{\partial x^2} - \left(\frac{I_5}{4R}\right)\frac{\partial^2 \ddot{\psi}_3}{\partial \theta^2} + RI_3\ddot{\psi}_3 + \left(-\frac{2RI_6}{3H^2}\right)\frac{\partial^2 \ddot{w}}{\partial x^2} + \left(-\frac{4I_6}{6RH^2}\right)\frac{\partial^2 \ddot{w}}{\partial \theta^2} + RI_2\ddot{w} + \left(-\frac{RI_6}{6}\right)\frac{\partial^2 \ddot{\theta}_3}{\partial x^2} - \left(\frac{I_6}{6R}\right)\frac{\partial^2 \ddot{\theta}_3}{\partial \theta^2}$$

$$+ RI_4\ddot{\theta}_3 + \left(\frac{I_4}{2} - \frac{4I_6}{6H^2}\right)\frac{\partial \ddot{\phi}_2}{\partial \theta}$$

$$\left(\frac{R}{3}\right)\frac{\partial^2 S_1}{\partial x^2} + \left(\frac{1}{3R}\right)\frac{\partial^2 S_2}{\partial \theta^2} + \left(\frac{2}{3}\right)\frac{\partial^2 S_6}{\partial x \partial \theta} - 2RM_3 - P_2 = \left(\frac{RI_4}{3}\right)\frac{\partial \ddot{u}}{\partial x} + \left(\frac{RI_5}{3} - \frac{4RI_7}{9H^2}\right)\frac{\partial \ddot{\phi}_1}{\partial x} + \left(\frac{I_4}{3} + \frac{4I_7}{9RH^2}\right)\frac{\partial \ddot{v}}{\partial \theta} \qquad \ldots\ldots (12)$$

$$-\left(\frac{RI_6}{6}\right)\frac{\partial^2 \ddot{\psi}_3}{\partial x^2} - \left(\frac{I_6}{6R}\right)\frac{\partial^2 \ddot{\psi}_3}{\partial \theta^2} + RI_4\ddot{\psi}_3 + \left(-\frac{4RI_7}{9H^2}\right)\frac{\partial^2 \ddot{w}}{\partial x^2} + \left(-\frac{4I_7}{9RH^2}\right)\frac{\partial^2 \ddot{w}}{\partial \theta^2} + RI_3\ddot{w} + \left(-\frac{RI_7}{9}\right)\frac{\partial^2 \ddot{\theta}_3}{\partial x^2} - \left(\frac{I_7}{9R}\right)\frac{\partial^2 \ddot{\theta}_3}{\partial \theta^2}$$

$$+ RI_5\ddot{\theta}_3 + \left(\frac{I_5}{3} - \frac{4I_7}{9H^2}\right)\frac{\partial \ddot{\phi}_2}{\partial \theta}$$

The resultants forces-displacement components relations are:

$$N_1 = \int \sigma_1 dz = \int \left(Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 + Q_{13}\varepsilon_3 + Q_{16}\varepsilon_6\right)dz \qquad \ldots\ldots (13)$$

From the constitutive relations of the kth lamina:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_6\end{Bmatrix} = \begin{bmatrix}\overline{Q}_{11}\overline{Q}_{12}\overline{Q}_{13}\overline{Q}_{16}\\ \overline{Q}_{21}\overline{Q}_{22}\overline{Q}_{23}\overline{Q}_{26}\\ \overline{Q}_{31}\overline{Q}_{32}\overline{Q}_{33}\overline{Q}_{36}\\ \overline{Q}_{61}\overline{Q}_{62}\overline{Q}_{63}\overline{Q}_{66}\end{bmatrix} \times \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_6\end{Bmatrix} \qquad \ldots\ldots (14)$$

And:

$$\begin{Bmatrix}\sigma_4\\ \sigma_5\end{Bmatrix}=\begin{bmatrix}\overline{Q_{44}}\,\overline{Q_{45}}\\ \overline{Q_{54}}\,\overline{Q_{55}}\end{bmatrix}\times\begin{Bmatrix}\varepsilon_4\\ \varepsilon_5\end{Bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Substituting the resultants forces in motion-equations and then the assumed displacement components according to Navier′s Solution **(Reddy J.N 2004)** (for simply supported boundary conditions).

$$u_0(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos\alpha x\sin\beta\theta e^{i\omega mn}$$

$$v_0(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}\sin\alpha x\cos\beta\theta e^{i\omega mn}$$

$$w_0(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}C_{mn}\sin\alpha x\sin\beta\theta e^{i\omega mn}$$

$$\phi_1(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}D_{mn}\cos\alpha x\sin\beta\theta e^{i\omega mn}$$

$$\phi_2(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}E_{mn}\sin\alpha x\cos\beta\theta e^{i\omega mn}$$

$$\psi_3(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}F_{mn}\sin\alpha x\sin\beta\theta e^{i\omega mn}$$

$$\theta_3(x,\theta,z,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}J_{mn}\sin\alpha x\sin\beta\theta e^{i\omega mn}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Where $\alpha=\left(\dfrac{m\pi}{L}\right)$, $\beta=n$

Developing mass matrix and stiffness matrix from solution of homogeneous equations, eignvalue equation is derived and the natural frequencies of vibration for simply supported cylindrical shell are obtained when solving the later equation:

$$\left|[C]-\omega^2[M]\right|\{A\}=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

The failure of laminates cylindrical shell under in-plane fatigue load is studied in this work using G.T.T. theory to calculate the critical (in-plane) fatigue load which cause the failure of fiber or matrix of laminate or both of them, and study the effect of this load on the natural frequencies of the shell. Using the same Navier′s solution (putting right side =0) for equations ((6)-(12)) and rearranging the obtained matrices we get:

$$\begin{bmatrix}C_{33}-\alpha^2N_x & C_{34} & C_{35} & C_{36} & C_{37} & C_{32} & C_{31}\\ C_{43} & C_{44} & C_{45} & C_{46} & C_{47} & C_{42} & C_{41}\\ C_{53} & C_{54} & C_{55} & C_{56} & C_{57} & C_{52} & C_{51}\\ C_{63} & C_{64} & C_{65} & C_{66} & C_{67} & C_{62} & C_{61}\\ C_{73} & C_{74} & C_{75} & C_{76} & C_{77} & C_{72} & C_{71}\\ C_{23} & C_{24} & C_{25} & C_{26} & C_{27} & C_{22} & C_{21}\\ C_{13} & C_{14} & C_{15} & C_{16} & C_{17} & C_{12} & C_{11}\end{bmatrix}\begin{Bmatrix}W\\ \phi_1\\ \phi_2\\ \psi_3\\ \theta_3\\ V\\ U\end{Bmatrix}=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Following the condensation of variables procedure to eliminate the displacement components ($U,V,\varphi^1,\varphi^2,\psi^3,\theta^3$), the critical value of (N_x) is obtained.

VALIDITY OF THE DEVELOPED EQUATIONS:

General Third Order Theory (G.T.T.) is employed to investigate its capability level for dynamic analysis of the symmetric and non-symmetric cross-ply laminated cylindrical shells, and compared with other theories used by other researchers such as HSDT, FSDT, CST.

The fundamental natural frequency of laminated closed cylindrical shell are listed for each (L/mR) ratio (for thin shell R/H=100) in **Table (1)** and compared with those obtained by using CST (but without neglecting (z/R)) in (Mohammad S. Qatu 2004), the results obtained by present work give excellent agreement with them, where maximum percentage error is (.127%) (error percentage is taken with relative to the reference results).

Table (2) lists the natural frequencies for [0-90] graphite/epoxy closed cylindrical shell with two different thickness ratio (R/H=20 & R/H=500). As can be seen from this table, thicker shells have lower frequency parameter than thinner shells for shorter shells (L/mR=1 and 2). It is interesting to note that the thickness ratio has minimal effects on natural frequencies when n=0 and 1. For n>1, the thickness ratio has a much greater effect. It is also observed that the fundamental frequency occurs at lower value for higher thickness ratios. In all calculation the material properties are as follows:

E_1=20E6 $E_2=E_3$=1.3E6 $G_{12}=G_{13}$=1.03E6 G_{23}=.9e6 $\nu_{12}=\nu_{13}=.3$ $\nu_{23}=.49$,

Frequency parameter= $\Omega=\left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$.

While in **Table (3),** minimum frequency parameter of laminated cylindrical shell for different lamination type are obtained and compared with other shell theories published in (Reddy and Liu 1985), a good agreement between results of theory GTT and HSDT, maximum relative percentage error is (3.7%), while with FSDT and CST are (3.7%, 17.2%) respectively, as presented in the table, also the frequency parameter is increased when number of layers is increased. For this results material properties are as follows: E_1=40 Gpa $E_2=E_3$=1Gpa $G_{12}=G_{13}$=.6Gpa G_{23}=.5Gpa $\nu_{12}=\nu_{13}=.3$ $\nu_{23}=.49$ (L/R) =2 (R/H) =5, frequency parameter= $\Omega=\left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$, percentage (%) in all tables $=\left|presentworkresults-refrenceresult/refrenceresults\right|*100$.

It is interesting to note that for thin shells the relative percentage error between GTT and FSDT or CST, is smaller then that for moderately thick shells, as shown in **Table (1)** (for thin shell) and **Table (3)** (for moderately thick shell), but fortunately GTT made fair with most accurate one of HSDT for both thin and moderately thick cylindrical shells. Also, each pairs of (m x n) values relates with seven successive bending frequencies by using GTT, and the increasing orders of these frequencies, for different combinations of (m, n) values, show no systematic trend, as being the case in rectangular plates where the fundamental frequency is always associated with the mode indices (1, 1) as being noted in **Table (3)** where minimum frequency parameter is found at indices (m=1, n=2).

In-plane load results are compared with that published in (Reddy J. N. and Liu C. F. 1985) as shown in **Table(4),** also GTT agree well with HSDT. Also the effect of (N_x) on the natural frequencies of cylinder, when it is compressive load it decreases natural frequency for cylindrical shell but when it tension load it increases it as shown in **Table (5).**

Table (1): Frequency parameters for [0-90] graphite/epoxy closed cylindrical shells, (R/H) =100.

(L/mR)	Reference	Frequency parameter= $\Omega = \left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$					
		n					
		0	1	2	3	4	5
2	Present	.35629	.239718	.145541	.09647	.070251	.059134
	(Mohammad S. Qatu 2004)	.35629	.23975	.14556	.09650	.07025	.05915
	Discrepancy (%)	0	.013	.013	.031	.001	.027
1	Present	.71258	.442059	.291537	.208603	.159365	.129270
	(Mohammad S. Qatu 2004)	.71259	.44205	.29163	.20869	.15935	.12928
	Discrepancy (%)	.001	.002	.031	.041	.009	.007
.5	Present	.7347119	.637573	.49575	.389404	.316127	.265658
	(Mohammad S. Qatu 2004)	.73565	.63822	.49616	.38973	.31622	.26575
	Discrepancy (%)	.127	.101	.082	.083	.029	.034

Table (2): Frequency parameters for [0-90] graphite/epoxy closed cylindrical shells.

(L/mR)	Frequency parameter= $\Omega = \left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$						
	n						
	Reference	0	1	2	3	4	5
(R/H)=20							
2	Present	.35647	.23789	.14591	.09626	.06763	.04991
	(Mohammad S. Qatu 2004)	.35629	.24015	.14590	.09623	.06761	.04991
	Discrepancy (%)	.05	.94	.0068	.03	.02	0
1	Present	.71294	.44170	.29620	.20845	.14758	.12397
	(Mohammad S. Qatu 2004)	.71258	.44245	.29191	.20838	.15753	.12393
	Discrepancy (%)	.05	.16	1.4	.03	6.3	.03
.5	Present	.73177	.63517	.49319	.38572	.31010	.25606
	(Mohammad S. Qatu 2004)	.73190	.63517	.49310	.38561	.31000	.25597
	Discrepancy (%)	.01	0	.01	.02	.03	.03
(R/H)=500							
2	Present	.35647	.24025	.14595	.11073	.12298	.16966
	(Mohammad S. Qatu 2004)	.35633	.23799	.14609	.11114	.12399	.17190
	Discrepancy (%)	.03	.94	.09	.36	.81	1.3
1	Present	.71294	.44258	.29202	.22644	.20670	.22753
	(Mohammad S. Qatu 2004)	.71266	.44283	.29705	.22751	.20855	.23095
	Discrepancy (%)	.03	.05	1.6	.47	.88	1.4
.5	Present	.77913	.68365	.55557	.47426	.43687	.43493
	(Mohammad S. Qatu 2004)	.78908	.69140	.56195	.48080	.44477	.44535
	Discrepancy (%)	1.2	1.1	1.1	1.3	1.7	2.3

Table (3): The effect of lamination type on minimum frequency parameters (m=1, n=2) of (graphite/epoxy) closed cylindrical shells.

Lamination	Theory	Frequency parameter= $\Omega = \left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$	Discrepancy (%)
(0/90)	Present theory(G.T.T.)	.156047	-
	HSDT (Reddy J. N. and Liu C. F. 1985)	.1566	.35
	FSDT (Reddy J. N. and Liu C. F. 1985)	.1552	.54
	CST (Reddy J. N. and Liu C. F. 1985)	.1630	4.2
(0/90/0)	Present theory(G.T.T.)	.171515	-
	HSDT (Reddy J. N. and Liu C. F. 1985)	.1777	3.4
	FSDT (Reddy J. N. and Liu C. F. 1985)	.1779	3.5
	CST (Reddy J. N. and Liu C. F. 1985)	.2073	17.2

Table (4): The dimensionless critical buckling loads of cross-ply circular cylindrical shell as predicted by various theories. (m=1, n=3, L/R=1, R/H=10, N=NL^2/100H^3E2)

Lamination	Theory	SS	Discrepancy (%)
[0-90]	HSDT (Reddy J. N. and Liu C. F. 1985)	.1687	3.9
	Present work GTT	.162065	
	FSDT (Reddy J. N. and Liu C. F. 1985)	.1670	2.9
	CST. (Reddy J. N. and Liu C. F. 1985)	.1817	10.8
[0-90-0]	HSDT (Reddy J. N. and Liu C. F. 1985)	.2794	1.7
	Present work GTT	.274399	
	FSDT (Reddy J. N. and Liu C. F. 1985)	.2813	2.4
	CST (Reddy J. N. and Liu C. F. 1985)	.4186	34.4

Table (5): The effect of the in-plane load (N=NL^2/100H^3E2=.25) on the dimensionless minimum frequencies of cross-ply closed circular cylindrical shells. $\bar{\omega} = \left(\frac{\omega L^2}{100H}\right)\sqrt{\frac{\rho}{E_2}}$

Lamination	Theory	Compressive in-plane load	Discrepancy (%)	Tension in-plane load
[0-90]	Present work (GTT)	.071682	-	.194108
	HSDT (Reddy J. N. and Liu C. F. 1985)	.0786	8.8	-
	FSDT (Reddy J. N. and Liu C. F. 1985)	.0761	5.8	-
	CST(Reddy J. N. and Liu C. F. 1985)	.0932	23.08	-
[0-90-0]	Present work (GTT)	.109846	-	.221492
	HSDT (Reddy J. N. and Liu C. F. 1985)	.1089	.86	-
	FSDT(Reddy J. N. and Liu C. F. 1985)	.1095	.315	-
	CST(Reddy J. N. and Liu C. F. 1985)	.1533	28.3	-
[0-90-0-90……] 10 layers	Present work (GTT)	.14733	-	.228323
	HSDT (Reddy J. N. and Liu C. F. 1985)	.1533	3.8	-
	FSDT (Reddy J. N. and Liu C. F. 1985)	.1531	3.7	-
	CST(Reddy J. N. and Liu C. F. 1985)	.1607	8.3	-

CONCLUSIONS:

In this work General Third order shell Theory (GTT) is developed to derive the governing equations for free vibration and obtaining in-plane critical load of simply supported cylindrical shells, for first time, and these equations are solved using Navier's solution. Good agreement between results obtained using GTT in present work with those obtained by other researchers using HSDT, FST for analyzing dynamic behavior of laminated cylindrical shells (maximum discrepancy with HSDT is 3.7%), however high order theory does not require the use of correction factor. Also the effect of the type of this in-plane load on the natural frequencies is studied, compressive axial load decrease natural frequencies, while tension in-plane load increase them, as proved by many other papers.

REFERENCES:

- Altan Kayran & Jack R. Vinson,' Free vibration analysis of laminated composite truncated circular conical shells', AIAAJ, vol. 28, no.7, 1990.
- Azam Tafreshi,'Delamination buckling and post buckling in composite cylindrical shells under combined axial compression and external pressure', Composite Structures, Article in press, 2005.
- Andrea Spagnoli,'Different buckling modes in axially stiffened conical shells', Engineering Structures, vol.23, 2001.
- Darvizeh M., Haftchenari H., Darvizeh A., Ansari R., Sharma C.B., 'The effect of boundary conditions on the dynamic stability of orthotropic cylinders using a modified exact analysis', Composite Structures, article in press, 2005.
- George J. Simitses & John S. Anastasiadis,'Shear deformable theories for cylindrical laminates-equilibrium and buckling with applications', AIAA J., vol.30, no.3, 1992.
- Geier B., Meyer-Piening H.-R., Zimmermann R.,'On the influence of laminate stacking on buckling of composite cylindrical shells subjected to axial compression', Composite Structures,vol.55, 2002.
- Hunt G.W., Lord G.J., Peletier M.A.,'Cylindrical shell buckling: a characterization of localization and periodicity', Discrete and continuous dynamical systems-series B, vol.3, no.4, 2003.
- Lee J. J., Yeom C.H., Lee I., ' Vibration analysis of twisted cantilevered conical composite shells', J. of Sound and vibration, vol. 255,no.5, 2002.
- Liyong Tong, 'Free vibration of composite laminated conical shells', Int. j. Mech. Sci., vol.35, no.1, 1993.
- Kabir H.R.H., Al-Khaleefi A.M., Chaudhuri R.A., 'Frequency response of a moderately thick antisymmetric cross-ply cylindrical panel using mixed type of Fourier solution functions', , J. of Sound and vibration ,vol.259, no.4,2003.

- Korjakin A., Rikards R., Chate A.,'Analysis of free damped vibrations of laminated composite conical shells', Compo.Stru. vol.41, 1998.
- Meyers C.A. &Hyer M.W.,'Response of elliptical composite cylinders to axial compression loading', Mechanics of Composite Materials and Structures, vol.6, 1999.
- Mohammad S. Qatu, 'Vibration of laminated shells and plates', Elsevier Academic Press, 2004.
- Narita Y., Ohta Y., Yamda G., Kobayashi Y., ' Analytical method for vibration of angle ply cylindrical shells having arbitrary edges', AIAA, vol.30, no.3, 1992.
- NG T.Y., Lam K.Y., Reddy J.N.,'Dynamic stability of cross-ply laminated composite cylindrical shells', Int. J. Mech. Sci., vol.40, no.8, 1998.
- NG T.Y., Lam K.Y., Reddy J.N,'Parametric resonance of a rotating cylindrical shell subjected to periodic axial loads', J. of Sound and vibration, vol.214, no.3, 1998.
- Pinto I.F. Correia, Motasoares C.M., Herskovits J., 'Analysis of laminated conical shell structures using higher order models', C. M. Mota Soares, 2001.
- Seishi Yamada & Croll G.J.A.,'Buckling and post-buckling characteristics of pressure-loaded cylinders', J. of Applied Mechanics, vol.60, 1993.
- Sofiyev A.H.,'the stability of compositionally graded ceramic-metal cylindrical shells under a periodic axial impulsive loading', Composite Structures, vol.69, 2005.
- Soedel W., 'Vibration of shells and plates', 2000.
- Romil Tanov, Ala Tabiei, George J. Simitses,'Effect of static preloading on the dynamic buckling of laminated cylinders under sudden pressure', Mechanics of Composite Materials and Structures, vol.6, 1999.
- Reddy J.N., 'Mechanics of laminated composite plates and shells', CRC Press 2004.
- Reddy J.N. and Liu C.F.,'A higher-order shears deformation theory of laminated elastic shells' I.J.Eng.Sci, vol.23, no.3, 1985.
- Tong T., 'Effect of axial load on free vibration of orthotropic truncated conical shells', ASME, J. of vib. & acous. vol.118,1996.
- Werner Hufenbach, Carsten Holste, Lothar Kroll, 'Vibration and damping behavior of multi-layered composite cylindrical shells', Comp.Stru. vol.58, 2002.
- Xi Z.C., Yam L.H. & Leung T.P., 'Free vibration of composite laminated shell of revolution: Effects of shear non-linearity', Int. J. Mech. Sci., vol.41, 1999.

- Young-Shin Lee, Myung-Hwan Choi, Jae Hoon Kim, 'Free vibrations of laminated composite cylindrical shells with an interior plate', J. of Sound and vibration, vol.265, 2003.

- Youngjin Chung,'Buckling of composite conical shells under combined axial compression, external pressure and bending ',Ph. D. thesis submitted to New Jersey Institute of Technology, 2001.

NOMENCLATURE:

a, b	Dimensions of shell.
A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}, J_{mn}	Arbitrary constants
{A}	Displacement vector
C_{ij}	stiffness matrix elements
E_1, E_2, E_3	Elastic Modulus components (Gpa)
G_{12}, G_{13}, G_{23}	Shear modulus components (Gpa)
H	Thickness (mm)
K	Kinetic energy
L	Cylinder length (mm)
[M]	Inertia matrix
m, n	indices
N_i ,M_i, P_i, S_i, Q_i, K_i (i=1,2,3,4,5,6)	Resultant reactions (N/mm),(N.mm)
N_x	Buckling load (N/mm)
Q_{ij}	Elastic stiffness coefficients
R	Cylinder radius (mm)
U	Potential energy (N.m)
u, v, w, φ_1, φ_2, ψ_1, ψ_2, ψ_3, θ_1, θ_2, θ_3	Displacement components (mm)
z	Distance from neutral axis (mm)
$\varepsilon_{1,2,3,4,5,6}$	Strain components in principle axes direction.
υ_{12}, υ_{13}, υ_{23}	Poisons ratio components
ρ	Density (Kg/m^3)
ω	Frequency (rad/s)
Ω	Frequency parameter for cylindrical shell.
$\sigma_{1,2,3,4,5,6}$	Stress components (Mpa), in principle axes direction.

ABBRIVATIONS:

CST	Classical shell theory
CLPT	Classical plate theory
ESL	Equivalent single layer
FSDT	First order shear deformation theory High shear
HSDT	deformation theory
GTT	General third order theory