COUPLED VERTICAL – TORSIONAL AND LATERAL FREE VIBRATION OF THIN-WALLED CURVED BEAM

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ABSTRACT

This study is concerned with the derivation of differential equation of motion for the free coupled vertical – torsional and lateral vibration of opened thin-walled curved beams. The curved beam to be considered in this study is of isotropic opened thin – walled (I) section with equal top and bottom flanges.

The derivation depends on Hamilton's principle which required finding the potential and kinetic energy of the curved beam section due to internal stresses and all types of movements (Vertical,Torsional and Lateral) .The effect of restrained warping displacement is also considered in this study.

Three differential equations are derived for vertical, torsional and lateral movement .and approximate solutions are developed by using the method of multiple scale via a perturbation technique. The resulting natural frequencies and modes for vertical , torsional and lateral movements are compared with those calculated by using finite element approach (STAAD Pro. 2007) and with the results other studies.

الخلاصة :

تعنى هذه الدراسة باشتقاق المعادلة التفاضلية للاهتزازات الحرة بالاتجاه العمودي والالتوائي والافقي (Vertical,Torsional and Lateral) للجسور المقوسة, وتختص هذه الدراسة بالجسور المقوسة التي تتكون من صفائح رقيقة تشكل مع بعضها بواسطة اللحام لتكون مقاطع مفتوحة بشكل الحرف (I).

اعتمد مبدا هاملتون (Hamilton's principle) في اشتقاق المعادلات التفاضلية بصورة رئيسة وقد تطلب هذا احتساب الطاقة الكامنة والحركية المتولدة في مقطع الجسر المقوس نتيجة الاجهادات المتولدة فيه ونتيجة لجميع أنواع الحركة الناتجة عن الاهتزاز الحر , وقد أخذ بالاعتبار تأثيرتقييد الحركة الالتوائية للمقطع (Warping Displacement) في هذه الدراسة .

توصل البحث الى اشتقاق ثلاث معادلات لاخطية للاهتزازات الحرة بالاتجاه ألعمودي و الالتوائي والأفقي وتم حل كل منها باستعمال طريقة (Perturbation Technique) و قورنت النتائج التي تم التوصل إليها (الترددات الطبيعية والأطوار) بمثيلاتها في دراسات أخرى .

KEYWORD
Thin-walled curved beams, Free vibration, Differential Equation of Motion

INTRODUCTION

Thin-walled beam represents an efficient case for most metal and some concrete structures. Wide development in modern structures, especially the needing for long span elements (with reduced self weight) increases the applications of thin-walled beams in the design due to it's substantial flexural rigidity more over this type of structure should be carefully analyzed because it has low resistance for torsional deformation.

History of thin-walled beams stared in 1961 when Vlasov [15], derived the linear theory of thin-walled beam by a set of four ordinary differential equations, and was later employed by Timoshington, S.P., and Gere. S. M [14] to develop the theory of torsional and torsional-flexural instability.

Culver 1967[4], studied the vibration of horizontally curved beams in a direction normal to the plane of curvature including the effecting of warping. The natural frequencies of prismatic and thin – walled opened sections were determined in this study. Rutenberg 1979[13], evaluated the natural frequencies of curved thin – walled beams with opened cross sections using two simple hand calculation methods. In this study the effecting of shear deformation and flexural rotary inertia were neglected and only the vibrations which is normal to a plane of curvature were considered..

Yoo and Fehrenbach 1981[19], determined the natural frequencies of thin – walled curved beams by using the means of variation procedure to formulate the stiffness relationship taking into consideration the effecting of warping contribution. Another study was represented by Yoo 1987[20], in which mass matrix of order (12x12) was obtained for curved beam element. Warping contribution to torsional behavior has been assessed (in this study) according to the magnitude of a cross – sectional parameter (L^2 G J / E Iw).

Simple closed form solution had been obtained by Roberts, T.M. 1987[12] for the lowest natural frequency of flexural, torsional and flexural-torsional vibration of strait thin-walled beams of opened cross sections. The beam in this study was under the effect of axial force and moment. The derivation depends on an assumption that the sum of potential and kinetic energy (V and KE, respectively) is constant.

Wekezer J.W. 1987[16], used the finite element method to analyze thin-walled beams of variable opened cross sections. The finite element which is considered in this study is a special case of membrane shell with internal constants (Vlasov's and Wagner's assumptions).

In 1989, Wekezer J.W [17], developed a general constant mass matrix for thin-walled curved beams of constant cross section for the case of small amplitude vibration. Finite elements process was used by more than one study (2,8,9) to determine the natural frequencies of curved beams. Ann N. A. 2002[1], developed two and three dimensional curved beam elements with six and seven degree of freedom per node (the seventh degree of freedom was accomplished for the explicit inclusion of warping). Lumped and consistent process were used in this study to developed the mass matrix of order (12x12) and (14x14) (for both six and seven degree of freedom, respectively) .

Genshu Tong, Qiang Xu. 2002[7], provided a detailed derivation of an exact theory for biaxial ending and torsion of thin-walled circularly curved beams with any open profile. The derivation is based on two well-accepted assumptions in the theory of thin-walled members. Exact expressions for longitudinal displacement, longitudinal normal stress and shear stress and their resultants are presented. Simplified theories are also given for practical applications.

Kim N. and Kim M. 2005[10], presented a curved beam theory based on centroid-shear center formulation for the spatially coupled free vibration and elastic analysis .For this, the displacement field is expressed by introducing displacement parameters defined at the centroid and shear center axes .The elastic strain and kinetic energies considering the thickness-curvature effect and the rotary inertia of curved beam were rigorously derived by degenerating the energies of the elastic continuum to those of curved beam and then the equilibrium equations and the boundary conditions are consistently derived for curved beams having non-symmetric thin-walled cross section.

BASIC ASSUMTIONS

- The formulation of the curved beam elements is depend on basic assumptions which are [19] :
- The element is prismatic.
- The cross-section maintains its original shape.
- The deformations are small with respect to the dimensions of the cross-section (linearized problem).
- The material is homogeneous, isotropic and obeys Hooke's low.
- The cross-section dimensions are small in relation to the radius of the curvature.
- Shear deformation in the middle surface vanishes for solid and considered linear across the thickness for tubular sections (for curved beam element without warping), and the position of the center may be considered solely as a geometric property.

FORMULATION OF THE DIFFERENTIAL EQUATION OF FREE VIBRATION

In order to derive the differential equation of free vibration, Hamilton's principle [3] given in eq (1) will be used;

$$\int_{t1}^{t2} \delta(T-V)dt + \int_{t1}^{t2} \delta W_{nc} dt = 0 \quad (1)$$

For the case of free vibration, there is no directly applied dynamic load and the curved beam is assumed to have elastic damping ($C = 2\xi\omega m$)[3] , thus there is no non- conservative force to be considered. Consequently, Hamilton's principle will take the form (eq (2)):

$$\int_{t1}^{t2} \delta(T-V)dt = 0 \quad (2)$$

An element of (dL) length will be considered to calculate the kinetic and potential energy that stored in the carved beam (see **Fig 1**) and method of integration over the whole length of the curved beam will be used to find the total energy (Kinetic or Potential) . The limits of integration were changed to be in term of (θ) in stead of (L) using the following equation:

$$dL = Rd\phi \quad (3)$$

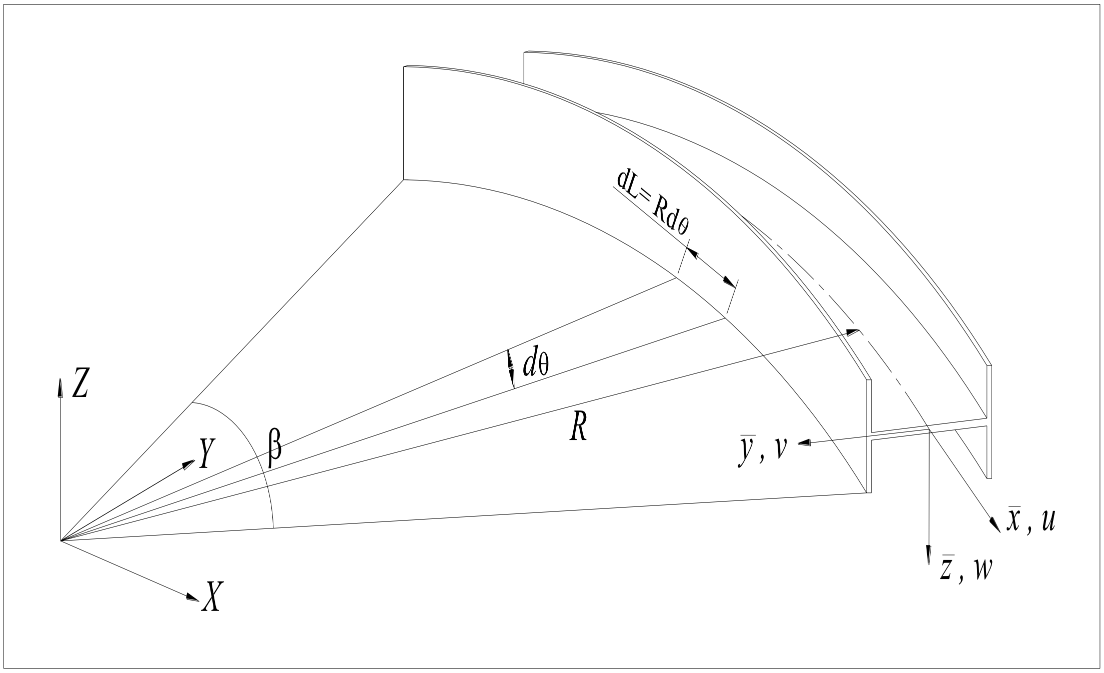


Figure 1: Thin-walled curved Beam layout and Coordinates

Kinetic Energy Of Thin-Walled Curved Beam

The kinetic energy of any system is given by [3];

$$\Pi = \frac{1}{2} mass (velocity)^2 \quad (4)$$

where:

$\Pi = Kinetic\ Energy$

Using equation (4), kinetic energy (stored in thin-walled curved beam) produced in free vibration will be calculated and it will include:

a- Kinetic energy produced due to vertical movement.

$$T_1(t) = \frac{R}{2} \int_0^{\beta} m (\frac{\partial w}{\partial t})^2 \, d\theta \quad (5)$$

b- Kinetic energy developed due to Lateral movement.

$$T_2(t) = \frac{R}{2} \int_0^{\beta} m \, (\frac{\partial v}{\partial t})^2 \, d\theta \quad (6)$$

c- Kinetic energy produced due to rotation of the beam section.

$$T_3(t)=\frac{R}{2}\int_0^{\beta} I_p\ (\frac{\partial\phi}{\partial t})^2\,d\theta \qquad (7)$$

Total Kinetic energy will be:

$$T=T_1+T_2+T_3 \qquad (8a)$$

$$=\frac{R}{2}\int_0^{\beta}(m(\frac{\partial w}{\partial t})^2+m(\frac{\partial v}{\partial t})^2+I_p(\frac{\partial\phi}{\partial t})^2)\,d\theta \qquad (8b)$$

Potential Energy of Thin-Walled Curved Beam.

It's well known that the potential energy is calculating by multiplying the applied force by the corresponding displacement. This concept will be applied to calculate the potential energy that produced in the thin-walled curved beam.

The strain energy produced in the thin-walled curved beam of an (I section) can be divided to :

a- Strain energy stored in the right and left flange due to both normal and flexural stresses (σ_t, σ_f) respectively (see **Fig** 2a&c)

$$V_1=\frac{R}{2}\int_0^{\beta}(\frac{A_f}{E}(\sigma_t+\sigma_f)^2+\frac{A_f}{E}(\sigma_t-\sigma_f)^2)\,d\theta \qquad (9a)$$

Or in terms of strain;

$$V_1=\frac{1}{R}\int_0^{\beta}(A_f E(\frac{\partial u}{\partial\theta})^2+I_{f\bar{z}}\,E(\frac{\partial w}{\partial\theta})^2)\,d\theta \qquad (9b)$$

b- Strain energy stored in the web due to longitudinal direct stress (σ_t) (see **Fig** 2a)

$$V_2=\frac{R}{2}\int_0^{\beta}\frac{A_w}{E}\,\sigma_t^{\ 2}\,d\theta \qquad (10a)$$

Or in terms of strain;

$$V_2=\frac{1}{2R}\int_0^{\beta}EA_w(\frac{\partial u}{\partial\theta})^2\,d\theta \qquad (10b)$$

c- Strain energy stored in the web and flanges due to shear stress (τ_w ,τ_f).(see **Fig** 2d)

$$V_3 = \frac{R}{2}\int_0^\beta \left(\frac{A_w}{G}\tau_w^2 + \frac{2A_f}{G}\tau_f^2\right)d\theta \qquad (11a)$$

Or in terms of strain;

$$V_3 = \frac{RG}{2}\int_0^\beta \left(A_w\gamma_w^2 + 2A_f\gamma_f^2\right)d\theta \qquad (11b)$$

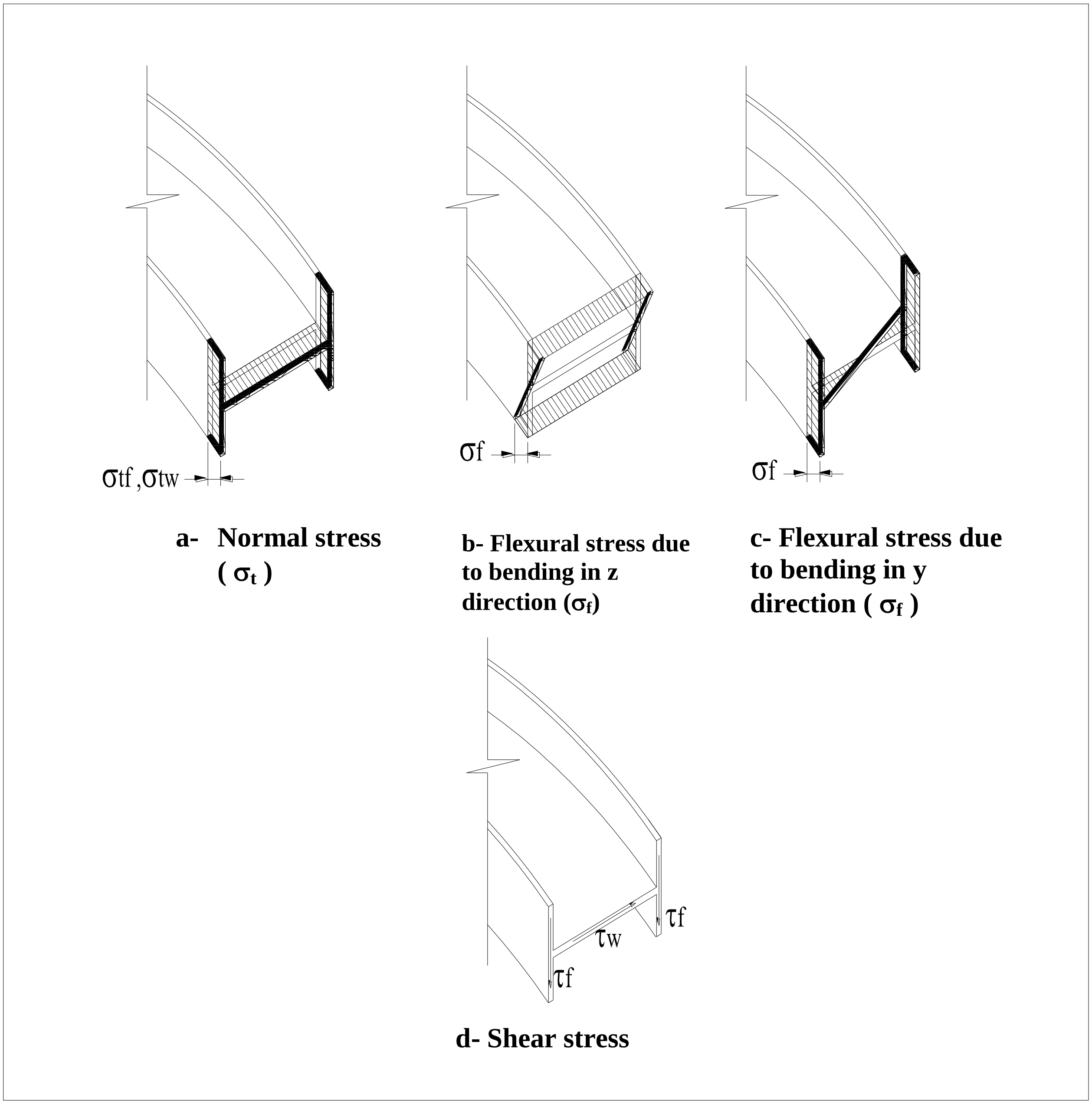


Figure 2 : Types of stress produced in curved thin-walled beam

Examining the estimated movements (vertical, longitudinal, lateral and torsional) for the section of thin-walled curved beam (Fig 3), shear strain for the flange and web can be written as;

$$\gamma_f = \frac{1}{R}\frac{\partial w}{\partial \theta} + \frac{b_f}{2R}\frac{\partial \varphi}{\partial \theta} - \frac{2u}{b_f} \quad (12a)$$

$$\gamma_w = \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{d_w}{2R}\frac{\partial \varphi}{\partial \theta} \quad (12b)$$

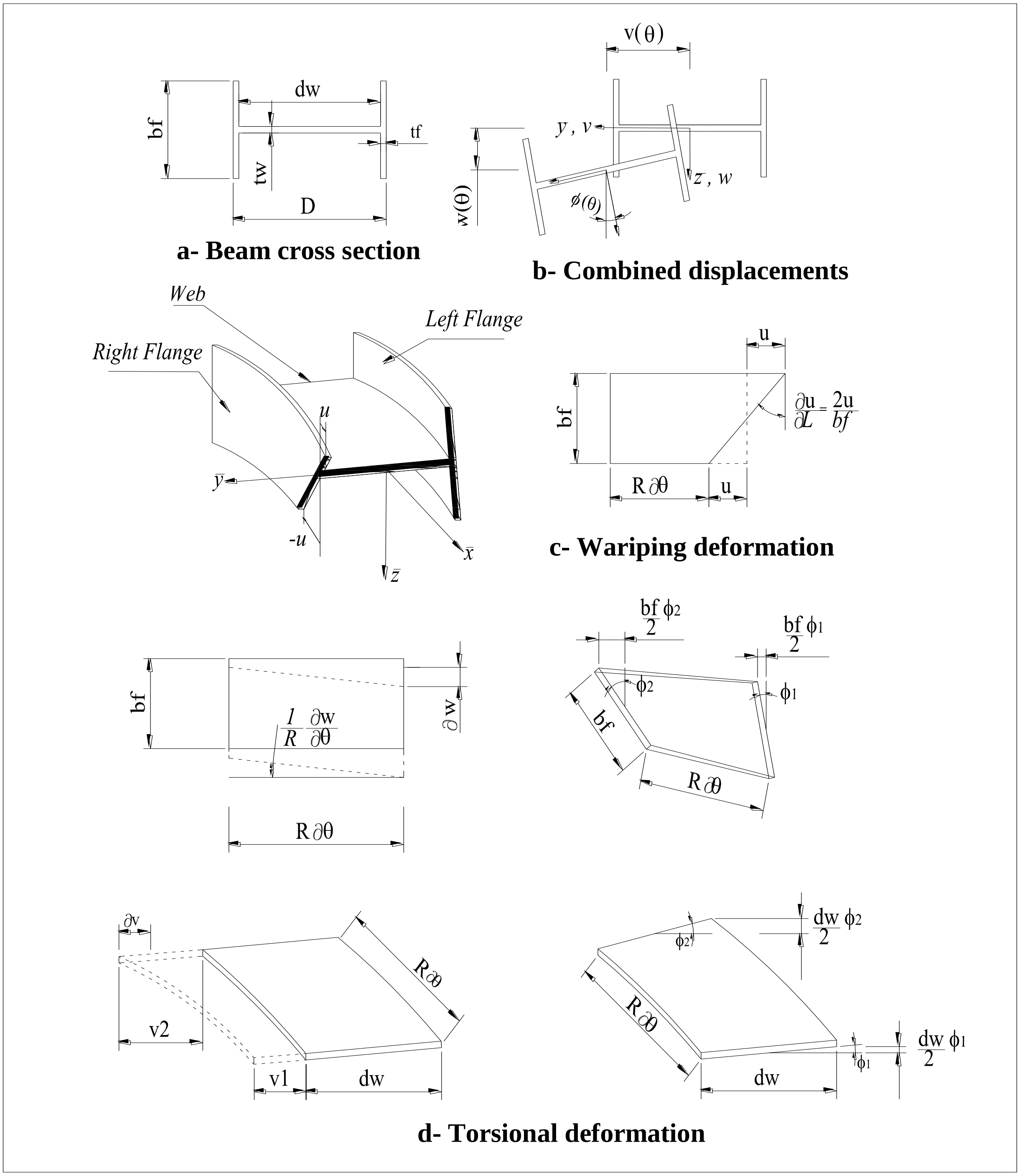


Figure 3 : Types of deformation in thin-walled curved beam

Substituted (Eq. 12) into (Eq. 11) and making the suitable simplification , (V_3) can be written as;

$$V_3 = \frac{RG}{2}\int_0^{\beta}\left(\begin{array}{l} \frac{A_w}{R^2}\left[(\frac{\partial v}{\partial\theta})^2 + d_w(\frac{\partial\phi}{\partial\theta})^2 + \frac{d_w}{2}(\frac{\partial v}{\partial\theta})(\frac{\partial\phi}{\partial\theta})\right] \\ + \frac{2A_f}{R^2}\left[(\frac{\partial w}{\partial\theta})^2 + \frac{b_f^2}{4}(\frac{\partial\phi}{\partial\theta})^2 + \frac{4u^2}{b_f^2} + b_f(\frac{\partial w}{\partial\theta})(\frac{\partial\phi}{\partial\theta}) + \frac{4Ru}{b_f}(\frac{\partial w}{\partial\theta}) + 2Ru(\frac{\partial\phi}{\partial\theta})\right]\end{array}\right)d\theta \qquad (13)$$

In order to replace all terms of (u) by an equivalent expression (in term of *v*,*w* and φ), the following relationships will be depends:

Using Hooke's low [(6)], the shear strain for the web (γ_w , γ_f) can be defined as;

$$\gamma_w = \frac{\tau_w}{G} = \frac{F_{\bar{y}}}{GA_w} \qquad (14a)$$

and

$$\gamma_f = \frac{\tau_f}{G} = \frac{F_{\bar{z}}}{2GA_f} \qquad (14b)$$

References (5,17,19) mentioned the relationship between internal force and the displacements at a point on the middle surface of the member (see **Fig** 4);

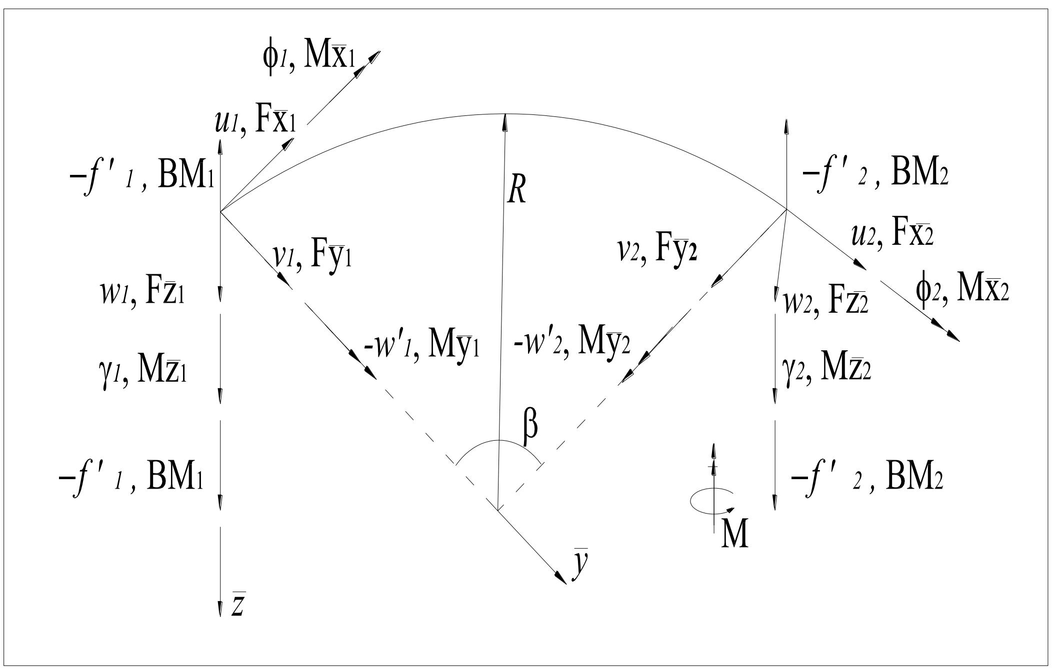


Figure 4 : Generalized forces and displacements for curved beams with warping[(1)]

$$F_{\bar z} = -M_{\bar y}' + \frac{M_{\bar x}'}{R} \qquad (15a)$$

and

$$F_{\bar y} = -M_{\bar z}' \qquad (15b)$$

where

$$M_{\bar y} = EI_{\bar y}\left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\phi}{R^2}\right) \qquad (16a)$$

$$M_{\bar z} = EI_{\bar z}\left(\frac{\partial^2 v}{\partial \theta^2} + \frac{v}{R^2}\right) \qquad (16b)$$

$$M_{\bar x} = GJ\left(\frac{\partial \phi}{\partial \theta} + \frac{1}{R}\frac{\partial w}{\partial \theta}\right) \qquad (16c)$$

Substituted (Eq. 15) into (Eq. 14) and making the suitable simplification , (γ_w , γ_f) can be written as;

$$\gamma_f = \frac{\partial \phi}{\partial \theta}\left(\frac{J}{2RA_f} - \frac{EI_y}{2GRA_f}\right) + \frac{J}{2R^2A_f}\frac{\partial w}{\partial \theta} - \frac{EI_y}{2GA_f}\frac{\partial^3 w}{\partial \theta^3} \qquad (17a)$$

and

$$\gamma_w = \frac{EI_{\bar z}}{2GA_f}\left(\frac{\partial^3 v}{\partial \theta^3} + \frac{1}{R^2}\frac{\partial v}{\partial \theta}\right) \qquad (17b)$$

Substituted value of (Eq. 18a) into (Eq. 13a) , the value of the warping displacement can get as;

$$u = b_f\left[\frac{\partial \phi}{\partial \theta}\left(\frac{J}{4RA_f} - \frac{EI_{\bar y}}{4GRA_f} - \frac{b_w}{4R}\right) + \frac{\partial w}{\partial \theta}\left(\frac{J}{4R^2A_f} - \frac{1}{R}\right) - \frac{EI_{\bar y}}{4GA_f}\frac{\partial^3 w}{\partial \theta^3}\right] \qquad (18a)$$

$$= b_f\left[K_1\frac{\partial \phi}{\partial \theta} + K_2\frac{\partial w}{\partial \theta} - K_3\frac{\partial^3 w}{\partial \theta^3}\right] \qquad (18b)$$

where

$$K_1 = \left(\frac{J}{4RA_f} - \frac{EI_{\bar y}}{4GRA_f} - \frac{b_w}{4R}\right) \qquad (19a)$$

$$K_2 = \left(\frac{J}{4R^2A_f} - \frac{1}{R}\right) \qquad (19b)$$

$$K_3 = \frac{EI_{\bar y}}{4GA_f} \qquad (19c)$$

and

$$\frac{\partial u}{\partial \theta} = b_f\left[K_1\frac{\partial^2 \phi}{\partial \theta^2} + K_2\frac{\partial^2 w}{\partial \theta^2} - K_3\frac{\partial^4 w}{\partial \theta^4}\right] \qquad (20)$$

)

Using the expression of warping displacement (Eq.18), the strain energy (V1,V2,V3 and V4) can be simplified to be in terms of (w,v and φ) as shown below .

$$V_1=\frac{1}{R}\int_0^{\beta}[EA_f\, b_f^2\,(K_1^2\,(\frac{\partial^2\phi}{\partial\theta^2})^2+K_2^2\,(\frac{\partial^2 w}{\partial\theta^2})^2+K_3^2\,(\frac{\partial^4 w}{\partial\theta^4})^2+2K_1K_2(\frac{\partial^2\phi}{\partial\theta^2}\frac{\partial^2 w}{\partial\theta^2})$$
$$-2K_1K_3(\frac{\partial^2\phi}{\partial\theta^2}\frac{\partial^4 w}{\partial\theta^4}-2K_3K_2(\frac{\partial^2 w}{\partial\theta^2}\frac{\partial^4 w}{\partial\theta^4}))+(EA_f b_f^2K_2^2+I_f E)(\frac{\partial w}{\partial\theta})^2]d\theta \tag{21}$$

$$V_2=\frac{EA_w b_f^2}{2R}\int_0^{\beta}\begin{bmatrix}K_1^2(\frac{\partial^2\phi}{\partial\theta^2})^2+K_2^2(\frac{\partial^2 w}{\partial\theta^2})^2+K_3^2\,(\frac{\partial^4 w}{\partial\theta^4})^2+2K_1K_2(\frac{\partial^2\phi}{\partial\theta^2}\frac{\partial^2 w}{\partial\theta^2})\\ -2K_1K_3(\frac{\partial^2\phi}{\partial\theta^2}\frac{\partial^4 w}{\partial\theta^4}-2K_3K_2(\frac{\partial^2 w}{\partial\theta^2}\frac{\partial^4 w}{\partial\theta^4})\end{bmatrix}d\theta \tag{22}$$

$$V_3=\frac{GA_w}{2R}\int_0^{\beta}\left(\left[(\frac{\partial v}{\partial\theta})^2+d_w(\frac{\partial\phi}{\partial\theta})^2+\frac{d_w}{2}(\frac{\partial v}{\partial\theta})(\frac{\partial\phi}{\partial\theta})\right]\right)d\theta \tag{23}$$

$$V_4=\frac{A_f G}{R}\int_0^{\beta}\left(\begin{bmatrix}(\frac{\partial w}{\partial\theta})^2+\frac{b_f^2}{4}(\frac{\partial\phi}{\partial\theta})^2+4\begin{bmatrix}K_1^2(\frac{\partial\phi}{\partial\theta})^2+K_2^2(\frac{\partial w}{\partial\theta})^2+K_3^2(\frac{\partial^3 w}{\partial\theta^3})^2+2K_1K_2(\frac{\partial\phi}{\partial\theta}\;\frac{\partial w}{\partial\theta})\\ -2K_1K_3(\frac{\partial\phi}{\partial\theta}\;\frac{\partial^3 w}{\partial\theta^3})-2K_2K_3(\frac{\partial w}{\partial\theta}\;\frac{\partial^3 w}{\partial\theta^3})\end{bmatrix}\\ +b_f(\frac{\partial w}{\partial\theta})(\frac{\partial\phi}{\partial\theta})+b_f(\frac{4R}{b_f}\frac{\partial w}{\partial\theta}+2R\frac{\partial\phi}{\partial\theta})(K_1\frac{\partial\phi}{\partial\theta}+K_2\frac{\partial w}{\partial\theta}-K_3\;\frac{\partial^3 w}{\partial\theta^3})\end{bmatrix}\right)d\theta \tag{24}$$

The total Potential energy will be:

$$V=V_1+V_2+V_3+V_4 \tag{25}$$

Applying Hamilton's principle and use it's role (The value of δ*v*, δ*w* and δφ are vanish at the limit of integration t_1 and t_2), the three equations of motion will be (method of integration by parts are used to simplified the equations and all terms of higher derivatives than four were neglected in the final result):

Vertical Vibration:

$$-\frac{\partial^4 w}{\partial\theta^4}\left[(Eb_f^2K_2^2)(A_f+\frac{A_w}{2})-4GA_fK_3(4K_2+R)\right]+\frac{\partial^2 w}{\partial\theta^2}(EA_fb_f^2K_2^2+EI_f+GA_f+4K_2^2GA_f+\frac{4RGA_fK_2}{b_f})$$
$$-\frac{\partial^4\phi}{\partial\theta^4}(Eb_f^2K_1K_2A)+\frac{\partial^2\phi}{\partial\theta^2}(8K_1K_2GA_f+GA_fb_f+4RK_1GA_f+2RK_2b_fGA_f)-m\frac{\partial^2 w}{\partial t^2}=0 \qquad (26a)$$

Lateral Vibration:

$$\frac{\partial^2 v}{\partial\theta^2}(\frac{GA_w}{2})+\frac{\partial^2\phi}{\partial\theta^2}(\frac{GA_wd_w}{4})-m\frac{\partial^2 v}{\partial t^2}=0 \qquad (26b)$$

Torsional Vibration:

$$-\frac{\partial^4\phi}{\partial\theta^4}(Eb_f^2K_1^2)(A_f+\frac{A_w}{2})+\frac{\partial^2\phi}{\partial\theta^2}(\frac{GA_wd_w}{2}+\frac{GA_fb_f^2}{4}+4GA_fK_1^2+2GA_fRK_1)$$
$$-\frac{\partial^4 w}{\partial\theta^4}(Eb_f^2K_1K_2A+8K_1K_3GA_f+2RK_3b_fA_fG)+\frac{\partial^2 w}{\partial\theta^2}(8K_1K_2GA_f+GA_fb_f+4RK_1GA_f+2RK_2b_fGA_f)$$
$$+\frac{\partial^2 v}{\partial\theta^2}(\frac{GA_wd_w}{4})-I_p\frac{\partial^2\phi}{\partial t^2}=0 \qquad (26c)$$

PERTURBATION TECHNIQUE

The three derived differential equation of motion (Eqs. 26) are solved analytically by expansion the solution to be in term of the normal modes of the linearized problem and using the perturbation technique (multiple scale method)[11] according to the following steps:

1- Assume the expression of the vertical, lateral and torsional displacement (w,v and ø) to be:

$$w(x,t)=\sum_{m=1}^{\infty}\Phi_{wm}(x)\;\;Z_{wm}(t)$$
$$v(x,t)=\sum_{m=1}^{\infty}\Phi_{vm}(x)\;\;Z_{vm}(t) \qquad (27)$$
$$\phi(x,t)=\sum_{m=1}^{\infty}\Phi_{wm}(x)\;\;Z_{wm}(t)$$

Where $\Phi_{wm}(x)$, $\Phi_{vm}(x)$ and $\Phi_{\phi m}(x)$ are the linear undamped natural vertical, lateral and torsional mth modes, respectively.

2- Substituting the assumed expressing of the three displacement into (Eqs. 26) , multiplying by (w_n,v_m and $ø_o$) and using the orthogonality properties of the linear mode, three coupled system of N-coupled second order ordinary differential equation for Z_{vn} , Z_{wm} and $Z_{øo}$ will obtained.

3- Mode by mode analysis will be used to solve the differential equation.

CASE STUDY :

The three derived differential equation of motion (Eqs. 26) are solved using perturbation technique ,the results (natural frequencies and modes) of the present study was compared with those calculated by Ann N. A. 2002[1], the geometry and material property of the case study will be the same of one used in problem 3 of Ref (1) (see **Fig** 5 and **Table** 1 & 2) .

Table 1 : Geometric properties of the study case

Sectional Property	
b_f = 1.00 m	A_w = 0.05 m^2
d_w = 1.45 m	J = 1.4583333E-4 m^4
$t_w = t_f$ = 0.05 m	$I_{\bar{y}}$ = 8.3489583E-3 m^4
A = 0.175 m^2	$I_{\bar{z}}$ = 7.033333E-2 m^4
A_f = 0.0725 m^2	R = 25.00 m

Table 2 : Material properties of the study case

Material Property
E = 200000 Mpa
G = 200 Mpa
m = 7834.6 kg/m^2

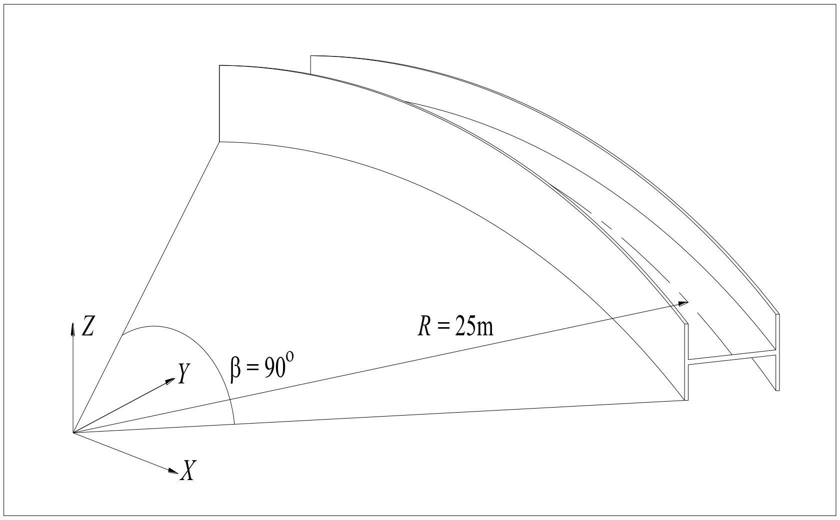
Table 3 : Natural frequency of the study case

Mode		Cyclie Frequency (Hz)			
Type	No.	STAAD Pro. F.E.	Study present in Ref (1)		Present study
			Lumped mass	Consistent mass	
FY	1	1.984	2.015906	1.998710	1.8851
FY & T	2	5.107	5.543756	5.273648	5.082
FY & T	3	9.558	11.850720	10.820084	9.483
FY & T	4	10.574	--	11.583677	10.324
FY & T	5	13.129	--	14.379131	13.100
FZ	1	17.055	18.215634	18.302639	17.104
FZ	2	24.368	25.096167	25.275968	24.233

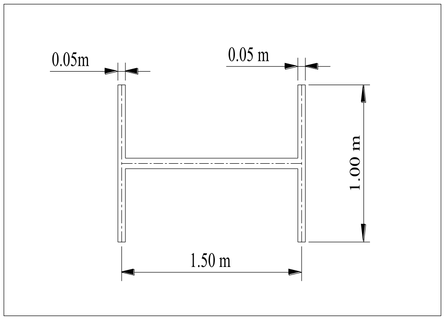
FY: Flexural mode in Y- direction .
T : Torsional mode .
FZ : Flexural mode in Z- direction .

Table 4 : Percentage of difference between the presented study and other theories

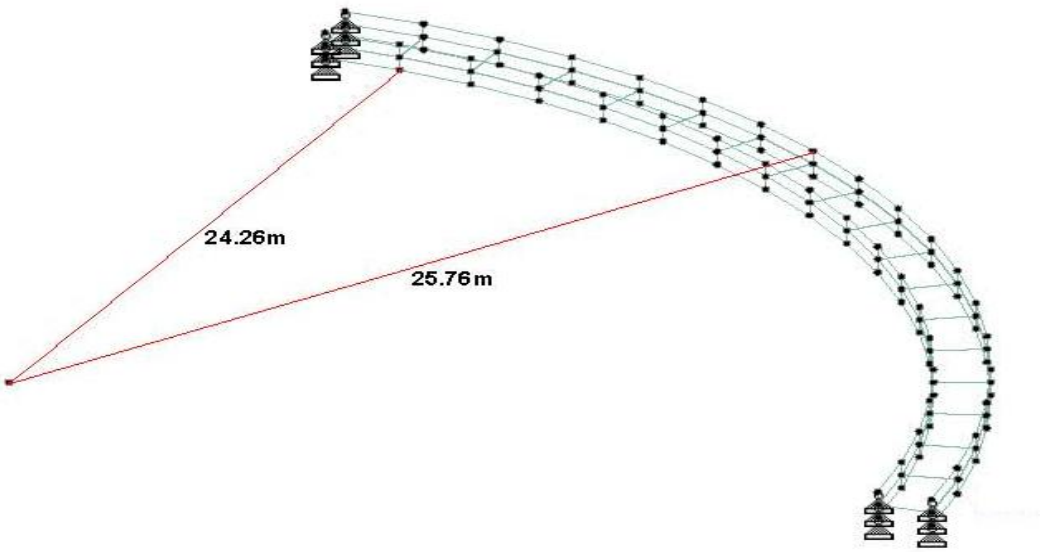
Mode		Percentage of difference		
Type	No.	STAAD Pro. F.E.	Study present in Ref (1)	
			Lumped mass	Consistent mass
FY	1	5.25	6.49	5.68
FY & T	2	0.49	8.33	3.63
FY & T	3	0.79	19.98	12.36
FY & T	4	2.42	---	10.87
FY & T	5	0.22	---	8.90
FZ	1	0.29	6.10	6.55
FZ	2	0.56	3.44	4.13



a- Layout of curved beam

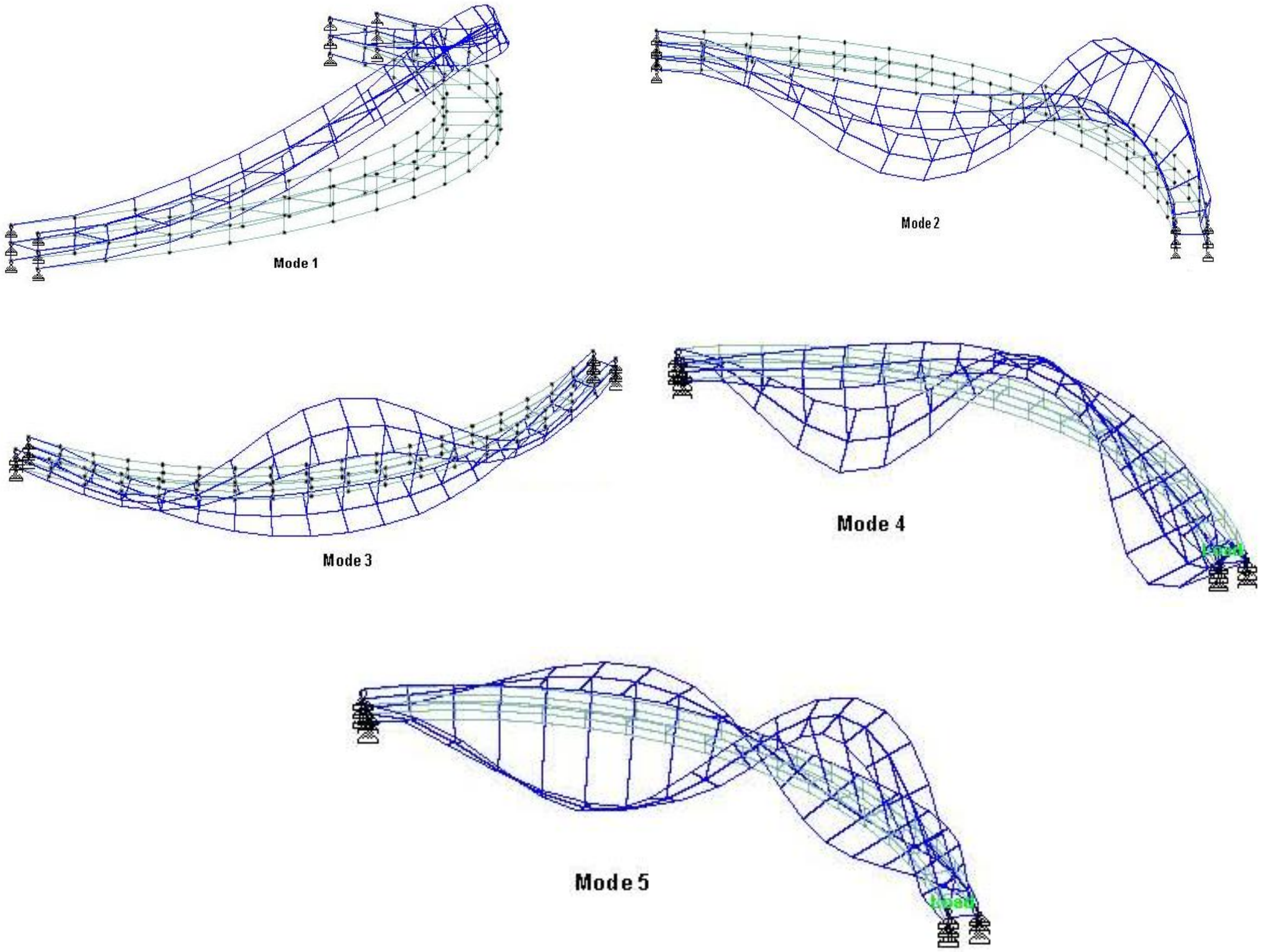


b- Cross section dimentions

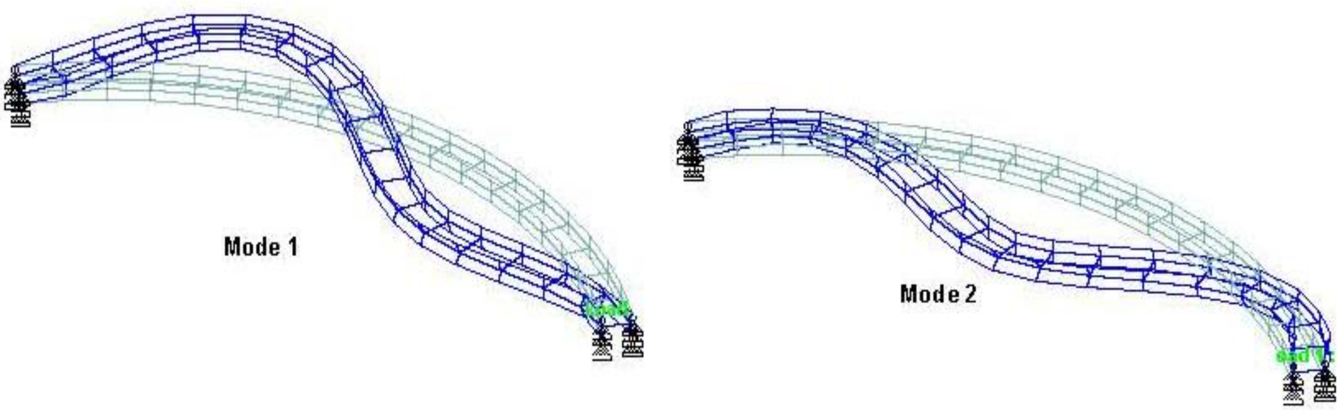


c – Finite element mesh

Figure 5: Case study No.1 of curved thin walled beam



a- Flexural and Torsional modes of out- of- plane action



b- Flexural modes of in- plane action

Figure 6 : Mode shapes of the study case

CONCLOSIONS

In the present study, the differential equations of motion (D.E.O.M) for combined vertical, torsional and lateral vibration of thin-walled curved beam are derived and solved. The calculated natural frequencies are compared with those calculated by the finite element approach (STAAD Pro. 2007) and with the results of the estimating element developed by Ann N. A. 2002[1].

Based on the result obtained from different case study that considered to study the effecting of different parameters on the natural frequencies, this study arrives with the following objectives and conclusion:

- Effecting of shear deformation:

The values of natural frequencies which obtained from the present study (Table 3) are differ from those obtained by an estimating element developed by Ann N. A. 2002[1] due to the effecting of shear deformation which is included in the present study while its ignored in the estimating element[1]. It's also clear that the effect of shear deformation can be neglected in deep thin – walled beam element

- Effecting of flange's width on the natural frequency :

The width of the flange has a direct and inverse effect on the value of the natural frequency for both vertical and lateral vibration respectively (see Fig 7 and 8) .Figure (7) shows the effect of the flange width on the frequency of vertical vibration and its seem that the relation is a direct relation, while figure (8) shows the effecting of the same parameter on the frequency of the lateral vibration. The previous conclusion is due to the influence of the flange width on the inertia of the curved beam in both vertical and lateral direction.

- Effecting of the web's width on the natural frequency :

This study detects an effecting for the web width on the vertical frequency differs from the one produced by the flange width (see Fig 9). The relationship between the web width and the vertical frequency can be divided into two parts, the first part (value of b_w/b_f approximately less than 60%) shows that b_w effects on the value of the vertical frequency directly , while the effecting will be inversely for the other value of b_w/b_f . b_w has also a direct action on the frequency of the lateral vibration (see Fig 10).

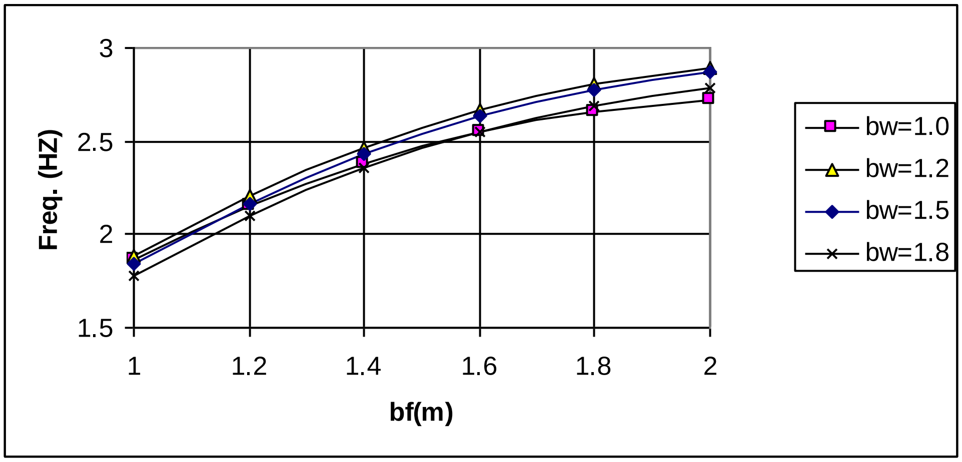


Figure 7 : Effects of b_f on the natural frequency of vertical vibration

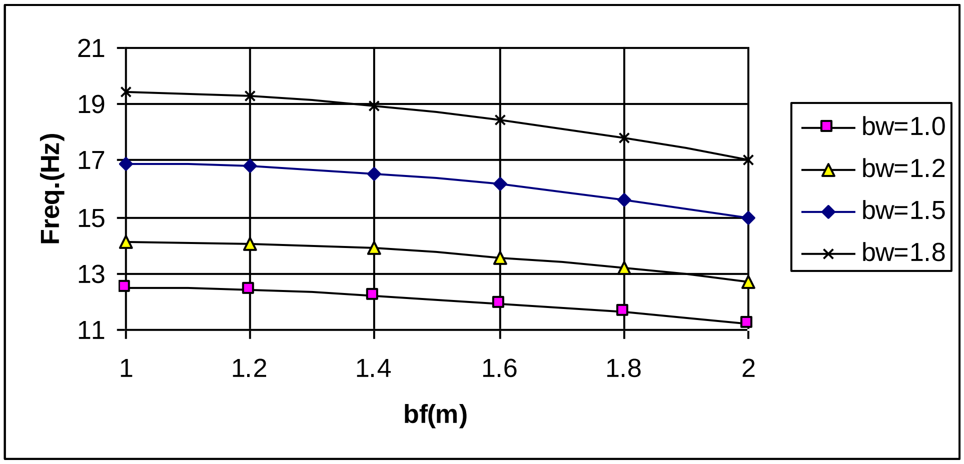


Figure 8 : Effects of b_f on the natural frequency of lateral vibration

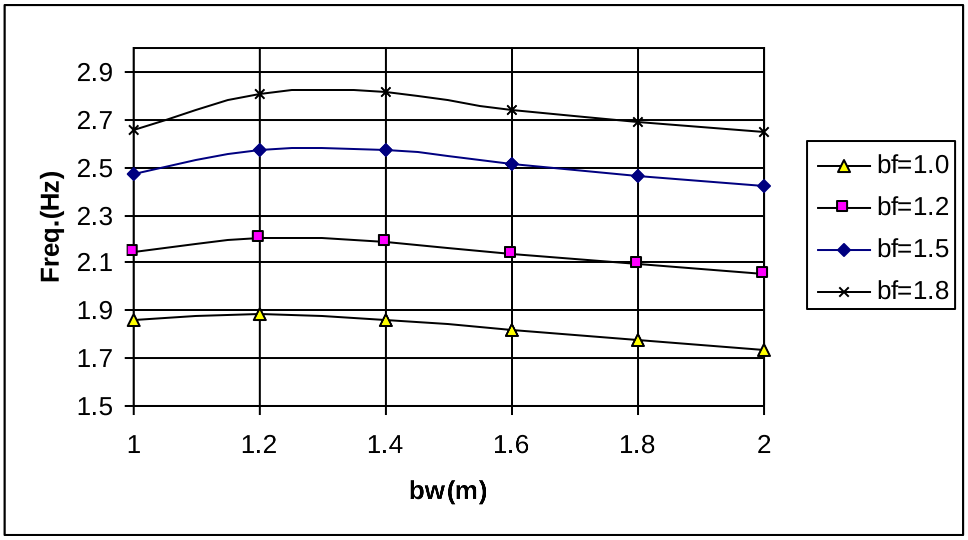


Figure 9 : Effects of b_w on the natural frequency of vertical vibration

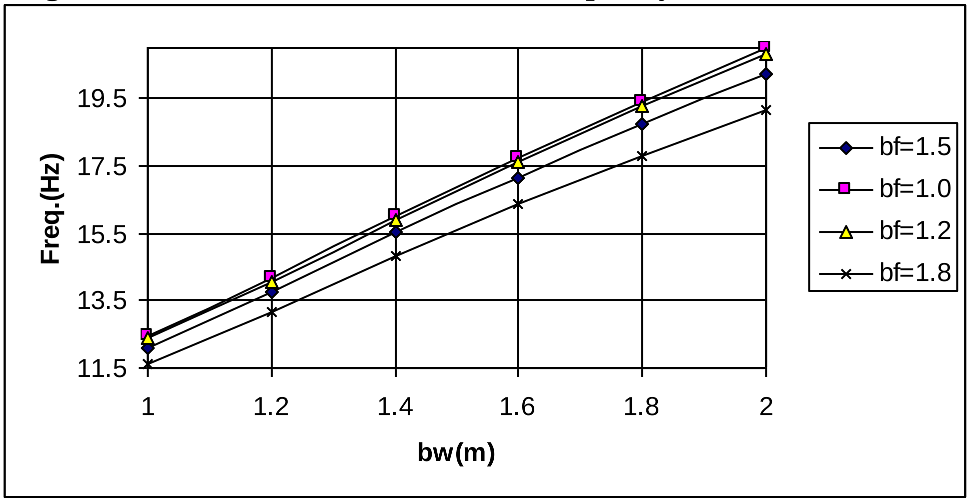


Figure 10 : Effects of b_w on the natural frequency of lateral vibration

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LIST OF SAMBOLS

A_f	Cross sectional area of the right or left flange.
A_w	Cross sectional area of the web.
b_f	Width of the flange.
D	Total depth of the thin-walled curved beam.
dw	Total depth of the web
E	Modulus of elasticity.
G	Shear Modulus of elasticity.
$I_{f\bar{z}}$	Second moment of inertia of the right or left flange about $\bar{z}$ axes.
I_p	Mass polar moment of inertia.
$I_{\bar{y}}$	Second moment of inertia about $\bar{y}$ axes.
$I_{\bar{z}}$	Second moment of inertia about $\bar{z}$ axes.
J	St. Venant's torsional constant.
m	Mass per unit length.
R	Radius of curvature .
t_f	Thickness of the of the right or left flange.
t_w	Thickness of the web.
T	Total kinetic energy.
u,w,v	Displacements in direction of $\bar{x},\bar{z}$ *and* $\bar{y}$ axes, respectively.
$\bar{x},\bar{z},\bar{y}$	Local curvilinear coordinate.
X,Y,Z	Global coordinate.
V	Potential energy.
Wnc	Non-Conservative work.
β	Angle of curvature for the thin-walled curved beam.
φ	Angle of twist.
σ_t	Direct longitudinal stress.

σ_f	Flexural stress.
τ_f	Shear stress produced in the left or right flange.
τ_w	Shear stress produced in the web.
γ_w	Shear strain of the web.
γ_f	Shear strain of the left or right flange.
δ	Variation operator taken during the indicated time interval