# DERIVATION OF THE LORENTZ-EINSTEIN TRANSFORMATION VIA ONE OBSERVER

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**ABSTRACT:**

Lorentz-Einstein transformation derived by Einstein in his theory of special relativity. Physical laws and principles are invariant in all Galilean reference frames under this transformation. The transformation in every day use in a host of contexts as in free solution of the Dirac equation in the modern field of heavy ion in atomic physics. Most books on theoretical physics and special theory of relativity and all research papers have derived the Eorentz-Einstein transformation using various propositions and employing two observers each located in Galilean system with relative motion receding the same events in the space-time manifold. This paper derives Lorentz-Einstein transformation by proposing just one observer using local coordinates of two Galilean system with relative motion following the track of a spherical pulse of light, which to our knowledge is not found in the literature.

**الخلاصة:**

انشتاين في نظريته النسبية الخاصة على تحويل يدعى الان تحويل لورانتز- انشتاين حيث تكون القوانين الفزيائية لازمة في جميع الانظمة الغاليلية في هذا التحويل. ان لهذا التحويل استخدامات متعددة كما هو الحال في معادلة ديراك في المجال الحديث للايون الثقيل في الفيزياء الذرية. يشتق تحويل لورانتز- انشتاين في معظم كتب الفيزياء النظرية وكتب النظرية النسبية وفي جميع البحوث بموجب فرضيات متباينة وبفرض وجود مشاهدين كل واحد منهما مستقر عند مرجع نظام غاليلي بينهما حركة نسبية ويحسب كل واحد منهما على انفراد نفس الحوادث في فضاء الثر مكان يهدف لأشتقاق تحويل لورانتز- انشتاين بفرض مشاهد واحد يستخدم احداثيات نظامين غاليليين بينهما حركة نسبية ويتبع المسار لنبضة ضوئية كروية.

**KEY WORDS:** Relativity theory, Lorentz-Einstein transformation.

## INTRODUCTION:

### - Four-Dimensional Manifold.

An observer observing the positions in three-dimensional Euclidean space of certain events records with reference to an orthogonal Cartesian system X of axes (x, y, z) or $(x^i)$ , i=l, 2, 3 the space coordinates. The observer using system of axes also measures by means of synchronized clocks stationary in the system, the time " t " at which the events occupying the observed positions thus an event is completely recorded by the $(x^i)$ , i=l, 2, 3, 4 where the $x^4$ denotes the variable time " t ", which is one-dimensional continuum " *The variable " t " is regarded to be independent not only of the space variables* $(x^i)$, *i=l, 2, 3 but also of the possible motion of the space reference system*" [1].

It is possible to think of $(x^i)$, i=l, 2, 3, 4 as a point in Four-dimensional manifold, which may be called the space -time manifold of four dimensions. The same observer who is observing the same events will similarly be able to record these events in terms of $(\bar{x}^i)$, i=l, 2, 3, 4 with reference to another orthogonal Cartesian system $\bar{X}$ of axes where $(\bar{x}^4)$, is the variable time t measured by clocks stationary in the system $\bar{X}$. Therefore, $(x^i)$ and $(\bar{x}^i)$, must be uniquely related to each other, since both record the same events in this four-dimensional manifold.

### - The Fundamental Laws of Newtanian Mechanics

The first fundamental laws of mechanics as stipulated by Newton (which is so-called the law of inertia) states that "*a material body continues to be in its state of rest or of uniform rectilinear motion unless it is acted by an external force*" [2].

This law holds only in a special reference frames, which are known as "Galilean system of reference (or inertial systems of reference)". All systems of reference are equivalent from point of view of mechanics, in any such system the three laws of Newtonian mechanics hold and are invariant, that is the laws of mechanics have the same form, which is called the "Galilean principle of relativity".

### - GALILEAN TRANSFORMATION

Consider a Galilean system of reference X, and a second system $\bar{X}$ which is moving with a constant velocity V with respect to the first system. Taking the space axes of the systems in such away that they are respectively parallel to one another and oriented so that the $x^1$ and $(\bar{x}^1)$, axes are parallel to the velocity V. The formulas for the change of axes are:

$$\left.\begin{aligned} \bar{x}^1 &= x^1 + Vx^4 \\ \bar{x}^2 &= x^2 \\ \bar{x}^3 &= x^3 \\ \bar{x}^4 &= x^4 \end{aligned}\right\} \qquad \ldots(1)$$

Where the origins of the two systems coincide at = $x^4$ =0
The set of equations (1) is called a "Galilean transformation ". This leads to the invariance of Newton laws of mechanics in $\bar{X}$, thus the system $\bar{X}$ is a Galilean system too, therefore the laws of inertia valid in X if it is valid in $\bar{X}$
From (1)

$$\frac{d\bar{x}^1}{d\bar{x}^4} = \frac{dx^1}{dx^4} + V \qquad \dots (2)$$

Therefore the velocity of a particle is not invariant when transformed from X to $\bar{X}$ under Galilean transformation. Hence, certainly a statement of any law that depends on the velocity relative to two Galilean systems, it will not be formally invariance when transformed from one Galilean system to another Galilean system. In particular the fundamental laws of electromagnetic are not invariant with respect to Galilean transformation because these laws depend on the velocity of propagation of light. To achieve the invariance of the fundamental laws of electromagnetic as well as mechanics, it is necessary to abrogate the hypotheses that the time is the same for all observer using Galilean systems independent of their relative motion, that is the time " t " is not universal and each system has its own time.

## - LORENTZ-EINSTEIN TRANSFORMATION

To resolve this crises in physics, in 1905, Einstein proposed two propositions:-

- physical laws and principles have invariant form in all Galilean systems.
- The speed of light in free space has the same constant value in all Galilean systems. [3]

An observer employing the two reference frames X and $\bar{X}$ as defined above finds that a particle acted by no force, will describe a straight line in the X reference frame, this means that its space coordinates ($x^i$), i=l, 2, 3 referred to the X-system will be linearly expressed in terms of the time $x^4$. According to the first proposition of Einstein, the same observer finds that the particle will also describe a straight line in the $\bar{X}$-system, consequently the space-time coordinate of $\bar{X}$ are linear functions of the space-time coordinates of X. Thus:

$$\left.\begin{aligned} \bar{x}^i &= a^i_j x^j + a^i_4 x^4 \\ \bar{x}^4 &= a^4_j x^j + a^4_4 x^4 \end{aligned}\right\} \qquad \dots (3) \text{ (summation on j and I, j=1, 2, 3)}$$

Where the a's are functions of the constant velocity V.

Now, the event $x^i$=0, coincides with the event $(\bar{x}^i)$=0, i=l, 2, 3, 4.

Since the relative motion is parallel to $x^1$ , the planes $x^2=0$ and $x^3 = 0$ must be identical.
Therefore:
$a_1^{\,2} = a_3^{\,2} = a_4^{\,2}=0$
Because the $x^1$, $(\bar{x}^1)$, axes in the direction of the velocity V, $x^2$ and $x^3$ cannot be involved in the equations of $x_2$and $x_4$, and therefore**,**
$a_2^{\,4} = a_3^{\,4}=0$
$a_2^{\,1} = a_3^{\,1}=0$
Thus,

$$\left.\begin{aligned} \bar{x}^1 &= a_1^1 x^1 + a_4^1 x^4 \\ \bar{x}^2 &= a_2^2 x^2 \\ \bar{x}^3 &= a_3^3 x^3 \\ \bar{x}^4 &= a_1^4 x^1 + a_4^4 x^4 \end{aligned}\right\} \quad \ldots (4)$$

All directions normal to V are equivalent, hence $\bar{x}^2 = \alpha(v)x^2$
Where $\alpha$ is a function of v. But it is also true $\bar{x}^2 = \alpha(-v)\bar{x}^2$
Hence $\therefore \alpha(v)= \alpha(-v)$
For directions normal to V , the sign of V unimportant $\therefore \alpha(v)=1$
Therefore

$$\left.\begin{aligned} \bar{x}^1 &= a_1^1 x^1 + a_4^1 x^4 \\ \bar{x}^2 &= x^2 \\ \bar{x}^3 &= x^3 \\ \bar{x}^4 &= a_4^1 x^1 + a_4^4 x^4 \end{aligned}\right\} \quad \ldots (5)$$

Put $a_1^1 = A, +a_4^1 = B, a_1^4 = E, a_4^4 = F$
Then (5) becomes

$$\left.\begin{aligned} \bar{x}^1 &= Ax^1 + Bx^4 \\ \bar{x}^4 &= Ex^1 + Fx^4 \end{aligned}\right\} \quad \ldots (6)$$

Taking differentials to obtain

$$dx^1 = Adx^1 + Bdx^4$$
$$dx^4 = Edx^1 + Fdx^4$$

$$\therefore \frac{d\bar{x}^1}{d\bar{x}^4} = \frac{A\dfrac{dx^1}{dx^4} + B}{E\dfrac{dx^1}{dx^4} + F} \quad \ldots (7)$$

Suppose that a spherical pulse of light sent out from the point $P(x^i)$, 1=1, 2, 3 of the system X at the time $x^4$. Light travels with constant velocity in all directions independent of the Galilean reference frame, [3, 4]. This means that:

$$\therefore \frac{dx^1}{dx^4} = C \quad \text{and} \quad \frac{d\bar{x}^1}{d\bar{x}^4} = C$$

Substituting in the above relationship, then

$$C = \frac{AC + B}{EC + F}$$

Or $EC^2+(F-A)C-B=0$ …(8)

Now for a particle not in motion in the X-system, it has the velocity V in the

$\bar{X}-$ system, hence $\frac{dx^1}{dx^4}=0, \quad \frac{d\bar{x}^1}{d\bar{x}^4}=V$

∴ Substituting in (7) then

$$V=\frac{B}{F} \quad \text{or} \quad B=EF \qquad \dots(9)$$

For the same reasons

$\frac{d\bar{x}^1}{d\bar{x}^4}=0, \qquad \frac{dx^1}{dx^4}=-V \qquad$ then (7) gives

$-AV+B=0 \quad \therefore B=AV \qquad \dots(10)$

From equations 8, 9 and 10

$$F=A, \qquad E=\frac{AV}{C^2},$$

After substituting in (6), the result is

$$\left.\begin{aligned} \therefore \bar{x}^1 &= A(x^1+x^4) \\ \bar{x}^4 &= A(x^1\frac{V}{C^2}+x^4) \end{aligned}\right\} \qquad \dots(11)$$

But for the spherical pulse,

$$(dx^1)^2-C^2(dx^4)^2=(d\bar{x}^1)^2-C^2(d\bar{x}^4)^2 \qquad \dots(12)$$

From (11) and (12), one may find that

$$A=\frac{1}{\sqrt{1-\frac{V^2}{C^2}}}$$

Hence (11) becomes

$$\left.\begin{aligned} \bar{x}^1 &= \frac{1}{\sqrt{1-\frac{V^2}{C^2}}}(x^1+Vx^4) \\ \bar{x}^4 &= \frac{1}{\sqrt{1-\frac{V^2}{C^2}}}(x^1+Vx^4) \end{aligned}\right\} \qquad \dots(13)$$

Equation (13) is what so called the Lorentz-Einstein transformation's, 6].

**CONCLUSIONS:**

This paper presents a method for driving the Lorentz-Einstein transformation by proposing one observer following a track of a spherical pulse of light in two Galilean reference frames in relative motion.

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