



HYDRODYNAMIC STUDIES OF BED EXPANSION IN LIQUID SOLID FLUIDIZED BED

Sawsan A. M. Mohammed

Chemical Engineering Department – College of Engineering – University of Baghdad – Iraq

ABSTRACT

Expanded bed behavior was modeled by using the Richardson-Zaki correlation between the superficial velocity of the feed stream and the void fraction of the bed at moderate Reynolds number. The terminal velocity expression was developed by introducing two empirical parameters, the effective diameter of the particles and an exponent for the $[(\rho_p - \rho_l)^2 / \rho_l \mu_l]$ term. The Richardson-Zaki exponent constant (n) was found to vary with the density ratio $(\rho_p - \rho_l) / \rho_l$ and diameter ratio d_p / D . It was noted that when the density ratio $(\rho_p - \rho_l) / \rho_l$ is less than one, there is no dense phase at the bottom of the test section. However, for density ratio $(\rho_p - \rho_l) / \rho_l$ greater than unity, there exists accelerating or dense regime at the bottom of the test section.

الخلاصة:

تم في هذا البحث تمثيل تصرف الاعمدة المتمددة باستخدام علاقة ريتشاردسن-زاكي بين السرعة الظاهرية للمجرى الداخل ونسبة الفراغ للعمود في حدود معتدلة من رقم رينولدز. تم استحداث علاقة جديدة للسرعة النهائية وذلك بادخال مقدارين تجريبيين وهما القطر المؤثر لحبيبات الحشوة ومقدار اسي للحد $[(\rho_p - \rho_l)^2 / \rho_l \mu_l]$ لقد وجد ان المقدار الاسي (n) لعلاقة ريتشاردسن - زاكي يتغير مع الحد $(\rho_p - \rho_l) / \rho_l$ والنسبة بين قطر الحبيبة وقطر العمود d_p / D . كذلك تم ملاحظة انه عندما تكون نسبة الكثافة $(\rho_p - \rho_l) / \rho_l$ اقل من واحد لم يظهر طور كثيف في اسفل العمود بينما في حالة كون نسبة الكثافة اكبر من واحد ظهر طور كثيف متسارع في اسفل العمود.

KEYWORDS: Liquid-solid fluidized bed, Bed expansion, Richardson-Zaki correlation. دراسة هيدروديناميكية لتمدد الاعمدة المسالة المتكونة من الصلب والسائل.

INTRODUCTION

Fluidized beds find extensive applications in chemical process industries as they provide large interfacial area, high degree of mixing, and temperature uniformity. In particular, liquid-solid fluidization beds are increasingly used in chemical processes such as fermentation, biological wastewater treatment, flow gas desulfurization, ore reduction etc. Although fluidization can be achieved either by liquid or gas as fluidizing medium, gas fluidized beds have gained more importance in scientific community due to its far more applications. This is in spite of the fact that possible uses of liquid fluidization in the mining industry were suggested as early as in 16th century as a means of separating solids of different sizes. Since the emergence of biosciences in the recent years and the adaptability of liquid fluidized bed for various applications, the importance of solid-liquid fluidized bed is receiving greater attention among scientists and researchers (Navaez et al., 1996, Yang and Renken, 2003, Bo and Yan, 2003). In recent years, the applications of liquid-solid fluidized beds are being extended for hydrometallurgy, food technology, biochemical processing, etc. Scientific research concerns with reference the hydrodynamic structure of liquid-particle, the equilibrium forces for fluid-particle interactions and heat or mass transfer properties in fluidized beds. Fluidization quality is closely related to the intrinsic properties of particles, e.g. particle density, particle size and size distribution, and also their surface characteristics. The expansion characteristic of solid particles in a liquid-solid fluidized bed is a function of superficial liquid velocity. A quantitative relationship linking the bed expansion with these parameters is necessary for a fundamental understanding of fluidization behavior and subsequent applications (Richardson and Jeronimo, 1979). The first important study of the forces acting on an immersed body moving relative to a viscous fluid was made in 1851 by Stokes who derived an equation for the viscous resistance to the motion of a single spherical particle in an infinite fluid.

$$F = 3\pi \mu_l V d_p \quad (1)$$

The terminal falling velocity V_t of the particle is obtained by equating the viscous drag as given by eq. (1), to the effective gravitational force.

$$3\pi \mu_l V_t d_p = \frac{\pi d_p^3}{6} (\rho_p - \rho_l) g$$

Hence

$$V_t = \frac{g d_p^2 (\rho_p - \rho_l)}{18\mu_l} \quad (2) \quad \text{for } Re_p < 0.4$$

In the flow region where Re_p varies between 2 and 500, which has been called the intermediate law, Lewis et al., 1952 defined the terminal velocity by the equation:



$$V_t = \frac{0.153g^{0.7}d_p^{1.14}(\rho_p - \rho_l)^{0.71}}{\rho^{0.29}\mu_l^{0.43}}$$

Wen and Yu, 1966 estimated the terminal velocity at moderate Reynolds number using the following equation:

$$V_t = \left[\frac{4(\rho_p - \rho_l)^2 g^2}{225 \rho_l \mu_l} \right]^{1/3} d_p \quad (3) \quad \text{for } 0.4 < \text{Re}_p < 500$$

Expanded bed technology lacks understanding about its hydrodynamic behavior as has been available for the conventional packed column technology. Therefore a method to describe the behavior of the expanded bed incorporating the properties of particles and feed stream should be provided.

MATHEMATICAL APPROACH

There are two methods to describe the expanded bed behavior. One is the dimensional analysis method, which has been successfully applied for the design of various fluid processing equipments. The other is the Richardson-Zaki equation, which predicts a linear correlation between logarithms of the superficial fluid velocity (V_0) and the void fraction of the bed ($(1 - \phi_s)$) for fluidized bed by

$$\log V_0 = n \log(1 - \phi_s) + \log V_t \quad (4)$$

where ϕ_s and V_t are the solid fraction of the expanded bed and the terminal settling velocity of the particle at infinite dilution ($\phi_s = 0$), respectively (Richardson and Zaki, 1954, Davidson and Keams, 1978).

The superficial velocity V_0 is determined by dividing the volumetric flow rate of the feed stream by the cross sectional area of the column.

The solid fraction ϕ_s of the expanded bed at height H can be obtained by

$$\phi_s = \phi_0 \frac{H_0}{H} \quad (5)$$

where ϕ_0 , and H_0 , are the solid fraction and height of the bed at zero flow stream, respectively. Combining eqs. (5) and (4) results

$$H/H_0 = \phi_0 / \left[1 - (V_0/V_t)^{1/n} \right] \quad (6)$$

An attempt was made to correlate values of n and V_t with the properties of particles and feed streams.

EXPERIMENTAL WORK

The schematic representation of experimental set-up is shown in Fig.1; it consists of a glass column of 2.45 cm internal diameter and a height of 75 cm. The column was packed with glass beads (0.4 – 0.6) mm in diameter, to a height of 8 cm. The fluidizing particles are supported by a wire mesh fitted at the column bottom. The liquid from the storage tank was pumped through a rotameter connected on the line. In order to change the solution properties, tap water and glycerol solutions (10, 20 and 30 wt percent) were used as feed streams. The viscosity of each glycerol solution was obtained by using a Fann V-G meter and density was measured by a standard hydrometer. The height of the bed was measured over a certain range of feed velocities.

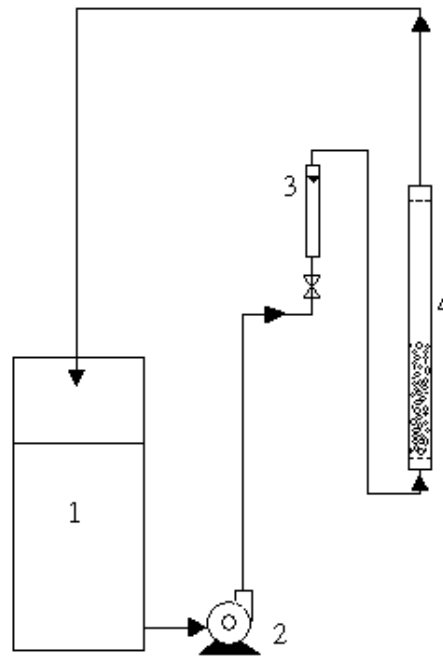


Fig.1: Schematic diagram of the experimental set up (1.liquid reservoir, 2.pump, 3.flow meter, 4.test section)

Another set of experiments were made using a variety of solid particles with tap water to estimate the effect of density ratio ($(\rho_p - \rho_l / \rho_l)$) and diameter ratio (d_p / D) on the Richardson-Zaki exponent constant (n). The solids used and therein characteristics are given in Table 1.

Table 1: Solid particles used and their characteristics

| Particles used | d_p , mm | ρ_p , kg/m ³ | V_{mf} , m/s |
|----------------|------------|------------------------------|----------------|
| Glass beads | 0.4 -0.6 | 2250 | 0.0018 |
| Perspex beads | 0.5-2 | 1600 | 0.0051 |
| Porcelain | 1.3 | 2100 | 0.0094 |



| | | | |
|------------------|-------|------|--------|
| PVC beads | 2.1 | 1750 | 0.0137 |
| Activated carbon | 1.8 | 1350 | 0.0058 |
| Silica gel | 1.5-2 | 1900 | 0.0124 |
| Sand | 0.66 | 2600 | 0.0039 |

The minimum fluidized velocity for the solid materials (V_{mf}) had been estimated using the following equation (Ramaswamy, 2008):

$$V_{mf} = \frac{\mu_l}{d_p \rho_l} \left[\sqrt{(33.7)^2 + 0.0408 \frac{d_p^3 \rho_l (\rho_p - \rho_l) g}{\mu_l^2}} - 33.7 \right] \quad (7)$$

RESULTS AND DISCUSSION

Modification of the terminal velocity expression using the Richardson-Zaki correlation:

The effects of superficial velocity (V_0) of the feed stream on the bed expansion (H/H_0) were measured. The bed expansion factor was limited below 3 as recommended by Reichent et al., 2001, for proper performance of expanded beds.

A dimensional analysis can be performed for the expansion of the bed. The bed expansion (H/H_0) can be expressed as a function of various operating conditions, such as the fluid velocity V , particle diameter d_p , solution viscosity μ_l , and fluid density ρ_l , by the following equation

$$H/H_0 = f(V, d_p, \mu_l, \rho_l) \quad (8)$$

Dimensional analysis of eq. (8) propose that the bed expansion may be represented by eq. (9) using the particle Reynolds number ($Re_p = d_p V \rho_l / \mu_l$) as a single combined parameter incorporating various operating conditions.

$$H/H_0 = g(Re_p) \quad (9)$$

Fig.2 shows the bed expansion as a function of particle Reynolds number, the expansion of the bed becomes greater as the solution viscosity increases at a fixed flow rate of the feed stream. The average diameter of the particles was used to calculate Re_p values.

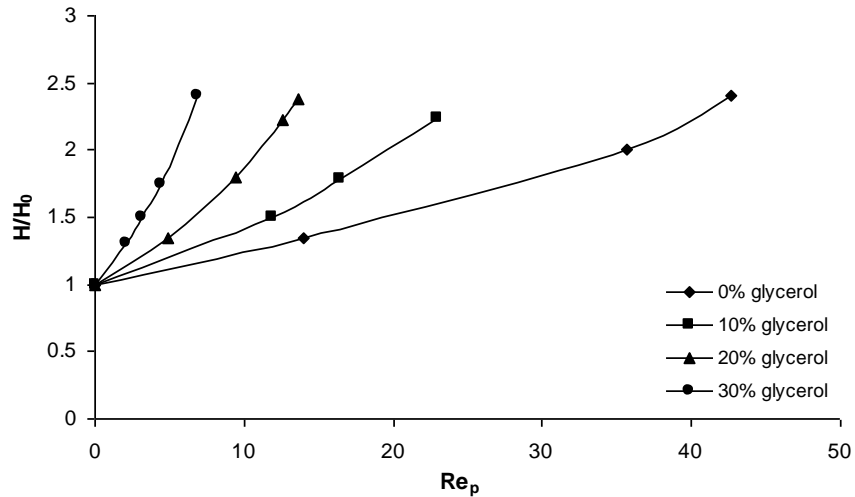


Fig.2 Bed expansion of the bed as a function of Reynolds number.

Although Coulson et al., 1991, provided $(\phi_s)_0$ values for spherical particles of various sizes, $(\phi_s)_0$ value for the bed was measured as follows. In the column the particles were settled in distilled water. The water contained inside the void fraction of the sedimented bed was drained off. The volume of the drained water was measured and divided by that of the total sedimented bed to determine the void fraction, ε of the bed. Finally, $(\phi_s)_0$ value determined by $1 - \varepsilon$ was 0.85.

Fig. 3 shows the Richardson – Zaki plots for the experimental data in Fig.2. All the lines in Fig.3 have approximately the same slope (n) values with different intercepts. The average value of the slopes, n_{av} , was 1.66.

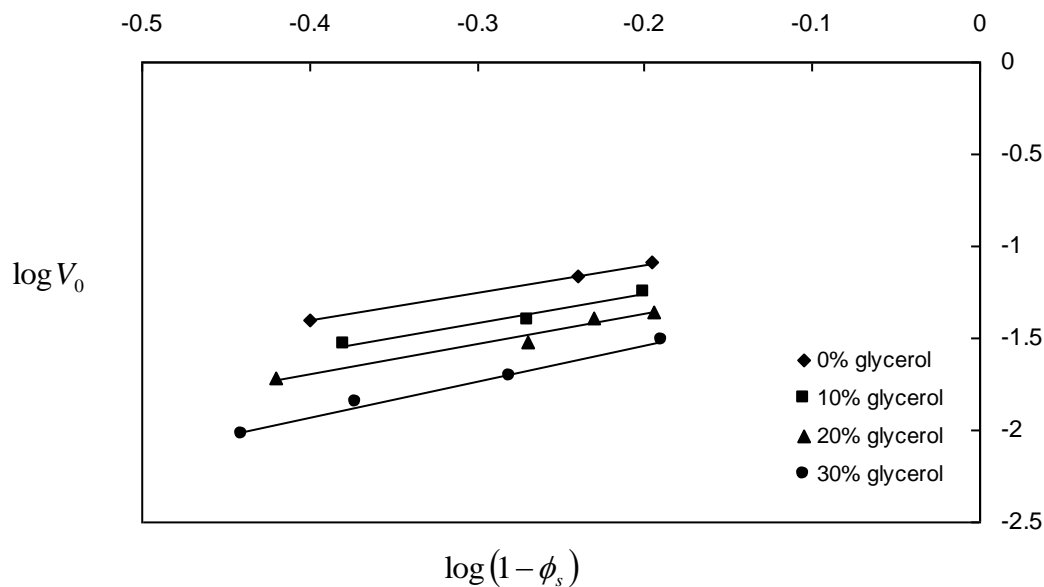


Fig.3 Richardson-Zaki plots for the bed expansion



The terminal velocity may be evaluated from bed expansion data by extrapolation at ϕ_s equals zero. Table 2 lists values of n and V_t in glycerol solutions along with their density and viscosity values.

Table 2. Physical properties of glycerol solutions at 20°C and the parameter values of Richardson-Zaki correlation.

| Glycerol (wt %) | Density, kg/m ³ | Viscosity, mPa.s | n | V _t , m/s |
|-----------------|----------------------------|------------------|-------|----------------------|
| 0 | 1000.0 | 1.005 | 1.513 | 0.1675 |
| 10 | 1022.1 | 1.310 | 1.56 | 0.1265 |
| 20 | 1046.9 | 1.760 | 1.60 | 0.0925 |
| 30 | 1072.7 | 2.500 | 1.97 | 0.0721 |

Eq. (3) implies that $\log V_t$ values can be linearly correlated with $\log[(\rho_p - \rho_l)^2 / \rho_l \mu_l]$ values. Fig. 4 shows that this is true. A modified expression for the terminal velocity is developed from the linear correlation in Fig.4 as follows

$$\log V_t = 0.386 \log \left[\frac{(\rho_p - \rho_l)^2}{\rho_l \mu_l} \right] - 3.261 \quad (10)$$

Comparing eqs. (10) and (3) enables the introduction of two empirical parameters, the effective particle diameter, d_{pe} and an exponent, a , to modify the terminal velocity expression as in the following general equation

$$V_t = \left(\frac{4g^2}{225} \right)^{\frac{1}{3}} \left(\frac{(\rho_p - \rho_l)^2}{\rho_l \mu_l} \right)^{\frac{a}{3}} d_{pe} \quad (11)$$

or

$$\log V_t = \log \left[\frac{4g^2}{225} \right]^{\frac{1}{3}} d_{pe} + \frac{a}{3} \log \left[\frac{(\rho_p - \rho_l)^2}{\rho_l \mu_l} \right] \quad (12)$$

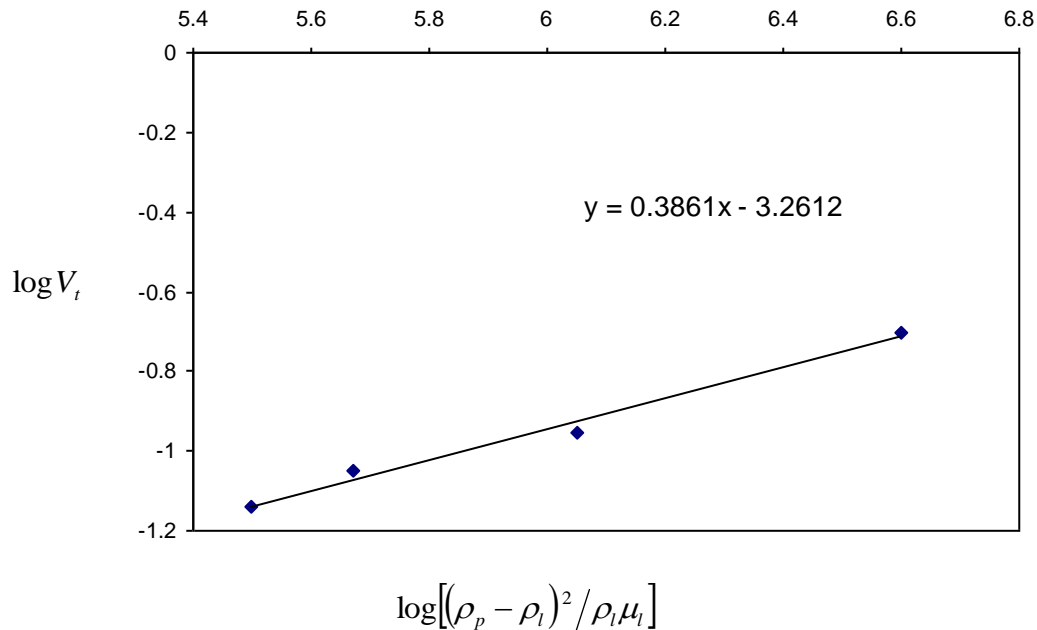


Fig.4 Linear correlation between $\log V_t$ values determined from Richardson-Zaki plots with $\log\left[\frac{(\rho_p - \rho_l)^2}{\rho_l \mu_l}\right]$ values.

By comparing eqs. (10) and (12), values of d_{pe} and a are calculated to be 0.458 mm and 1.158, respectively. It is very reasonable that the effective diameter is close to the diameter of the smallest particles used, since the smallest particles would be at the top of the expanded bed, and therefore should be used as a basis in modeling the behavior of the expanded bed.

Fig. 5 demonstrates that the calculated bed expansion, $(H/H_0)_{cal}$ using the modified expression eq. (10) and eq. (6) are in good agreement with experimentally measured values of the bed expansion $(H/H_0)_{exp}$. In this case the n values of eq. (6) are replaced by the average, n_{av} of n values, the slopes of the linear plots in Fig. 3 for all glycerol solutions.

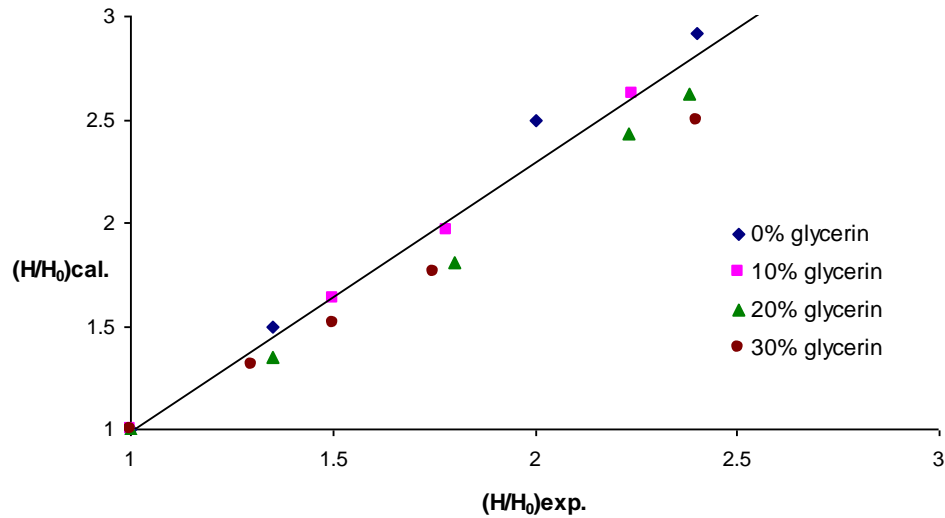


Fig.5 Comparison of the bed expansion calculated using the modified terminal velocity expression, $(H/H_0)_{cal}$, with those of experimentally determined values, $(H/H_0)_{exp}$.

EFFECT OF DENSITY RATIO AND DIAMETER RATIO ON THE RICHARDSON- ZAKI EXPONENT CONSTANT (N):

The effect of density ratio on the values of n is shown in Fig. 6. The exponent n decreases with an increasing density ratio as long as the value of $(\rho_p - \rho_l)/\rho_l$ is less than unity. In the case the ratio is unity, the exponent value is also unity. For the case where $(\rho_p - \rho_l)/\rho_l$ is greater than unity, the decreasing trend in exponent value reverses i.e. increasing. In a similar manner, the variation of exponent n has been plotted against particle to column diameter ratio (d_p/D) in Fig. 7. The exponent was found to decrease with increasing particle to column diameter ratio for both the cases when $(\rho_p - \rho_l)/\rho_l < 1$ and $(\rho_p - \rho_l)/\rho_l > 1$.

$$n = f \left[\left(\frac{\rho_p - \rho_l}{\rho_l} \right), \frac{d_p}{D} \right]$$

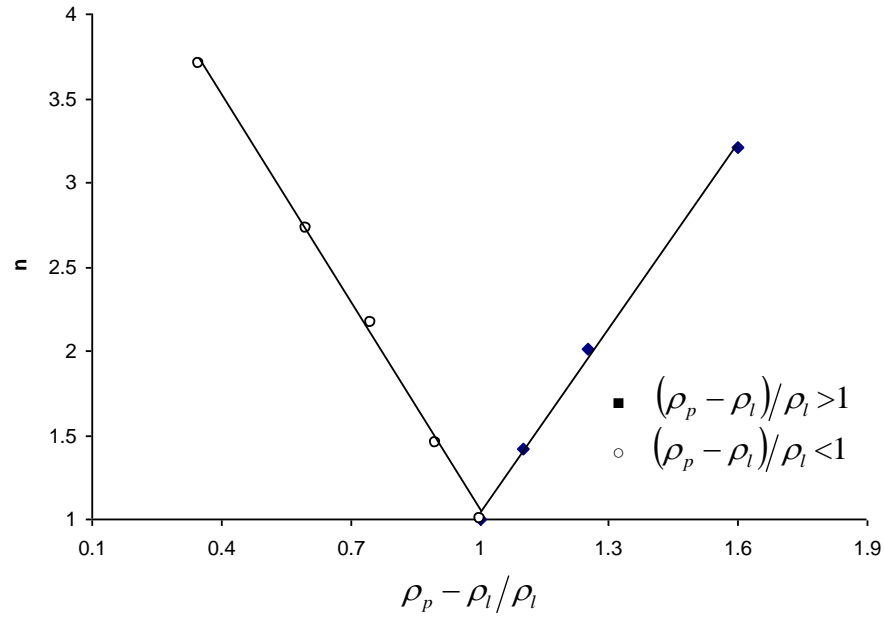


Fig. 6 Exponent variation with relative density to fluid density ratio

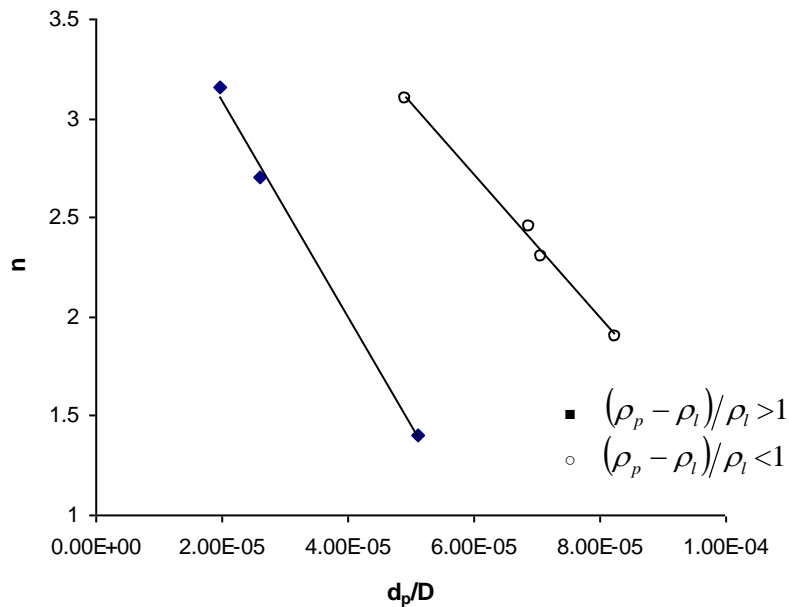


Fig. 7 Exponent variation with particle to bed diameter ratio.

It was noted that when the density ratio $(\rho_p - \rho_l) / \rho_l$ is less than one, there is no dense phase at the bottom of the test section. However, for density ratio $(\rho_p - \rho_l) / \rho_l$ greater than unity, there exists accelerating or dense regime at the bottom of the test section.



CONCLUSION

- The expression of the Richardson- Zaki correlation in combination with the modified Wen and Yu expression for terminal velocity, developed in this study, can be successfully used to model the behavior of expanded beds at moderate Reynolds number.
- For beds composed of particles of mixed sizes it is observed that the smallest particles collect near the top. The same result was obtained by McCune and Wilhelm, 1949 and Lewis and Bowerman, 1952.
- The Richardson-Zaki exponent constant n varies with $(\rho_p - \rho_l)/\rho_l$ and d_p/D . These variations have been found to be of two types, namely, $(\rho_p - \rho_l)/\rho_l < 1$ and $(\rho_p - \rho_l)/\rho_l > 1$.
- It was observed that for the case of high density particles, there was a considerable height of dense phase at the bottom of the test section. In the case of lower density particles, no dense phase was noted at the bottom of the test section. The reason of this type of behavior can be explained as follows: for heavy solid particles, the gravitational force is more predominant and particles have to accelerate so as to reach the fully developed regime since the contribution of drag is obtained by the gravitational component on particles. For lesser density particles, the buoyancy force is more predominant as compared to the gravitational force, so, the particles soon reach the fully developed regime with smooth and homogeneous type of fluidization. The same behavior was observed by Rao, 2005.

NOMENCLATURE

| | |
|----------|--|
| d_p | Particle diameter (m) |
| d_{pe} | Effective particle diameter (m) |
| D | Column diameter (m) |
| F | Drag force (N) |
| g | Acceleration due to gravity (m/s^2) |
| H | Height of the expanded bed (m) |
| H_0 | Height of bed at zero flow stream (m) |
| n | Richardson- Zaki exponent constant (—) |
| Re_p | Particle Reynolds number (—) |
| V | Particle velocity (m/s) |
| V_{mf} | Particle minimum fluidization velocity (m/s) |
| V_0 | Superficial velocity (m/s) |
| V_t | Particle terminal velocity (m/s) |

Greek letters

- ρ_l Density of fluid (kg/m³)
 ρ_p Density of particle (kg/m³)
 μ_l Dynamic viscosity of fluid (kg/m.s)
 ϕ_s Solid fraction of the expanded bed (—)
 ϕ_0 Solid fraction of the bed at zero flow stream (—)
 ε Void fraction of the bed (—)

REFERENCES

- Bo, X., and Yan, S., 2003, “*Modeling of Sedimentation of Polydisperse Spherical Beads with a Broad Size Distribution*”, 58, pp 1531-1543.
- Coulson, J. M., Richardson, J. F., Backhurst, J. R. and Harker, J. H., 1991, *Chemical Engineering Vol. 2*, 4th Edn. Pergamon Press, Oxford.
- Davidson, J.F. and Keairs, D.L., 1978, *Fluidization*, Cambridge University Press.
- Lewis, E. W., and Bowerman, E. W., 1952, “*Fluidization of Solid Particles in Liquids*”, Chem. Engng. Progr., 48, pp 603-609.
- McCune, L. K., and Wilhelm, R. H., 1949, “*Mass and Momentum Transfer in Solid-Liquid System*”, Industr. Engng. Chem., 41, pp 1124-1134.
- Narvaes, I., Orio, A., Aznar, M.p. and Corella, J., 1996, “*Biomass Gasification with Air in an Atmospheric Bubbling Fluidized Bed. Effect of Six Operational Variables on the Quality of the Produced Raw Gas*”, Ind. Eng.Chem. Res., 35, pp 2110-2120.
- Ramaswamy, E., Sirinivasakannan, C. and Balasubramaniam, N., 2008, “*Bed Expansion Characteristics of Liquid-Solid Fluidized Bed with Internals*”, Modern Applied Science, 2(2), pp 84-92.
- Rao, V.V.B., 2005, “*Hydrodynamic Predictions in Liquid Solid Circulating Fluidized Bed*”, IE Journal- CH, 85, pp 64-68.
- Reichent, U., Knieps, E., Slusarczyk, H., Kular, N. R. and Thommes, J., 2001, “*Isolation of a Recombinant Format Dehydrogenase by Pseudo- Affinity Expanded Bed Adsorption*”, J. Biochem. Biophys. Methods, 49, pp 533.
- Richardson, J.F. and Zaki, W.N., 1954, “*Sedimentation and Fluidization. Part I*”, Trans. Instn Chem. Engrs., 32, pp 35-53.
- Richardson, J.F., and Jeronimo, M.A.D.S., 1979, “*Velocity-Voidage Relations for Sedimentation and Fluidization*”, Chem. Eng. Sci., 34, pp 1419-1422.



- Wen, C.Y. and Yu, Y.H., 1966, “A *Generalized Method for Predicting the Minimum Fluidization Velocity*”, *AICHEJ*, 12, pp 610-612.
- Yang J. and Renken A., 2003, “A *Generalized Correlation for Equilibrium of Forces in Liquid – Solid Fluidized Beds*”, *Chem. Eng. J.*, 92(7), pp 7-14.