



## A NEW METHOD FOR DESCRIBE THE EFFECT OF E.H.D LUBRICATION ON THE PRESSURE DISTRIBUTION IN COLD ROLLING

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### ABSTRACT

The present study has been carried out to investigate the effect of ElastoHydrodynamic Lubrication (E.H.L) on the pressure distribution of cold rolling strip by taking new conditions, which give new results in the theoretical analysis. This analysis takes into consideration the variation of the yield stress of the material, the position of the neutral angle ( $\phi_n$ ), and the shear stress along the direction of rolling.

The results obtained from the numerical method are in good agreement with the previous experimental result.

### الخلاصة

إن هذا البحث يدرس عملية تأثير الزيت على توزيع الضغط في مناطق التشكيل للمعدن المدرفل على البارد وذلك باستخدام نظام التزييت الهيدروديناميكي المرن (E.H.L) وباستخدام طريقة تحليل جديدة تأخذ بنظر الاعتبار التغيير في اجهاد الخضوع (yield stress) للمعدن اثناء التشوه وتغيير موقع زاوية المنطقة المحايدة (neutral angle) مع مقدار نسبة التشوه وكذلك تغيير اتجاه اجهادات القص على طول منطقة التشوه، هذا بالاضافة الى استخدام طريقة جديدة في التحليلات البرمجية للمعادلات الرياضية.

وقد تم مقارنة النتائج النظرية المستحصلة من هذه الدراسة مع النتائج العملية التي اجريت من قبل بحوث سابقة واعطت تطابق جيد مع النتائج العملية.

### KEY WORDS

The present of the effect of ElastoHydrodynamic Lubrication in cold strip rpling.

### INTRODUCTION

To study the mechanism of lubrication in cold rolling, it is very important to know the effect of lubricant on the pressure distribution, the shear stresses and the coefficient of friction along the contact length of the strip with the roller. The two objectives of this study are:

- 1- The minimum pressure and then the optimum load could be used for the same material and the same percentage reductions.

2- Reduction of the shear stresses and the friction between the roll and strip by using proper lubricant or different metal forming processes.

The application of ElastoHydrodynamic Lubrication (E.H.L) theory in the rolling process was made by (Cheng, 1966) who published a theory of plastrohydrodynamic lubrication and calculated the film thickness using E.H.D theory.

Bedi and Hillier (1968) and Avitzur (1972) have used the energy approach to calculate the hydrodynamic film thickness but did not explored the effect of viscosity changes with pressure and temperature.

Wilson and Walwoit (1971) developed an isothermal model for the lubrication of rolling which used an inlet analysis to calculate the lubricant film thickness. Dow et al (1975), Wilson and Murch (1976) developed a thermal model for the lubrication of rolling using an inlet analysis to calculate the lubricant film thickness. Yuan and Chern (1990) developed a thermal hydrodynamic lubrication analysis, which takes account of temperature - dependent viscosity variation along the film thickness in cold strip rolling.

In this paper a new method is used in analyzing the pressure and shear stress distribution along the film thickness in the work zone of cold strip rolling using the thermal hydrodynamic lubrication analysis of Yuan and Chern (1990) in the inlet zone.

These results are compared with the experimental work of (Dow et al, 1975).

### THEORETICAL ANALYSIS

The region between the rollers and workpiece is divided into three zones:

- 1- Inlet Zone: It is the zone where the workpiece is considered to be rigid. In this zone the Reynold equation can be written as, (Dow et al, 1975):

$$\frac{dp}{dx} = 6\eta(u_1 + u_2) \left( \frac{h_1 - h}{h^3} \right) \quad (1)$$

Where  $\eta = \eta_0 e^{\gamma p}$

$$h_1 = \frac{3\eta_0 \gamma (u_1 + u_2)}{(1 - e^{-\gamma(\sigma_1 - \sigma_i)})} \sqrt{\frac{R}{r y_1}}$$

eq. (1) can be solved to find the pressure distribution in the inlet zone.

- 2- Work Zone: In this zone the plastic deformation of the workpiece takes place. The momentum equation used in lubrication analysis is given by:

$$\frac{dp}{dx} = \frac{d\tau_{xy}}{dy} \quad (2)$$

Integration of eq. (2) gives, assuming  $p=p(x)$

$$\tau_{xy} = \tau_1 + y \frac{dp}{dx} \quad (3)$$

for Newtonian fluid:

$$\tau_{xy} = \eta \left( \frac{du}{dy} \right) \quad (4)$$

$$\frac{du}{dy} = \tau_{xy} / \eta \quad (5)$$

substituting of eq. (3) into eq. (5) and integrating gives:

$$u = \int_0^y \left( \frac{\tau_1}{\eta} + \frac{y}{\eta} \frac{dp}{dx} \right) dy + c$$

$$u = \frac{\tau_1}{\eta} y + \frac{y^2}{2\eta} \frac{dp}{dx} + c \quad (6)$$

The boundary conditions for this equation are:

at  $y=0$ ,  $u=u_1$  [ $u_1$ =the velocity of the lower surface].

at  $y=h$ ,  $u=u_2$  [ $u_2$ =the velocity of the upper surface].

Substituting the boundary conditions in eq. (6) gives:

$$u_2 = u_1 + \frac{\tau_1}{\eta} h + \frac{h^2}{2\eta} \frac{dp}{dx} \quad (7)$$

$$\therefore u_1 = u_2 - \frac{\tau_1}{\eta} h - \frac{h^2}{2\eta} \frac{dp}{dx} \quad (8)$$

Assuming very thin film thickness, so that:

$$\tau = \mu.p \quad (9)$$

the velocity of each point on the workpiece can be found by discretization of eq. (8):

$$u_1(i) = u_2 - \frac{\mu.p(i)}{\eta(i)} . h(i) - \frac{h(i)^2}{2\eta(i)} \frac{p(i) - p(i-1)}{x(i) - x(i-1)} \quad (10)$$

where  $x(i) - x(i-1) = R(\sin \phi(i) - \sin \phi(i-1))$

where  $\eta(i)$  is the viscosity of lubricant which is only a function of pressure of the form

$$\eta(i) = \eta_0 e^{\gamma p(i)}$$

3- Outlet Zone: The outlet zone is quite narrow as in the inlet zone and the pressure distribution along this zone could be found by solving Reynolds equation.

### SOLUTION PROCEDURE

1- Assume the pressure distribution in the work zone as in (Bland and Ford, 1948):

$$P^+ = \frac{k.y}{y_0} \left( 1 - \frac{\sigma_0}{k_0} \right) . e^{\mu.H}$$

$$P^- = \frac{k.y}{y_i} \left( 1 - \frac{\sigma_i}{k_i} \right) . e^{\mu.(H_i - H)}$$

Where  $\mu$  = the coefficient of friction between roll surfaces and material with lubrication.

The value of  $\mu$  can be measured by the same procedure used by (Whitton and Ford, 1955):

$$H = 2 \sqrt{\frac{R'}{y_0}} \arctan \left( \sqrt{\frac{R'}{h_0}} . \phi \right)$$

2- From the pressure distribution total film thickness which is in two parts, can be found by:

a- The elastic deformation of the roller material which could be derived from E.H.L theory (Dowson and Higginson, 1966):

$$h_{\text{elastic}} = x^2 / 2R - \frac{1}{\pi E'} \int_{x_{\text{in}}}^{x_{\text{out}}} p(s) \ln(x-s)^2 ds$$

where  $R$ =radius of the roller.

$$E' = \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)$$

$\nu_1$  = Material Poisson's ratio.

$\nu_2$  = Roller Poisson's ratio.

b- The plastic deformation of the strip could be obtained from the following:  
from the constant volume flow rate:

$$y(i).u(i) = y(n).u(n)$$

where  $n$  represents the neutral point.

From above equation:

$$u(i) = u_n \cdot y(n) / y(i)$$

$$\therefore u_n = R \cdot \omega \cdot \cos \phi_n$$

$$\therefore u(i) = R \cdot \omega \cdot \cos \phi_n \cdot y(n) / y(i)$$

$$\therefore u(i) = R \cdot \omega \cdot \cos \phi_n \cdot y(n) / y(i) (1 / \cos \phi)$$

Where:  $w$ = Angular velocity (rad/sec).

$R$ = Roller radius (m).

$\phi_n$  = Neutral angle (rad).

then the horizontal displacement:

$$\Delta S_{(hor)}(i) = [R \cdot \omega \cdot \cos \phi_n \cdot y(n) / y(i)] \cdot \Delta t$$

$$= R \cdot d\phi \cdot \cos \phi_n \cdot y(n) / y(i)$$

then the vertical displacement is :

$$\Delta S_{(ver)}(i) = R \cdot d\phi \cdot \cos \phi_n \cdot y_n / y(i) (\sin \phi / \cos \phi)$$

and total film thickness is:

$$h(i)_{total} = x^2 / 2R - \frac{1}{\pi E'} \int_{x_{in}}^{x_{out}} p(s) \ln(x-s)^2 ds + \Delta S_{(ver)}(i) \quad (11)$$

3- From point 2 we could substitute the values of  $h(i)$  in eq. (1) to find the pressure distribution in the inlet and outlet zones.

In the work zone, the values of  $h(i)$  and  $p(i)$  are substituted in eq. (10) to find the new values of  $u_1(i)$  along the strip and new position of  $\phi_n$  [where  $u_1 = u_2$ ].

4- The new pressure distribution in the work zone could be obtained from Reynolds equation which given as in (Venner et al,1990):

$$\frac{d}{dx} \left( \epsilon \frac{dp}{dx} \right) - \frac{d(\rho h)}{dx} = 0$$

(taking  $ph$  = max pressure in the neutral axis).

5- The new values of  $p(i)$  from step (4) are compared with the old one so that the pressure convergence criterion  $\sum |P_{new} - P_{old}| / \sum P_{new}$  must be less than 0.005

6- If the convergence criterion  $> 0.005$  then a new value for the pressure must be found as:

$$P_{new}(i) = w_p (P_{new}(i) - P_{old}(i)) + P_{old}(i)$$

7- The new values of pressure from step (6) with the new values of  $u_1(i)$  and  $\phi_n$  are used again to find the film thickness as:



$$h(i)_{total} = x^2 / 2R - \frac{1}{\pi E'} \int_{x_{in}}^{x_{out}} p(s) \ln(x-s)^2 ds + u_1(i) \frac{d\phi}{\omega} \sin \phi \quad (12)$$

and the program returns to step (3).

- 8- Then the pressure in the work zone must satisfy the expression which derived by (Cheng, 1966) from equilibrium considerations on a section of the workpiece and von Misses criterion so become:

$$y \frac{dp}{dx} - \frac{d(y\sigma)}{dx} + 2\tau = 0 \quad (13)$$

By solving this expression using Runge-Kutta method to find the pressure distribution in the contact zone which are compared with the pressures found from step (5) by using pressure convergence criterion. If the convergence criterion  $> 0.005$  then the program returns to step (6).

It must be noted that in step (8) of the procedure of solution the yield stress of the strip material is obtained from the tables used by (Dow et al, 1975).

### NUMERICAL ANALYSIS

The numerical solution method is outlined as shown in **Fig. (1)**. Three very important notes to be made in our solution:

- 1- The discretized Reynolds equation in node on a grid with mesh size  $\Delta$ :

$$\frac{1}{\Delta^2} (\epsilon_{i-1/2} p_{i-1} - (\epsilon_{i-1/2} + \epsilon_{i+1/2}) p_i + \epsilon_{i+1/2} p_{i+1}) - \frac{1}{\Delta} (\rho_i h_i - \rho_{i-1} h_{i-1}) = 0$$

where

$$\epsilon = ph_i^3 / \eta \lambda, \lambda = 6\eta_0 U_s R^2 / b^3 \cdot ph; U_s = u_1 + u_2, \Delta = \phi_{i+1} - \phi_i$$

- 2- The discretization of elastic film thickness equation results is:

$$h(i) = (x_i^2 / 2) + \left( 1 / \pi \sum_{j=1}^n k_{ij} \cdot p_j \right)$$

where:

$$k_{ij} = (i - j + 1/2) \Delta \left( (\ln|i - j + 1/2| \Delta) - 1 \right) - (i - j - 1/2) \Delta \left( (\ln|i - j - 1/2| \Delta) - 1 \right)$$

- 3- The value of the yield stress used in solving the equation of step (8) of the procedure of solution was taken from **Table (1)**, which was predicted by (Dow et al, 1975). It is also shown that the lubricant used in this research are two kinds of a viscous mineral oil designed as oil H and oil C as shown in **Table (2)**.
- 4- The inlet and outlet zones are each divided into 15 nodes (coarse grid) while for detailed study of the work zone it is divided into 90 nodes (fine grid) to find the pressure spike in many situations. The details of this method used in (Chang et al, 1989).

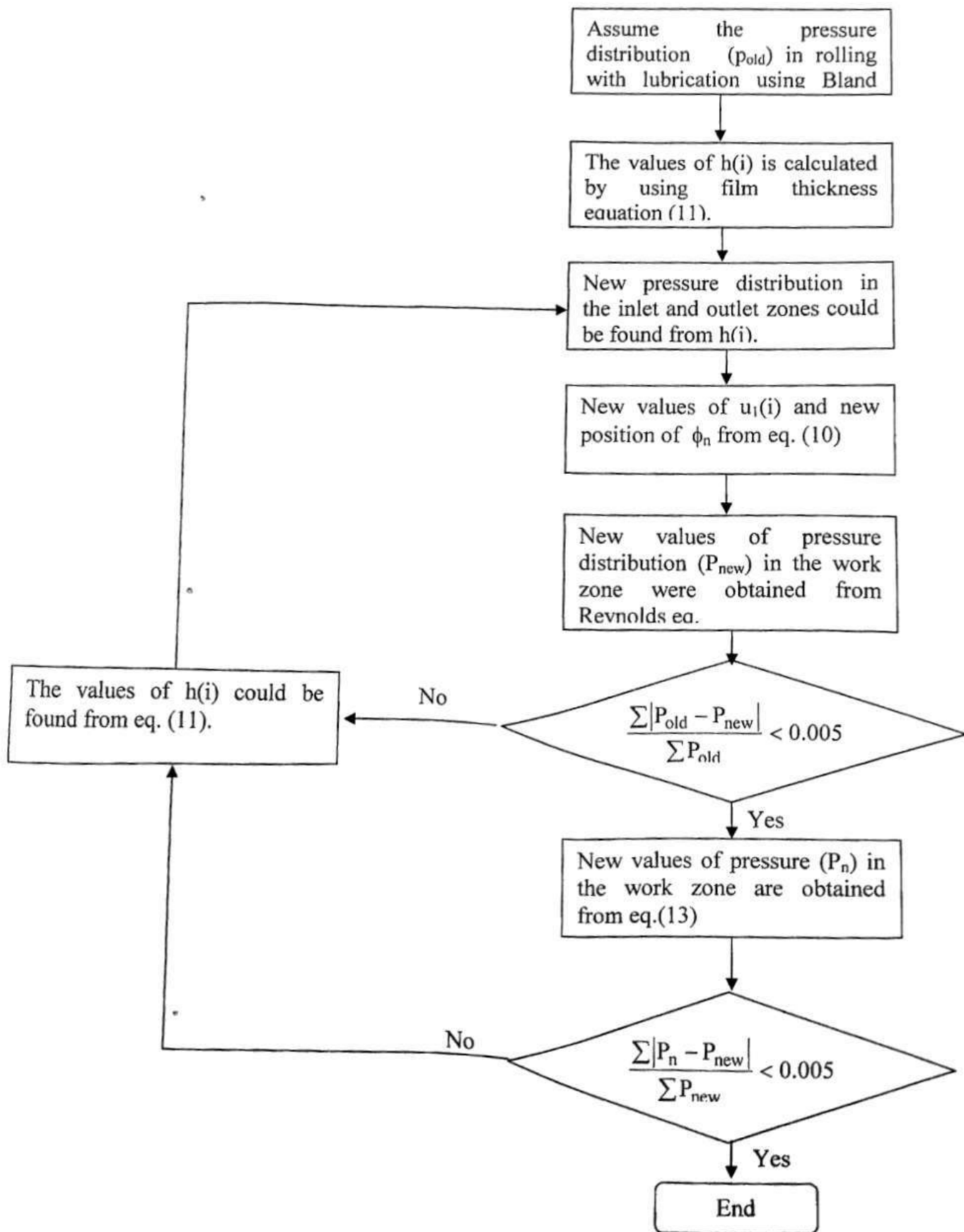


Fig. (1) Flow Chart of the Solution

## RESULTS

It must be known that the theoretical model of the rolling process developed in the preceding section will be compared with the experimental results that has been discussed by (Dow et al, 1975). In addition that the pressure distribution in the work zone for all conditions used in this paper are begins with the yield point for steel which is shown in **Table (2)**.

The pressure distribution in **Fig. (2-a)** illustrate that by using E.H.D lubrication in the cold rolling gives results much higher than the results that obtained from (Dow et al, 1975) who used hydrodynamic condition for solving the equations of lubrication. In this research we don't take into consideration the effect of the temperature variation along the work zone and assuming that the lubricant to behave as a Newtonian fluid, although that the values of the coefficient of friction between the roll surface and strip material with lubrication which were obtained from (Whitton and Ford, 1955) have very high effects on the first assumption of the pressure distribution which were obtained from (Bland and Ford, 1948) equations.

But the new procedure used in this research gives confidence to the results obtained from it because the profile of the pressure distribution in the theoretical results are behaves much closely to the experimental work rather than that of the pressure distribution which obtained from hydrodynamic equations, as seen in **Fig. (2-b)**.

**Fig. (3)** is the theoretical and experimental pressure distribution for 14 percent reduction of steel strip as in the **Fig. (2)** but by using low viscosity oil (C).

In **Fig. (3-a)** it is shown that the results is much closes to the experimental results **Fig. (3-b)** than we obtained in **Fig. (2)** is of low viscosity which is seems to be more available for E.H.D lubrication than for high viscosity oil (H).

**Fig.(4)** shows the theoretical and experimental results of the pressure distribution by using the same oil (C) used in Fig.(3) but for reduction 30 percent which is double the reduction used in **Fig.(3)**. Because of high reduction and low viscosity oil used in this case then the pressure will be very high and so that the results obtained by using E.H.D lubrication are very closed to the experimental condition.

In general the position of the neutral point (which defined here as the point where the strip speed equals the roll speed) is near the outlet of the strip, but for constant roll speed the position of the neutral point affected by two factors:

- 1- With increasing the viscosity of the oil the neutral point becomes near the inlet zone, as seen in **Fig. (2)**.
- 2- With increasing the percent of reduction, the neutral point becomes more close to the inlet zone, as seen in **Fig.(4)**.

In **Fig.(5-a)** represent the theoretical pressure distribution which obtained from applying our new method of E.H.D lubrication on the strip of aluminum for 16 percent reduction and using oil (H). A comparison between theoretical and experimental one **Fig.(5-b)** shows clearly some differences between them. This is because of low percent reduction, high viscose oil and the fact that the yield point of aluminum could not be defined well as for steel (for steel the yield point could be seen very well from stress-strain curve) which depend our first assumption of pressure distribution in the work zone on the value of the yield stress of the material.

## CONCLUSIONS

A new formulation of E.H.D lubrication has been obtained for analyzing the pressure distribution in cold rolling. The theoretical results obtained from applying this new formula on strip of steel and aluminum are compared with the experimentally measured values. It seems from this comparison that:

- 1- This method give result which has a profile very closed to the profile of the experimental one. This has not been seen when applying hydrodynamic lubrication.

- 2- The results become very close to the experimental one with increasing the percent reduction at the strip and decrease the viscosity of the oil.
- 3- The neutral point is become near the inlet zone by increasing the viscosity of the oil and increasing the percent reduction of the strip rolling.
- 4- The lubricant behavior assumed to be a Newtonian fluid and the effect of temperature are not taken into consideration.

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**NOMENCLATURE**

English Symbols	Units
B = The distance from the inlet to the outlet axis.	(m)
h = Film thickness.	(m)
$h_1$ = Film thickness at the edge of the inlet zone.	(m)
K = Yield stress with initiated speed.	(N/m <sup>2</sup> )
$K_i, k_o$ = Yield stress with inhibited speed at inlet and outlet side respectively.	(N/m <sup>2</sup> )
$p_n$ = The pressure at neutral axis.	(N/m <sup>2</sup> )
P = Pressure.	(N/m <sup>2</sup> )
r = Reduction.	%
R = Roll radius.	(m)
$R'$ = Radius of deformed arc of contact.	(m)
t = Time for rolling.	(sec)
$u_1$ = Strip velocity.	(m/sec)
$u_2$ = Roll speed.	(m/sec)
$w_p$ = Pressure under relaxation factor from 0.02 to 0.7.	----
x = Dimension along arc of contact.	(m)
$y_1$ = Initial strip thickness.	(m)
Y = Thickness of strip.	(m)
$Y_o$ = Thickness of strip at exit.	(m)
$Y_1$ = Thickness of strip at inlet.	(m)
The suffix n denotes the neutral point.	

**GREEK SYMBOLS**

$\eta_o$ = Base viscosity of lubricant at ambient temperature.	(N/m.sec)
$\gamma$ = Lubricant pressure coefficient of viscosity.	----
$\eta$ = Viscosity of lubrication.	(N/m.sec)
$\sigma_1$ = Yield stress of strip material.	(N/m <sup>2</sup> )
$\sigma_1$ = Back tension.	(N)
$\sigma_o$ = Front tension.	(N)
$\tau_{xy}$ = Shear stress.	(N/m <sup>2</sup> )
$\tau_1$ = Shear stress at the surface of the strip.	(N/m <sup>2</sup> )
$\mu$ = Coefficient of friction between roll surface and material.	----
$\phi$ = Angular coordinates.	(rad)
$\Delta t$ = The duration of time for grid size ( $\Delta x$ ).	(sec)
$\Delta s$ = Displacement of the strip roll deformation.	(m)
$\Delta s_{(hor.)}$ = Displacement of the strip roll deformation in the x-direction.	(m)
$\Delta s_{(ver.)}$ = Displacement of the strip roll deformation in the y-direction.	(m)
$\epsilon$ = Coefficient in equation.	----
$\rho$ = Density.	(Kg/m <sup>3</sup> )
$\omega$ = Angular speed of the roller.	(rad/sec)

Table (1) Properties of Lubricants from (Dowson and Higginson, 1966)

Lubricant	Lubricant	Base				Pressure
		Viscosity, cp	Temperature		Coefficient of Viscosity	
Designation	Lubricant	T=77 coefficient of viscosity				N/m <sup>2</sup>
		$\gamma(10^{-5}/10^8/\text{Deg F } \delta(1/F)^{(a)}(1/K) \text{ psi})^{(b)}$				
C	A naphthenic industrial oil, fully formulated	93	0.0347	0.0625	19.9	(2.9)
H	A high viscosity naphthenic industrial oil, fully formulated	3738	0.0594	0.107	17.0	(2.5)

Table (2) Tensile Strength for 10101 Steel of  
(Dowson and Higginson, 1966)

Reduction	Tensile Strength, N/m <sup>2</sup> (psi)	Flow Strength (Von Mises Criteria) N/m <sup>2</sup> (psi)
0	2.27*10 <sup>8</sup> (33.000)	2.62*10 <sup>8</sup> (38.000)
10	3.65*10 <sup>8</sup> (53.000)	4.2*10 <sup>8</sup> (61.000)
20	4.75*10 <sup>8</sup> (69.000)	5.44*10 <sup>8</sup> (79.000)
30	5.16*10 <sup>8</sup> (75.000)	5.92*10 <sup>8</sup> (86.000)

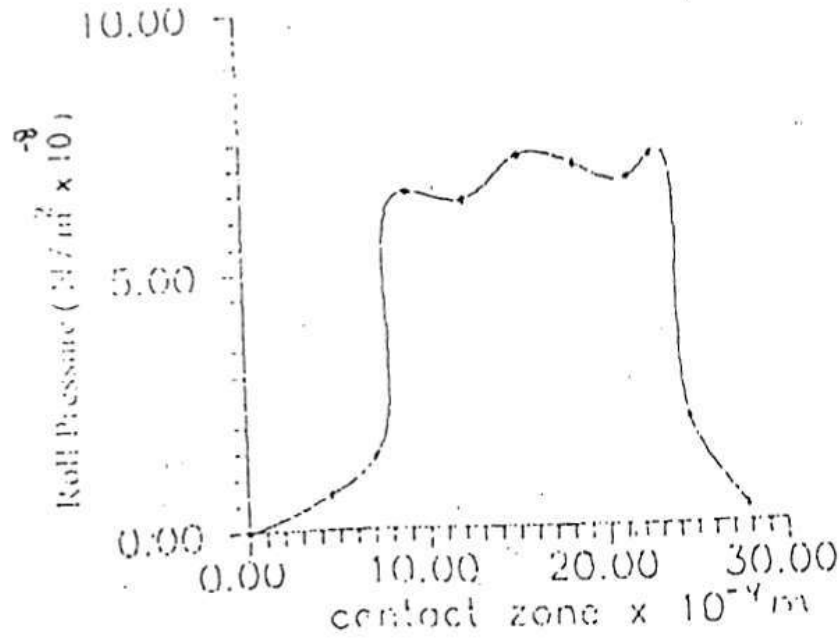


Fig. (2-a) A theoretical pressure distribution for 14 percent reduction of steel strip using oil H (Dow et al, 1975)

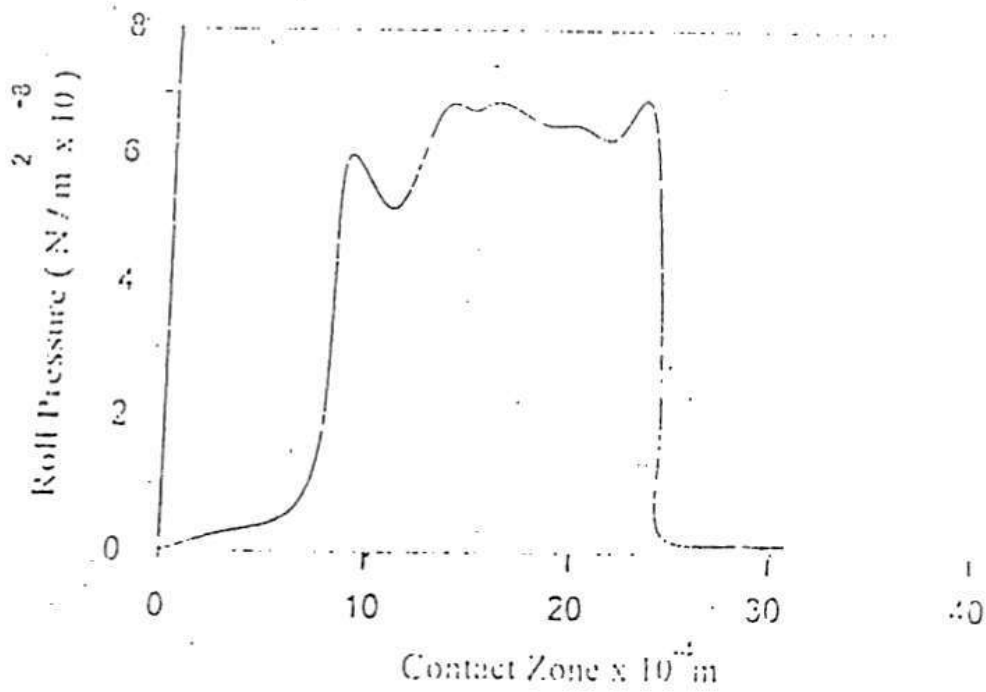


Fig. (2-b) A theoretical and experimental pressure distribution for 14 percent reduction of steel strip using oil H (Dow et al, 1975)

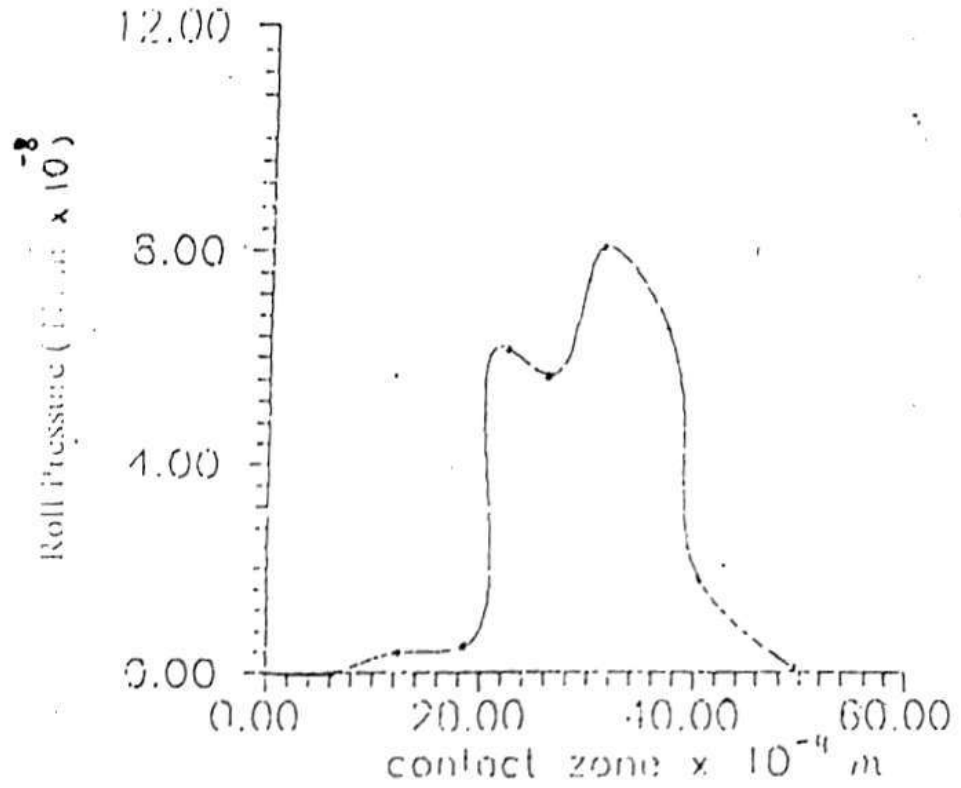


Fig. (3-a) A theoretical pressure distribution for 14 percent reduction of steel strip using oil C (Dow et al, 1975)

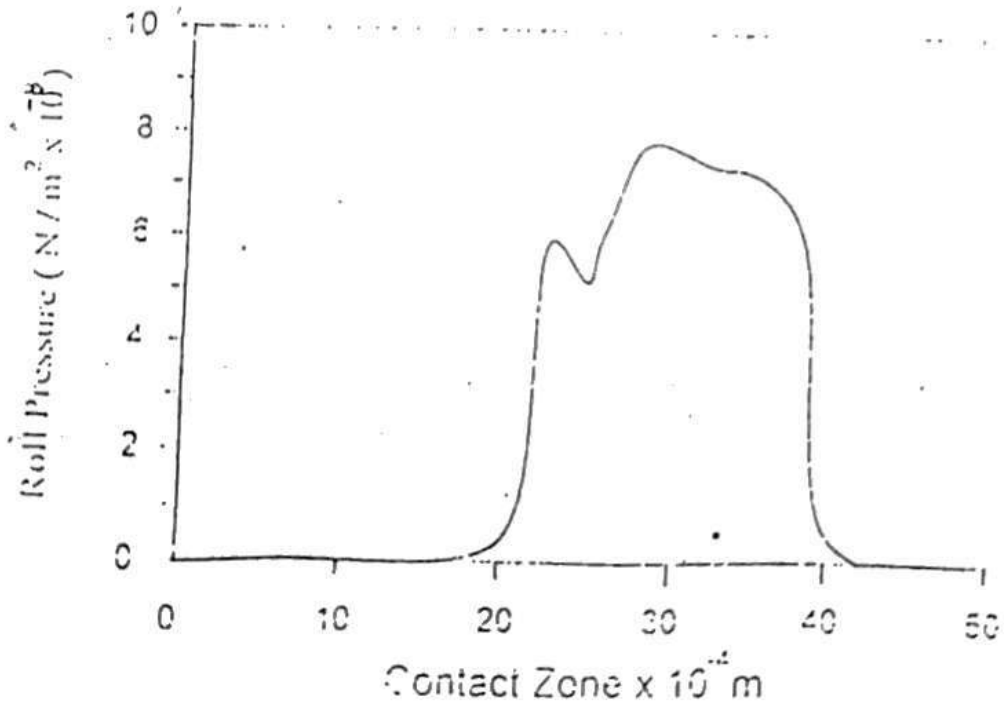


Fig. (3-b) A theoretical and experimental pressure distribution for 14 percent reduction of steel strip using oil C (Dow et al, 1975)

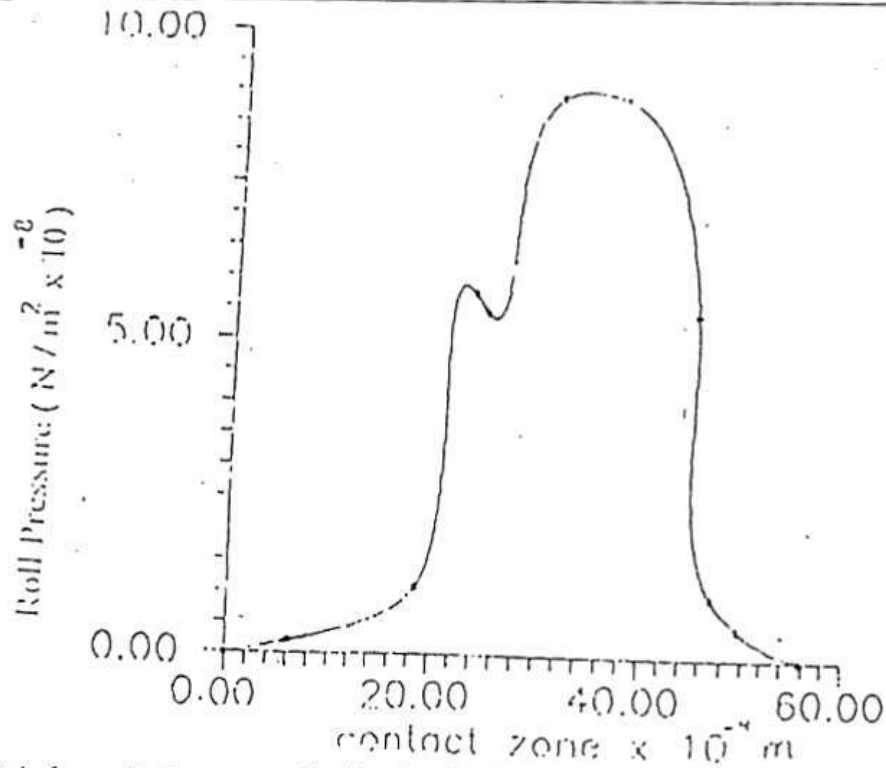


Fig. (4-a) A theoretical pressure distribution for 14 percent reduction of steel strip using oil C (Dow et al, 1975)

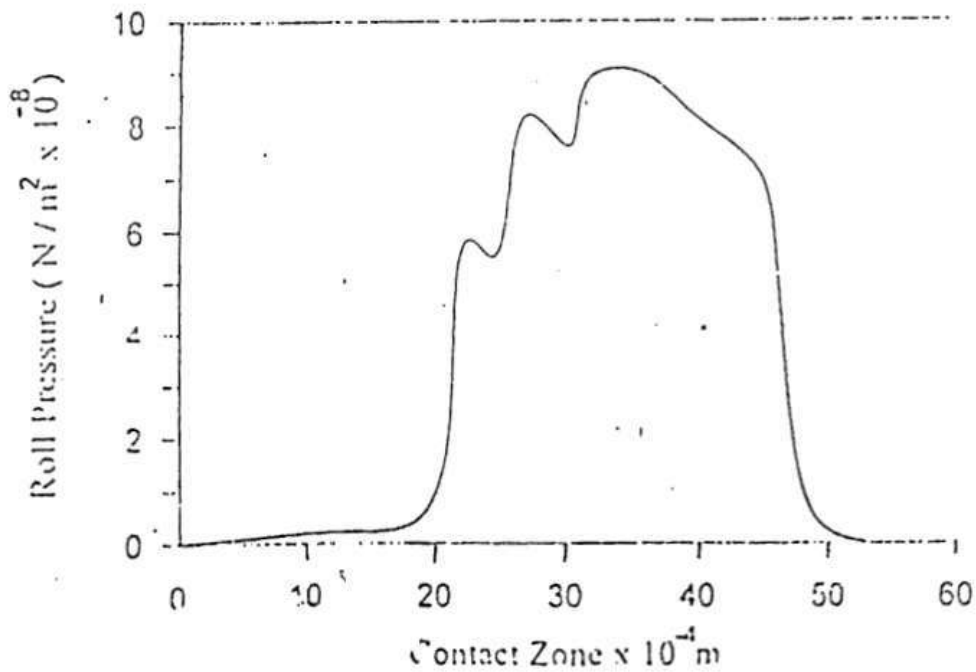


Fig. (4-b) A theoretical and experimental pressure distribution for 30 percent reduction of steel strip using oil C (Dow et al, 1975)

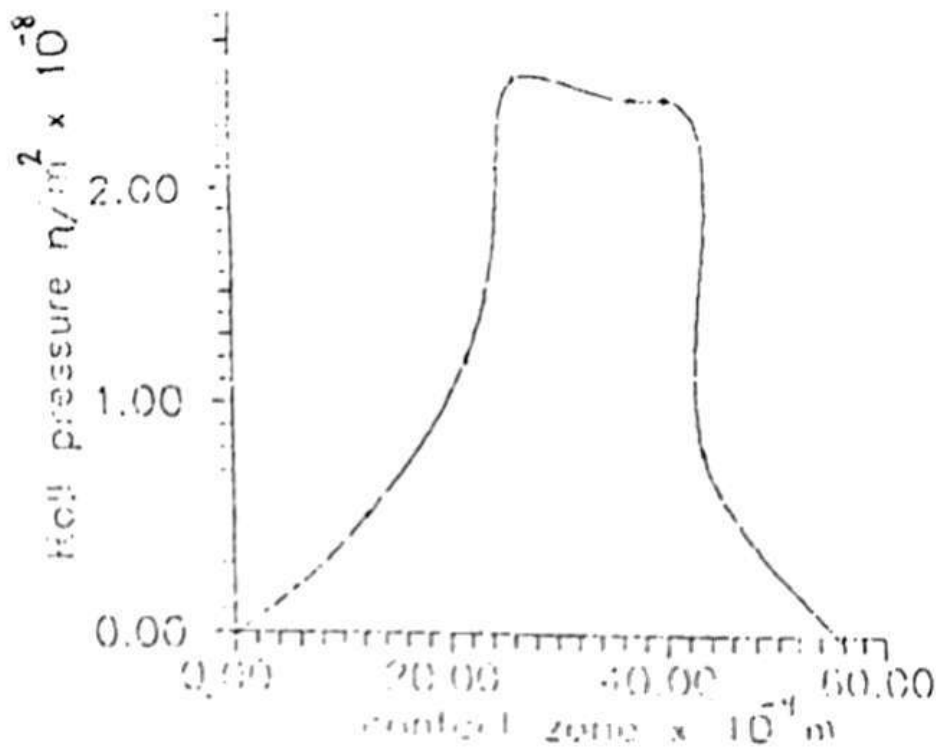


Fig. (5-a) A theoretical pressure distribution for 16 percent reduction of steel strip using oil H (Dow et al, 1975)

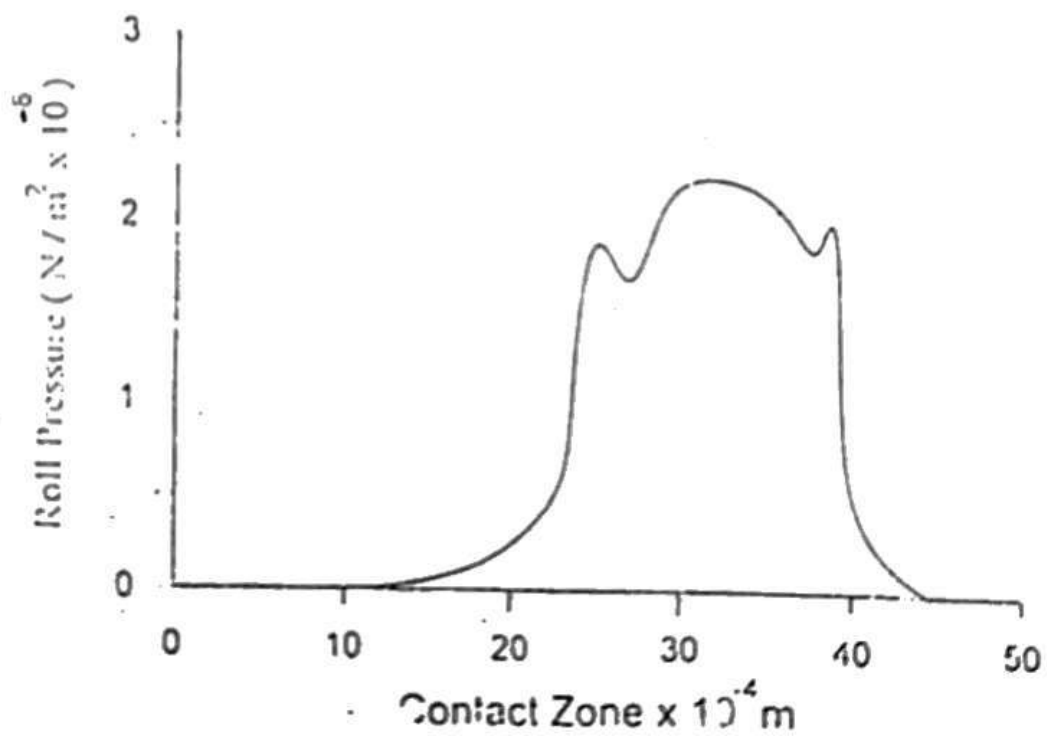


Fig. (5-b) A theoretical and experimental pressure distribution for 30 percent reduction of steel strip using oil H (Dow et al, 1975)