DETERMINATION OF DEPTH OF PLACEMENT OF TUNNELS AND CAVITIES BY THE BOUNDARY ELEMENT METHOD

Dr. Omar Al-Farouk Al-Damluji
Assistant Professor and Head,
Department of Civil Engineering,
University of Baghdad.

Dr. Mohammed Yousif Fattah
Lecturer,
Department of Building and Construction, University of Technology

Rana A. J. Al-Adhamii
Formerly Graduate student,
Department of Civil Engineering,
University of Baghdad.

ABSTRACT

A boundary element numerical algorithm has been developed for the determination of stresses and deformations around cavities and tunnels. A study of the influence of depth below the ground surface on the distribution of stresses and deformations around cavities and tunnels is presented in this paper. The soil is assumed to behave linearly elastic.

A computer program has been built to perform the numerical computations. The results show that with increasing the depth of placement of tunnel or opening below the ground surface, the settlements decrease. The maximum stresses occur at the haunches of the tunnel rather than at the crown.

For the circular cavity that is considered in this paper, it was found that with increasing the depth below the ground surface (depth/tunnel diameter > 3), the surface settlements do not exceed 6 % from those obtained for the case of no-cavity condition.

الخلاصة

لقد طورت طريقة العناصر الحدودية كخوارزمية عددية لايجاد الاجهادات و الازاحات حول الانفاق و الفتحات الارضية. في هذا البحث أجريت دراسة حول تاثير عمق الفتحة أو النفق تحت سطح الارض على توزيع الاجهادات و الازاحات حول الانفاق أو الفتحات الارضية. و قد أعتبر تصرف التربة مرنا خطيا.

و قد بني برنامج حاسبة الكترونية لاجراء الحسابات العدديه. وبينت النتائج أنه مع زيادة عمق الفتحه أو النفق تحت سطح الارض تقل الاجهادات و الهبوطات و أن الاجهادات القصوى تحدث عند الحافات الجانبية للنفق بدلا من قمته.

و في حالة الفتحة الدائرية التي تم تحليلها في هذا البحث، وجد أنه مع زيادة العمق تحت سطح الأرض، (العمق \ قطر النفق > 3)، لا تتجاوز الازاحات السطحية 6% من تلك التي تحصل في حالة عدم وجود فتحة.

KEY WORDS

Tunnels, Cavities and Boundary Element Method.

INTRODUCTION

Rapid growth in urban development has resulted an increased demand for the construction of water supply, sewage disposal and transportation systems. Tunnels are an essential component of these systems and constitute a major portion of project expenditure.

Recent advances in tunnelling technology reduce construction time with consequent decrease in cost. However, even with modern equipment, experience has shown that designing of tunnels must include dealing with three important problems:

1- Maintaining stability of face and wall of the tunnel before supported by lining.
2- Predicting displacements caused by excavation of the tunnel on the surface and throughout the adjacent ground mass.
3- Predicting the magnitude and distribution of earth pressure acting on the tunnel.

   So, there is an urgent need for reliable means to estimate the extent and nature of the movements and disturbance occuring in areas above and adjacent to tunnels. These deformations may significantly affect nearby structures and need to be considered during design.

The object of this paper is to provide, as far as possible, a picture of the stress distribution around cavities and tunnels in an isotropic medium. Also, provide at least a temporary expedient for estimating the settlements to be expected at varying distances laterally from the centre line of a cavity or a tunnel.

PREVIOUS STUDIES

Although the finite element techniques have been used in so many practical problems, the boundary formulations appear as an alternative technique that, in many cases, can provide more reliable or economical analysis. Even with automatic mesh generation techniques, the finite element method has not found widespread application to tunneling problems because of the data preparation problems and considerable computer time requirements.

The input data requirements of the boundary element method (BEM) are considerably less than these of the finite element method (FEM) since only the boundary needs to be discretized. Unlike the FEM, the BEM can model the boundaries at infinity without truncating the outer boundary at some arbitrary distance from the region of interest.

In the boundary element method, the unknowns appear only on the boundaries of a domain, so the number of the unknowns may be reduced compared to the three-dimensional finite element method. This condition is well suited to tunnels, where the most significant unknown, the surface subsidence appears on the boundary.

The research already conducted on tunneling problems or soil-structure interaction using the BEM can be summarized as follows:

1- Brady and Bray (1978a and b) have described a boundary element method for determining the distribution of stress and induced displacements around long, narrow, parallel - sided openings in an elastic medium. A good agreement was found between the results of the boundary element analyses and those obtained from analytical solutions. A BEM of stress analysis was also developed for the solution of complete plane strain problems and applied to determine the stress distribution around openings with irregular cross sections having any arbitrary orientation in a triaxial stress field. The displacements induced by the excavation are also included.
2- Venturini and Brebbia (1981) have described for the first time, the extension of the BEM to no tension materials such as those present in underground and surface excavations.
3- Ito and Histake (1982)* treated generally, a three-dimensional problem of an advancing shallow tunnel in an elastic and non-elastic ground by the boundary element method. The tunnel advance velocity and the position of the face were taken into consideration. The method has been illustrated and verified on two sites where subsidence measurements were taken simultaneously.

The disadvantage of this method is that it does not deal with displacements inside the ground nor with the corresponding changes in stresses.

4- Gioda, Carini and Cividini (1984)** discussed a boundary integral equation technique for the visco-elastic stress analysis of underground openings. The results of a test problem were presented concerning a shallow circular tunnel. These results show that an acceptable accuracy of the numerical solution is obtained even when adopting a relatively small number of free variables

5- Again, Venturini and Brebbia in (1984)** have proposed a boundary element formulation to analyze plane strain problems with possible displacements at the third direction. An algorithm to model nonlinear behavior is presented including an initial stress process. The study of an unlined opening was carried out illustrating that tunnels whose axes do not coincide with the original principal stress direction can not be analyzed assuming plane strain conditions only.

THE BOUNDARY ELEMENT METHOD

There are many engineering problems for which it is possible to represent the governing equations by a system of boundary integral equations (BIEs); that is , the integrated unknown parameters, in such equations, appear only in integrals over the boundary of the problem domain. There are many numerical approaches for the solution of such equations, and each approach gives the solution of such equations, and each one of them may be called a boundary integral equation method (BIEM).

Characteristics of the Boundary Element Method

The boundary element method (BEM) is considered nowadays the most popular numerical technique for the direct solution of BIEM. It is based upon piecewise discretization of the problem boundary in terms of sub–boundaries, known as boundary elements, in a way similar to that employed for the finite element method. The main advantages of the BEM compared with domain numerical techniques can be summarized in the following statements: -

1- For many applications, the dimensionality of the problem is reduced by one, resulting in a considerable reduction in the data and computer CPU time required for the analysis.

2- The BEM is ideal for problems with infinite domains, such as problems of soil mechanics, fluid mechanics and acoustics.

3- No interpolation errors inside the domain.

4- Boundaries at infinity can be modeled conveniently without truncating the outer at some arbitrary distance from the region of interest.

5- Surface problem, such as those of elastic fracture mechanics, or elastic contact, is dealt with more efficiently and economically with the BEM.

6- Valuable representation for stress concentration problems.

7- The BEM offers a fully continuous solution inside the domain, and the problem parameters can be evaluated directly at any point.

The boundary element method has also disadvantages and they can be outlined as follows, (EL-Zafrany 1992):

1- The derivation of the governing BIEs may require a level of mathematics higher than that with other methods, but the procedure of the BEM itself is not different from that of the FEM.

2- It leads to fully populated matrices for the equations to be solved, thus it is not possible to employ the elegant FEM solvers such as the banded or frontal solvers with the BEM.

3- The BIEs of nonlinear problems may have domain integrals which require the use of domain elements for their evaluation, thus losing the main advantage of the dimensionality reduction mentioned above.

4- The method is not accurate for problems within narrow strips or curved shell structures.

The Governing Equations

In the elastic stress analysis of a plane-stress, or a plane strain engineering component, there are eight basic independent parameters to be determined, namely: the displacements u and v, strains ε_x, ε_y and γ_{xy} and stresses σ_x, σ_y and τxy. They are governed, at any point inside the component, by eight partial differential equations, which can be deduced for homogeneous isotropic materials from equations given in the last section.

Strain-displacement relationships

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{1}$$

Stress-strain relationships

$$\left.\begin{aligned} \sigma_x &= d_{11}\varepsilon_x + d_{12}\varepsilon_y \\ \sigma_y &= d_{21}\varepsilon_x + d_{22}\varepsilon_y \\ \tau_{xy} &= d_{33}\,\gamma_{xy} \end{aligned}\right\} \tag{2}$$

where:

$$\left.\begin{aligned} d_{11} &= d_{22} = 2G(1-p)/(1-2p) \\ d_{12} &= d_{21} = 2Gp/(1-2p) \\ d_{33} &= G \end{aligned}\right\} \tag{3}$$

G = shear modulus

P = ν (Poisson's ratio) for plane strain problems

$= \dfrac{\nu}{1+\nu}$ for plane stress problems.

Equations of equilibrium

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y &= 0 \end{aligned}\right\} \tag{4}$$

with the following equations, at any point on the boundary:

$$\left.\begin{aligned} T_x &= l\sigma_x + m\tau_{xy} \\ T_y &= l\tau_{xy} + m\sigma_y \end{aligned}\right\} \tag{5}$$

where: T_x and T_y are the traction components in x- and y- directions.
l and m are directional cosines in x- and y-directions, respectively.

Two-dimensional equations in terms of displacement

Substituting Equations (1) into (2), then the stress components may be expressed in terms of displacement components. Substituting the resulting equations into the equations of equilibrium

(Equations 4), then the governing equations are reduced to the following elliptic partial differential equations in terms of displacement components u and v:

$$\left.\begin{aligned} \nabla^2 u + \frac{1}{1-2p}\frac{\partial}{\partial x}(\nabla.\vec{q}) + f_x/G = 0 \\ \nabla^2 v + \frac{1}{1-2p}\frac{\partial}{\partial y}(\nabla.\vec{q}) + f_y/G = 0 \end{aligned}\right\} \qquad (6)$$

where $\vec{\mathbf{q}} = \mathbf{u}\hat{\mathbf{i}} + \mathbf{v}\hat{\mathbf{j}}$, which is the displacement vector.

Biharmonic representation

Gelerkin introduced strain functions G_x and G_y which may be expressed in terms of a vector known as the Gelerkin vector, i.e. (EL-Zafrany 1992):

$$\vec{\mathbf{G}} = \mathbf{G_x}\,\hat{\mathbf{i}} + \mathbf{G_y}\,\hat{\mathbf{j}} \qquad (7)$$

such that (Little 1973):

$$\vec{\mathbf{q}} = \nabla^2\vec{\mathbf{G}} - \frac{1}{2(1-\mathbf{p})}\nabla(\nabla.\vec{\mathbf{G}}) \qquad (8)$$

Writing the partial differential equations (6) in the following vectorial form:

$$\nabla^2\vec{\mathbf{q}} + \frac{1}{2(1-\mathbf{p})}\nabla(\nabla.\vec{\mathbf{q}}) + \vec{\mathbf{f}}/\mu = 0 \qquad (9)$$

then from the definition of the Gelerkin vector, the previous equation can be modified as follows:

$$\nabla^2(\nabla^2\vec{\mathbf{G}}) + \vec{\mathbf{f}}/\mu = 0$$

which can be rewritten explicitly in terms of the following Biharmonic equations:

$$\begin{aligned} \nabla^4\mathbf{G_x} + \mathbf{f_x}/\mu = 0 \\ \nabla^4\mathbf{G_y} + \mathbf{f_y}/\mu = 0 \end{aligned} \qquad (10)$$

FUNDAMENTAL SOLUTION OF SOLID CONTINUUM PROBLEMS

Fundamental Displacement

A two-dimensional solid continuum problem is considered in a semi-infinite domain, with the x-y plane in a state of loading defined by a concentrated force acting at point (x_i,y_i) with a uniform distribution, in the z direction, over a thickness t, which has a constant value for the whole domain. The applied force is represented by the following vector (Fung 1965):

$$\vec{\mathbf{F}} = \mathbf{t}(\mathbf{e_x}\hat{\mathbf{i}} + \mathbf{e_y}\hat{\mathbf{j}}) \qquad (11)$$

where e_x and e_y are the x and y- components of the applied force per unit thickness.

From the definition of the two-dimensional Dirac delta function, a domain distribution of the load intensity equivalent to the applied force, may be expressed as follows (Fung 1965):

$$\left.\begin{aligned} f_x^* = e_x\delta(x-x_i, y-y_i) \\ f_y^* = e_y\delta(x-x_i, y-y_i) \end{aligned}\right\} \qquad (12)$$

Using Equations (6) and (7), the governing partial diferential equations for the above case may be written in the following displacement form:

$$\left.\begin{aligned} \nabla^2 u^* + \frac{1}{1-2p}\frac{\partial}{\partial x}\left(\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}\right)+\frac{f_x^*}{G}=0 \\ \nabla^2 v^* + \frac{1}{1-2p}\frac{\partial}{\partial y}\left(\frac{\partial u^*}{\partial x}+\frac{\partial v^*}{\partial y}\right)+\frac{f_y^*}{G}=0 \end{aligned}\right\} \tag{13}$$

and the solution to such expressions is known as the fundamental solution.
If the displacement components (u*, v*) are expressed in terms of the componants (G_x^*, G_y^*) of Galerkin's vector, such that:

$$\left.\begin{aligned} u^* = \nabla^2 G_x^* - \frac{1}{2(1-p)}\frac{\partial}{\partial x}\left(\frac{\partial G_x^*}{\partial x}+\frac{\partial G_y^*}{\partial y}\right) \\ v^* = \nabla^2 G_y^* - \frac{1}{2(1-p)}\frac{\partial}{\partial y}\left(\frac{\partial G_x^*}{\partial x}+\frac{\partial G_y^*}{\partial y}\right) \end{aligned}\right\} \tag{14}$$

then, equations (13) can be reduced to the following biharmonic equations:

$$\left.\begin{aligned} \nabla^4 G_x^* + e_x \delta(x-x_i, y-y_i)/G = 0 \\ \nabla^4 G_y^* + e_y \delta(x-x_i, y-y_i)/G = 0 \end{aligned}\right\} \tag{15}$$

The previous equations lead to the conclusion that the parameters G_x^*, G_y^* can be defined in terms of one functions: $G_x^* = g^* e_x$, $G_x^* = g^* e_x$ (16)

Hence, Equations(15) may be reduced to the following equation:

$$\nabla^4 g^* + \delta(x-x_i, y-y_i)/G = 0 \tag{17}$$

Defining another function ϖ^* such that: $\nabla^2 g^* = \varpi^* / G$ (18)

Then Equation (17) can be rewritten in terms of the following Poisson's partial differential equation:

$$\nabla^4 \varpi^* + \delta(x-x_i, y-y_i) = 0 \tag{19}$$

which has the following solution:

$$\varpi^* = \frac{1}{2\pi}\left[\log(1/r) + C_1\right] \tag{20}$$

Substituting the above expression into equation (18), and using direct integration, it can be shown that:

$$g^* = \frac{r^2}{8\pi\mu}\left[\log(1/r) + C_1 + 1\right] + C_2 \tag{21}$$

where C_1 and C_2 are arbitrary integration constants.
Then, equations (14) become as:

$$\mathbf{u}_{\alpha}^{*}(x-x_i, y-y_i) = G_{\alpha 1}(x-x_i, y-y_i)e_x + G_{\alpha 2}(x-x_i, y-y_i)e_y \tag{22}$$

where the fundamental solution parameter $G_{\alpha\beta}$ is expressed as follows:

$$G_{\alpha\beta}(x-x_i, y-y_i) = \nabla^2 g^* \delta_{\alpha\beta} - \frac{1}{2(1-p)}\left(\frac{\partial^2 g^*}{\partial x_\alpha \partial x_\beta}\right) \tag{23}$$

All explicit expressions for the fundamental solution parameters given in this paper are found in (al-Adthami, 2003).

Fundamental Strain

The components of Cauchy's strain tensor can be defined for the previous case, as follows (Desai and Siriwardane 1984):

$$\varepsilon_{\alpha\beta}^{*} = \frac{1}{2}\left(\frac{\partial u_\beta^*}{\partial x_\alpha} + \frac{\partial u_\alpha^*}{\partial x_\beta}\right) \tag{24}$$

and using equation (22), the previous equation may be written in the following form:

$$\varepsilon_{\alpha\beta}^{*} = A_{\alpha\beta 1}^{*} e_X + A_{\alpha\beta 2}^{*} e_y \tag{25}$$

where

$$A_{\alpha\beta\gamma}^{*} = \frac{1}{2}\left(\frac{\partial G_{\beta\gamma}}{\partial x_\alpha} + \frac{\partial G_{\alpha\gamma}}{\partial x_\beta}\right) \tag{26}$$

All fundamental solutions given in this paper are functions of $(x-x_i, y-y_i)$.

Fundamental Stress

Substituting the fundamental strain tensor defined by equation (25) into the stress-strain relationships, then it can be proved that:

$$\sigma_{\alpha\beta}^{*} = D_{\alpha\beta 1}^{*} e_X + D_{\alpha\beta 2}^{*} e_y \tag{27}$$

Fundamental Traction

If the fundamental stress components defined above are employed in equations (5), then the corresponding components of fundamental tractions can be expressed in the following form:

$$\left.\begin{aligned} T_x^* &= F_{11} e_X + F_{12} e_y \\ T_y^* &= F_{21} e_X + F_{22} e_y \end{aligned}\right\} \tag{28}$$

Boundary Integral Equations

The governing boundary integral equations are usually obtained by employing fundamental solutions as weighting functions in inverse weighted - residual expressions. For linear elastic problems, the Maxwell-Betti reciprocal theorem may also be used for direct derivation of boundary integral equations.

Boundary Integral Equations of Displacement

Substituting the fundamental loading parameters defined by equations (12) into the inverse expression, and using Dirac delta properties, it can be deduced that:

$$\left.\begin{aligned} & C_i u_i e_x + C_i v_i e_y + \oint_\Gamma (T_x^* u + T_y^* v) d\Gamma = \\ & \oint_\Gamma (T_x u^* + T_y v^*) d\Gamma + \iint_\Omega (f_x u^* + f_y v^*) dxdy \end{aligned}\right\} \tag{29}$$

where: $\mathbf{u_i = u(x_i, y_i)}, \quad \mathbf{v_i = v(x_i, y_i)}$

Employing fundamental displacements (equation 22), and fundamental tractions (equation 28), for arbitrary values of e_x, e_y, then equation (19) can be split into the following boundary integral equations which are defined with respect to the source point (x_i, y_i):

$$C_i u_i + \oint_\Gamma (F_{11} u + F_{21} v) d\Gamma = \oint_\Gamma (G_{11} T_x + G_{21} T_y) d\Gamma + U(x_i, y_i) \tag{30}$$

$$C_i v_i + \oint_\Gamma (F_{12} u + F_{22} v) d\Gamma = \oint_\Gamma (G_{12} T_x + G_{22} T_y) d\Gamma + V(x_i, y_i) \tag{31}$$

where:

$$U(x_i, y_i) = \iint_\Omega (G_{11} f_x + G_{21} fy) dxdy \tag{32}$$

$$V(x_i, y_i) = \iint_\Omega (G_{12} f_x + G_{22} fy) dxdy \tag{33}$$

which represent domain loading terms.

If the source point (x_i, y_i) is inside the domain, then C_i=1, and equations (30) and (31) may be modified as follows:

$$u(x_i, y_i) = U(x_i, y_i) + \oint_\Gamma (G_{11} T_x + G_{21} T_y) d\Gamma - \oint_\Gamma (F_{11} u + F_{21} v) d\Gamma \tag{34}$$

$$v(x_i, y_i) = V(x_i, y_i) + \oint_\Gamma (G_{12} T_x + G_{22} T_y) d\Gamma - \oint_\Gamma (F_{12} u + F_{22} v) d\Gamma \tag{35}$$

The analysis given in the remaining subsections will be limited to cases with source points being inside the domain.

Boundary Integral Equations of Strain

Equations (34) and (35) can be differentiated partially with respect to x_i and y_i; that is, Cauchy's strain components may be defined at an internal point (x_i,y_i) as follows (Banerjee 1994):

$$\varepsilon_{xx}(x_i, y_i) = \frac{\partial u_i}{\partial x_i}, \quad \varepsilon_{yy}(x_i, y_i) = \frac{\partial v_i}{\partial y_i}, \quad \varepsilon_{xy} = \frac{1}{2}\left[\frac{\partial u_i}{\partial y_i} + \frac{\partial v_i}{\partial x_i}\right] \tag{36}$$

When employing displacement equations (equations 34 and 35) in the previous expressions of strain components, integral terms are to be differentiated with respect to x_i and y_i. Then, the boundary integral equation for Cauchy's strain tensor may be expressed in the following form:

$$\begin{aligned} & \varepsilon_{\alpha\beta}(x_i, y_i) = \iint_\Omega (A_{\alpha\beta 1} f_x + A_{\alpha\beta 2} f_y) dxdy + \oint_\Gamma (A_{\alpha\beta 1} T_x + A_{\alpha\beta 2} T_y) d\Gamma - \\ & \oint_\Gamma (B_{\alpha\beta 1} u + B_{\alpha\beta 2} v) d\Gamma \end{aligned} \tag{37}$$

where: $A_{\alpha\beta\gamma} = -A_{\alpha\beta\gamma}$, and $\quad B_{\alpha\beta\gamma} = -\frac{1}{2}\left[\frac{\partial F_{\gamma\beta}}{\partial x_\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x_\beta}\right]$ (38)

Boundary Integral Equations of Stress

Substituting the strain tensor defined by the boundary integral equation (37) into the stress-strain relationships, then a boundary integral equation for the stress tensor at the internal source point (x_i, y_i) can be described, and expressed in the following form (Banerjee 1994):

$$\sigma_{\alpha\beta}(x_i, y_i) = \iint_{\Omega} (D_{\alpha\beta1} f_x + D_{\alpha\beta2} f_y) dxdy + \oint_{\Gamma} (D_{\alpha\beta1} T_x + D_{\alpha\beta2} T_y) d\Gamma - \oint_{\Gamma} (E_{\alpha\beta1} u + E_{\alpha\beta2} v) d\Gamma \tag{39}$$

where: $D_{\alpha\beta\gamma} = -D_{\alpha\beta\gamma}$ (40)

Numerical Treatment of the Boundary Integral Equations

The boundary element method, as described in the previous sections, is based upon dividing the boundary into a suitable number of boundary elements, and approximating the boundary distributions of field function parameters such as displacements and tractions by interpolating them in terms of their nodal values within each element. Discretizing the boundary Γ of a two-dimensional elasticity problem into n_e boundary elements, the boundary integral equations (equations 30 and 31) with respect to the source point may be rewritten as follows:

$$\left.\begin{aligned} C_i u_i + \sum_{e=1}^{n_e} \left[\oint_{\Gamma_e} \{ F_{11} u(\Gamma_e) + F_{21} v(\Gamma_e) \} d\Gamma \right] &= \\ \sum_{e=1}^{n_e} \left[\oint_{\Gamma_e} \{ G_{11} T_x(\Gamma_e) + G_{21} T_y(\Gamma_e) \} d\Gamma \right] &+ U(x_i, y_i) \end{aligned}\right\} \tag{41}$$

$$\begin{aligned} C_i v_i + \sum_{e=1}^{n_e} \left[\oint_{\Gamma_e} \{ F_{12} u(\Gamma_e) + F_{22} v(\Gamma_e) \} d\Gamma \right] &= \\ \sum_{e=1}^{n_e} \left[\oint_{\Gamma_e} \{ G_{12} T_x(\Gamma_e) + G_{22} T_y(\Gamma_e) \} d\Gamma \right] &+ V(x_i, y_i) \end{aligned} \tag{42}$$

where each parameter in the form of $f(\Gamma_e)$ represents a field function parameter approximated over the boundary Γ_e of the eth element.

A Computer Program for Two-Dimensional Solid Continuum Problems

A computer program based upon the theory of the two-dimensional solid continuum mechanics problems of the boundary element method with constant elements is coded in FORTRAN 77 and introduced herein. The program can deal with plane-stress and plane strain problems with surface and domain loading.

In the design of tunnels to be constructed in urban areas, it is necessary to estimate the magnitude and distribution of the stresses and settlements that are likely to occur due to a particular design and

construction technique. Also, the effect of these stresses and movements upon existing surface and buried structures has to be studied.
The main factors that greatly affect the stresses and deformations around tunnels and underground excavations are the shape, dimensions, depth of opening below the ground surface, distance between the openings and the kind of supports (gap parameters). Therefore, the influence of the depth of the tunnel below the ground surface is conducted herein by considering a cavity of 4 meters diameter under a constant surcharge load of 50 KN/m^2.
The computer program is used for the determination of the stress and deformation fields around one cavity. The soil is assumed to be homogeneous, isotropic and a linearly elastic medium containing one opening representing the cavity dimensions and positions. The chosen discretization boundary element mesh is shown in **Fig. (1).**

INFLUENCE OF DEPTH BELOW THE GROUND SURFACE:

<u>Case of a Single Cavity:</u>

Fig. (2) shows a schematic representation of the problem to be studied for 6 values of depth/diameter ratios (Z_o/D = 1, 1.5, 2, 2.5, 3 and ∞)
Figs (3) and (4) show the vertical and horizontal displacements (U_y and U_x) along the ground surface. It can be noticed from these figures that as ($Z_o/D>3$), the disturbing influence on the ground surface does not exceed 5% from the case of no-cavity condition.
Fig. (5) shows the variation of vertical stresses over a line passing through the centerline of the surface loading and the center of cavities (line I-I) in **Fig. (3).** The stresses are normalized by dividing the values by the applied load. From this figure, it can be seen that the vertical stress distributions increase with the increase of Z_o/D ratio, reaching to maximum values as $Z_o/D \rightarrow \infty$ (case of no cavity).
Fig. (6) shows the variation of horizontal stresses over a line passing through the centerline of the surface loading and the center of cavities ((line I-I) in **Fig. (3)**). The stresses are normalized by dividing the values upon the applied load, P. From this figure, it can be seen that the maximum value of horizontal stress decreases as Z_o/D increases, and the point of maximum compressive horizontal stress lies between the ground surface and 0.5D below it, depending on the position of the cavity.
Fig. (7) shows the variation of vertical stresses along a vertical line (II-II) (in **Fig. (3)**) at a distance of 0.625D from the cavity's centerline (where D is the diameter of the cavity). It is evident from this figure that the maximum values of σ_y occur at the point lying on the horizontal level of the centerline of the cavities.
Fig. (8) shows the variation of the horizontal stresses along the same line (as described above). From this figure, it can be seen that the value of σ_x increases to a maximum compressive value above the centerline of the cavity then reverses back to a maximum tensile value on the spring level. Afterwards, it decreases asymptotically to a minimum value as $Z_o/D \rightarrow \infty$.

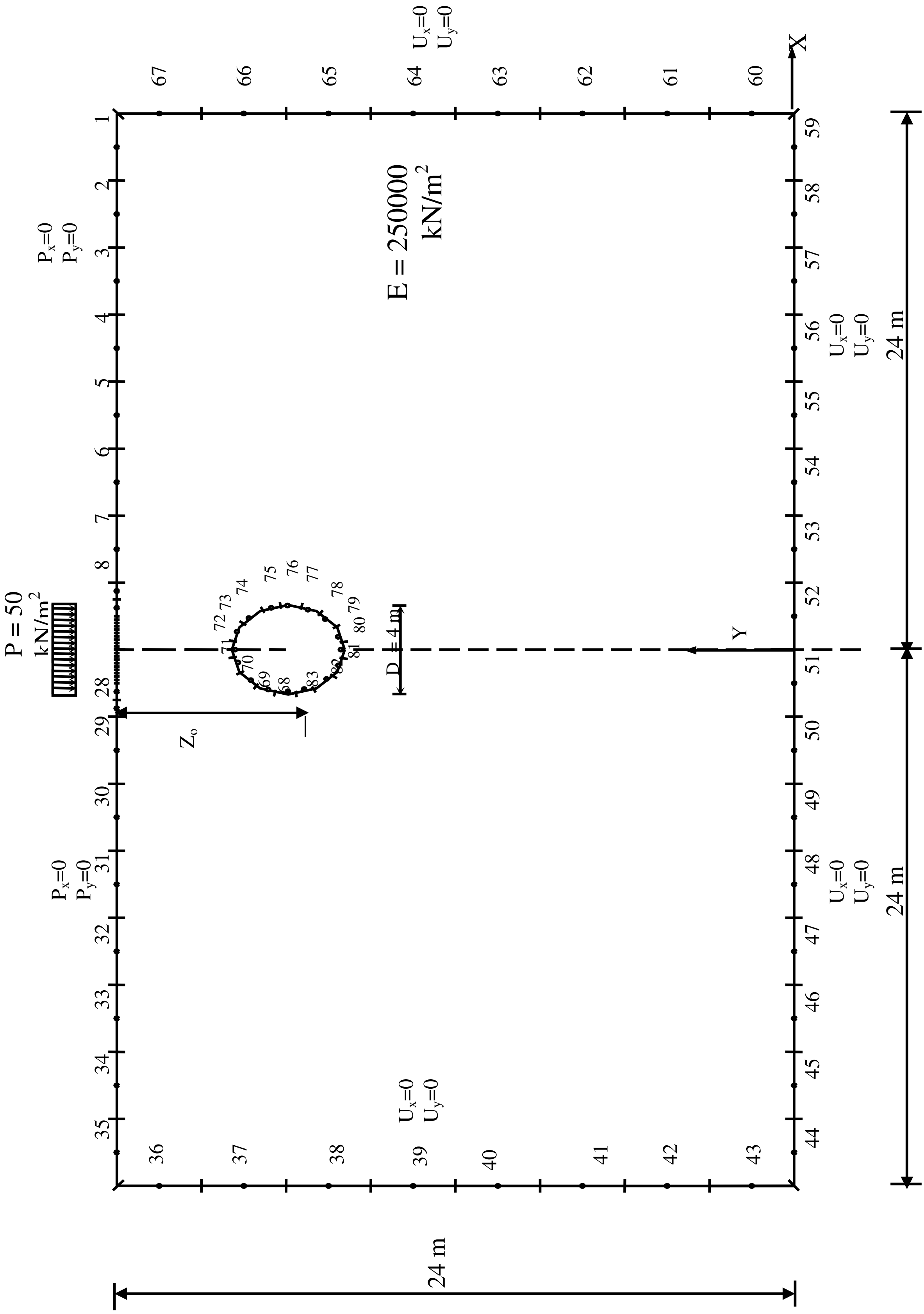


Fig. (1)-Constant boundary element discretization mesh for soil-cavity system.

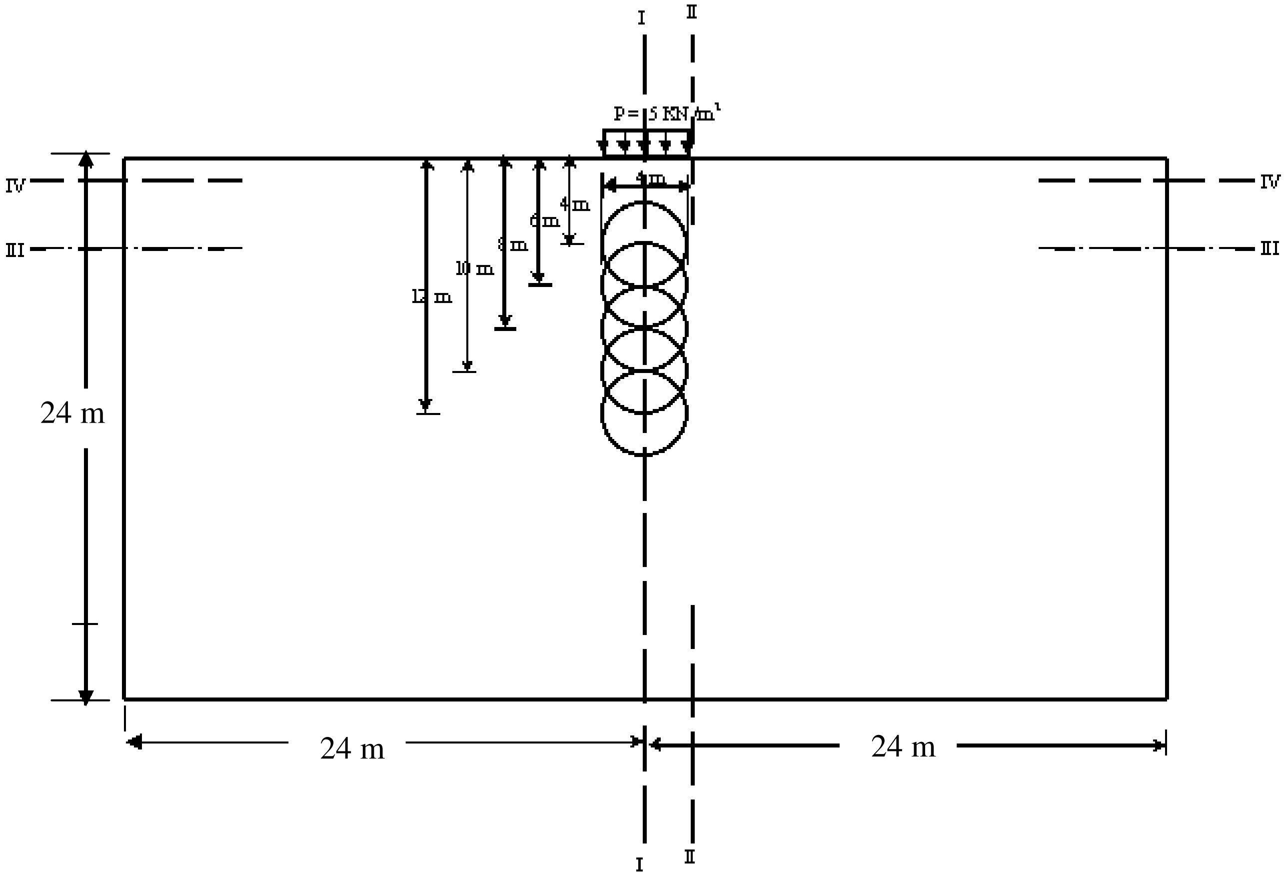


Fig. (2)-Schematic views of surface load-soil-cavities system.

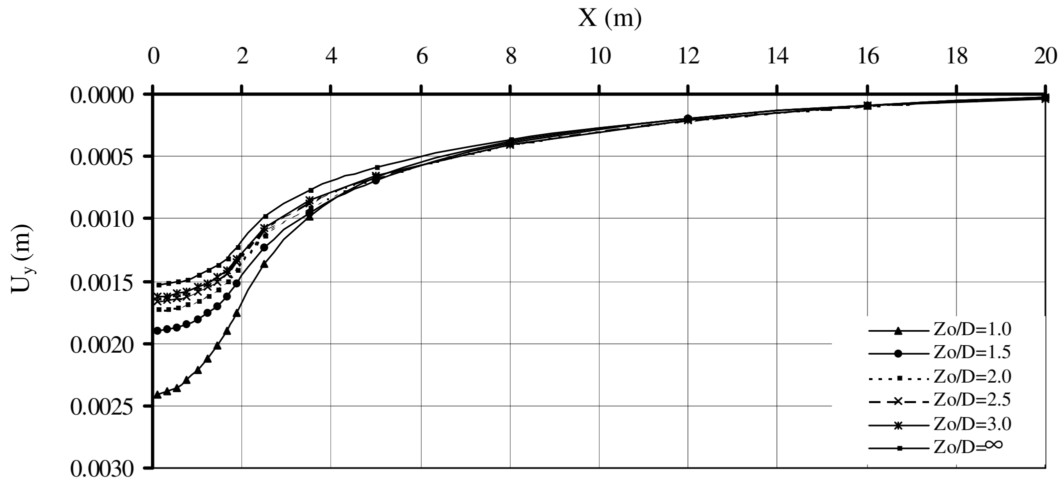


Fig. (3) – Vertical displacements on the surface

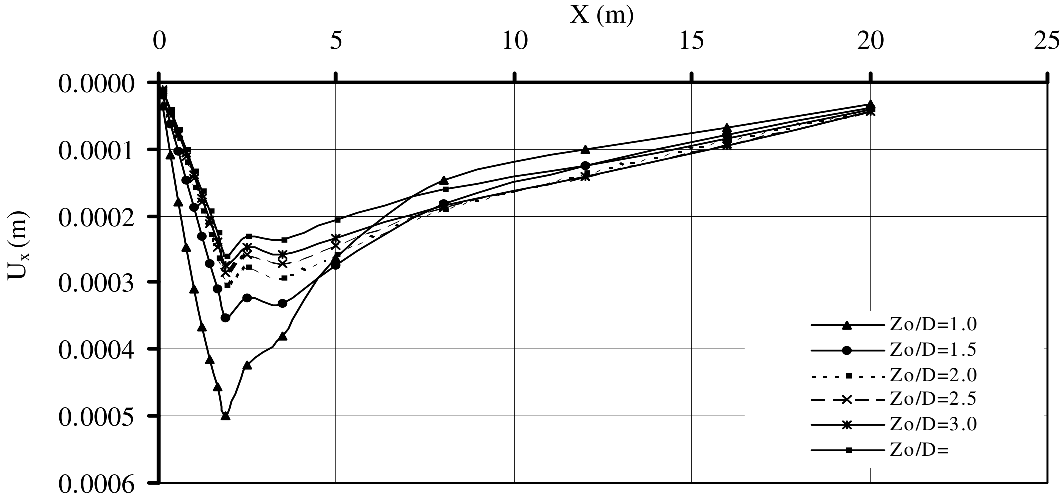


Fig. (4) – Horizontal displacements on the surface

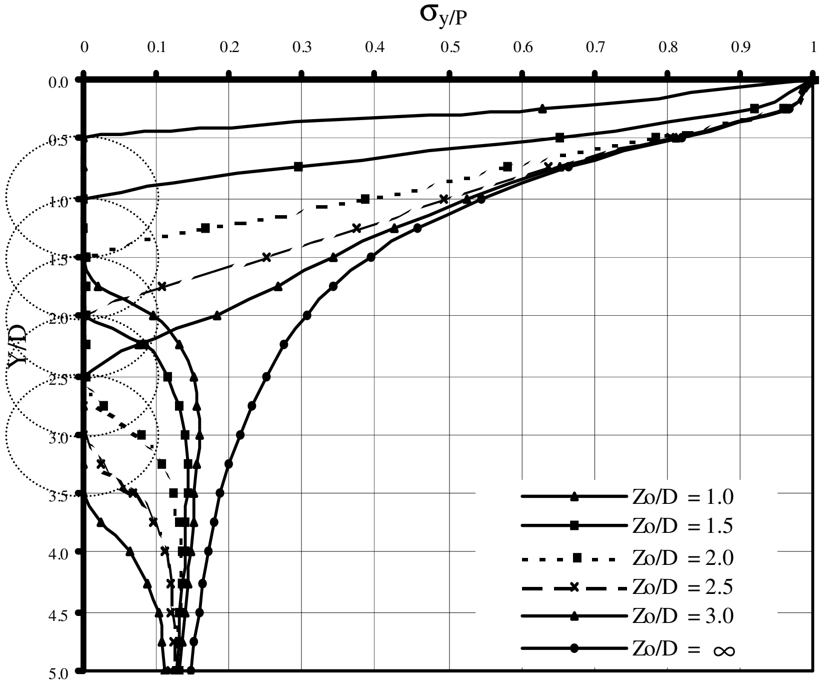


Fig. (5)-Variation of vertical stresses along line I-I.

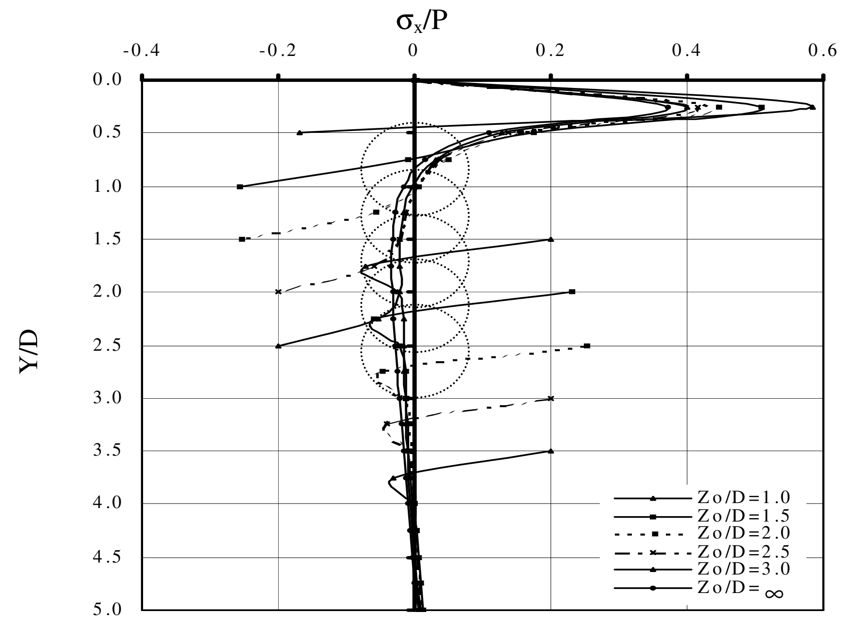


Fig. (6)-Variation of horizontal stresses along line I-I.

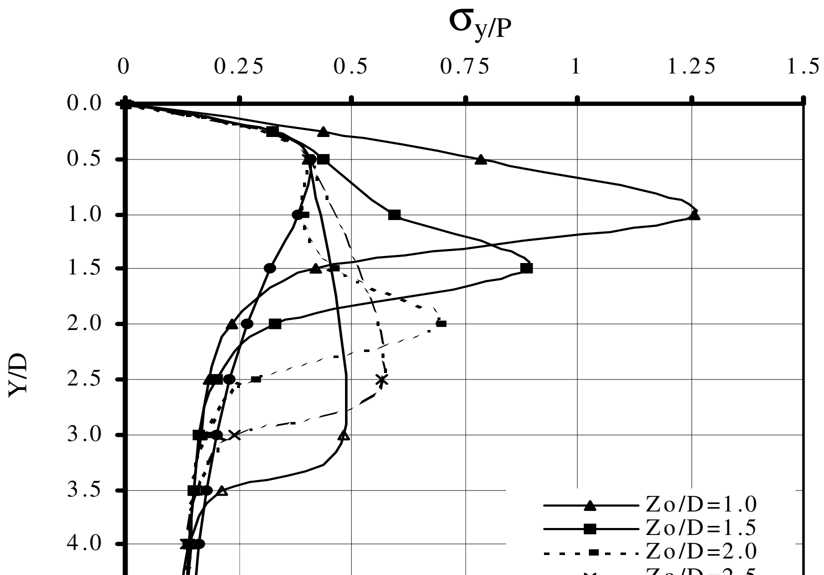


Fig. (7)-Variation of vertical stresses along line II-II.

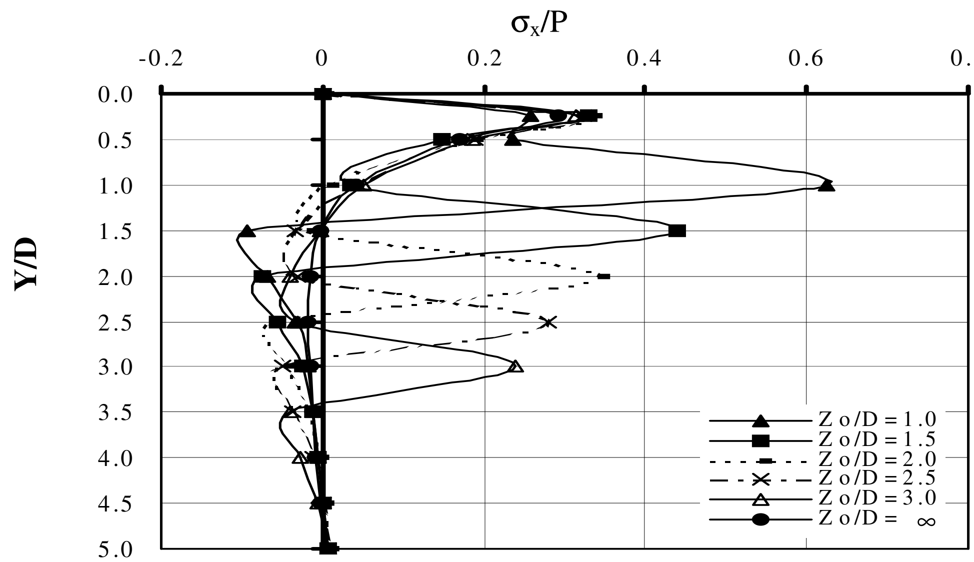


Fig. (8)-Variation of horizontal stresses along line II-II.

Fig. (9) shows the distribution of vertical stresses over a horizontal line 4.0 meters below the ground surface, namely (III-III in **Fig.(3)**), which may represent the raft foundation level of some buildings. It is obvious that by increasing values of Z_o/D, the corresponding σ_y values increase for the region $|X|<D/2$ and then take an opposite trend for $|X| \geq D/2$.

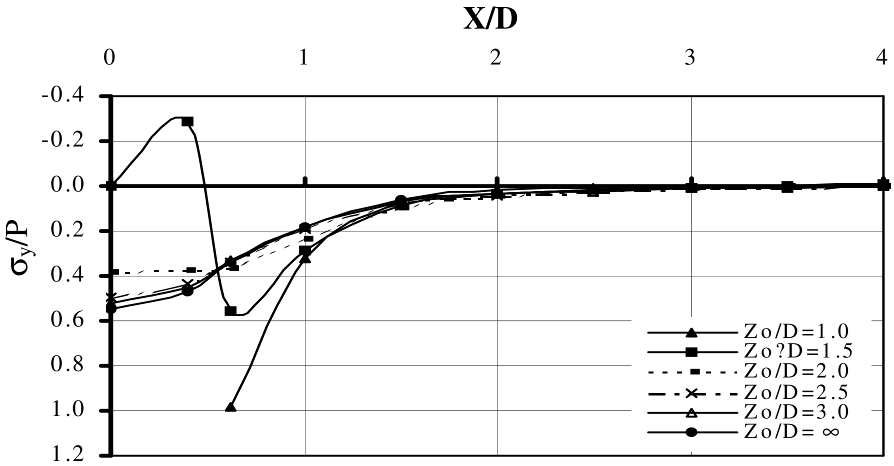


Fig. (9)-Vertical stress distribution on line III-III.

Figs. (10) and (11) show the vertical and horizontal displacements on the same line (III-III). It is noticed that their values increase with the decrease of Z_o/D ratio.

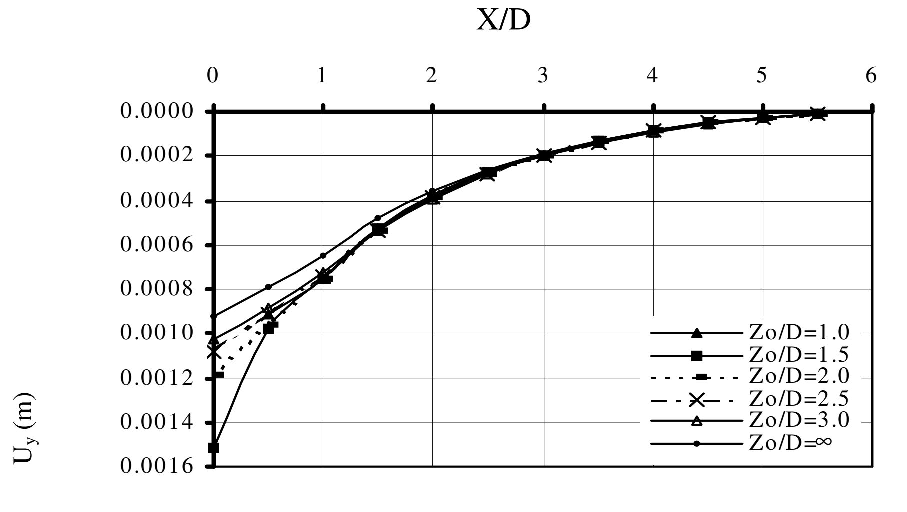


Fig. (10) -Vertical displacements along line III-III.

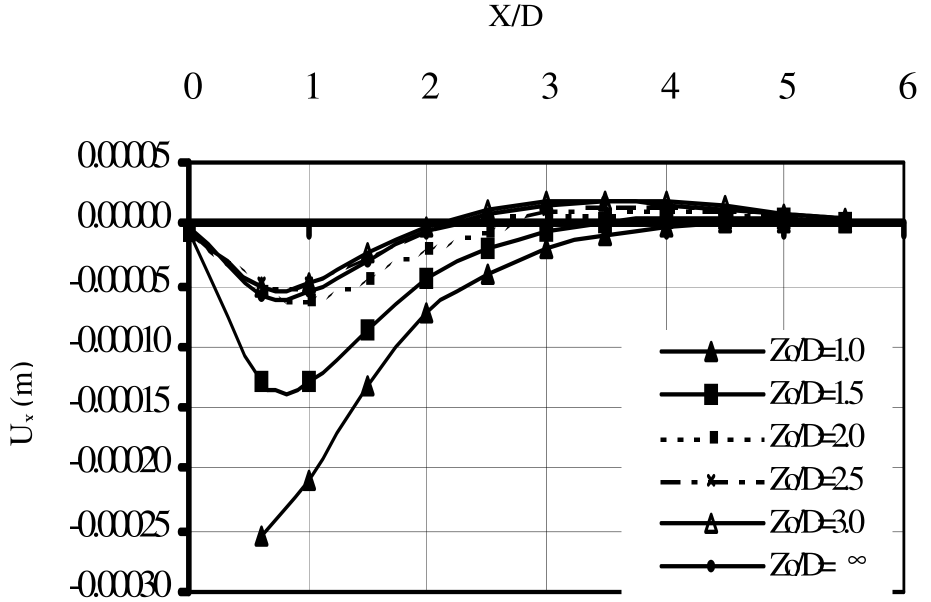
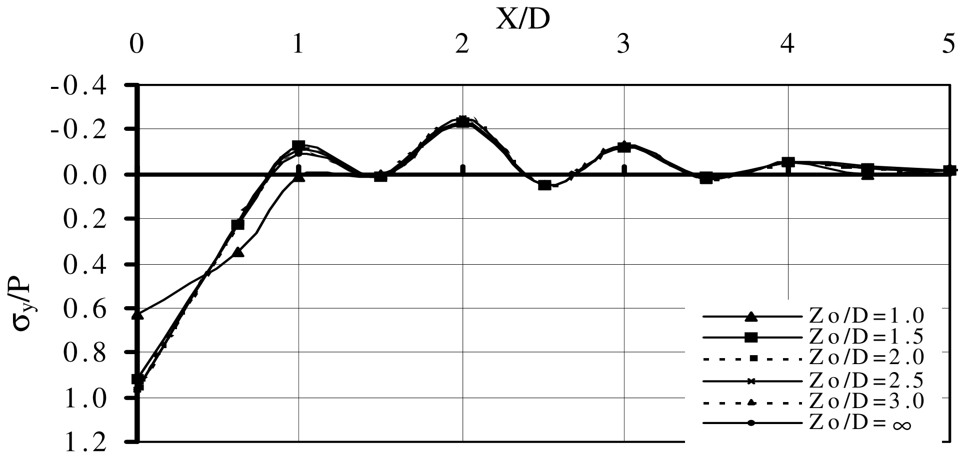


Fig. (11)-Horizontal displacement along line III-III.

Fig. (12) shows the vertical stresses over a horizontal line 1.0 meter below the ground surface (IV-IV) (**Fig. (2)**) which may represent the foundation level of many isolated footings. It is noticed that the heave effect starts to appear at a distance equal to d from the centerline of the surface loading.



\

Fig. (12)-Vertical stress distribution along line IV-IV.

Fig. (13) shows the vertical displacements along the same line above (IV-IV). It is noticed that for the values of $Z_o/D < 3$, the displacements can be significantly more and the cavity effect has to be considered. For the values of $Z_o/D > 3$, the displacements do not exceed those from the case of no-cavity by more than 6% and then the effect of cavity can be neglected.

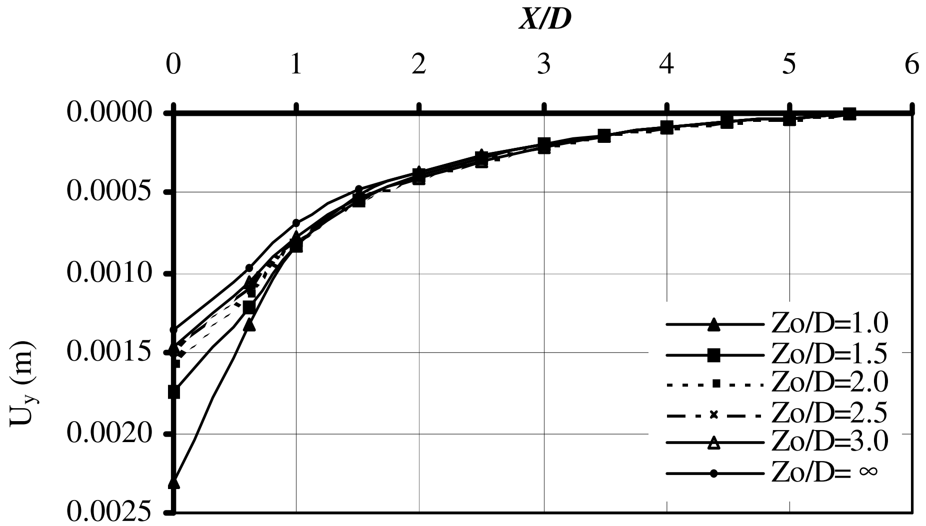


Fig. (13)-Vertical displacements along line IV-IV.

CONCLUSIONS

1- The boundary element method is a practical numerical tool that can be used to obtain solutions to a number of geotechnical problems of considerable complexity.
2- For two-dimensional solid continuum problems, the boundary element method presents the same advantage concerning the discretization of only the boundaries and reduction of the time for preparation of data.
3- A marked increase of stresses is found as the cavity approaches the ground surface and the stress distribution is very sensitive to the depth variation compared with the case of no-cavity conditions.

4- The maximum stresses occur at the haunches of the tunnel rather than at the crown.
5- For the circular cavity that is considered in this paper, it was found that with increasing the depth below the ground surface (depth/tunnel diameter > 3), the surface settlements do not exceed 6 % from those obtained for the case of no-cavity condition.

REFERENCES

Al-Adthami, R. A. J., (2003), Applications of the Boundary Element Method to Soil Media Containing Cavities, M.Sc. thesis, University of Baghdad.

Banerjee, P. K., (1994), The Boundary Element Method in Engineering, McGraw – Hill (UK), London.

Brady, B. H. G. and Bray, J. W., (1978a), The Boundary Element Method for Elastic Analysis of Tabular Orebody Extraction Assuming Complete Plane Strain, International Journal of Rock Mechanics, Mineralogical Science in Geomechanics, Vol. 15, PP. 29 - 37.

Brady, B. H. G. and Bray, J. W., (1978b), The Boundary Element Method for Determining Stresses and Displacements around Long Opening in a Triaxial Stress Field, International Journal of Rock Mechanics, Mineralogical Science in Geomechanics, Vol. 15, PP. 21-28.

Brebbia, C. A., (1984), Boundary Elements VI, Proceedings of the sixth Internation Conference, On Board the Linear, the Queen Elizabeth 2, Springer-Verlag..

Desai, C. S. and Siriwardane, H. J., (1984), Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials, Prentice-Hall, Inc., Englewood Cliffs, N.

El-Zafrany, A., (1992), Techniques of the Boundary Element Method, Ellis Horwood, New York.

Fung, Y. C., (1965), Foundations of Solid Mechanics, Prentice-Hall, Inc., Englewood, Cliffs.

Little, R. W., (1973), Elasticity, Prentice Hall, Englewood Cliffs, New Jersey.

Venturini, W. S. and Brebbia, C. A., (1981), The Boundary Element Method for the Solution of No – Tension Materials, Boundary Element Method, Proceeding of the 3rd International Seminar Irvine, California, edited by Brebbia C. A., pp. 371-390, Springer, Verlage, Berlin.

Note: The superscript (**) refers to a reference cited by Brebbia (1984).