OPTIMUM SATELLITE LAUNCHER TRAJECTORY GUIDED WITH PROPORTIONAL NAVIGATION PLUS GRAVITY COMPENSATION GUIDANCE

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ABSTRACT

The optimum trajectory of a single or multi-stage satellite launcher guided with proportional navigation guidance (PNG) is addressed. The PNG is extended to compensate for the gravity effect. For the trajectory optimization problem, the launcher is modeled as a mass point flying around the center of the Earth. To provide a completely valid analysis, all known influences on the launcher trajectory have been considered; Empirical equations have been used in order to model the Earth standard atmosphere in SI units. A computer program had been constructed in order to simulate the trajectory of such launcher from the available initial conditions. Pegasus launcher is used as a hypothetical example. The simulator results show that the proportional navigation plus gravity compensation guidance gives fairly accurate results.

الخلاصة

في هذا البحث, تمت دراسة المسار الأمثل لناقل أقمار صناعية (مرحلة واحدة أو متعدد المراحل) موجه بطريقة الملاحة التناسبية وتم تطوير قانون الملاحة التناسبية للتعويض عن تأثير الجاذبية الأرضية على مسار الناقل. لغرض ايجاد المسار الامثل, تم فرض الناقل كجزيئة تطير حول مركز الكرة الارضية. ولغرض توفير تحليل رصين, تم اعتبار كل العوامل المؤثرة على مسار الناقل بما فيها تأثير دوران الكرة الأرضية ، وكذلك تم وضع معادلات تجريبية عالية الدقة لوصف تغير خصائص الغلاف الجوي للأرض مع ارتفاع الناقل بالوحدات العالمية الموحدة. لقد تم أعداد برنامج لمحاكاة مسار هكذا ناقل من خلال توفير المعلومات الأوليةلاطلاق الناقل ، وقد تم استخدام بيغاسوس كمثال لتطبيق موضوع البحث. وقد اضهرت النتائج آن استخدام طريقة الملاحة التناسبية مع التعويض عن الجاذبية الأرضية يعطي نتائج مقبولة الدقة.

KEYWORDS

Modeling of Flight System, Proportional Navigation Guidance Law, Gravity Compensation, Linearization, and Optimization.

INTRODUCTION

New concept for space transportation are proposed and investigated in various countries as a means for improving the space transportation capability and for reducing costs. The main task of guidance process is to determine the vehicle position and velocity, computation of control actions necessary to properly adjust position and velocity, and delivery of a suitable adjustments command to the vehicle control system to achieve the correct trajectory. **(Zarchan, 1990 and Bong Wie 1998).**

PNG is accepted as a celebrated guidance law for many guided missile applications like the surface-to-air, air-to-air, air-to-surface missile encounters, standoff weapon delivery, and space rendezvous. The guided point (launcher) is assumed to move towards a target point in a plane containing the velocity vectors of the two points. The PNG technique is defined such that the

velocity vector (heading) of the launcher is turned at a rate proportional to the rotation rate of the line joining the launcher and the target, which is the line of sight (LOS). The PNG principle helps to estimate the magnitude of the lateral acceleration that is perpendicular to LOS as a function of LOS turn rate **(Zarchan, 1990 and Asher & Yaesh 1998).**

The trajectories of rocket vehicle have three successive phases. In the first phase, which is called the boost phase (initial phase), the rocket engine (or engines if the rocket is multi-stage) provide the precise amount of propulsion required to place the rocket on a specific trajectory. Then the engine quits, and the final stage of the rocket (payload) coasts in the second phase that is called midcourse phase, and finally the terminal phase (or gravity turn trajectory)

Guidance operations may occur in the initial, midcourse, or terminal phase of flight. Ballistic missiles are commonly guided only during the initial flight phase, while the rocket engines are burning. A cruise type of missile, such as the Shark or Matador, uses midcourse guidance, operating continuously during cruising flight. Air-to-air missile such as Sidewinder employ terminal guidance systems that lead the missile directly to the target on the basis of measurements on the target itself.

Errors in accuracy for rocket vehicles trajectories are generally expressed as launch point errors, guidance en-route errors or aim point error. Both launch and aim-point errors can be corrected by surveying the launch and the target areas more accurately. Aim errors on the other hand, must improve the rocket's design particularly its guidance systems. A missile's circular error probability (CEP) and bias usually measure aim errors. CEP uses the mean point of impact of missile test firings, usually taken at maximum range, to calculate the radius of a circle that would take in 50 percent of the impact points. Bias measures the deviation of the mean impact point from the actual aim point. An accurate missile has both a low CEP and low bias **(Encyclopedia Britannica, 2002).**

There are no single set of initial conditions required to arrive at a specified target, but rather there are an infinite number of possible free flight paths originating at points in space in the vicinity of some nominal starting point which terminate at the desired destination. For each such point there is a corresponding proper velocity. It is the task of the guidance system to cause the rocket to take up any one of these free flight paths.

LAUNCHER DYNAMIC MODEL

For the trajectory optimization problem the usual mass point modeling is applied for describing the flight system dynamics. With reference to the rotating spherical Earth, the equations of motion can be expressed as **Fig.(1) (Mayrhofer & Sachs 1997)**

$$\dot{V} = \left(\frac{T\cos(\alpha+\delta)-D}{m}\right) - g\sin\gamma + \omega_E^2(R_E+h)\cos\Phi(\sin\gamma\cos\Phi - \cos\gamma\sin\Phi\cos\psi) \quad (1)$$

$$\dot{\gamma} = \left(\frac{T\sin(\alpha+\delta)+L}{mV}\right)\cos\varphi + \left(\frac{V}{R_E+h} - \frac{g}{V}\right)\cos\gamma + 2\omega_E\cos\Phi\sin\psi + \frac{\omega_E^2(R_E+h)}{V}\cos\Phi (\cos\gamma\cos\Phi + \sin\gamma\sin\Phi\cos\psi) \quad (2)$$

$$\dot{\psi} = \left(\frac{T\sin(\alpha+\delta)+L}{mV\cos\gamma}\right)\sin\varphi + \left(\frac{V}{R_E+h}\right)\cos\gamma\sin\psi\tan\Phi - 2\omega_E(\tan\gamma\cos\Phi\cos\psi - \sin\gamma) + \left(\frac{\omega_E^2(R_E+h)}{V\cos\gamma}\right)\sin\Phi\cos\Phi\sin\psi \quad (3)$$

$$\dot{\Phi} = \frac{V\cos\gamma\cos\psi}{(R_E + h)} \quad (4)$$

$$\dot{\Lambda} = \frac{V\cos\gamma\sin\psi}{(R_E + h)\cos\Phi} \quad (5)$$

$$\dot{h} = V\sin\gamma \quad (6)$$

The aerodynamic model can be described as:

$$L = C_L qS \quad (7)$$

$$D = C_D qS \quad (8)$$

Where C_L and C_D are function of α and M

The model of the main rocket propulsion is described as

$$T = \dot{m} g_0 I_{SP} \quad (9)$$

PROPORTIONAL NAVIGATION GUIDANCE LAW

This guidance method is based on the requirement **(Zarchan, 1990)**

$$\frac{d\gamma}{dt} = N'\frac{d\lambda}{dt} \quad (10)$$

And the command acceleration will be

$$a_c = V_c\dot{\gamma} = V_c N'\dot{\lambda} \quad (11)$$

Guidance Equations

According to (**Asher & Yaesh 1998).**

$$\dot{r}_{TM} = V_c = V_T\cos\varepsilon_T - V\cos\varepsilon \quad (12)$$

$$\dot{\lambda} = \frac{1}{r_{TM}}(V_T\sin\varepsilon_T - V\sin\varepsilon) \quad (13)$$

$$\dot{\gamma} = N'\dot{\lambda} \quad (14)$$

But for satellite launcher, the target is a fixed point in the space, so $(V_T = 0)$. Therefore, command acceleration expressed as

$$a_c = \frac{N'V^2}{2r_{TM}}\sin 2\varepsilon = \frac{N'V^2}{2r_{TM}}\sin 2(\lambda - \gamma) \quad (15)$$

The target can be defined as a fixed point in the missile local plane (ρ, z) as shown in **Fig.(2)** measured from the launcher , then the calculations of related quantities gives

$$\rho_M = \int_{t_0}^{t_f} V\cos\gamma dt \quad (16)$$

$$z_M = \int_{t_0}^{t_f} V\sin\gamma dt \quad (17)$$

$$\rho_{TM} = \rho_T - \rho_M \quad (18)$$

$$z_{TM} = z_T - z_M \quad (19)$$

$$r_{TM} = \sqrt{\rho_{TM}^2 + z_{TM}^2} \quad (20)$$

$$\lambda = \tan^{-1}\left(\frac{z_{TM}}{\rho_{TM}}\right) \quad (21)$$

LINEARIZATION

In order to allow better ways of analyses and optimization of the guidance command acceleration it is important to leave the non-linear missile target simulation and find a simpler model, as it is usual in engineering practice. We are going to linearize the acceleration command equations allowing the application of powerful analytical techniques **(Zarchan, 1990).**

As defined previously z_{TM} is the relative separation between the rocket and the target in local vertical plane, then the relative acceleration can be expressed as

$$\ddot{z}_{TM} = -a_c \cos\lambda \tag{22}$$

And the expression for the LOS angle will be

$$\sin\lambda = \frac{z_{TM}}{r_{TM}} \tag{23}$$

If we assume that the LOS angle is small, then **eq.(22)** and **eq.(23)**

$$\ddot{z}_{TM} - a_c \tag{24}$$

$$\lambda = \frac{z_{TM}}{r_{TM}} \tag{25}$$

In linearized analyses we treat the closing velocity as a positive constant and equal to the missile velocity. Since closing velocity has also been previously defined as the negative derivative of the range from the missile to target, and since the missile-target separation distance must go to zero at the flight, we can also linearized the range equation with the time varying relationship **(Zarchan, 1990 and Asher & Yaesh 1998).**

$$r_{TM} = V_c\left(t_f - t\right) \tag{26}$$

Since the missile-target separation distance goes to zero at the end of flight by definition, the linearized miss distance is taken to be the relative separation between missile and target z_{TM}, at the end of flight:

$$Miss = z_{TM}\left(t_f\right) \tag{27}$$

This linearized model will give very high accuracy, where its results are the same as those obtained from the non-linear model for fixed or non-maneuvering targets (the case of our search), and for heading error case, and will give overestimations from the non-linear model in the case of maneuvering targets **(Zarchan, 1990).**

Linearization of PNG Law

Substituting **eq.(26)**, into **eq.(24)** we get

$$\lambda = \frac{z_{TM}}{V_c\left(t_f - t\right)} \tag{28}$$

The derivative of **eq.(28)** will give the LOS rate by

$$\dot{\lambda} = \frac{z_{TM} + \dot{z}_{TM}\left(t_f - t\right)}{V_c\left(t_f - t\right)^2} = \frac{z_{TM} + \dot{z}_{TM} t_{go}}{V_c t_{go}^2} \tag{29}$$

Thus we can express the PNG law as

$$a_c = N' V_c \dot{\lambda} = \frac{N'\left(z_{TM} + \dot{z}_{TM} t_{go}\right)}{t_{go}^2} \tag{30}$$

The expression in the parentheses of **eq.(30)** represents the future separation between missile and target. More simply, the expression in parentheses is the miss distance that would result if the missile made no further corrective acceleration and the target did not maneuver. This expression is referred to as the zero effort miss (ZEM). Therefore, we can also think of PN as a guidance law in which commands acceleration are issued inversely proportional to the square of time to go and directly proportional to the ZEM **(Zarchan, 1990 and Asher & Yaesh 1998).**

OTMIZATION OF PNG LAW

We seek to find a guidance law that is a function of the system states. There are an infinite number of possible guidance laws; thus, it is necessary to state in mathematical terms what the guidance law should do. Certainly we would like to hit the target; therefore, one feature of the guidance law should be a zero miss distance requirement. In addition, we would like to hit the target in an efficient manner. In other words, we desire to use minimal total acceleration. A poplar and mathematically convenient way of stating the guidance problem to be solved is that we desire to achieve zero miss distance and to minimize the integral of the square of the command acceleration **(Zarchan, 1990 and Asher & Yaesh 1998)** i.e.

$$z_{TM}(t_f) = 0 \tag{31}$$

Subject to minimizing

$$\int_0^{t_f} a_c^2(t)dt \tag{32}$$

Unfortunately, if we minimize a more meaningful performance index such as the integral of the absolute value of a_c, the solution would be mathematically intractable. Typically this type of problem with a quadratic performance index is solved using techniques from optimal control theory. However, this class of problems can be solved more easily using Schwartz inequality **(Zarchan, 1990).**

Now, we will construct from the previous equations the satellite launcher flight in state space form as

$$\begin{bmatrix} \dot{\mathbf{z}}_{\mathbf{TM}} \\ \ddot{\mathbf{z}}_{\mathbf{TM}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{z}_{\mathbf{TM}} \\ \dot{\mathbf{z}}_{\mathbf{TM}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ -\mathbf{1} \end{bmatrix} \mathbf{a_c} \tag{33}$$

Before going in any further analyses, we must first check the controllability of the state space that we had performed. The general state space form

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{34}$$

by comparing **eq.(33)** and **eq.(34)** we find that

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{1} \end{bmatrix}$$

We must find the matrix $[\mathbf{B} \,\vdots\, \mathbf{AB}]$

$$[\mathbf{B} \,\vdots\, \mathbf{AB}] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \text{nonsigular}$$

The system is therefore completely state variable **(Ogata, 2002).**

The solution of the state space vector deferential equation is given at the final time t_f by the vector relationship **(Zarchan, 1990).**

$$\mathbf{x(t)} = \mathbf{\Phi(t_f - t)x(t)} + \int \mathbf{\Phi(t_f - \tau)Bu(\tau)d\tau} \tag{35}$$

Where $\mathbf{\Phi(t)} = \begin{bmatrix} \mathbf{1} & \mathbf{t} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

Using the above fundamental matrix in the solution for the state space vector deferential equation and only looking at the first state, we get

$$z_{TM}(t_f) = z_{TM}(t) + \dot{z}_{TM}(t)(t_f - t) - \int_t^{t_f} (t_f - \tau) a_c(\tau) d\tau \tag{36}$$

For convenience, let us define the terms

$$f_1(t_f - t) = z_{TM}(t) + \dot{z}_{TM}(t)(t_f - t) \tag{37}$$

$$h_1(t_f - \tau) = t_f - \tau \tag{38}$$

Then we can say that

$$z_{TM}(t_f) = f_1(t_f t) - \int_t^{t_f} h_1(t_f - \tau) a_c(\tau) d\tau \tag{39}$$

For the conditions in which we have zero miss distance $z_{TM}(t_f) = 0$, we can rewrite **eq.(39)**

$$f_1(t_f - t) = \int_t^{t_f} h_1(t_f - \tau) a_c(\tau) d\tau \tag{40}$$

If we apply the Schwartz inequality to **eq.(40)** we get the relationship **(Zarchan, 1990).**

$$f_1^2(t_f - t) \le \int_t^{t_f} h_1^2(t_f - \tau) d\tau \int_t^{t_f} a_c^2(\tau) d\tau \tag{41}$$

Then $\int_t^{t_f} a_c^2(\tau) d\tau \ge \dfrac{f_1^2(t_f - t)}{\int_t^{t_f} h_1^2(t_f - \tau) d\tau}$ (41)

The integral of the square of the command acceleration will be minimized when the equality sign holds in the preceding inequality. According to the Schwartz inequality, the equality sign holds when

$$a_c(\tau) = k h_1(t_f - \tau) \tag{42}$$

This means that the integral of the squared acceleration is minimized when

$$k = \frac{f_1(t_f - t)}{\int_t^{t_f} h_1^2(t_f - \tau) d\tau} \tag{43}$$

Therefore, the command acceleration is given by

$$a_c = \left[\frac{f_1(t_f - t)}{\int_t^{t_f} h_1^2(t_f - \tau) d\tau} \right] h_1(t_f - t) \tag{44}$$

Substituting yields to the feedback control guidance law

$$a_c = \frac{3(z_{TM} + \dot{z}_{TM} t_{go})}{t_{go}^2} \tag{45}$$

By comparing **eq.(30)** with **eq.(45)** we find that the optimum value of effective navigation ratio for the satellite launcher is $N' = 3$.

PNG LAW WITH GRAVITY COMPENSATION

One of the major effects that prevent to obtain a straight-line missile's trajectory is the gravity effect. Changes in the missile velocity due to gravity caused the LOS to rotate. The PNG law responds to the apparent LOS rate with command acceleration. If we have knowledge of gravitational acceleration, it seems reasonable that it might be possible to compensate for unnecessary acceleration via the guidance law. **(Zarchan, 1990).**

Fortunately in our particular case the target did not have gravitational acceleration. The launcher gravitational acceleration can be expressed as **(Meriam, & Kraige, 1998).**

$$g = \frac{Gm_o}{r^2} \tag{46}$$

The component of the gravity acceleration perpendicular to the LOS for the launcher is

$$g_{M_{PLOS}} = g\cos\lambda \tag{47}$$

And the component of the gravity acceleration perpendicular to the LOS for the target is zero.

The gravitational acceleration difference between the target and the missile can be treated as an additional terms in the ZEM. Therefore, we can modify the PNG law to account for gravity. The resultant law is **(Zarchan, 1990 and Asher & Yaesh 1998).**

$$a_c = N'V_c\dot{\lambda} + \frac{N'}{2}\left(g_{T_{PLOS}} - g_{M_{PLOS}}\right) \tag{48}$$

For the case of the satellite launcher, **eq.(48)** can be reduced to

$$a_c = N'V_c\dot{\lambda} - \frac{N'g\cos\lambda}{2} \tag{49}$$

HYPOTETICAL EXAMPLE

Pegasus is used for showing PNG and PN plus gravity compensation guidance. Pegasus is a small commercial launch vehicle developed by orbital science. It is provided with solid propellant booster and wings, and is lunched from an aircraft. Pegasus mission is to inject the satellite at altitude of (123 km), with velocity of (7790 m/sec), and zero flight path angle (FPA). Pegasus is released from its carrier with an initial velocity of about (0.8 Mach) and zero initial FPA with free flight duration of (5 sec). At the free flight, delta wing of the Pegasus is capable of achieving a zero FPA at the start of the first stage burning. **(Isakowitz, & Hopkins Jr, & Hopkins, 1999).**

RESULTS AND DISCUSSION

Three cases are simulated. The first case when Pegasus launched without guidance, the second when Pegasus guided with optimum PNG, and the third Pegasus guided with PN plus gravity compensation guidance. The altitude, velocity, and FPA are shown in **Figs.(3), Figs.(4),** and **Figs.(5)** respectively.

Pegasus without guidance is rapidly falling due to negative values of the FPA, which cause the thrust force to push down. This means that Pegasus cannot fly without guidance.

For the second case the large drop in altitude, and FPA during the second stage flight lead the rocket be unable to reach its target which is very far from the rocket, although the altitude and FPA increase, the velocity continue to decrease then starts to increase before the end of flight.

For the third case, the altitude of this case is highly accurate, and we can see from **Figs.(5)** that launcher is exceed the target and return to it because of the guidance, i.e. the launcher is reach to altitude of (136 km) at (267 sec) then return to attitude of (132 km) at the end of flight. At the end of flight for this case the launcher velocity about (5.62 %) less than the required velocity and the FPA is about (-3.9°).

From comparing the Pegasus trajectory and the final mission requirements as given in **(Isakowitz, & Hopkins Jr, & Hopkins, 1999),** and that obtained from the PN plus gravity compensation guidance, we can say that the PN plus gravity compensation guidance is fairly applicable for satellite launcher case.

CONCLUSION

The following concluding remarks are drawn from the present work:

1. The optimum value of the effective navigation ratio of the proportional navigation guidance for a satellite launcher application is $N' = 3$.
2. The proportional navigation plus gravity compensation guidance advised to use for the satellite launcher application.

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NOMENCLATURE

(SI units are used, unless otherwise stated)

a_c	Command acceleration
C_D	Drag coefficient
C_L	Lift coefficient
D	Drag force
G	Universal constant
g	Local Earth gravitational acceleration
g_o	Earth gravitational acceleration at sea level
g_{PLOS}	Gravitational acceleration component perpendicular to LOS
h	Altitude of the launcher vehicle
I_{SP}	Specific impulse
L	Lift force
M	Mach number
m	Total launcher vehicle mass
$\dot{m}$	Fuel mass flow rate
m_o	Mass of the Earth
N'	Proportional navigation constant
q	Dynamic pressure
r	Radial distance between the center of the Earth and the launcher center
R_E	Mean radius of the Earth

r_{TM}	Target-Missile separation distance
S	Launcher reference area
T	Total thrust of the launcher
t_f	Final flight time
t_{go}	Time to go
t_0	Initial flight time
V	Launcher vehicle velocity
V_c	Closing velocity between the missile and the target
z_{TM}	Target-Missile separation distance in local z direction
α	Attack angle
ψ	Azimuth angle
ε	Launcher lead angle
Φ	Geocentric latitude angle
φ	Roll (Bank) angle
ϕ	Sight angle
γ	Flight path angle
Λ	Geographic longitude
λ	Local line of sight angle
ω_E	Angular velocity of the Earth rotation
(ρ, z)	Local coordinates of launcher vehicle
θ	Pitch or (Elevation) angle

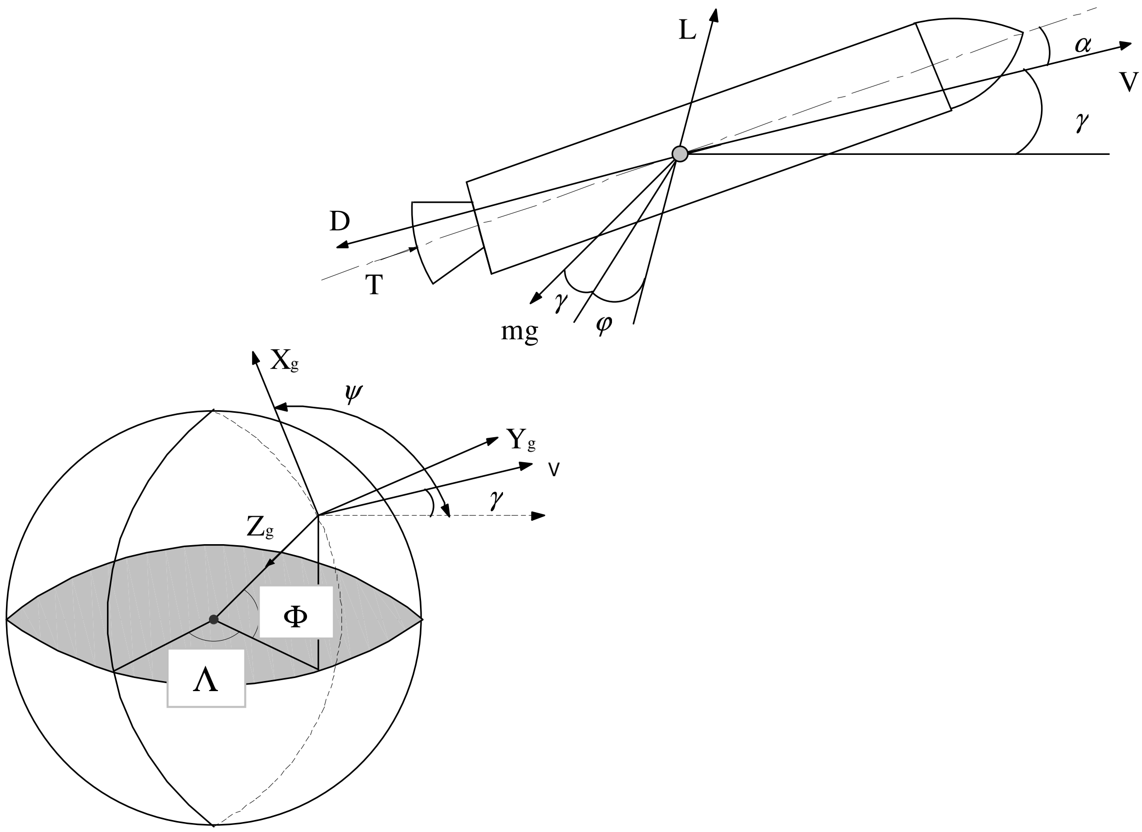


Fig.(1)

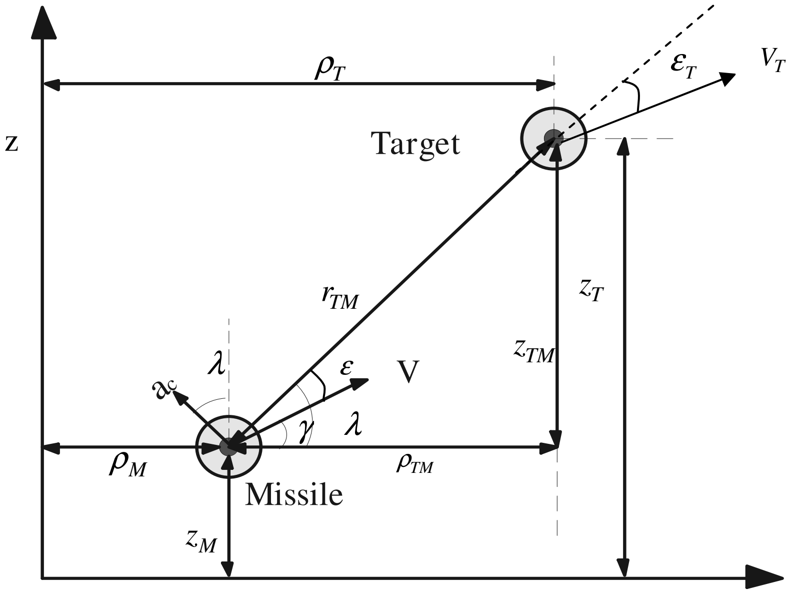
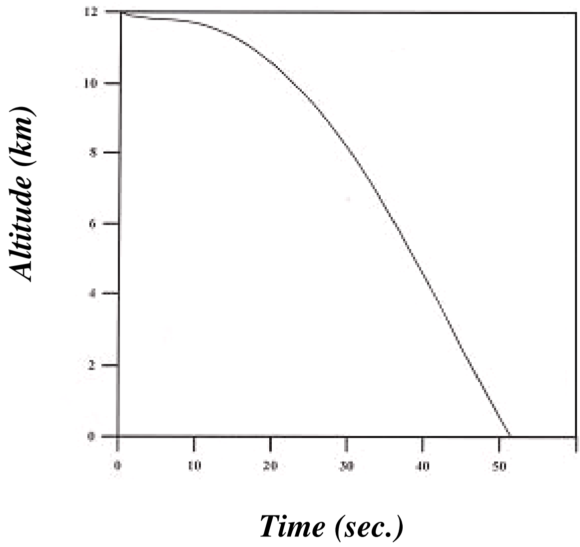
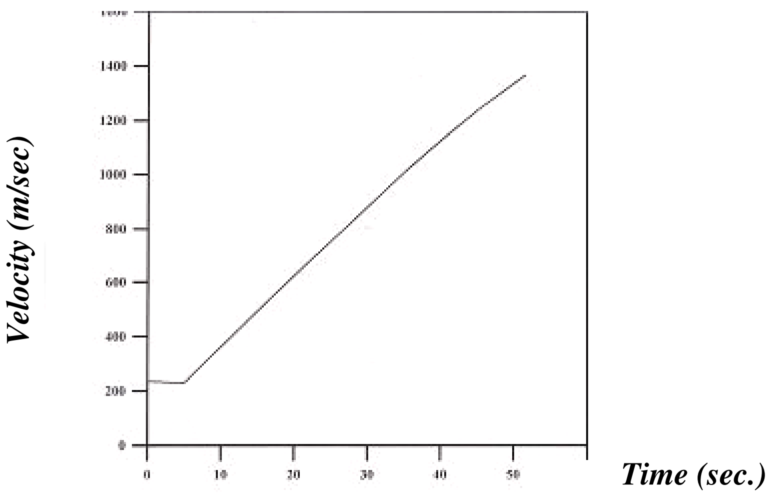
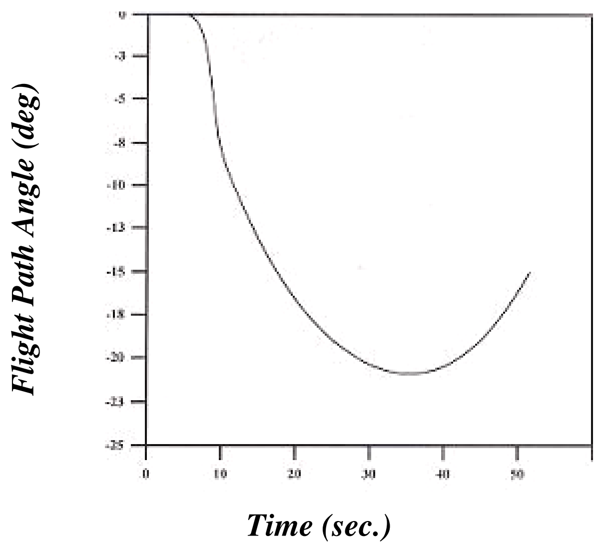

z
ρ
Target
Missile
V
ε
λ
γ

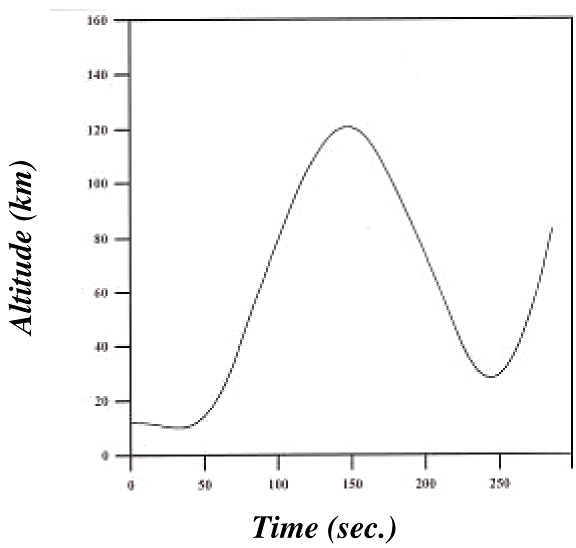
Fig.(2)

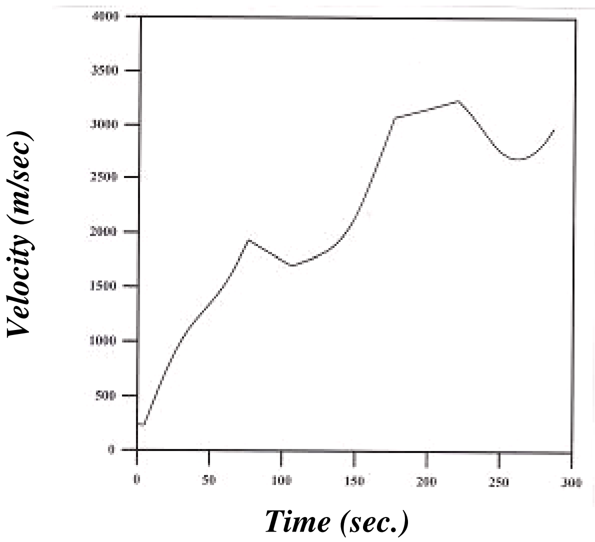

Altitude (km)
Time (sec.)

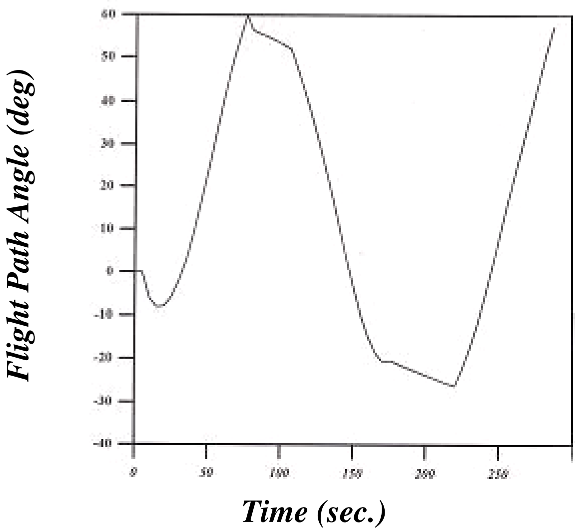

Velocity (m/sec)
Time (sec.)



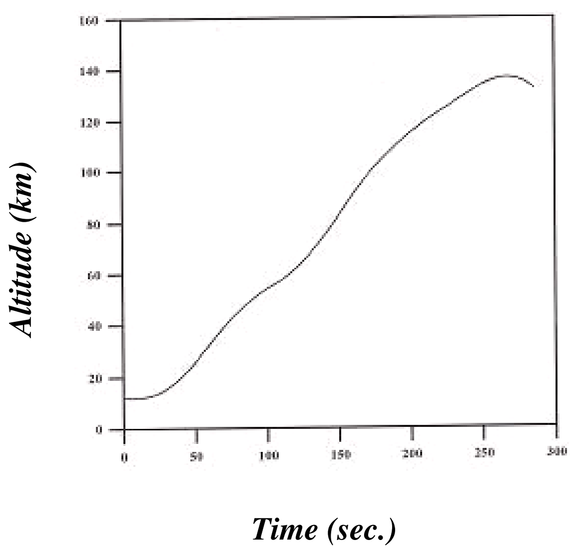
Figs.(3a,b,c) respectively
Launcher without guidance

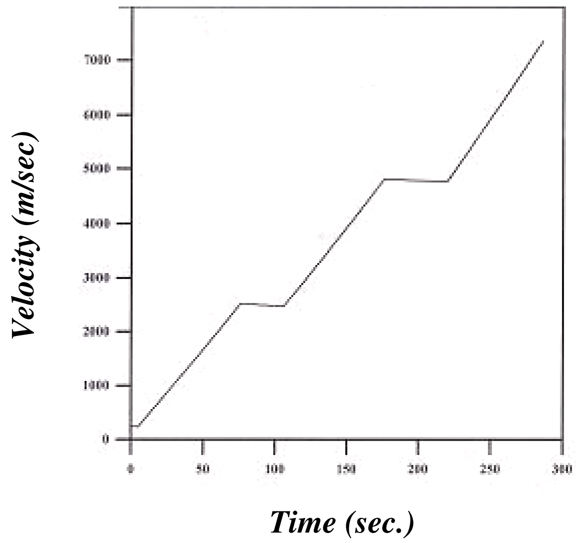


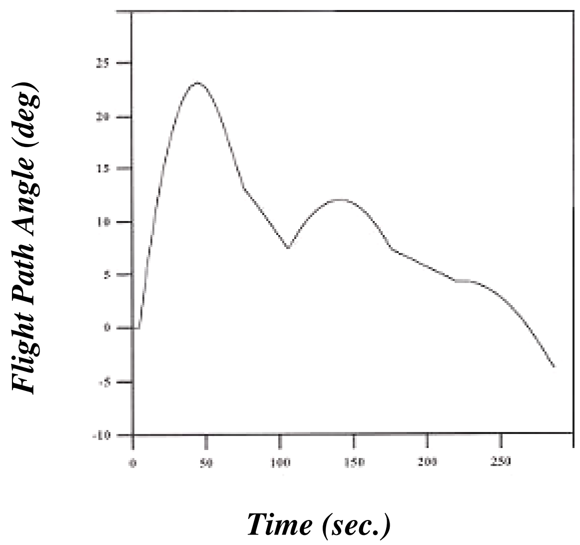




Figs.(4a,b,c) respectively
Launcher with PN guidance







Figs.(5a,b,c) respectively
Launcher with PN plus gravity compensation guidance