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Modeling Seepage Control in Hydraulic Structures

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ABSTRACT.

This paper brings together random field generation and finite element technique to model steady state seepage through two dimensional soil domain in which the hydraulic conductivity is randomly distributed in space. The analysis treats the hydraulic conductivity as a spatially random property with specified mean, variance and spatial correlation length. The results of the combined model used in an optimization procedure to obtain the optimum dimension of the hydraulic structure seepage control techniques.

الخلاصه

في هذا البحث تم ربط نموذجين لتوليد القيم العشوائية الحقلية مع تقنية العناصر المحددة لنمذجة التسرب المستقر خلال مجال تربة ثنائي البعد حيث التوصيلية الهيدروليكية موزعة عشوائياً. التحليل عالج التوصيلية الهيدروليكية كخاصية موزعة توزيعاً مكانياً بثبوت قيم المعدل والتباين و طول الارتباط المكاني . نتائج النموذج المركب أُدخلت في نموذج لاستخراج الحل الأمثل لإيجاد البعد الأمثل لتراكيب السيطرة على التسرب في المنشات المائيةً .

INTRODUCTION.

The work presented in this paper brings together finite element analysis for analyzing the seepage phenomena and random field theory to simulate the foundation beneath hydraulic structures to be anisotropic hetrogeneous media. This model coupled with optimization procedure to obtain the optimum dimensions of hydraulic structure seepage control techniques with minimum cost of these techniques. As it is known, the dimensions of seepage control techniques have different effects on the factors of safety against piping and uplift pressure. For example increasing the length of d/s cutoff will increase the factor of safety against piping and decreases that for uplift pressure. Hence a model is required to find the optimum dimension of such length and other control techniques .The constrains adopted was the factors of safety against uplift pressure and piping phenomena. The objective function adopted was the cost of those seepage control techniques.

Smith and Freeze (1979) were the first to study the problem of confined flow through a stochastic medium using finite differences method, in which two dimensional (2D) examples of flow between parallel plates and beneath a single sheetpile were presented. Recent developments in random field and finite element

methodology have led to further studies by Griffiths and Fenton (1994) for two dimensional seepage beneath water retaining structures and Griffiths and Fenton

(1997) for three dimensional seepage problem under sheetpile.

SEEPAGE EQUATION.

In order to perform a seepage analysis, a general model describing the phenomena of seepage must be available supplied with specific boundary conditions and soil properties. This model can be used to determine head and flow distribution. The field equation is the mathematical basis for several models or methods used in seepage analysis. The hydraulic conductivity in the two-dimensional case is so oriented that maximum and minimum values occur along the preferred axes, which are called principal axes. Other directions through the domain yield values of hydraulic conductivity which are between the principal values and are distributed as an ellipse, which the principal values making the major and minor axes.

The theory of seepage flow through anisotropic heterogeneous porous media is found in many researches. The following derivation was taken from Harr (1962). Consider a domain of flow under hydraulic structure at anisotropic soil foundation, where the domain of flow is oriented in the (x, y) coordinate axes will makes an angle θ with the principal axes of hydraulic conductivity of the porous media. The coordinate system (μ, λ) is designed with the principal axes of the hydraulic conductivity. The general seepage equation in heterogeneous anisotropic porous media in two dimensional space is:

$$\frac{\partial}{\partial x}[(k_\mu \cos^2\theta + k_\lambda \sin^2\theta)\frac{\partial H}{\partial x}] + \frac{\partial}{\partial x}[(k_\mu - k_\lambda)\sin\theta\cos\theta)\frac{\partial H}{\partial y}] \text{......................(1)}$$

$$+\frac{\partial}{\partial y}[(k_\mu - k_\lambda)\sin\theta\cos\theta)\frac{\partial H}{\partial x}] + \frac{\partial}{\partial y}[(k_\mu \cos^2\theta + k_\lambda \sin^2\theta)\frac{\partial H}{\partial y}] = 0$$

consider the term in equation above as a simple form :

$K_{xx} = (K_\mu \cos^2\theta + k_\lambda \sin^2\theta)$

$K_{xy} = (K_\mu - k_\lambda) \sin\theta \cos\theta)$

$K_{yy} = (K_\mu \sin^2\theta + k_\lambda \cos^2\theta)$

In this research, the piezometric head represents the differences of piezometric head, thus:

$$H = \frac{H - H1}{H1 - H2} \text{..(2)}$$

Where:

$H_{o:}$ difference ratio of piezometric head

H_1, H_2: the piezometric head in upstream and downstream of the structure respectively.

H: the piezometric head at any point in the flow domain.

Basically there are two kinds of boundary condition to deal with, the first, is a specific head along a given boundary, an important special case for this boundary condition is where the specified head is a constant along the boundary called an equipotential line.

In mathematical term H = Ho on boundary S1

Where H is the peizometric head on (S1) hence all reservoir boundaries represent equipotential lines.

A second type of boundaries is where the normal component of the gradient is specified, along the boundaries. Thus, we use the term flux boundary or more commonly-flow boundary-; a special type of flow boundary is a no-flow boundary that is expressed mathematically.

$\partial H/\partial N = 0$

Or in general

$$(k_{xx}\frac{\partial H}{\partial x} + k_{xy}\frac{\partial H}{\partial y})L_x + (k_{xy}\frac{\partial H}{\partial x} + k_{yy}\frac{\partial H}{\partial y})L_y = 0 \text{..(3)}$$

Where L_x and L_y are a direction cosine of normal vector on the surface with direction X and Y respectively. These boundaries represent a streamline of constant stream function. In modeling two-dimensional seepage beneath hydraulic structure where the lateral boundary of the model typically are no-flow boundary, experience suggested that the distance to these lateral boundary at least three times the depth of the flow system for isotropic media .In anisotropic medium the boundary should be located at a distance of $3(k_{xx} / k_{yy})^{1/2}$ times the depth of the flow system.

BRIEF DESCRIPTION OF RANDOM FIELD MODEL.

In this paper a random field generator known as the Local Average Subdivision Method (LAS) devised by Fenton (1990). The reliability of many fields in presence of uncertainty has been a crucial factor in their analysis and design. Several of these fields are inherently random and can be modeled as a random processes, in seepage modeling one can use Monte Carlo random generation method to generate a field permeability through porous media to simulate a heterogeneous field hydraulic conductivity.

Field measurements of hydraulic conductivity have indicated an approximately lognormal distribution as seen before. The same distribution has therefore been adopted for the simulations generated in this research. Essentially the field's hydraulic conductivity obtained through the transformation.

$$K_i = \exp.\,(\mu_{lnk} + \sigma_{lnk}\, G_i\,) \qquad (4)$$

Where K_i is the permeability assigned to the ith elements, G_i is the local average of a standard gaussian random field G over all the domain of the ith elements, μ_{lnk} and σ_{lnk} are the mean and standard deviation of the logarithm of K obtained from the prescribed mean and standard deviation μ_k and σ_k via the transformation

$$\mu_k = \exp\left\{\mu_{\ln k} + \tfrac{1}{2}\sigma^2_{\ln k}\right\} \qquad (5)$$

$$\sigma_k = (\mu_k)^2\{\exp(\sigma^2_{\ln k} - 1\} \qquad (6)$$

This technique generate realizations of the local average G_i that are derived from the isotropic random field G having zero mean and unit variance and Gauss markove spatially correlation function

$$\rho(\tau) = \exp\{-\frac{2}{\theta_k} \mid \tau \mid\} \qquad (7)$$

Where $\mid \tau \mid$ is the distance between points in the field and θ_k is the scale of fluctuation . The term realization in the context refers to the single generation of random field and subsequent finite element analysis of that field.

Loosely speaking, the scale of fluctuation is the distance over which points in the field are significantly correlated.

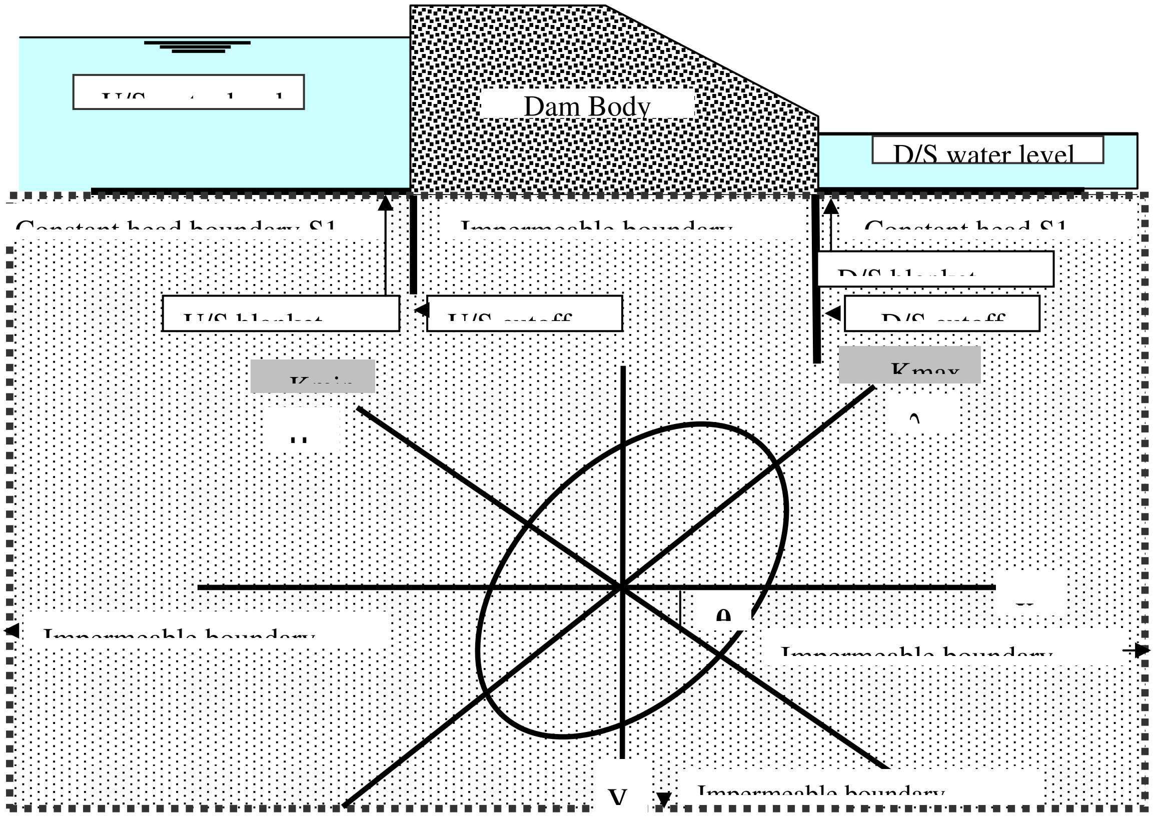


Fig. (1). Soil's hydraulic conductivity ellipsoid and boundary condition.

Brief Description of Finite Element Model.

In this paper a random field generator is combined with the power of finite element method for modeling spatially varying soil properties. The problem chosen for study is a simple boundary value problem of steady seepage beneath hydraulic structure with two cutoffs, one in u/s side and the other in the d/s side.

Finite Element Formulation of Governing Equation.

The behavior of field variable on each element is defined approximately by a function depending on nodal value.

$$H^e = \sum_{i=1}^{n} Ni\,Hi \quad \ldots\ldots(8)$$

H^e : approximate solution of field variable in the element
Hi : nodal value of (H) of element (e)
n : number of nodes for the element (e)
Ni : shape function of the element (e)
One can write the equation in matrix form
$H^e = [Ni]\{Hi\}$
Where; Hi : vector matrix of nodal value, Ni : shape function matrix of element (e)
And for overall domain the approximate solution will be

$$H = \sum_{e=1}^{ne}\sum_{i=1}^{n} Ni\,Hi \quad \text{or} \quad \sum_{e=1}^{ne}[Ni]\{Hi\} \quad \ldots\ldots(9)$$

Where (ne) is the total No. of elements.

GALERKIN PROCEDURE OF WEIGHTED RESIDUAL METHOD.

The weighted residual method is a technique used to result a solution of a linear differential equations. If this method is used, the finite element equation is directly derived from the governing differential equation of the problem.

Let A be the field domain, H the field variable then the governing equation can be written as

$F(H) = 0$ at (A)

Letting Ha as the approximate solution, by substituting in above equation.

$F(Ha) = R \neq 0$ in the domain (A)

Where R is the residual obtained to the use of the approximate solution Ha.

The aim is to make the weighted residual to be minimum. In order to perform this aim the weighted residual should be integrated on the problem domain using suitable weighting function and equated to zero.

$$\int_A Wj\, R\, d\, A = 0$$

and in the element form:-

$$\sum_1^n \int_{Ae} Wj\, Re\, d\, A = 0 \quad \ldots\ldots(11)$$

Where : Wj: weighted function, R^e: element residual

Galerkin's method, which applied to weighted residual procedure, assumed that the weighted function is the same as the (shape function) which describe the variable variation within the element.

$Wj = Nj$

A quadratic element has curved edges varying as a parabolas. For curved boundaries it is more desirable to use a quadratic element, which is known as "a higher order Isoparametric element". a two-dimensional eight nodes quadratic Isoparametric element are used here.

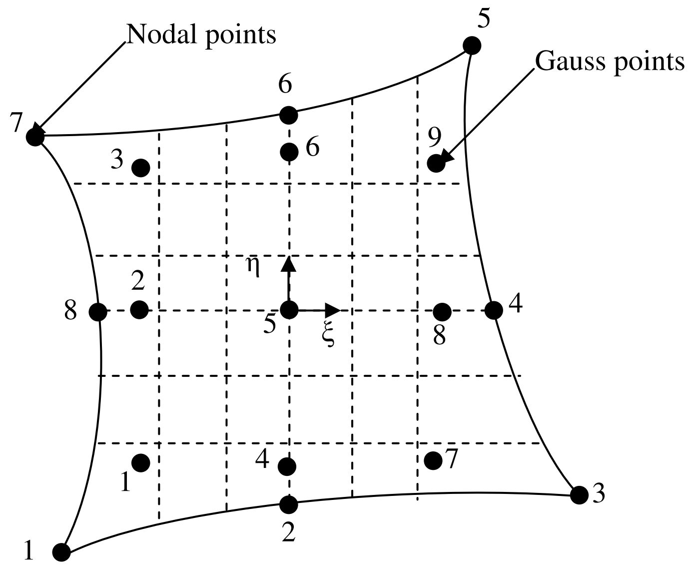


Fig (2). Quadratic Isoparametric Element

BRIEF DESCRIPTION OF OPTIMIZATION MODEL.

The objective function and decision variables.

Decision variables in the optimization model used herein consist of the variables that define by the objective function which is

$F = c_1 s_1 + c_2 s_2$

Where (F) as above is the objective function which represent the total cost of the seepage control techniques component. The optimization procedure will minimize it to obtain the minimum cost and result the optimum dimension of seepage control techniques (s_1, s_2) where:

s_1: the u/s cutoff dimension , s_2: the D/s cutoff dimension

The costs (c_1, c_2) is the real in time costs and transformed as a unit by dividing one into another to solve with unit less cost, therefore, the solution will be more general in this case.

Table (1). Costs of hydraulic structure seepage control devices by ID in year of 2000 from [١]

	Seepage Control Component	Cost
1	Sheet pile + related costs	150000 ID /m^2

The result factor after dividing on sheet pile cost will be:

$c_1 = 1$, $c_2 = 1$

CONSTRAINS:

The objective function defined before is subject to the following constrains:

1. Design constraints: The factor of safety against piping (Fs_i) and factor of safety against floating (Fs_f) is fixed along the optimization procedure as recommended in design texts. Factor of safety against piping (Fs_i) taken as 3 as recommended, and Cedergren as reported in, and the factor of safety against floating (Fs_f) processed as (4) as suggested by soil mechanics experiences as Terzaghi and Peck in formulas form.

   $Fs_i \geq 3$

   $Fs_f \geq 4$

   $V = \frac{1}{3}$ *(uplift pressure with no seepage control techniques)

   Where V is the assumed force down which fixed during the optimization process to find the factor of safety against floating (Fs_f). The third constrain was adopted according to the USBR recommendation. They recommend that the weight of the small hydraulic structures assumed to be {1/3 *(uplift pressure with no seepage control techniques)}.
2. Geological constraints the use of partial cutoffs in shallow impervious foundation is not recommended and if a complete cutoff satisfy, there is no need to a blanket as a seepage effect control techniques in function form.

   $s1,s2 \leq 95\%$ T

   Where T is the thickness of impervious layer

OPTIMIZATION PROCEDURE:

Searching-iteration optimization method was used to explore the minimum value of objective function (F) "that is the minimum cost"). The optimization procedure started from an initial value for all decision variables (s1, s2), and using these initial values with FEM for seepage analysis, the exit gradient and uplift pressure for these values of seepage control techniques dimensions will be performed. After that the factor of safety against floating (Fs_f) and factor of safety against piping (Fs_i) can be found. If these factors satisfy the constraints, the objective function (F) "cost function" will be evaluated and stored in a matrix with the decision variables “seepage control techniques dimensions”. If the constrains are not satisfied the four dimensions values will be changed. These four dimensions will be changed using nested loops. Four loops were needed with small increment. These loops associated with these small increments were used to cover all possible sets of the four variables. The set that minimizes F will be adopted as the optimum dimensions. However the selection of the minimum values will be according to the designer judgment.

SUMMARY OF RESULTS FROM COMBINED MODEL.

The results of the combined model presented. The model results shown in form of charts with different angle of orientation (θ).Structure with two cutoffs only was adopted, Different anisotropy ratio (k_{max} / k_{min}), for three different types of soils: sandy soil, silt soil and clayey soil presented.

Random generation of stochastic soil properties (k_x, k_y) was used to model the heterogeneity of soils. For each type of soil five random series of k-values was generated for three standard deviations, every set of trials has the same mean and standard deviation.

Various cases were investigated to determine the effect of angle of anisotropy ellipse (θ) for a various types of soil foundation.

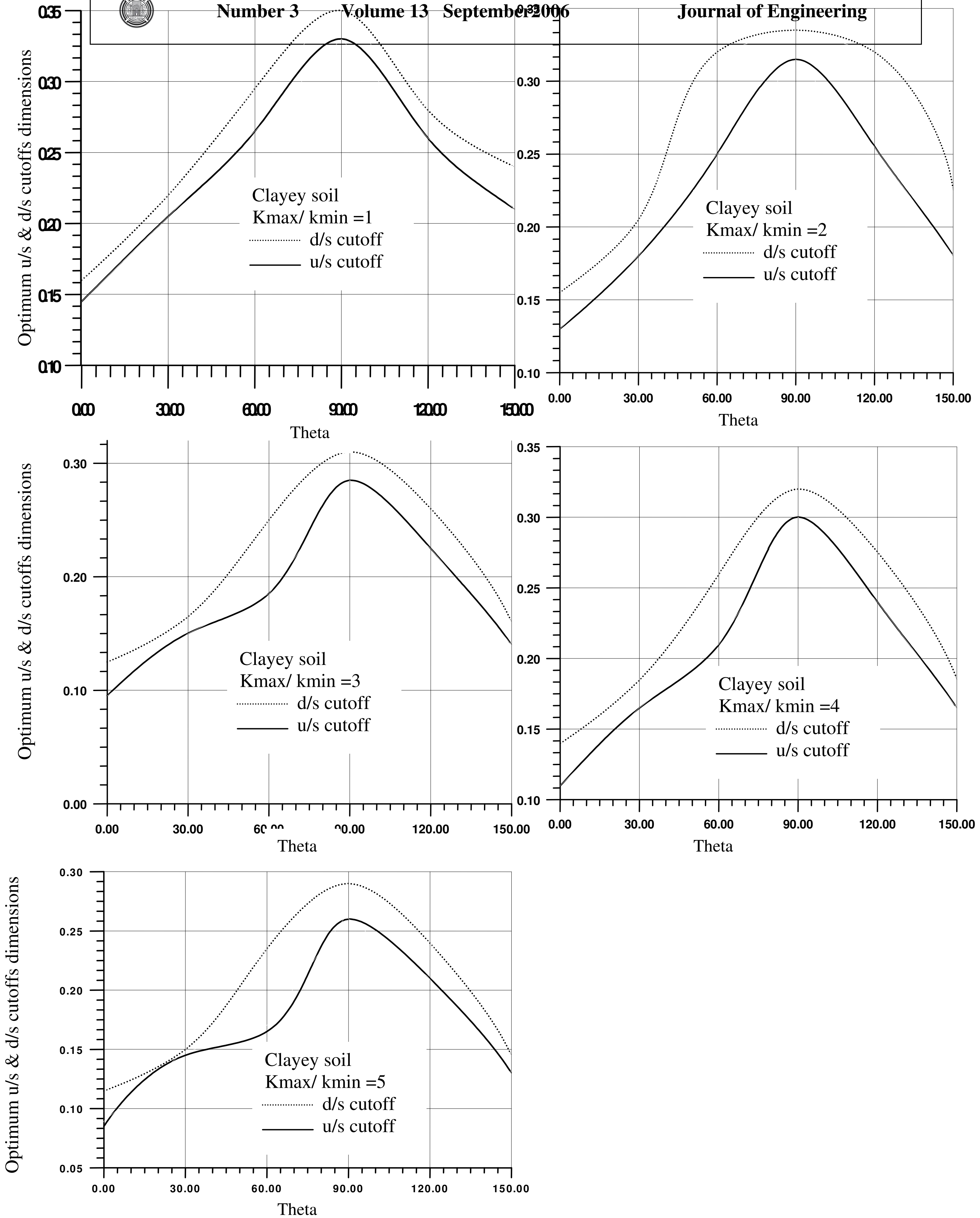
Figures (3,4 and 5) illustrates the variation of the optimum dimensions with the angle of anisotropy (θ) with k_{max} / k_{min} =1, 2, 3, 4, 5. These figures show s1/b s2/b optimum values with no u/s and d/s blankets in sandy, clayey and silty soil respectively. It is shown that the optimum dimensions of s1/b and s2/b increase with the increase of the angle of anisotropy when theta < 90. The maximum optimum dimensions of s1/b and s2/b were found when theta =90. This behavior caused by the streamlines directed more toward the floor of the hydraulic structure leading to increasing in uplift pressure and exit gradient, particularly near the floor .the optimum dimensions then decreased as (θ) increased beyond θ=90.

The effect of soil anisotropy ratio is seen from Figures (5 ,6 and 7) which indicate the values of optimum dimensions for different values of k_{max} / k_{min}, for k_{max} / k_{min} = 1 the optimum dimensions is the greatest. This is attributed to the relatively large hydraulic conductivity along the cutoff causing an increase in uplift pressure and the exit gradient. It is also shown that, for k_{max} / k_{min} = 5, the value of optimum dimensions is smallest. In fact, the effect of anisotropy ratio is small compared with the effect of the angle of orientation.

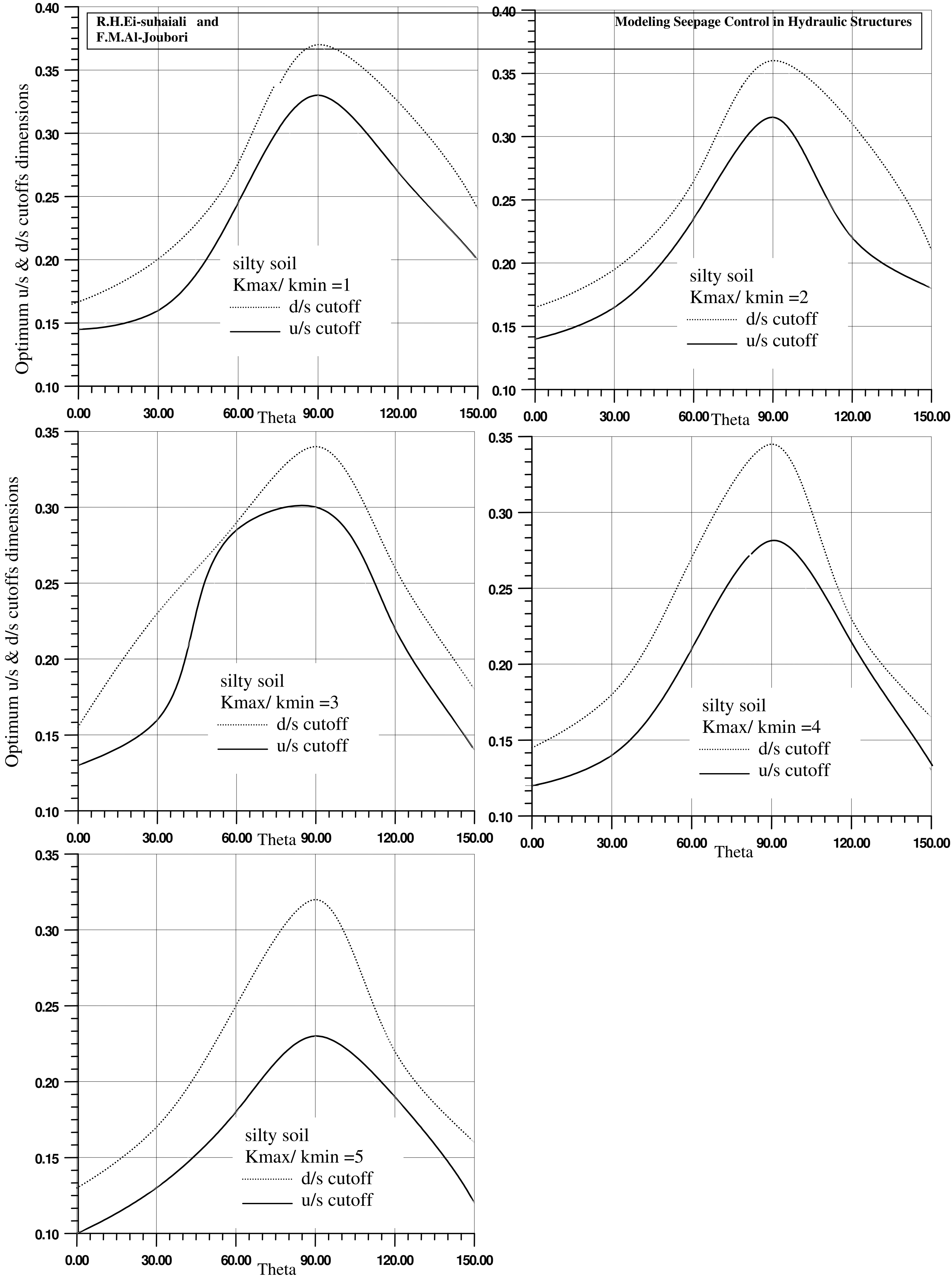
Three types of soil foundation were adopted in this research. Namely, clayey soil with permeability value from ***$1*10^{-12}$ – $4.7*10^{-9}$*** m/s, silty soil with permeability value from ***$1*10^{-9}$ to $2*10^{-6}$*** m/s and sandy soil have a permeability value from ***$2*10^{-6}$ – $2*10^{-3}$*** m/s. Those values were taken as a mean for random 7generation of k- values in order to simulate heterogeneity.

From **Fig**(5 ,6 and 7) , it seen that the optimum dimension decrease with decrease in soil permeability, that is, for clayey soil optimum dimension less than for sandy soil as shown below in table (2).

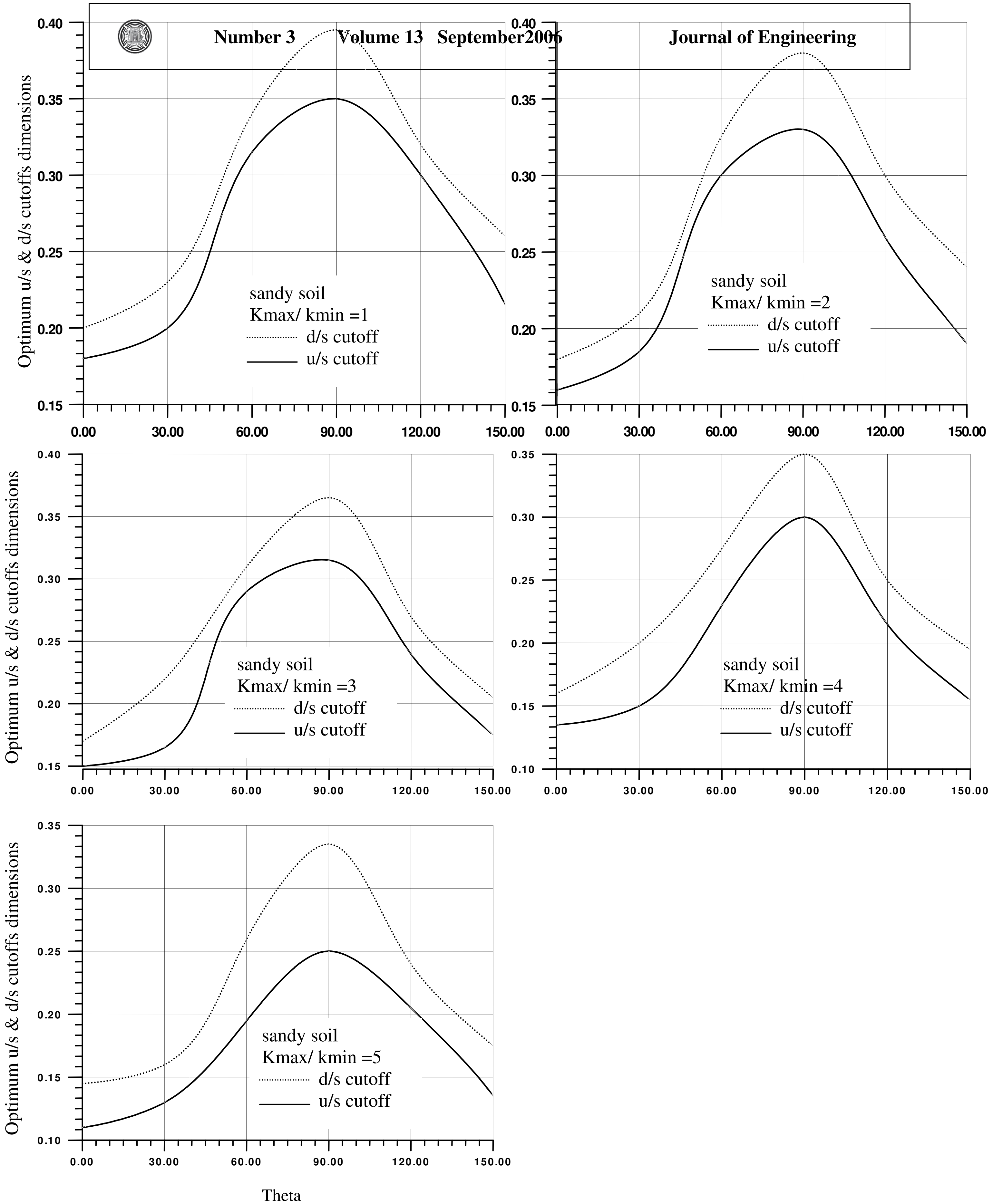
Fig (6 (a, b, c)) illustrate the effect of random hydraulic conductivity distribution along the field. In these figures five random series was generated for the clayey soil which is log-normally frequency distribution, with fixed standard deviation and mean value and for three different standard deviation as shown in table (4-1). It is shown that the difference between the five trials taken for a certain type of the soil (clay) and a structure with two cutoffs and anisotropy ratio k_{max} / k_{min} = 5 was small for the u/s cutoff dimension. that is, in fact, shows the view of the random generation as a tool to simulate the field in acceptable realization and more trust with model results. The optimum dimension was decrease with increase in covariance (***Cov***) value specially from (***0.5***) to(***0.9***) which was (***13.97 %***) in the other hand the optimum dimension was increased with covariance (***Cv***) increased from (***0.9***) to (***1.1***) by (***0.35 %***) which can be ignored.



Fig(3). relationship between angle of orientation (θ) and the u/s and d/s optimum dimensions in clayey soil



Fig(4). relationship between angle of orientation (θ) and the u/s and d/s optimum dimensions in silty soil



Fig(5). relationship between angle of orientation (θ) and the u/s and d/s optimum dimensions in sandy soil

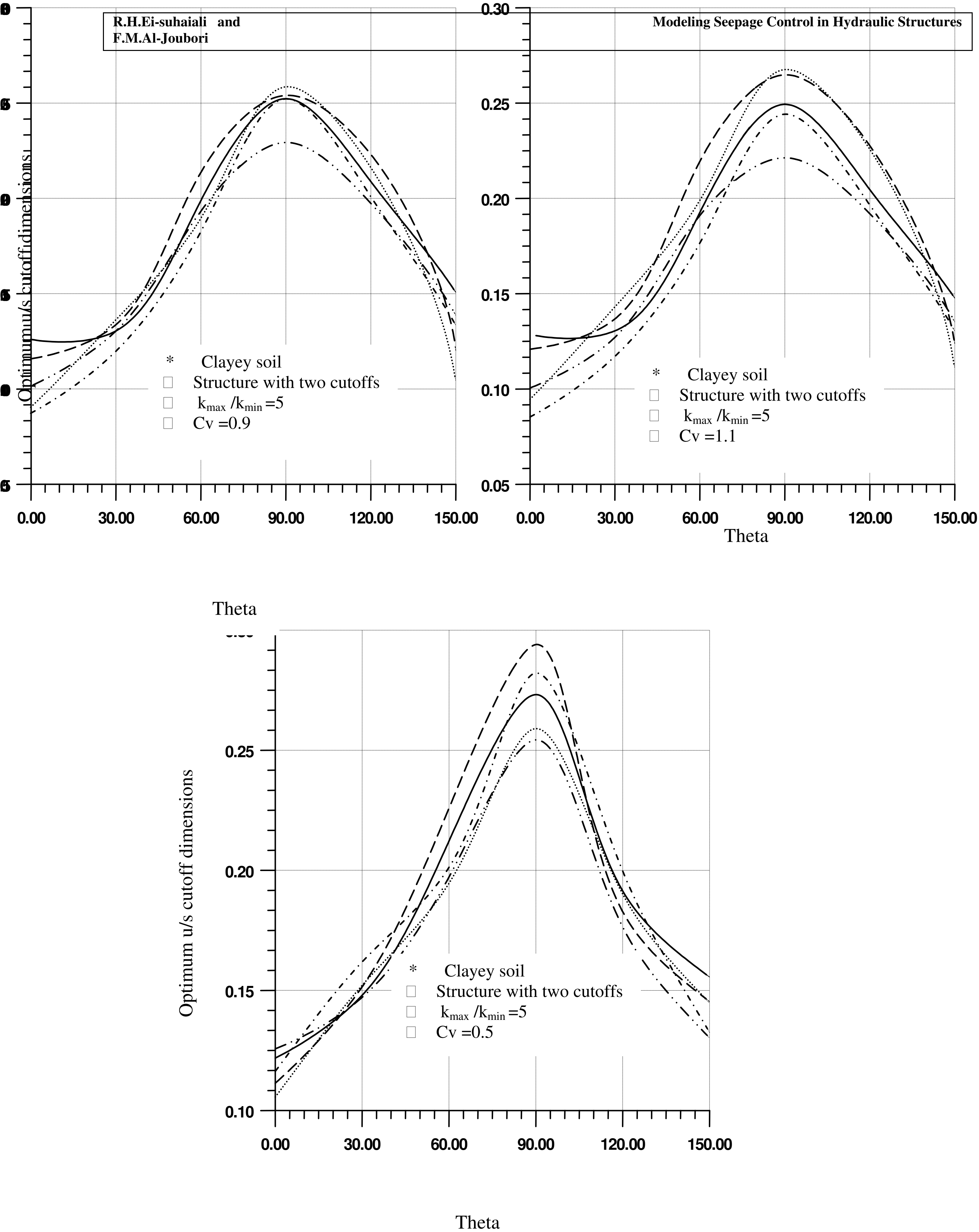


Fig (6). Effect of random generation of hydraulic conductivity with fixed mean and standard deviation on model results for clayey soil

Table (2). Average optimum dimension of seepage control devices for clayey, sandy and silty soils.

Struct. Type	With2- Cutoffs Only	
soil type	s2/b	s1/b
clay	0.232	0.203
sand	0.256	0.218
silt	0.254	0.210

Table. (3) . Soil types, standard deviations and mean for sandy soil, silty soil and clayey soil and for three different values of standard deviation.

Soil Type	Mean	Standard Deviation		
Sandy soil	$5* 10^{-5}$ m/s	$2.5* 10^{-5}$ m/s	$4.5* 10^{-5}$	$5.5* 10^{-5}$ m/s
Silty soil	$5* 10^{-8}$ m/s	$2.5* 10^{-8}$ m/s	$4.5* 10^{-8}$	$5.5* 10^{-8}$ m/s
Clayey soil	$5* 10^{-12}$ m/s	$2.5 * 10^{-12}$ m/s	$4.5 * 10^{-12}$	$5.5 * 10^{-12}$ m/s

CONCLUDING REMARKS.

In this paper , 2D random field generator and Finite Element models have been combined with optimization model to result a design model give the optimum dimensions of hydraulic structure seepage control techniques .

The study shows that the optimum dimensions of u/s and d/s cutoffs for hydraulic structures with two cutoffs only increase with (θ) beyond (θ) < 90 , when (θ) =90 the value is maximum after (θ) >90 the value of optimum dimensions will decrease.

The results illustrate that the anisotropy ratio have small effect on results compare with the effect of the angle of orientation (θ) .

From results of the model, it seen that the use of random generation to generate the hydraulic conductivity of soil beneath hydraulic structure was be a good way to simulate the field when Cv ≈ 1.

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Appendix A: List of Symbols

Symbol	Definition	Dimension
[N^e]	Number of Element in the Problem Domain	—
c1,c2	Cost Factors	—
F	Objective Function	ID
Fs_f	Factor of Safety Against Floating	—
Fs_i	Factor of Safety Against Piping	—
G_i	Local Average of Standard Gaussian Random field G	—
H	Pressure Head at Any Point in the Problem Domain	L
H1	Piezometric Head Upstream the Structure	L
H2	Piezometric Head Downstream the Structure	L
H_o	Difference Ratio of Piezometric Head	L
k	Hydraulic Conductivity	L / T
k_λ	Manor principal coefficient Hydraulic Conductivity	L / T
k_μ	Major principal coefficient Hydraulic Conductivity	L / T
L_x , L_y	Direction Cosines	—
N	k_{max} / k_{min}	—
n	Number of Nodes Per Element	—
Ni	Shape Function Matrix of Element e	—
R^e	Element Residual	—
s1	Length of Upstream Cutoff	L
P1	Constant Head Boundary	L
s2	Length of Downstream Cutoff	L
P2	Impervious Boundary	L
T	Depth of Pervious Layer	L
Wj	Weighted Residual Function	—
x,y	Coordinate Axis in the Real Region	—
ζ , η	Local Coordinate	—
θ	Orientation of Soil Ellipsoid	o
μ , λ	Coordinate Axis Paralleled to Direction of Maximum and Minimum Coeff. of Hydraulic Conductivity .	—
μ_{lnk} , σ_{ink}	Mean and Standard Deviation of the Logarithm of k	L / T