Number 3 Volume 13 September2006 Journal of Engineering

A MODIFIED METHOD FOR DETERMINATION OF SCALE FACTOR OF THE PROJECTED GEODESIC

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ABSTRACT

Conformal projection is one of the most important aspects that geodesy dealing with. The determination of the scale factors in the meridian, the parallel and projected geodesic directions are the final result of the conformal projection. Methods for determining the scale factors in the meridian and the parallel directions have a quite sufficient accuracy. While methods for determining the projected geodesic have different accuracy and computation complicity.

This research adopts a modified method for computing the exact value of scale factor in geodesic direction. In this method the scale factor is obtained by determining the true and projected distances of the geodetic line. In the traditional methods for determining the projected distance it is usual to use the 1/3 Simpson's rule in the computations while the modified method the 3/8 Simpson's rule is used.

Computations and mathematical tests were carried out to obtain the scale factors using the traditional methods and comparison was made with modified method.
By applying the developed method and the traditional methods to calculate the scale factor, it was found that the modified method is more accurate and the projected distances can be obtained exactly.

الخلاصة

يعد استخدام المساقط المعدلة من المواضيع المهمة في الجيوديسي , حيث يمكن حساب معامل التشويه الخطي باتجاه خطوط الطول ودوائر العرض كناتج نهائي اضافة الى حسابه باتجاه الخط الجيوديسي المسقط , حيث تتباين دقة طرق حساب معامل التشويه باتجاه خطوط الطول ودوائر العرض . أما طرق حسابه للخط الجيودسي المسقط فتختلف حسب الدقة والتعقيد.
يتبنى هذا البحث طريقة محسنة تؤدي إلى استنتاج القيمة الأدق لمعامل تشويه الخط الجيوديسي المسقط من خلال حساب المسافة الحقيقية والمسقطة للخط الجيوديسي, حيث يستخدم طريقة (3/8 Simpson's rule) لحساب معامل تشويه الخط الجيوديسي بدلاً من الطريقة الشائعة (1/3 Simpson's rule). إن دراسة سلوك الخط الجيوديسي المسقط يتطلب حسابات معقدة وإجراء اختبارات رياضية لإقرار أدق الطرق في الحسابات.
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KEY WORDS
Surveying; Geodesy; Geodesic; Mathematical Cartography; Scale Factor

GEODESIC ON ELLIPSOID

In the field of the geodesy there are at least three distinct geodetic curves. These curves are: the great elliptic arc, the normal sections, and the geodesic curve. Each curve furnishes different distances and different azimuths for any one reference ellipsoid, as shown in **Fig (1).**

The great elliptic arc is a curve lies in a plane which contains the center of the reference ellipsoid. Mathematically, this curve doesn't represent the shortest connection between two points. The meridians and the equator are special cases can be considered as great elliptic arcs.

The normal section is a curve lies in a plane which contains the normal at the one point and passes through the other point.

The Geodesic conceders the principle path as a representation of the shortest distance in a unique way between any two given points on the ellipsoidal earth, which has double curvature.

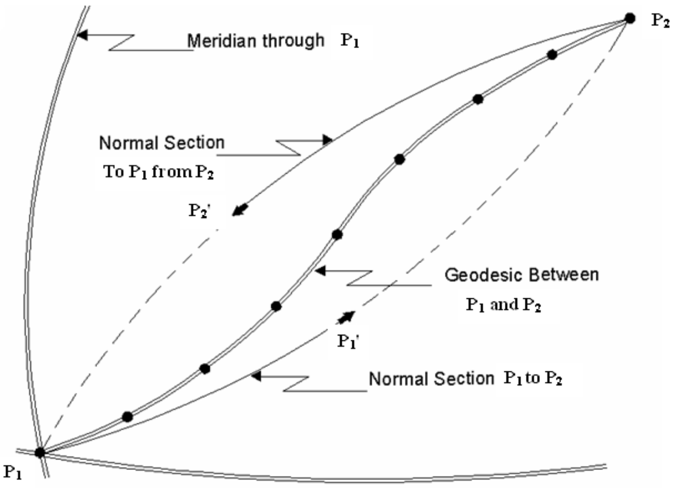

Fig.1: Explains geodetic curves on the ellipsoid

From the definition the geodesic arc, the principle normal of the geodesic at any point will coincide with the ellipsoid normal at that point. This property doesn't exist in the normal section. This property imposes the product of the parallel radius times the sine of the geodesic azimuth, at each point on the geodesic is, [Hooijberg, 1997], this property is known as "Clairaut's equation, that is:

$$N_1.\cos\varphi_1.\sin A_1 = N_2.\cos\varphi_2.\sin A_2 = \text{constant} \quad (1)$$

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PROJECTED GEEODESIC AND SCALE FACTOR

In conformal projection, although angles are transformed undistorted it remains in many instances necessary to calculate corrections to ellipsoidal elements in order to obtain corresponding elements in the map, and vice versa. The plane situation is shown in **Fig. (2)**

The geodesic on the ellipsoid of length (S), will projects on a plane to be a projected geodesic, this curve however does not as a rule project as a straight line, but transforms into the curve of a length (s).

In order to obtain the true distance (S), from the projected distance (s) derived from grid coordinates, or alternatively to convert a true distance to grid distance, it is necessary to calculate the scale factor and apply it in the correct sense.

So, the length of projected geodesic (s) is determined in terms of the ellipsoidal length (S) and the scale factor (K):

$$K = \partial s / \partial S \qquad \Rightarrow \qquad s = \int_0^S K.\partial S \tag{2}$$

Therefore, the scale factor is the ratio of infinitesimal projected linear distance in any direction at a point on the plane grid to the corresponding true distance on the ellipsoid.

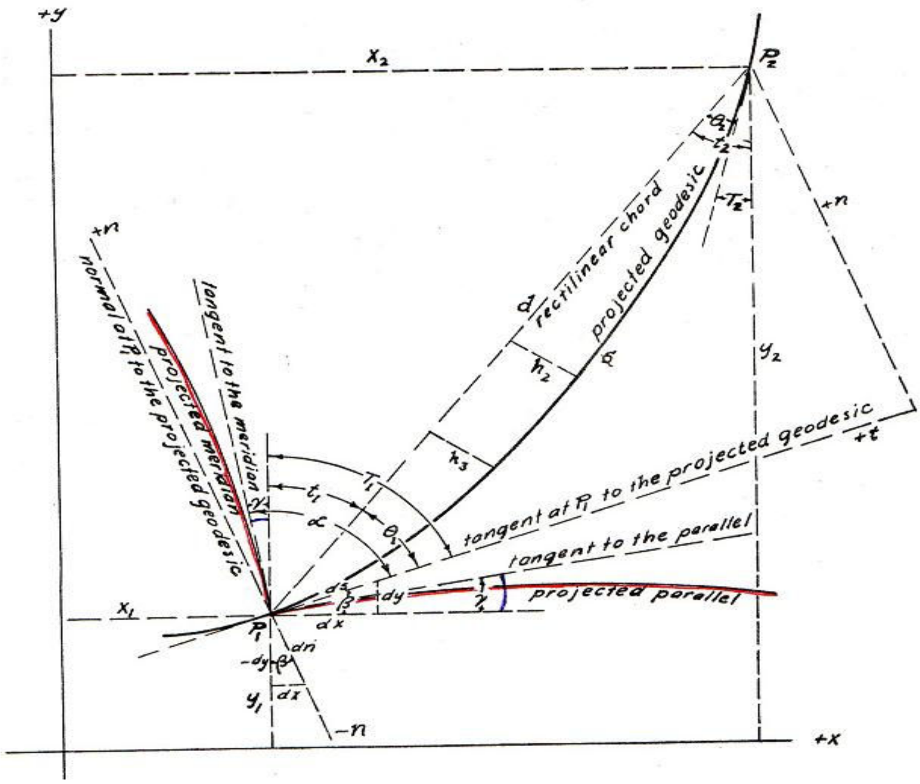

Fig.2: Explains map elements on conformal projection

COMPUTATION OF PROJECTED COORDINATES AND SCALE FACTOR

The most practical forms of the Universal Transverse Mercator, UTM, projection, which is a conformal projection for geodetic computations, is expressed as a set of a series approximations in powers of ($\Delta\lambda$), [Snyder, 1987], that is:

$$\mathrm{x} = K_o^*.N\left[A + (1 - T + C).\frac{A^3}{6} + (5 - 18.T + T^2 + 72.C - 58.e'^2).\frac{A^5}{120} + ...\right] \tag{3}$$

$$Y = \left\{K_o^*.\left\{(M_P - M_o) + N.\tan\varphi.\left[\frac{A^2}{2} + (5 - T + 9C + 4C^2).\frac{A^4}{24} + \right.\right.\right.$$

$$+(61-58T+T^2+600C-330e'^2).\frac{A^6}{720}+........\Bigg]\Bigg]\Bigg\} \qquad (4)$$

$$K=\Bigg\{K_o^*.\Bigg[1+(1+C).\frac{A^2}{2}+(5-4T+42C+13C^2-28.e'^2)\frac{A^4}{24}+$$

$$+(61-148T+16T^2).\frac{A^6}{720}+.........\Bigg]\Bigg\} \qquad (5)$$

$$M_P=a\Bigg[(1-\frac{e^2}{4}-\frac{3e^4}{64}-\frac{5e^6}{256}-....).\varphi-(\frac{3.e^2}{8}+\frac{3.e^4}{32}+\frac{45.e^6}{1024}+....)\sin 2\varphi+$$

$$+(\frac{15.e^4}{256}+\frac{45.e^6}{1024}+...).\sin 4\varphi-(\frac{35.e^6}{3072}+.....).\sin 6\varphi+.....\Bigg] \qquad (6)$$

$$e'^2=\frac{e^2}{1-e^2} \qquad (7)$$

$$N=\frac{a}{\sqrt{(1-e^2.\sin^2\varphi}} \qquad (8)$$

$$T=\tan^2\varphi \qquad (9)$$

$$C=e'^2.\cos^2\varphi \qquad (10)$$

$$A=(\lambda-\lambda_o).\cos\varphi \qquad (11)$$

Where (x, y) are the projected plane coordinates,

(K_o^*) is the scale on the central meridian, (K_o^*=0.9996 for UTM, K_o^* =1.000 for Transverse Mercator projection.)

(M_P) is the true distance along the central meridian from the equator to the (φ)

(φ, λ) is the geodetic latitude, geodetic longitude and (λ_o) is the longitude of the central meridian

(a, e^2) are the characteristic of the spheroid and ($e^{'2}$) is the second eccentricity.

(K) is a point scale factor.

Along the geodetic line, the scale factor will differ from point to another. So the computation needed another approach to apply a correction to an ellipsoidal distance that converted to plane grid distance, it is necessary to use the value of all points.

Equation (5) represent the scale factor at a point, to obtain the scale factor for a line, it is traditional to apply the 1/3 Simpson's rule. That is :

$$\overline{K} = \frac{1}{6}.(K_1 + 4K_m + K_2) \tag{12}$$

In which K_1 and K_2 are the scale factors at the ends of each line, where K_m is the scale factor at the mid point of that line.

IMPROVED METHOD

Utilization the (3/8) Simpson's rule instate of (1/3) Simpson's in obtaining the geodetic scale factor, distances at equal intervals are determined by dividing the whole length of geodesic into a multiple of three line elements are determined. Then the scale factor will be:

The computed area ………………… $A = \frac{3}{8} * x_i (y_1 + 3y_2 + 3y_3 + y_4)$ (13)

The compute the scale factor ……….. $\overline{K} = \frac{3}{8} * \frac{1}{3}(K_1 + 3K_2 + 3K_3 + K_4)$ (14)

The denominator takes (3) related to the number of divisions distances to whole line.

To examine the validity of equation (14), the computation of true distance and projected distances are needed to be compared with that obtained from this equation.

COMPUTATION OF POSITIONS

If a geodesic, length (S) is divided into (n) equal small elements (δS) in length, then the problem can be reliably solved with excellent accuracy by adding together the lengths of small elements (δS) of the length of geodetic line (S).

The direct problem involves computation of the position of ending point (P_2) whose distance and azimuth from a given initial starting point (P_1) are known:-

- given point (P_1) : (φ_1, λ_1), A_{12}, S_{12}
- to compute (P_2) : (φ_2, λ_2)

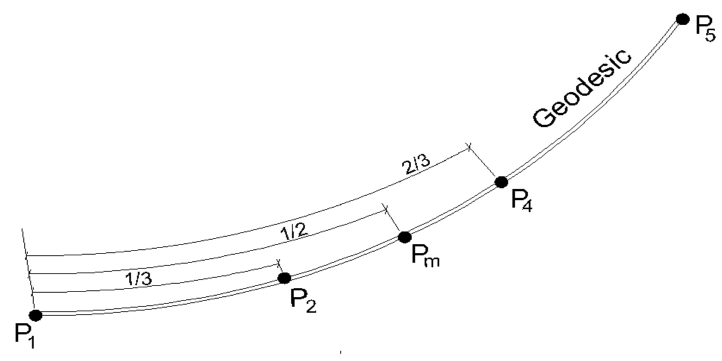

Fig.(3): division of geodesic to compute coordinates.

$$\begin{aligned} \varphi_2 &= \varphi_1 + \Delta\varphi \\ \lambda_2 &= \lambda_1 + \Delta\lambda \\ A_2 &= A_1 + \Delta A \end{aligned} \tag{15}$$

The precise computation of the position is computed according to Legender Solution, that is:

$$\Delta\varphi'' = \frac{\rho''}{M}(S.\cos A_1) - \frac{3.\rho''.e'^2.\sin\varphi_1.\cos\varphi_1}{2.M_1.N_1}.(S^2.\cos^2 A_1) - \frac{\rho''.\tan\varphi_1}{2.M_1.N_1}.(S^2.\sin^2 A_1) + ... \tag{16}$$

$$\Delta\lambda'' = \frac{\rho''}{N.\cos\varphi}(S.\sin A_1) + \frac{\rho''.\tan\varphi_1}{N_1^2.\cos^2\varphi_1}.S^2.\sin A.\cos A + ... \tag{17}$$

$$\Delta A'' = \frac{\rho''.\tan\varphi_1}{N_1}.S.\sin A_1 + \frac{\rho''(1+2.\tan^2\varphi_1 + e'^2.\cos^2\varphi_1)}{2.M_1.N_1}.S^2.\sin A_1.\cos A_1 + ... \tag{18}$$

To check the results by using Eq.(1) :-

Now, the geodetic positions of all points along the geodesic can be computed, so the corresponding positions on projection can also be determined (by equations (3)-(5)), then accurate distances and linear scale factor can be computed (table 2).

To satisfy the mathematical calculation we must distinguish between the arc and the chord on both surfaces: spheroid and plane projection, beside the relationship between them.

COMPUTATION OF THE ACCURATE LENGTH OF PROJECTED GEODESIC

It's possible to compute the true length of geodesic (S) on spheroid, but how to compute the length of the projected arc (s) on the map, Equation (2), to conclude the linear scale factor. (K) may be expressed as a function of the map coordinates and the map coordinates may be expressed as function of the arc(s) of the projected curve, after integration we have: - [Thomas, 1952]

$$s = K_o.S + \frac{1}{2}K_o.K_o^{'}.S^2 + \frac{1}{6}(K_o^2 K_o^{''} + K_o K_o^{'2})S^3 + \frac{1}{24}(K_o^3 K_o^{'''} + 4K_o^2 K_o^{'} K_o^{''} + K_o K_o^{'3})S^4 + ... \tag{19}$$

Where $K_o^{'}, K_o^{''}, K_o^{'''}$ are the derivatives of the scale ratio K, with respect to (s) and evaluated at (s =0).

(K) Was achieved by eq. (5), but can be also computed from rectangular coordinates as:-

$$K = 1 + \frac{A}{2}(\frac{x}{N_f})^2 + \frac{B}{24}(\frac{x}{N_f})^4 + \frac{1}{720}(\frac{x}{N_f})^6 + ... \tag{20}$$

$$K^{'} = \frac{\partial K}{\partial x}.\cos\beta + \frac{\partial K}{\partial y}.\sin\beta \tag{21}$$

$$K''=\frac{\partial^2 K}{\partial x^2}.\cos^2\beta+\frac{\partial^2 K}{\partial y^2}.\sin^2\beta+\frac{\partial^2 K}{\partial x\partial y}\sin^2\beta-K\sigma^2 \tag{22}$$

Where $A=1+\eta_f^2$ (23)

$$B=1+6.\eta_f^2+9.\eta_f^4+4.\eta_f^6-24.t_f^2.\eta_f^4-24.t_f^2.\eta_f^6 \tag{24}$$

$$\eta_f^2=e'^2.\cos^2\varphi_f \;:\; t_f^2=\tan\varphi_f \tag{25}$$

(σ) is the expression for the curvature of the projected geodesic at a given point in terms of the scale factor at that point and the angle (β) is: -

$$\sigma=\frac{\partial\beta}{\partial s}=\frac{1}{K}.(\frac{\partial K}{\partial x}.\frac{\partial y}{\partial s}-\frac{\partial K}{\partial y}.\frac{\partial x}{\partial s}) \tag{26}$$

But $\frac{\partial x}{\partial s}=\cos\beta$ and $\frac{\partial y}{\partial s}=\sin\beta$ (27)

So, $\sigma=\frac{1}{K}.(\frac{\partial K}{\partial x}.\sin\beta-\frac{\partial K}{\partial y}.\cos\beta)$ (28)

$$\beta=\tan^{-1}\frac{\Delta y}{\Delta x} \text{ or } \beta=\tan^{-1}\frac{\Delta x}{\Delta y} \tag{29}$$

Formula (22) gives the value of (K) for T.M projection, and it is function of (x) coordinate alone, hence formula (28) becomes:- (all the variables and equations can be computed in the next example)

$$\sigma=\frac{1}{K}.\frac{\partial K}{\partial x}.\sin\beta \tag{30}$$

$$K'=\frac{\partial K}{\partial x}.\cos\beta \quad\text{and}\quad \frac{\partial K}{\partial x}=\frac{A}{2N_f^2}(2x)+\frac{B}{24N_f^4}(4x^3)+\frac{1}{720N_f^6}(6x^5)+.... \tag{31}$$

$$K''=\frac{\partial^2 K}{\partial x^2}.\cos^2\beta-K\sigma^2 \tag{32}$$

$$\frac{\partial^2 K}{\partial x^2}=\frac{A}{N_f^2}+\frac{B}{2N_f^4}(x)^2+\frac{1}{24N_f^6}(x)^4+.... \tag{33}$$

Also (N_f) related to "foot point" (φ_f) of latitude φ, [Thomas, 1952]

$$N_f=\frac{a}{\sqrt{(1-e^2.\sin^2\varphi_f}} \tag{34}$$

$$\varphi_f=\mu+(\frac{3}{2}e_1-\frac{27}{32}e_1^3+..)\sin 2\mu+(\frac{21}{16}e_1^2-\frac{55}{32}e_1^4+..)\sin 4\mu+(\frac{151}{96}e_1^3+..)\sin 6\mu+(\frac{1097}{512}e_1^4+..)\sin 8\mu+.....$$

$$\mu=\left[(M_p)/a(1-\frac{e^2}{4}-\frac{3e^4}{64}-\frac{5e^6}{256})\right]*\frac{180}{\pi} \tag{35}$$

$$e_1=\frac{1-\sqrt{1-e^2}}{1+\sqrt{1-e^2}} \tag{36}$$

$$M_p = M_o + \frac{y}{K_o^*} \tag{37}$$

(M_p, M_o) can be computed by eq. (6), it seen that, all the elements of eq.(21) are computed, that leads to calculate the accurate length of projected geodesic (s).
The relationship between projected geodesic (s) and its rectilinear chord (d) is:

$$d = s - s^3.\frac{\sigma_o^2}{24} - s^4.\frac{\sigma_o^2.\sigma_o^{'}}{24} - \frac{s^5}{5760}.(72.\sigma_o.\sigma_o^{''} + 64.\sigma_o^{'2} - 3.\sigma_o^4) \tag{38}$$

Where (σ_o) is the curvature of the projected geodesic, ($\sigma_o^{'}$) the first derivative, and ($\sigma_o^{''}$) the second derivative achieved for the starting point of projected geodesic (s=0)

Also when the projected coordinates of points (P_1) and (P_2) are known, then the length of chord (d) is determined, the difference between arc and chord will be concluded.

$$d = \sqrt{\Delta x^2 + \Delta y^2} \tag{39}$$

$$\begin{aligned} \Delta x &= x_2 - x_1 \\ \Delta y &= y_2 - y_1 \end{aligned} \tag{40}$$

Where (x_i, y_i) are the projected coordinates. With value of (d) computed from above formulas, we then compute (s) from eq. (41) by substitution the value of (d) instead of (s), then the difference in length will be calculated and added to (d) to find the exact value of (s).

The relationship between the geodetic distance (S) and its projected (s) on TM projection **(Fig.2)** is:-

$$s = S + \frac{S}{2R^2_m}\left[\left(\frac{x_1 + x_2}{2}\right)^2 + \frac{1}{3}\left(\frac{x_2 - x_1}{2}\right)^2\right] \tag{41}$$

$$R_m = \sqrt{(M_1.N_1)} \tag{42}$$

RELATIOINSHIP BETWEEN ARC AND CHORD ON SPHEROID

The relationship between arc and chord on spheroid can be determined as the following:-
When (S) is known between two points (p_1) and (p_2) its chord is:

$$D = S - \frac{S^3}{24.M_1^2} + \frac{e^2.\cos A_z.\sin 2\varphi}{(1-e^2)}.\frac{S^4}{16.N.M_1^2} \tag{43}$$

Or, (D) can be determined by Cartesian coordinates:

$$D = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2} \tag{44}$$

$$\begin{aligned} \Delta X &= X_2 - X_1 & & X_i = N_i.\cos\varphi_i.\cos\lambda_i \\ \Delta Y &= Y_2 - Y_1 & : \quad & Y_i = N_i.\cos\varphi_i.\sin\lambda_i \\ \Delta Z &= Z_2 - Z_1 & & Z_i = N_i.(1-e^2).\sin\lambda_i \end{aligned} \tag{45}$$

RESULT AND DISSCUSION

Four geodetics of different distances were used to compute the difference between the arcs and the chords on spheroid Clark 1880 spheroid, (a=6 378 249.145 m.), (e^2 = 0.006 803 481 196 02), the projected distances on T.M projection. As shown in Table (1), the difference between the geodetic distance and its chord is very small, so the curvature of geodesic will not be obvious, unless substituting large distances.

Table (1): The differences between arc and chord on both spheroid and projection.

S_i	On Spheroid Clark 1880			On Projection T.M		
	(S) in m.	**(D) in m.**	**(S-D) m.**	**(s) in m.**	**(d) in m.**	**(s-d) m.**
S_1	51 000.00	50 999.8633	0.1367	51 007.512477	51 007.51242	0.000057
S_2	105 000.0	104 998.8074	1.1926	105 019.67054	105 019.66935	0.00119
S_3	150 000.0	149 996.5232	3.4768	150 033.76998	150 033.76688	0.00310
S_4	210 000.0	209 990.4605	9.5395	210 059.17548	210 059.16770	0.00778

The same distances mentioned above were used to apply the modified method in computing the projected distances then to compute the scale factor. Computations were made based on Clark 1880 spheroid when the starting point is P_1 (30˚, 10˚), and the azimuth (30˚).While projected computations based on (λ_o= 09˚) as central meridian. , Computations details are shown in **Tables (2) and (3). Table (2)** shows computations of TM projection that has (K_o^*=1.000) and **Table (3)** shows computations of UTM projection that has (K_o^*=0.9996). The computations of linear scale factor presented in these tables are designated as follow:

$\overline{K}_1$: By division projected distance to actual distance.
$\overline{K}_2$: By Simpson' 1/3 rule.

$\overline{K}_3$: By Simpson' 3/8 rule.

As shown in these tables, the computed projected distances obtained using scale factor of the modified method is more accurate than that of the traditional method when compared with the exact distances.

Table (2): Related to the computations on the spheroid and dealing with TM projection that has ($K_o^* = 1.000$)

	P_i	On Spheroid				On projection		
		(S_i) in m.	φ	λ	Back Az.	x_i in (m.)	y_i in (m.)	K_i
S_1	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	096691.84344	3320275.13957	1.00011482643166
	P_2	17 000	**30°** 07' 58.03309"	**10°** 05' 17.55696"	**210°** 02' 39.0972"	105063.81284	3335073.19441	1.00013561308082
	P_m	25 500	**30°** 11' 56.96583"	**10°** 07' 56.65415"	**210°** 03' 59.0446"	109249.65516	3342472.45875	1.00014665393964
	P_4	34 000	**30°** 15' 55.84242"	**10°** 10' 35.96488"	**210°** 05' 19.2588"	113435.39726	3349871.88978	1.00015812641574
	P_5	51 000	**30°** 23' 53.42628"	**10°** 15' 55.23020"	**210°** 08' 00.4902"	121806.56551	3364671.27745	1.00018236599608
S_2	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	096691.84344	3320275.13957	1.00011482643166
	P_2	35 000	**30°** 16' 23.94185"	**10°** 10' 54.72139"	**210°** 05' 28.7133"	113927.83059	3350742.42244	1.00015950449207
	P_m	52 500	**30°** 24' 35.55514"	**10°** 16' 23.44205"	**210°** 08' 14.7681"	122545.17699	3365977.14043	1.00018458764498
	P_4	70 000	**30°** 32' 46.92748"	**10°** 21' 53.07879"	**210°** 11' 01.9631"	131162.04711	3381212.65068	1.00021149954654
	P_5	105 000	**30°** 49' 08.94161"	**10°** 32' 55.12909"	**210°** 16' 39.7980"	148394.22338	3411686.27050	1.00027080743828
S_3	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	096691.84344	3320275.13957	1.00011482643166
	P_2	50 000	**30°** 23' 25.33938"	**10°** 15' 36.42604"	**210°** 07' 50.9763"	121314.15601	3363800.70502	1.00018089236152
	P_m	75 000	**30°** 35' 07.27506"	**10°** 23' 27.42997"	**210°** 11' 49.9433"	133623.91773	3385565.80715	1.00021952447976
	P_4	100 000	**30°** 46' 48.71394"	**10°** 31' 20.32220"	**210°** 15' 51.2533"	145932.61898	3407332.67162	1.00026188732617
	P_5	150 000	**31°** 10' 10.07868"	**10°** 47' 11.85588"	**210°** 24' 00.9745"	170546.44782	3450872.33944	1.00035779843341
S_4	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	096691.84344	3320275.13957	1.00011482643166
	P_2	70 000	**30°** 32' 46.92748"	**10°** 21' 53.07879"	**210°** 11' 01.9631"	131162.04711	3381212.65068	1.00021149954654
	P_m	105 000	**30°** 49' 08.94161"	**10°** 32' 55.12909"	**210°** 16' 39.7980"	148394.22338	3411686.27050	1.00027080743828
	P_4	140 000	**31°** 05' 29.96871"	**10°** 44' 00.93000"	**210°** 22' 22.2677"	165624.09057	3442163.72739	1.00033742359791
	P_5	210 000	**31°** 38' 08.99857"	**11°** 06' 24.01838"	**210°** 34' 01.3139"	200075.81791	3503131.93533	1.00049256035241

S_i	On Spheroid		On Projection		Calculation Of the Scale Factor		
	(S) in m.	(D) in m.	(s) in m.	(d) in m.	K_1	K_2	K_3
S_1	51 000.0	50 999.8633	51 007.512477	51 007.51242	1.00014730347709	1.00014730136438	1.00014730136468
S_2	105 000.0	104 998.8074	105 019.67054	105 019.66935	1.00018733848947	1.00018733074164	1.00018733074822
S_3	150 000.0	149 996.5232	150 033.76998	150 033.76688	1.00022513321539	1.00022512046402	1.00022512049102
S_4	210 000.0	209 990.4605	210 059.17548	210 059.16770	1.00028178803703	1.00028176942286	1.00028176952718

Table (3): Related to the computations on the spheroid and dealing with UTM projection that has ($K_o^* = 0.9996$)

	P_i	On Spheroid: (S_i) in m.	On Spheroid: φ	On Spheroid: λ	On Spheroid: Back Az.	On projection: x_i in (m.)	On projection: y_i in (m.)	On projection: K_i
S_1	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	596453.16669	3318947.02951	0.999714780501085
	P_2	17 000	**30°** 07' 58.03309"	**10°** 05' 17.55696"	**210°** 02' 39.0972"	604821.78731	3333739.16513	0.999735558835585
	P_m	25 500	**30°** 11' 56.96583"	**10°** 07' 56.65415"	**210°** 03' 59.0446"	609005.95530	3341135.46976	0.999746595278066
	P_4	34 000	**30°** 15' 55.84242"	**10°** 10' 35.96488"	**210°** 05' 19.2588"	613190.02309	3348531.94102	0.999758063165176
	P_5	51 000	**30°** 23' 53.42628"	**10°** 15' 55.23020"	**210°** 08' 00.4902"	621557.84288	3363325.40893	0.999782293049677
S_2	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	596453.16669	3318947.02951	0.999714780501085
	P_2	35 000	**30°** 16' 23.94185"	**10°** 10' 54.72139"	**210°** 05' 28.7133"	613682.25946	3349402.12546	0.999759440690273
	P_m	52 500	**30°** 24' 35.55514"	**10°** 16' 23.44205"	**210°** 08' 14.7681"	622296.15892	3364630.74957	0.999784513809921
	P_4	70 000	**30°** 32' 46.92748"	**10°** 21' 53.07879"	**210°** 11' 01.9631"	630909.58228	3379860.16562	0.999811414946718
	P_5	105 000	**30°** 49' 08.94161"	**10°** 32' 55.12909"	**210°** 16' 39.7980"	648134.86569	3410321.59599	0.999870699115301
S_3	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	596453.16669	3318947.02951	0.999714780501085
	P_2	50 000	**30°** 23' 25.33938"	**10°** 15' 36.42604"	**210°** 07' 50.9763"	621065.63034	3362455.18473	0.999780820004573
	P_m	75 000	**30°** 35' 07.27506"	**10°** 23' 27.42997"	**210°** 11' 49.9433"	633370.46816	3384211.58082	0.999819436669966
	P_4	100 000	**30°** 46' 48.71394"	**10°** 31' 20.32220"	**210°** 15' 51.2533"	645674.24593	3405969.73855	0.999861782571239
	P_5	150 000	**31°** 10' 10.07868"	**10°** 47' 11.85588"	**210°** 24' 00.9745"	670278.22923	3449491.99050	0.999957655314034
S_4	P_1	00	**30°** 00' 00.00000"	**10°** 00' 00.00000"	**210°** 00' 00.0000"	596453.16669	3318947.02951	0.999714780501085
	P_2	70 000	**30°** 32' 46.92748"	**10°** 21' 53.07879"	**210°** 11' 01.9631"	630909.58228	3379860.16562	0.999811414946718
	P_m	105 000	**30°** 49' 08.94161"	**10°** 32' 55.12909"	**210°** 16' 39.7980"	648134.86569	3410321.59599	0.999870699115301
	P_4	140 000	**31°** 05' 29.96871"	**10°** 44' 00.93000"	**210°** 22' 22.2677"	665357.84093	3440786.86189	0.999937288628476
	P_5	210 000	**31°** 38' 08.99857"	**11°** 06' 24.01838"	**210°** 34' 01.3139"	699795.78758	3501730.68256	1.000092363328270

S_i	On Spheroid: (S) in m.	On Spheroid: (D) in m.	On Projection: (s) in m.	On Projection: (d) in m.	Calculation Of the Scale Factor: $\overline{K}_1$	Calculation Of the Scale Factor: $\overline{K}_2$	Calculation Of the Scale Factor: $\overline{K}_3$
S_1	51 000.0	50 999.8633	50 987.10947	50 987.10942	0.999747244555695	0.999747242443838	0.999747242444130
S_2	105 000.0	104 998.8074	104 977.66267	104 977.66148	0.999787263554074	0.999787255809345	0.999787255815920
S_3	150 000.0	149 996.5232	149 973.75647	149 973.75337	0.999825043162106	0.999825030415830	0.999825030442819
S_4	210 000.0	209 990.4605	209 975.15181	209 975.14404	0.999881675321816	0.999881656715093	0.999881656819367

CONCLUUTIONS

1.It is obvious from the previous discussion that the computation of linear scale factor depending on the exact calculation of both geodetic distance and its projection.

2.Any error in computations of geodetic position will effect the computation of the linear scale factor, so the precise computations will be needed.

3.Logically, the arch length is greater than the chord length for the same two points and the difference between the geodetic distance (S) and its chord (D) is very small, so the curvature of geodesic will not be obvious, unless substituting large distances.

4.The difference value between arch and chord (S-D) on spheroid is greater than the corresponding difference value (s-d) on projection.

5.There are minimum differences between the three methods that used to compute the linear scale factor.

6. The computed projected distances obtained using scale factor of the modified method is more accurate than that of the traditional method

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NOMENCLATURE

S: the true length of geodesic line on spheroid (arc)
D: the length of chord related to geodesic (S)
s: the length of projected geodesic on conformal projection.
d: the rectilinear chord of projected geodesic.
φ: the geodetic latitude.
λ: the geodetic longitude.
λ_o: the longitude of the central meridian.
K : the point scale factor.
K_o^*: scale factor on central meridian.
K_o : scale factor at starting point when (s=0).

$\overline{K}i$: the linear scale factor.
(a, e^2) characteristic of the spheroid
UTM: Universal Transverse Mercator conformal projection.
TM: Transverse Mercator conformal projection.