Study the Dynamic Behavior of Rotor Supported on a Worn Journal Bearings

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ABSTRACT

In this paper, the effect of wear in the fluid film journal bearings on the dynamic behavior of rotor bearing system has been studied depending on the analytical driven of dynamic stiffness and damping coefficients of worn journal bearing. The finite element method was used to modeling rotor bearing system. The unbalance response, critical speed and natural frequency of rotor bearing system have been studied to determine the changes in these parameters due to wear. MATLAB software was used to find the analytical values of dynamic coefficients of journal bearing. The results of rotor mounted on fluid film journal bearings showed that the wear in journal bearing increases the amplitude of unbalance response and decrease critical speed, stability and the natural frequencies.

Key words: Rotor, Journal Bearing, Wear, Critical Speed,

دراسة السلوك الديناميكي لدوار مستند على كراسي تحميل متآكلة

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الخلاصة

في هذه ألورقة تم دراسة تأثير ألتآكل في كراسي ألتحميل ذات ألغشاء ألسائل على ألسلوك ألديناميكي لمنظومة ألدوار وكرسي ألتحميل بالاعتماد على ألحساب النظري لمعاملات الجساءة والتخميد الديناميكية لكرسي تحميل متآكل. أُستخدمت طريقة العناصر المحددة لتمثيل منظومة الدوار وكرسي التحميل. تم دراسة الاستجابة لعدم ألاتزان, السرعة الحرجة والتردد الطبيعي لمنظومة الدوار وكرسي التحميل لتحديد التغيرات في هذه العوامل نتيجة للتآكل. تم استخدام برنامج الماتلاب لايجاد القيم التحليلية للمعاملات الديناميكية لكراسي التحميل. أظهرت النتائج ان التآكل في كراسي التحميل يزيد سعة الاستجابة لعدم الاتزان ويقلل السرعة الحرجة والأستقرارية والتردد الطبيعي للدوارات المستندة على كراسي تحميل ذو غشاء سائل متآكلة.

الكلمات الرئيسية : الدوار, المحمل الأسطواني, التآكل, السرعة الحرجة.

1. INTRODUCTION

The response of a rotor mounted on the fluid film bearings is depended on the material characteristics and geometric of the rotor, the reaction forces and the dynamic coefficients of the journal bearings which support the rotor. The lubrication of the fluid film bearings is described by the Reynolds equation. The wear in the bearing metals is a usual situation in the systems. This wear occurs due to overloads which leading to contact between journal and bearing or due to dry friction between them during the startup or shutdown of the system. In such cases the wear changes the oil thickness and then the pressure distribution, the equilibrium position of the rotor's center and finally the dynamic coefficients of the journal bearing and the response amplitude of the system.

Dufrane, et al., 1983. inspected the worn hydrodynamic journal bearings used in steam turbine generators and established a model of wear geometry for use in analytical studies of the bearings. **Hashimoto, et al., 1986.** examined theoretically and experimentally the effects of geometric change due to wear on the hydrodynamic lubrication of journal bearings in both laminar and turbulent regimes. **Nikolakopoulos and Papadopoulos, 1994**. calculated the full stiffness and damping matrices of a misaligned wear pattern in a fluid film bearing including higher order nonlinear terms. **Kumar and Mishra, 1996**.studied numerically the influences of geometric change due to wear on stability of hydrodynamic journal bearings. **Kumar and Mishra,1996**, studied the non-circular worn journal bearings under condition of non-laminar lubrication regimes, and the influence of wear on friction and load carrying capacity. **Bouyer and Fillon, 2004.** investigated the effect of wear on the thermohydrodynamic performance of a plain journal bearing and they found out that the temperature of lubrication oil is lowering due to wear in journal bearings**. Nikolakopoulos, et al., 2005.** developed and presented a numerical method to identify the clearance defect due to wear in a rotating flexible rotor supported on two misaligned journal bearing, by response identification at any arbitrary point. **Nikolakopoulos and Papadopoulos, 2007** developed an analytical model in order to find out the relationship between the friction coefficient, the misalignment angles, and wear depth. **Gertzos, et al.,2011.** developed a graphical detection method to identify the wear depth correlating with the measured dynamic bearing characteristics. **Chasalevris, et al., 2013.** analyzed a rotor bearing system supported on worn fluid film bearings in order to estimate the progress of specific frequency components in the rotor response during resonance due to the action of an external excitation force. **Desjardins, 2013.** used special software known the COMSOL software to study the Effect of Wear on the maximum Oil Pressure and Load-Carrying Capacity of a Plain Journal Bearing. **Robbersmyr, et al.,2014**. investigated the metallic contact degree and thus wear and tear between bearing bushes and rotating shafts due to the oil whirl and oil whips in oil film journal bearings by measuring electric currents which pass through the oil film.

2. MATHEMATICAL MODEL OF WEAR IN FLUID FILM JOURNAL BEARING

2.1 Wear Model

The wear model used in this work is proposed by **Dufrane, et al., 1983.** The shape of wear model is shown schematically in **Fig 1.**

The region of positive pressure in the state of worn journal bearing can be divided to three sub region as following:

1. First non- worn region where $(0 \leq \theta \leq \theta_s)$ .In this region the lubricant oil fluid thickness at equilibrium position can be described by the following equation, **Cameron, 1981**:

$$h_o = c + e\cos\theta \quad (1)$$

For small amplitude motions about the equilibrium journal position the fluid film thickness can be rewritten as shown in the following equation and this represents a new approach adopted in this paper to determine the wear effect on the fluid film thickness

$$h = C + e(t)\cos(\theta) ; \quad (2)$$

Where,
$\theta = \gamma - \emptyset_0$, γ is an angle measured from y axis.
$e(t) = e_0 + \Delta e(t)$; $\emptyset(t) = \emptyset_0 + \Delta\emptyset(t)$, where Δe and $\Delta\emptyset$ are small radial and angular displacement quantities, respectively. Eq. (2) can be rewritten as in the following equation

$$h = C + (e_0 + \Delta e)\{cos\theta\, cos\Delta\emptyset + sin\theta\, sin\,\Delta\emptyset\}, \quad (3)$$

And, for small amplitude motions, $cos(\Delta\emptyset) \sim 1, sin(\Delta\emptyset) \sim \Delta\emptyset$. Small values products such as $(\Delta e \Delta\emptyset)$ should be neglected (considered equal to zero)

$$h = C + e_0\, cos\theta + \Delta e\, cos\theta + e_0\,\Delta\emptyset\, sin\theta = h_0 + h_1 \quad (4)$$

Where,

$$h_0 = C + e_0\, cos\theta ;\ h_1 = \Delta e\, cos\theta + e_0\,\Delta\emptyset\, sin\theta$$

2. Worn region where $(\theta_s \le \theta \le \theta_f)$.The thickness of lubricant oil fluid (h_{ow}) in the journal bearing wear region at equilibrium position can be described by the following equation **Dufrane, et al., 1983.**

$$h_{ow} = d_0 + e\cos\theta - c cos(\theta + \emptyset_o) \quad (5)$$

Where, c is the journal bearing radial clearance.
By using the same procedures mentioned above for non worn region the thickness fluid film for worn region (h_w) can be written as following

$$h_w = do + (e_0 + \Delta e)\cos(\gamma - \emptyset_0 - \Delta\emptyset) - c\cos(\theta + \emptyset_0)$$

$$h_w = do + e_0\cos\theta - c\cos(\theta + \emptyset_0) + e_0\Delta\emptyset\sin\theta + \Delta e\cos\theta$$

$$= h_{0w} + h_1 \quad (6)$$

3. Second non-worn region where $(\theta_f \le \theta \le \pi)$. In this region the thickness of lubricant oil fluid as in the first non-worn region.

The starting and final angles of the region where the wear take place $(\theta_s\ \ and\ \theta_f)$ are given by the solution of the following equation **Nikolakopoulos and Papadopoulos, 2009**

$$\cos(\theta + \emptyset_o) = \frac{d_0}{c} - 1 = \delta - 1 \quad (7)$$

Where $\delta = \frac{d_0}{c}$

So the values of starting and final angles of the region where the wear take place are

$$\theta_s = \pi - \emptyset_o - arcos(1-\delta)\ ,\ \theta_f = \pi - \emptyset_o + arcos(1-\delta) \tag{8}$$

2.2 Reynolds Equation

For a laminar flow and isoviscous incompressible fluid the Reynolds equation with some necessary assumptions **Kramer, 1993.** And for short journal bearing can be written as below

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right) = \frac{\partial h}{\partial t} + \frac{\Omega}{2}\frac{\partial h}{\partial \theta} \tag{9}$$

By integrating Eq. (9) with respect to the journal bearing length (z), the pressure distribution for non worn region can be described by the following equation

$$P(\theta, z, t) = \frac{6\mu}{h^3}\left(\frac{\partial h}{\partial t} + \frac{\Omega}{2}\frac{\partial h}{\partial \theta}\right)\left(Z^2 - \frac{L^2}{4}\right) \tag{10}$$

And for worn region in journal bearing above equation becomes

$$P_w(\theta, z, t) = \frac{6\mu}{h^3}\left(\frac{\partial h_w}{\partial t} + \frac{\Omega}{2}\frac{\partial h_w}{\partial \theta}\right)\left(Z^2 - \frac{L^2}{4}\right) \tag{11}$$

Where P_w and h_w are the pressure and fluid film thickness in the worn region of journal bearing respectively.

2.3 Calculation of Dynamic Coefficients of Worn Journal Bearings

The radial and tangential components of fluid film journal bearing force are **Chen and Gunter, 2007**

$$\begin{Bmatrix} F_r \\ F_t \end{Bmatrix} = 2\int_0^{L/2}\int_0^{\pi} P(\theta, z, t) R \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} d\theta dz \tag{12}$$

According to the above divided of positive pressure in journal bearing, by using Eq. (10) and Eq.(11) for pressure in non worn region and worn region of fluid film journal bearing respectively , Eq. (12) becomes

$$\begin{Bmatrix} F_r \\ F_t \end{Bmatrix} = \frac{12\mu}{h^3}\int_0^{L/2}\left\{\int_0^{\theta_s}\left(\frac{\partial h}{\partial t} + \frac{\Omega}{2}\frac{\partial h}{\partial \theta}\right) + \int_{\theta_s}^{\theta_f}\left(\frac{\partial h_w}{\partial t} + \frac{\Omega}{2}\frac{\partial h_w}{\partial \theta}\right) + \int_{\theta_f}^{\pi}\left(\frac{\partial h}{\partial t} + \frac{\Omega}{2}\frac{\partial h}{\partial \theta}\right)\right\}\begin{pmatrix} cos\,\theta \\ sin\,\theta \end{pmatrix} \left(Z^2 - \frac{L^2}{4}\right) d\theta dz \tag{13}$$

Substitute the derivative of fluid film thickness for worn and non worn region of journal bearing in the Eq. (13) and integrate it with respect to the bearing length (Z) with limits (0 to L/2), and let $\mathrm{H} = \frac{\mathrm{h}}{\mathrm{c}}$, $\mathrm{H_w} = \frac{\mathrm{h_w}}{\mathrm{c}}$ therefore Eq. (13) can be rewritten as following

$$\begin{Bmatrix} F_r \\ F_t \end{Bmatrix} = -\frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\int_0^{\theta_s}\begin{Bmatrix} -e_0 \sin\theta\cos\theta \\ -e_0 \sin^2(\theta) \end{Bmatrix} d\theta - \frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\int_{\theta_f}^{\pi}\begin{Bmatrix} -e_0 \sin\theta\cos\theta \\ -e_0 \sin^2(\theta) \end{Bmatrix} d\theta$$
$$-\frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H_w^3}\int_{\theta_s}^{\theta_f}\begin{Bmatrix} -e_0 \sin\theta\cos\theta + c\sin(\theta+\emptyset_0)\cos\theta \\ -e_0 \sin^2(\theta) + c\sin\theta\sin(\theta+\emptyset_0) \end{Bmatrix} d\theta$$
$$-\frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\left[\int_0^{\theta_s}\begin{Bmatrix} -\sin\theta\cos\theta \\ -\sin^2\theta \end{Bmatrix} d\theta\right]\Delta e - \frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\left[\int_0^{\theta_s}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right] e_0\Delta\emptyset$$
$$-\frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H_w^3}\left[\int_{\theta_s}^{\theta_f}\begin{Bmatrix} -\sin\theta\cos\theta \\ -\sin^2(\theta) \end{Bmatrix} d\theta\right]\Delta e - \frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H_w^3}\left[\int_{\theta_s}^{\theta_f}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right] e_0\Delta\emptyset$$
$$-\frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\left[\int_{\theta_f}^{\pi}\begin{Bmatrix} -\sin\theta\cos\theta \\ -\sin^2\theta \end{Bmatrix} d\theta\right]\Delta e - \frac{\Omega}{2}\frac{\mu L^3 R}{c^3 H^3}\left[\int_{\theta_f}^{\pi}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right] e_0\Delta\emptyset$$
$$-\frac{\mu L^3 R}{c^3 H^3}\left[\int_0^{\theta_s}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right]\Delta\dot{e} - \frac{\mu L^3 R}{c^3 H^3}\left[\int_0^{\theta_s}\begin{Bmatrix} \sin\theta\cos\theta \\ \sin^2\theta \end{Bmatrix} d\theta\right] e_0\Delta\dot{\emptyset}$$
$$-\frac{\mu L^3 R}{c^3 H_w^3}\left[\int_{\theta_s}^{\theta_f}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right]\Delta\dot{e} - \frac{\mu L^3 R}{c^3 H_w^3}\left[\int_{\theta_s}^{\theta_f}\begin{Bmatrix} \sin\theta\cos\theta \\ \sin^2\theta \end{Bmatrix} d\theta\right] e_0\Delta\dot{\emptyset}$$
$$-\frac{\mu L^3 R}{c^3 H^3}\left[\int_{\theta_f}^{\pi}\begin{Bmatrix} \cos^2\theta \\ \sin\theta\cos\theta \end{Bmatrix} d\theta\right]\Delta\dot{e} - \frac{\mu L^3 R}{c^3 H^3}\left[\int_{\theta_f}^{\pi}\begin{Bmatrix} \sin\theta\cos\theta \\ \sin^2\theta \end{Bmatrix} d\theta\right] e_0\Delta\dot{\emptyset}$$

(14)

The solution of Eq. (14) has the following form **Andres, 2006**.

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = \begin{bmatrix} F_{ro} \\ F_{to} \end{bmatrix} - \begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix}\begin{bmatrix} \Delta e \\ e_0\,\Delta\emptyset \end{bmatrix} - \begin{bmatrix} C_{rr} & C_{rt} \\ C_{tr} & C_{tt} \end{bmatrix}\begin{bmatrix} \Delta\dot{e} \\ e_0\,\Delta\dot{\emptyset} \end{bmatrix} \tag{15}$$

So as to solve equation (14) a first order Tyler series expansion has been used

$$H^{-3} = H_0^{-3} - 3H_0^{-4}.H_1 \quad , \; H_w^{-3} = H_{0w}^{-3} - 3H_{0w}^{-4}.H_1 \tag{16}$$

After long algebraic operations, the solution of Eq. (14) can be described as in the following equation

$$\begin{Bmatrix} F_r \\ F_t \end{Bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \begin{bmatrix} I_3 & I_4 \\ I_5 & I_6 \end{bmatrix}\begin{Bmatrix} \Delta e \\ e\Delta\emptyset \end{Bmatrix} - \begin{bmatrix} I_7 & I_8 \\ I_9 & I_{10} \end{bmatrix}\begin{Bmatrix} \Delta\dot{e} \\ e\Delta\dot{\emptyset} \end{Bmatrix} \tag{17}$$

Where

$$I_1 = c.F_s\begin{Bmatrix} \int_0^{\theta_S}\frac{\varepsilon_o\sin\theta\,\cos\theta}{H_O^3}d\theta + \int_{\theta_s}^{\theta_f}\frac{\varepsilon_o\sin\theta\,\cos\theta - \cos\theta\,\sin(\theta+\emptyset_o)}{H_{0w}^3}d\theta \\ + \int_{\theta_f}^{\pi}\frac{\varepsilon_o\sin\theta\,\cos\theta}{H_O^3}d\theta \end{Bmatrix}$$

$$I_2 = c.F_s \left\{ \begin{array}{c} \int_0^{\theta_S} \frac{\varepsilon_o (sin\theta)^2}{H_O^3} d\theta + \int_{\theta_s}^{\theta_f} \frac{\varepsilon_o (sin\theta)^2 - sin\theta\ sin(\theta+\emptyset_o)}{H_{0w}^3} d\theta \\ + \int_{\theta_f}^{\pi} \frac{\varepsilon_o (sin\theta)^2}{H_O^3} d\theta \end{array} \right\}$$

$$I_3 = F_s \left\{ \begin{array}{c} -\int_0^{\theta_S} \frac{sin\theta\ cos\theta}{H_O^3} d\theta + 3.\varepsilon_o.\int_0^{\theta_S} \frac{sin\theta\ (cos\theta)^2}{H_O^4} d\theta \\ -\int_{\theta_s}^{\theta_f} \frac{sin\theta\ cos\theta}{H_{0w}^3} d\theta + 3.\varepsilon_o.\int_{\theta_s}^{\theta_f} \frac{sin\theta\ (cos\theta)^2}{H_{0w}^4} d\theta - 3.\int_{\theta_s}^{\theta_f} \frac{(cos\theta)^2\ sin(\theta+\emptyset_o)}{H_{0w}^4} d\theta \\ -\int_{\theta_f}^{\pi} \frac{sin\theta\ cos\theta}{H_O^3} d\theta + 3.\varepsilon_o.\int_{\theta_f}^{\pi} \frac{sin\theta\ (cos\theta)^2}{H_O^4} d\theta \end{array} \right\}$$

$$I_4 = F_s \left\{ \begin{array}{c} \int_0^{\theta_S} \frac{(cos\theta)^2}{H_O^3} d\theta + 3.\varepsilon_o.\int_0^{\theta_S} \frac{cos\theta\ (sin)^2}{H_O^4} d\theta \\ +3.\varepsilon_o.\int_{\theta_s}^{\theta_f} \frac{(sin\theta)^2\ cos\theta}{H_{0w}^4} d\theta + \int_{\theta_s}^{\theta_f} \frac{(cos\theta)^2}{H_{0w}^3} d\theta - 3.\int_{\theta_s}^{\theta_f} \frac{cos\theta\ sin(\theta+\emptyset_o)}{H_{0w}^4} d\theta \\ +\int_{\theta_f}^{\pi} \frac{(cos\theta)^2}{H_O^3} d\theta + 3.\varepsilon_o.\int_{\theta_f}^{\pi} \frac{cos\theta\ (sin\theta)^2}{H_O^4} d\theta \end{array} \right\}$$

$$I_5 = F_s \left\{ \begin{array}{c} -\int_0^{\theta_S} \frac{(sin\theta)^2}{H_O^3} d\theta + 3.\varepsilon_o.\int_0^{\theta_S} \frac{cos\theta\ (sin)^2}{H_O^4} d\theta \\ +3.\varepsilon_o.\int_{\theta_s}^{\theta_f} \frac{(sin\theta)^2\ cos\theta}{H_{0w}^4} d\theta - \int_{\theta_s}^{\theta_f} \frac{(sin\theta)^2}{H_{0w}^3} d\theta - 3.\int_{\theta_s}^{\theta_f} \frac{sin\theta\ cos\theta\ sin(\theta+\emptyset_o)}{H_{0w}^4} d\theta \\ -\int_{\theta_f}^{\pi} \frac{(sin\theta)^2}{H_O^3} d\theta + 3.\varepsilon_o.\int_{\theta_f}^{\pi} \frac{cos\theta\ (sin\theta)^2}{H_O^4} d\theta \end{array} \right\}$$

$$I_6 = F_s \left\{ \begin{array}{c} \int_0^{\theta_S} \frac{sin\theta\ cos\theta}{H_O^3} d\theta + 3.\varepsilon_o.\int_0^{\theta_S} \frac{(sin)^3}{H_O^4} d\theta \\ +3.\varepsilon_o.\int_{\theta_s}^{\theta_f} \frac{(sin)^3}{H_{0w}^4} d\theta + \int_{\theta_s}^{\theta_f} \frac{sin\theta\ cos\theta}{H_{0w}^3} d\theta - 3.\int_{\theta_s}^{\theta_f} \frac{(sin\theta)^2\ sin(\theta+\emptyset_o)}{H_{0w}^4} d\theta \\ +\int_{\theta_f}^{\pi} \frac{sin\theta\ cos\theta}{H_O^3} d\theta + 3.\varepsilon_o.\int_{\theta_f}^{\pi} \frac{(sin)^3}{H_O^4} d\theta \end{array} \right\}$$

$$\mathrm{I}_7 = \frac{2\mathrm{F_s}}{\Omega} \left\{ \int_0^{\theta_\mathrm{S}} \frac{(\cos\theta)^2}{\mathrm{H_O^3}} d\theta + \int_{\theta_\mathrm{s}}^{\theta_\mathrm{f}} \frac{(\cos\theta)^2}{H_{0w}^3} d\theta + \int_{\theta_\mathrm{f}}^{\pi} \frac{(\cos\theta)^2}{\mathrm{H_O^3}} d\theta \right\}$$

$$\mathrm{I}_8 = \frac{2\mathrm{F_s}}{\Omega} \left\{ \int_0^{\theta_\mathrm{S}} \frac{\sin\theta\ \cos\theta}{\mathrm{H_O^3}} d\theta + \int_{\theta_\mathrm{s}}^{\theta_\mathrm{f}} \frac{\sin\theta\ \cos\theta}{H_{0w}^3} d\theta + \int_{\theta_\mathrm{f}}^{\pi} \frac{\sin\theta\ \cos\theta}{\mathrm{H_O^3}} d\theta \right\}$$

$$I_9 = \frac{2F_s}{\Omega} \left\{ \int_0^{\theta_S} \frac{sin\theta\ cos\theta}{H_O^3} d\theta + \int_{\theta_s}^{\theta_f} \frac{sin\theta\ cos\theta}{H_{0w}^3} d\theta + \int_{\theta_f}^{\pi} \frac{sin\theta\ cos\theta}{H_O^3} d\theta \right\}$$

$$I_{10} = \frac{2F_s}{\Omega} \left\{ \int_0^{\theta_S} \frac{(sin\theta)^2}{H_O^3} d\theta + \int_{\theta_s}^{\theta_f} \frac{(sin\theta)^2}{H_{0w}^3} d\theta + \int_{\theta_f}^{\pi} \frac{(sin\theta)^2}{H_O^3} d\theta \right\}$$

$$\mathrm{F_s} = \frac{\mu \Omega R L^3}{2c^3} \quad ,$$

By comparison between Eq. (15) and Eq. (17) get the values of dynamic coefficients in rotating coordinates system

$F_{ro} = I_1$, $F_{to} = I_2$, $K_{rr} = I_3$, $K_{rt} = I_4$, $K_{tr} = I_5$, $K_{tt} = I_6$

$C_{rr} = I_7$, $C_{rt} = I_8$, $C_{tr} = I_9$, $C_{tt} = I_{10}$

For dynamic coefficients in fixed coordinate system the following equation can be used to convert from the rotating coordinates to the fixed coordinates **Andres, 2006**.

$$\begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} = \begin{bmatrix} \sin\emptyset_o & \cos\emptyset_o \\ -\cos\emptyset_o & \sin\emptyset_o \end{bmatrix} \begin{bmatrix} K_{rr} & K_{rt} \\ K_{tr} & K_{tt} \end{bmatrix} \begin{bmatrix} \sin\emptyset_o & -\cos\emptyset_o \\ \cos\emptyset_o & \sin\emptyset_o \end{bmatrix}$$

(18)

$$\begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} = \begin{bmatrix} \sin\emptyset_o & \cos\emptyset_o \\ -\cos\emptyset_o & \sin\emptyset_o \end{bmatrix} \begin{bmatrix} C_{rr} & C_{rt} \\ C_{tr} & C_{tt} \end{bmatrix} \begin{bmatrix} \sin\emptyset_o & -\cos\emptyset_o \\ \cos\emptyset_o & \sin\emptyset_o \end{bmatrix}$$

The integrals in Eq. (17) can be calculated by using table of journal bearings integrals **Booker, 1965.** for integrals of non worn regions and by using Simpsons 1/3 rule with multiple divisions for integrals in the worn regions. Many Matlab scripts have been written to calculate dynamic coefficients for non worn journal bearing and worn journal bearings. The dynamic coefficients of non worn journal bearings can be calculated by using the same equations of worn journal bearings except the value of wear depth (do) equal zero.
The following steps were followed to calculate dynamic coefficients for journal bearings in state of non worn and worn case

1. Writing Matlab script to calculate iteratively (using Newton – Raphson method) the eccentricity ratio of journal bearing by using the balancing between the film reaction force components and applied static load which represented in the following equation **Friswell, et al, 2010**.

$$\varepsilon^8 - 4\varepsilon^6 + \left(6 - S_s^2(16-\pi^2)\right)\varepsilon^2 - (4+\pi^2 S_s^2)\varepsilon^2 + 1 = 0 \tag{19}$$

Where S_s is modified Sommerfeld Number $S_s = \frac{\mu\Omega L^3 R}{4c^2 F_0}$ and R, L, **Ω**, μ,c and F_0 are journal radius , bearing length, rotor rotational speed, lubricant oil viscosity, radial clearance and external applied load (the part of rotor weight supported by every bearing) respectively.

2. Writing Matlab script is used to calculate iteratively the attitude angle in state of worn journal bearing by using the following equation

$$\left({F_{ro}}^2 + {F_{to}}^2\right)^{0.5} - F_0 = 0 \tag{20}$$

3. Writing Matlab script is used to calculate dynamic coefficients for non worn journal bearing by using Eq.(17) with (substitute do = 0 in Eq.(6) then substitute resultant equation in Eq.(17))
4. Writing Matlab script is used to calculate dynamic coefficients for worn journal bearing by using Eq. (17). For bearing number one as shown in the **Fig.1** while bearing number two still without wear.

3. ROTOR BEARING SYSTEM ANALYSIS USING ANSYS

3-D Solid model rotor bearing system gives more accurate results than in the case of one dimensional beam model as well as there are many advantages in adopting this model **Rao and Sreenivas, 2003.** Therefore it is used in this work, Solid187 element has been used to model shaft and disk as shown in the **Fig.2. (a),** as well as comi214 element used to model journal bearing **ANSYS Guide, 2012**. The eight dynamic coefficients of journal bearing are depending on the rotational speed therefore when these coefficients represent in ANSYS must be changed with rotor speed. The dimensions of rotor and bearings which used in this work are shown in **Fia.2. (b),** and **Table.1.**

4. RESULTS AND DISCUSSION

4.1 Effect of Wear on the Dynamic Coefficients of Journal Bearings

The eccentricity ratio for fluid film journal bearing decreases with the increasing the modified Sommerfeld number (S_s) and with increasing the rotor spin speed due to the inverse relationship between modified Sommerfeld number and rotational speed of rotor and then with eccentricity ratio as shown in the **Fig.3**.

Fig.4 and **Fig.5** describe the variation of the intact rotor dimensionless stiffness and damping coefficients respectively with the modified Sommerfeld number (duty number) and rotational speed of rotor, these figures showed that the direct dynamic coefficients (stiffness and damping) become somewhat fixed value when the rotor spin speed reach about 2000 rpm, because the eccentricity ratio becomes somewhat fixed and eccentricity ratio represents the important parameter in the dynamic coefficients calculations.

Fig .6 shows the variation of attitude angle with wear depth and eccentricity ratio, the attitude angle under the constant eccentricity ratio decreases with increasing in the wear depth parameter.

Fig.7 demonstrated that the direct dynamic coefficients ($K_{xx}, K_{yy}, C_{xx}, C_{yy}$) have fixed value when rotational speed of the rotor becomes higher than the 2000 rpm approximately, also cross coupling dynamic coefficients ($K_{xy}, K_{yx}, C_{xy}, C_{yx}$) have no significant changes when rotor speed becomes higher than the 2000 rpm this is because the change in the value of the eccentricity ratio becomes small at this speed.

Fig .8 and **Fig.9** clearly show that the dynamic coefficients decrease with the increasing of wear depth parameter except the direct stiffness coefficient in the y- direction increases with the increasing in wear depth parameter that means the stability of rotor supported on worn journal bearing will decreases with increasing in the wear depth parameter.

4.2 Effect of Wear on the Dynamic Behavior of Rotor

The critical speed of rotor supported on fluid film journal bearings is speed dependence, **Childs, 1993.** Therefore the location of the critical speed is at maximum unbalance response, **Rao, 1996.** The ANSYS results showed that the using of fluid film journal bearing in the rotor bearing system increases the instability at the critical speed due to existence of cross coupling coefficients as shown in the **Fig.10**. The wear in the journal bearing with the wear depth parameter ($\delta = 0.2\ , do = 0.02\ mm$) decreases the critical speed by about 2.3% and increases the amplitude by about 50% at the critical speed as shown in the **Fig.11** this is due to the high decreasing in the cross coupling (K_{xy}) with low decreasing in the direct damping coefficients and less increasing in the cross coupling damping coefficients but this state increases the stability of rotor after critical speed as shown in the **Fig.11** where there is smooth decay beyond maximum response.

The additional increase in the wear depth parameter ($\delta = 0.4\ , do = 0.04\ mm$) led to decrease the critical speed by about 8.87 % and decrease the amplitude of unbalance response by 8.9 % due to high increasing in the K_{yy} as well as high increasing in the cross coupling damping coefficients as shown in the **Table 2**., but The important point that deserves to be mentioned here the system become more instable as shown in the **Fig.12,** and this is undesirable case

5. CONCLUSIONS

1. Wear in journal bearing is generally decreasing the critical speed of rotor bearing system and that means decreasing the natural frequency.
2. Low wear depth is increasing the stability of rotor bearing system but in the same time increasing unbalance response amplitude.
3. High wear depth led to increasing in the instability of the rotor bearing system.
4. Measuring of unbalance response can be used to on line wear detection in the rotor journal bearing system and develop a monitoring system for rotating machine can be applied in the virtual system at early wear detection and prevent the sudden failure that could lead to full system break down.

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NOMENCLATURE

c = radial clearance, m
do = wear depth, m
e = eccentricity between journal center and bearing center, m
e_0 = eccentricity between journal center and bearing center at equilibrium position, m
F_0 = journal bearing external applied force = weight of rotor applied on every bearing , N
F_r = radial component of fluid film journal bearing force, N
F_t = tangential component of fluid film journal bearing force, N
F_{r0} = radial component of fluid film journal bearing force at equilibrium position, N
F_{t0} = tangential component of fluid film journal bearing force at equilibrium position, N
K_{ij} = dynamic coefficients in rotating coordinates , i,j = r,t
K_{ij} = dynamic coefficients in fixed coordinates , i,j = x,y
h = fluid film thickness non- worn region, m
H = nondimensional fluid film thickness of non- worn region =h/c
h_0 = non- worn fluid film thickness at equilibrium position, m
h_w = worn fluid film thickness, m
H_w = nondimensional fluid film thickness of worn region = h_w/c
h_{0w} = worn fluid film thickness at equilibrium position, m
L = bearing length, m
P = lubricant oil pressure of non- worn region, pa
P_w = lubricant oil pressure of worn region, pa
R = journal radius, m
γ = circumferential coordinate from Y-axis, **Fig. 1**
δ = wear depth parameter = do/c
ε = eccentricity ratio = e/c
θ = circumferential coordinate, **Fig 1.**
θ_s= circumferential location of the starting point of the worn region, **Fig. 1**
θ_f= circumferential location of the final point of the worn region, **Fig. 1**
μ = viscosity pa s
$\emptyset_0$ = attitude angle, **Fig. 1**
Δe = small displacement at equilibrium position
$\Delta\emptyset$ = small variation of attitude angle
Ω = rotational speed of rotor, rad/s

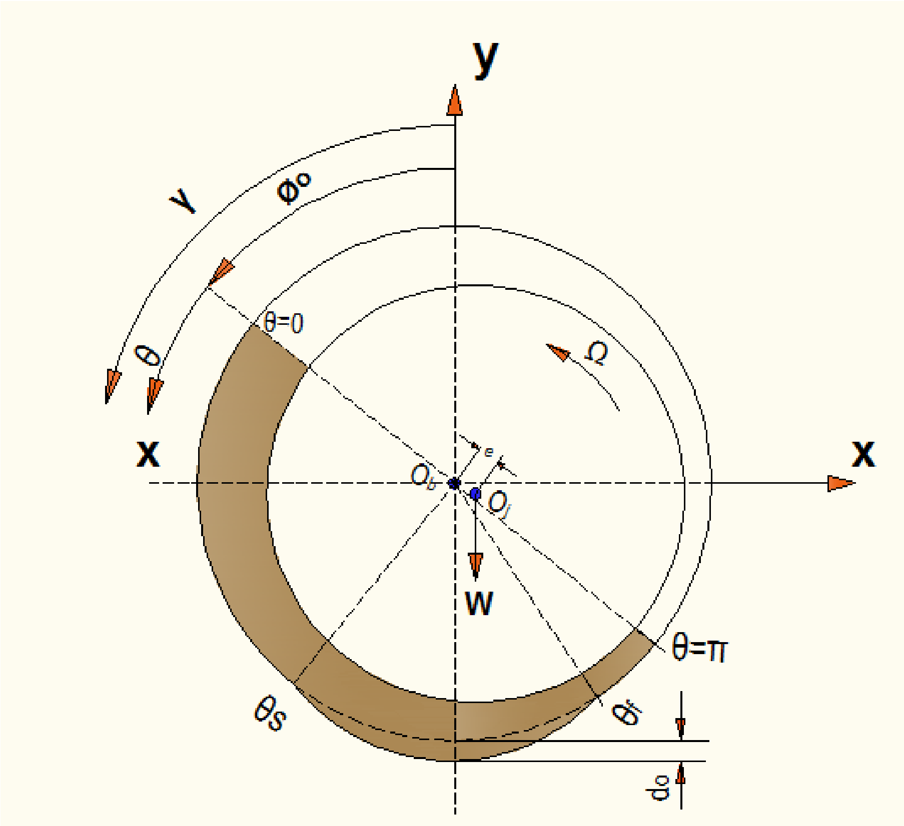
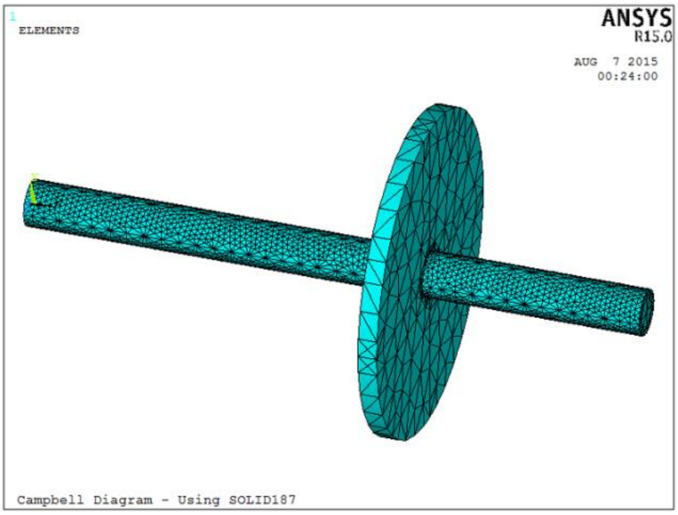
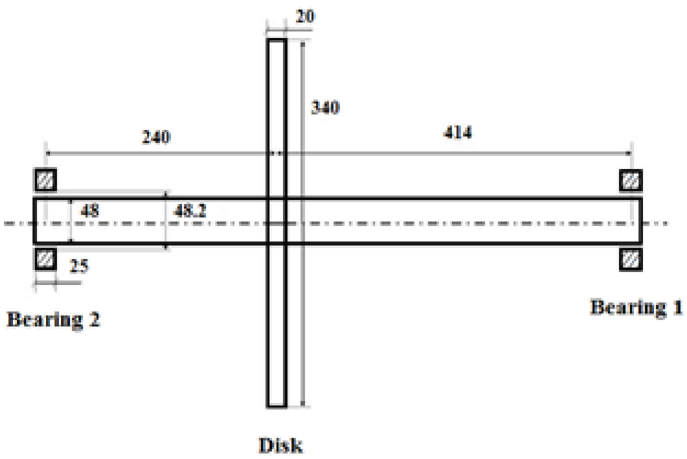


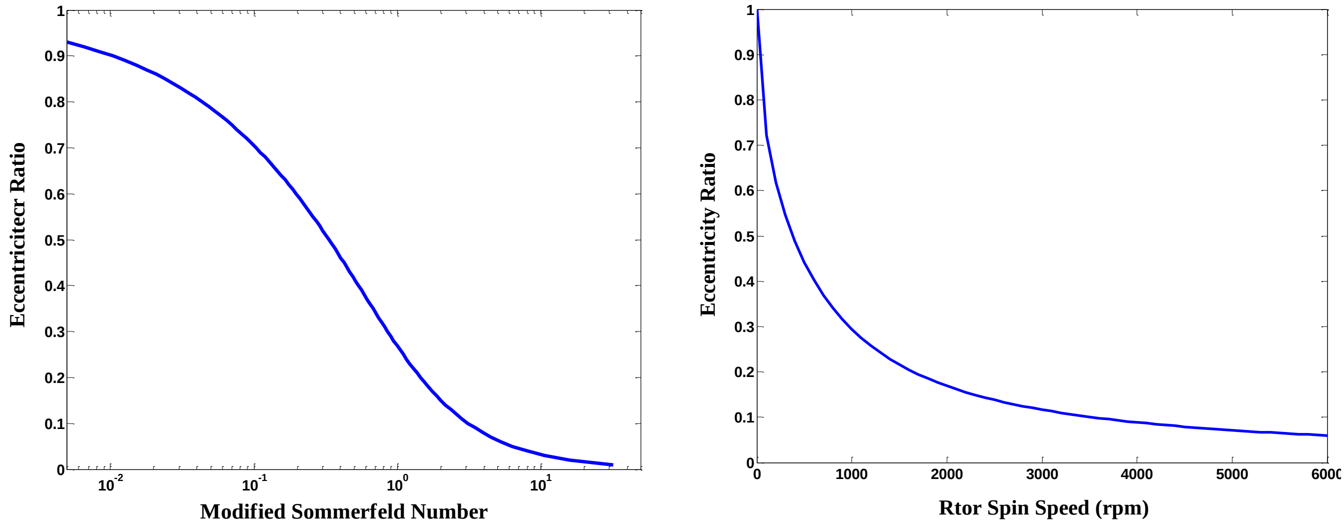
Figure 1. Worn journal bearing.





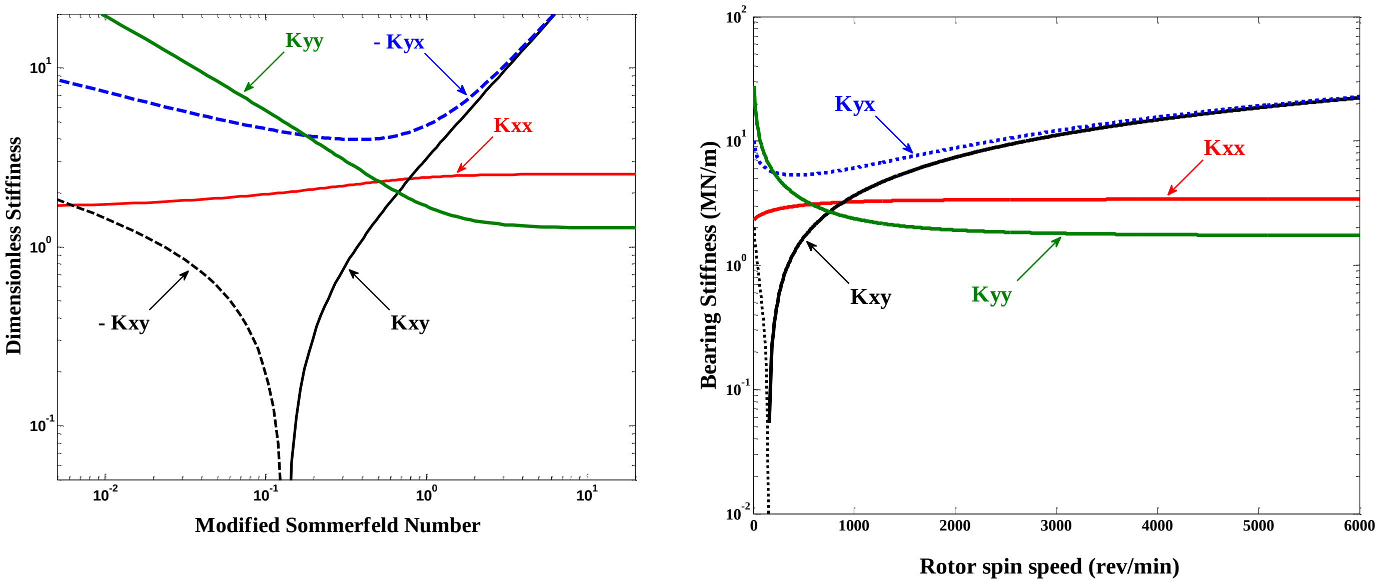
(a) (b)

Figure 2. (a) Ansys rotor model (b) Mechanical drawing of present rotor bearing system.



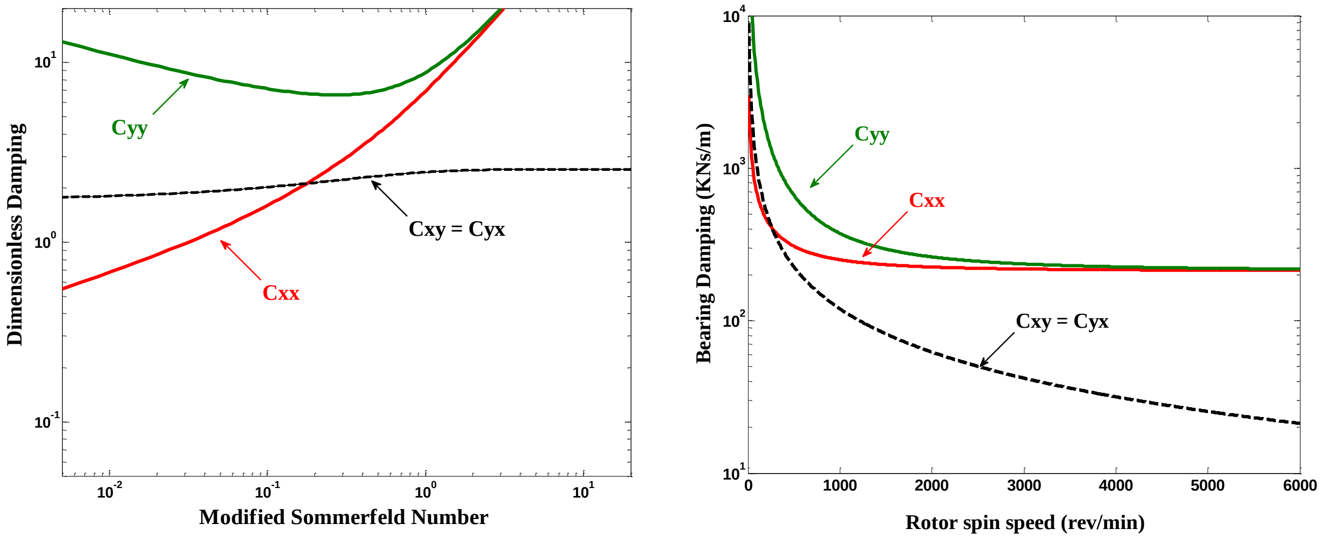
(a) (b)

Figure 3. Bearing eccentricity ratio (a) as a function of the modified Sommerfeld number (b) as a function of the Rotor Spin Speed.



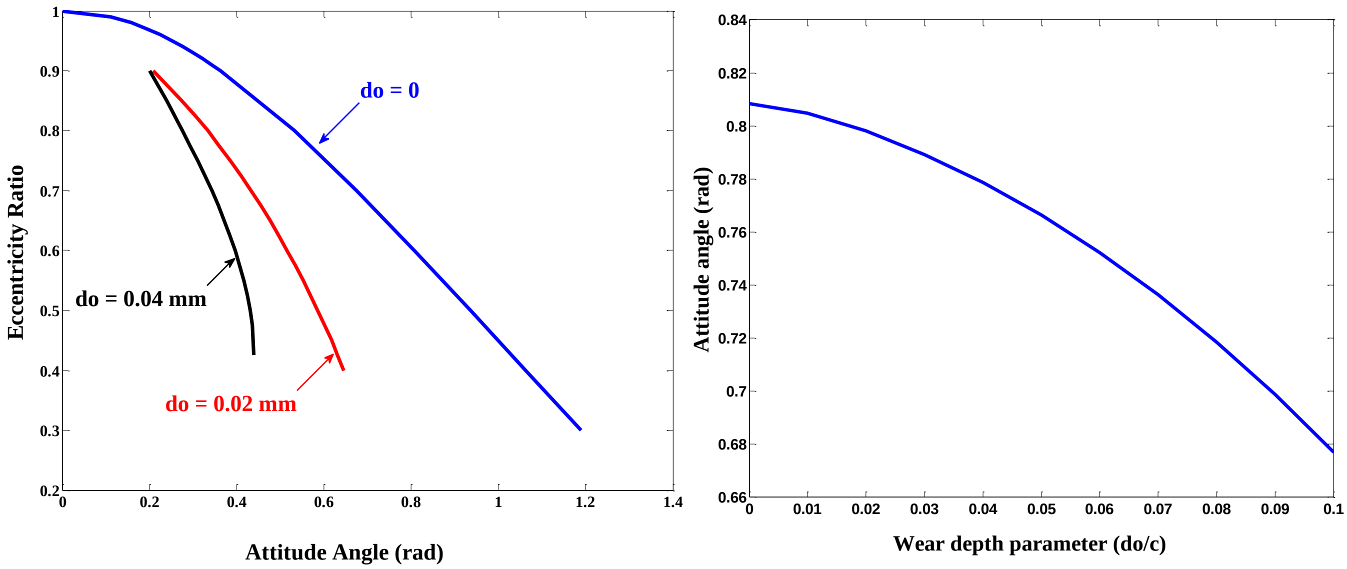
(a) (b)

Figure 4. Nondimensional stiffness of an intact fluid film bearing (a) as a function of the modified Sommerfeld number (b) as a function of the rotor spin speed, (negative coefficients are shown as dashed lines).



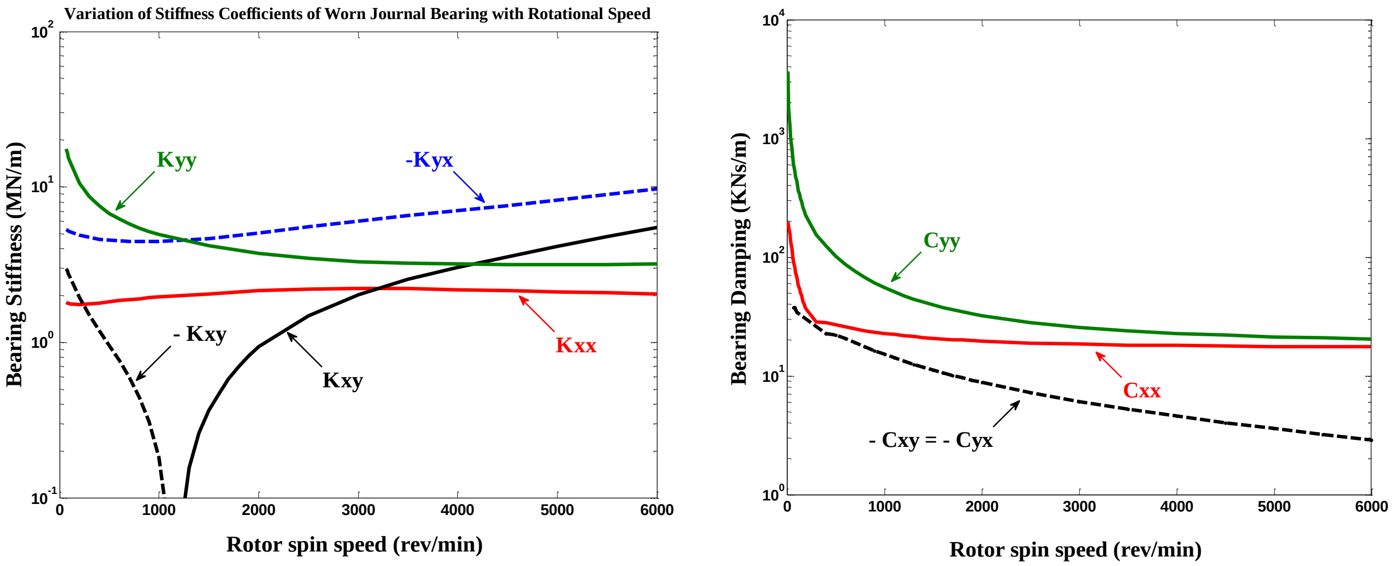
(a) (b)

Figure 5. Nondimensional damping of an intact fluid film bearing (a) as a function of the modified Sommerfeld number (b) as a function of the rotor spin speed, (negative coefficients are shown as dashed lines).



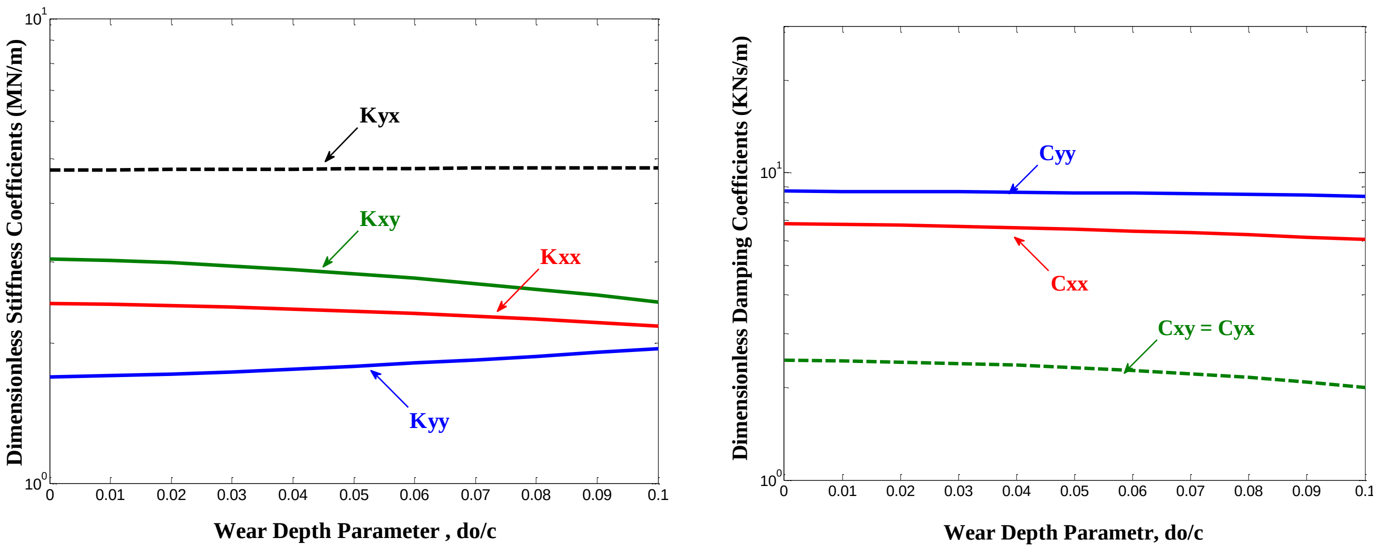
(a) (b)

Figure 6. Variation of attitude angle (a) with the eccentricity ratio and depth wear parameter, (b) with depth wear parameter (do/c).



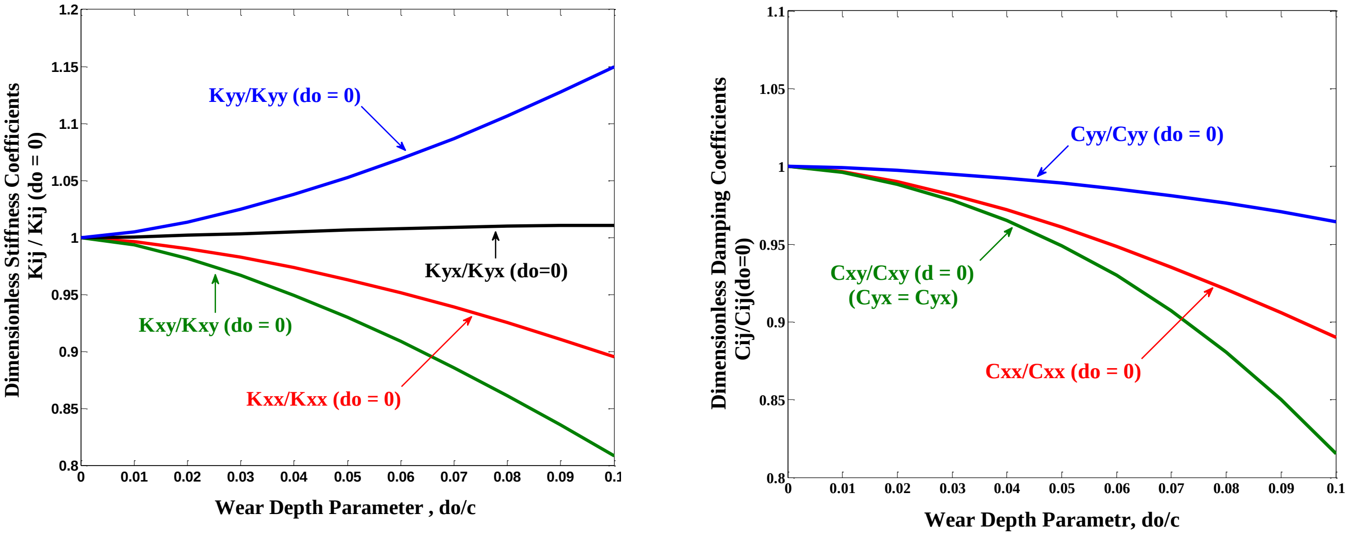
(a) (b)

Figure 7. Nondimensional dynamic coefficients of a worn fluid film bearing (a): stiffness as a function of the rotor spin speed (b): damping as a function of the rotor spin speed, (negative coefficients are shown as dashed lines).



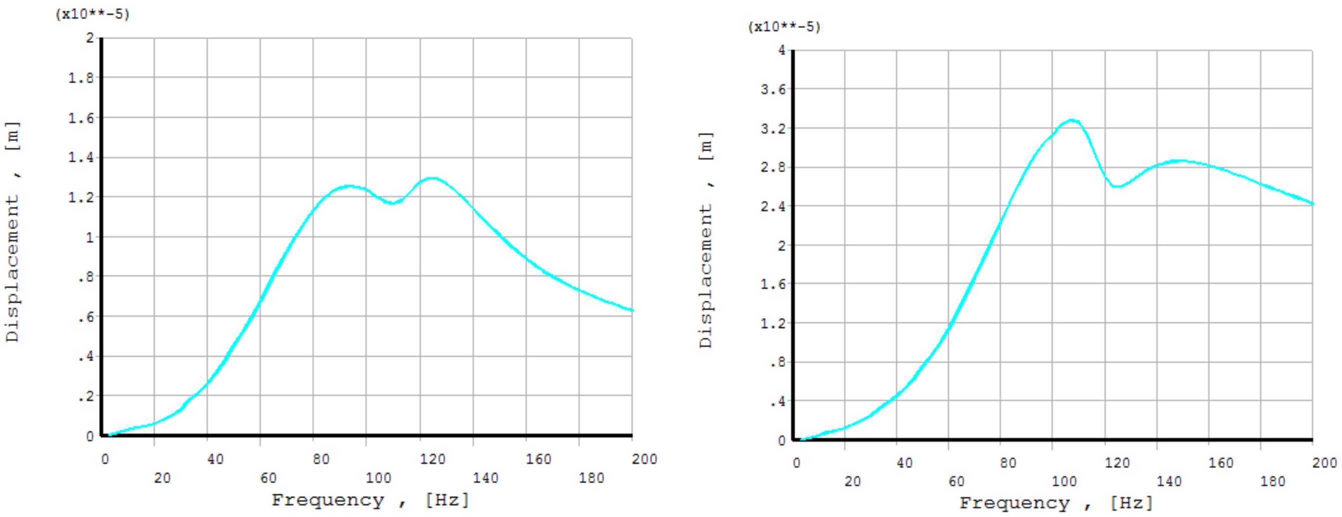
(a) (b)

Figure 8. Nondimensional dynamic coefficients of a worn fluid film bearing (a): stiffness as a function of the wear depth parameter (b) damping as the wear depth parameter, (negative coefficients are shown as dashed lines).



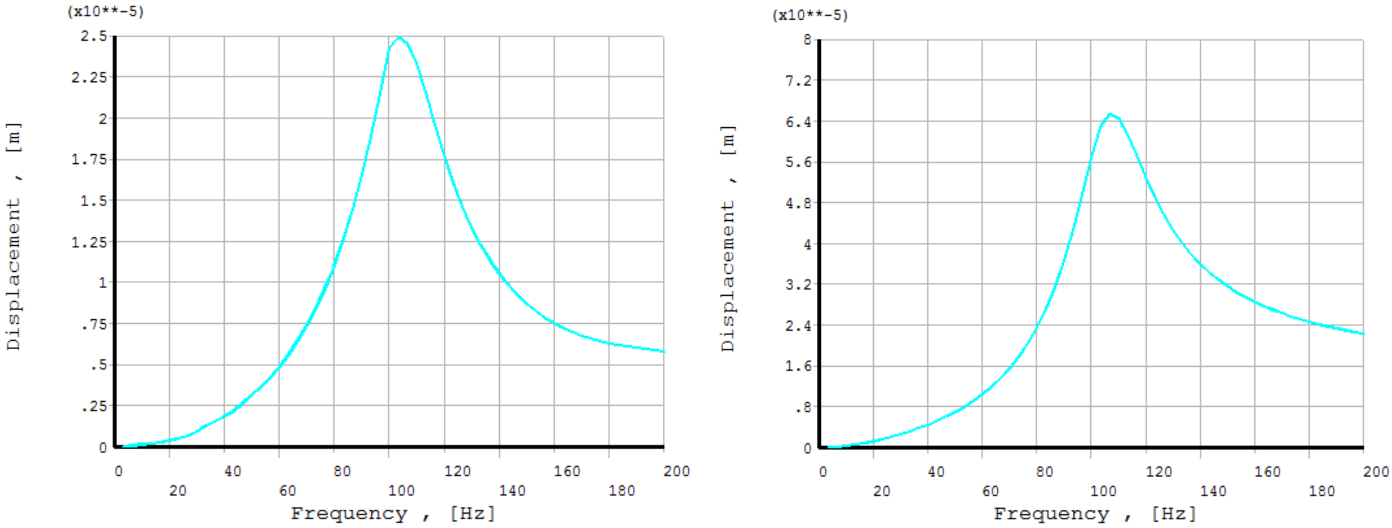
(a) (b)

Figure 9. Nondimensional dynamic coefficients ratio of a worn fluid film bearing (a): stiffness ratio as a function of the wear depth parameter (b) damping ratio as the wear depth parameter, (negative coefficients are shown as dashed lines).



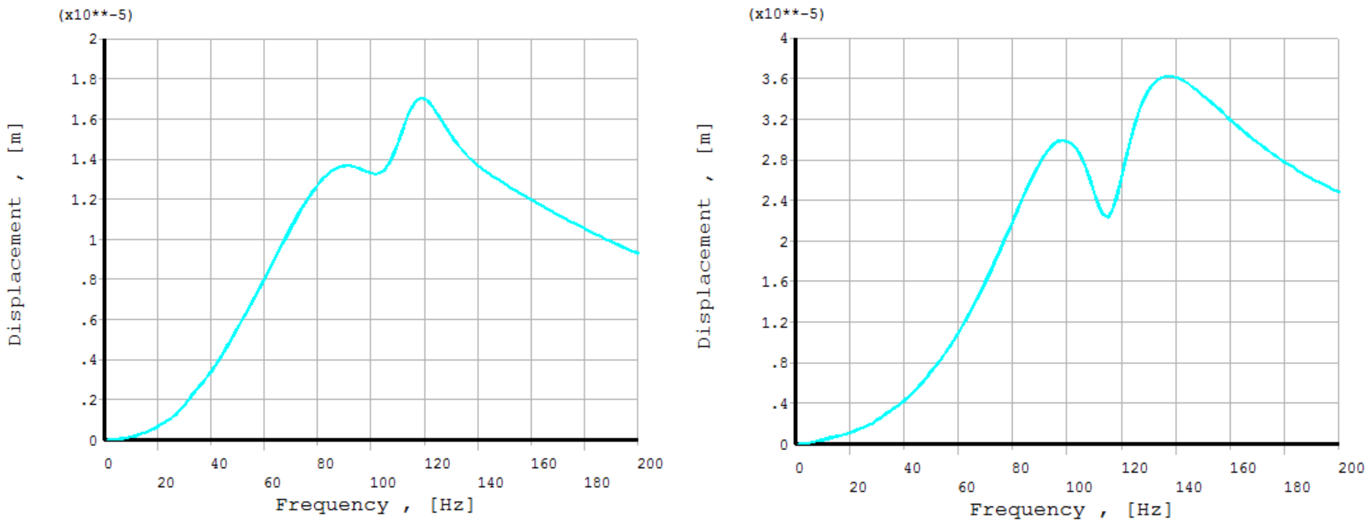
(a) (b)

Figure 10. Unbalance response of rotor supported on fluid film journal bearings (a): at bearing No.1 (b): at disc center.



(a) (b)

Figure 11. Unbalance response of rotor supported on worn journal bearings (do= 0.02 mm) (a): at bearing No.1 (b): at disc center.



(a) (b)

Figure 12. Unbalance response of rotor supported on worn journal bearings (do= 0.04 mm) (a): at bearing No.1 (b): at disc center.

Table 1. Shows rotor material and lubricant oil specifications

Shaft length m	Shaft diam. m	Disc diam. m	Disc thickness m	Modulus of Elasticity pa	Shaft and Disk Density Kg/m^3	Lubricant oil viscosity Pa s	Unbalance force Kg - m
o.654	0.048	0.34	0.02	$2.1x10^{11}$	7850	0.032	$0.323x10^{-6}$

Table .2 Variation of journal bearing coefficients with wear depth

δ	K_{xx} $x10^6$	K_{xy} $x10^6$	K_{yx} $x10^6$	K_{yy} $x10^6$	C_{xx} $x10^4$	C_{xy} $x10^4$	C_{yx} $x10^4$	C_{yy} $x10^4$
0	2.4	5.9	-6.81	1.34	1.94	-0.38	-0.38	2.1
0.2	2.1	3.93	-6.86	1.6	1.69	-0.25	-0.25	1.7
0.4	1.5	3.37	-7.77	2.2	1.94	0.176	0.176	1.59