AEROTRIANGULATION BY COPLANARITY

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ABSTRACT
Before corresponding points in photos taken with two cameras can be used to recover distances to objects in a scene, one has to determine the position and orientation of one camera relative to the other. This is the classic photogrammetric problem of aerotriangulation. Iterative methods for determining X,Y,Z ground positions for unknown points using aerotriangulation process, were developed long ago; without them we would not have most of the topographic maps we do today.
Described here in this research a simple iterative scheme for recovering relative orientation process then applying intersection problem (vector method) using the condition of coplanarity, out of the usual for photogrammetrists in using the familiar condition of collinearity. The data required is a pair of bundles of corresponding rays from the two projection centers to points in the scene. It is well known that at least five pairs of rays are needed, because, each object point gives only one equation. The results were amazing according to the variances that have been obtained for the angular orientation elements. The programs have been written using Matlab software ver. 5.3.

الخلاصة
قبل أن يكون بامكان زوج النقاط الصورية المتناظرة والملتقطة بكامرتين جويتين ان تستخدم لحساب المواقع الارضية والمسافات الحقيقية بين نقاط مجهولة الموقع الارضي الحقيقي في المشهد المصور ، يجب ان يحسب موقع وتوجيه احدى الكامرتين في الفضاء نسبة الى الاخرى ، هذه الخطوتين المتلاحقتين تعرف بعملية التثليث الجوي. هناك العديد من الطرق المستخدمة لحساب المواقع الارضية للنقاط المجهولة باستخدام عملية التثليث الجوي والتي استخدمت منذ فترات طويلة والتي لولاها لما كنا قد حصلنا على معظم الخرائط الطوبوغرافية التي نمتلكها ونتداولها اليوم.
في هذا البحث تم وصف وتطبيق طريقة بسيطة لحساب عناصر التوجيه الخارجي لاحدى الكامرتين نسبة الى الاخرى ثم تطبيق معادلات التقاطع الامامي (طريقة الـ vectors) لحساب المواقع الارضية للنقاط المجهولة في المنطقة المراد تكثيفها وذلك باستخدام معادلات الـ coplanarity condition equations بدل استخدام الطريقة المألوفة والمستخدمة من قبل معظم باحثي وعلماء المسح التصويري وهي معادلات الـ collinearity condition equations وذلك لغرض اختبار النتائج المستحصلة باستخدام هذه المعادلات (coplanarity) ومقارنتها مع النتائج المستحصلة من معادلات الـ (collinearity). المدخلات المطلوبة لاستخدام هذه المعادلات تتطلب زوج من حزم من الاشعة المتناظرة والقادمة من مركزي الاسقاط الى النقاط المعنية في المشهد المصور. وكما هو معلوم فانه على الاقل لابد من ان يتوفر خمسة ازواج من هذه الاشعة

واللازمة لايجاد قيمة المجاهيل وذلك لان كل هدف من هذه النقاط الارضية المختارة يعطينا معادلة واحـــدة فقط. النتائج التي تم الحصول عليها في هذا البحث كانت مذهلة نسبة الى الاختلافات التي تم الحصول عليها لعناصر التوجيه الزاوي. البرامج المستخدمة في هذا البحث تم اعدادها باستخدام لغة الــــــMatlab المرنـــة والتي توفر تصور واضح للقارئ عن نجاح تجربة البحث.

KEY WORDS
Photogrammetry, Analytical Photogrammetry, Analytical Aerotriangulation, Coplanarity Condition

INTRODUCTION

Aerotriangulation is the term most frequently applied to the process of determining X,Y & Z ground coordinates of individual points based on measurements from photographs. Aerotriangulation process can be applied using different techniques, such as analogue, semianalytical, analytical and of course digital techniques. Analytical aerotriangulation tends to be more accurate than analogue or semi analytical aerotriangulation, largely because analytic techniques can more effectively eliminate systematic errors. Several different variations in analytical aerotriangulation techniques have evolved. Basically, however, all methods consist of writing condition equations, which express the unknown elements of exterior orientation of each photo. The equations are solved to determine the unknown orientation parameters and the ground coordinates of unknown points. The most commonly used methods enforce one of two conditions: collinearity or coplanarity. In coplanarity method (that used in this research), one equation may be written for each object point whose images appear on both photos of the stereopair. The coplanarity equations do not contain object space coordinates as unknown; rather, they contain only the elements of exterior orientation of the two photos of the stereopair. Therefore; after solving for the elements of exterior orientation, object point coordinates are calculated, by solving the space intersection problem by colliniarity,or using the vectors method that have been used in this research.

COPLANARITY CONDITION

The coplanarity condition equation illustrated in **Fig (1)** is fundamental to relative orientation. When relative orientation is achieved, the vector $\vec{R}_{1i}$ from O_i to P_i will interest the vector $\vec{R}_{2i}$ from O_2 to P_i , and these two vectors together with air base vector, $\vec{b}$, will be coplanar.

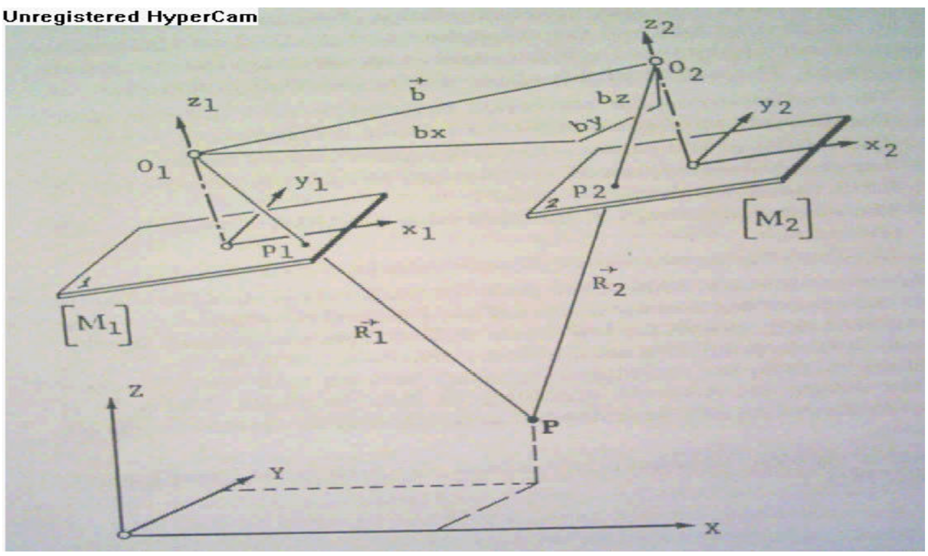

Fig. (1) The coplanarity condition

Hence, their scalar triple product is zero. That is

$$F_1 = \vec{b} \cdot \vec{R}_{1i} \times \vec{R}_{2i} = 0 \qquad (1)$$

Where F_1 is the mathematical model. Furthermore,

$$\vec{b} = \begin{bmatrix} b_X \\ b_Y \\ b_Z \end{bmatrix} = \begin{bmatrix} X_{O2} - X_{O1} \\ Y_{O2} - Y_{O1} \\ Z_{O2} - Z_{O1} \end{bmatrix}$$

$$\vec{R}_{1i} = \begin{bmatrix} X_{1i} \\ Y_{1i} \\ Z_{ii} \end{bmatrix} = K_1 M_1^T \begin{bmatrix} x_{1i} - x_{c1} \\ y_{1i} - y_{c1} \\ -f \end{bmatrix} = K_1 M_1^T \cdot \vec{r}_1$$

$$\vec{R}_{2i} = \begin{bmatrix} X_{2i} \\ Y_{2i} \\ Z_{2i} \end{bmatrix} = K_2 M_2^T \begin{bmatrix} x_{2i} - x_{c2} \\ y_{2i} - y_{c2} \\ -f \end{bmatrix} = K_2 M_2^T \cdot \vec{r}_2$$

K_1 and K_2 are scale factors, $\vec{r}_1$ and $\vec{r}_2$ are the corresponding location vectors in camera space.

$$M^T = \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\kappa & -\cos\phi\sin\kappa & \sin\phi \\ \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & -\sin\omega\cos\phi \\ \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa & \cos\omega\cos\phi \end{bmatrix}$$

The assumptions made about the rotation matrix M are:

- ❖ The rotations are in a right-handed system.
- ❖ The rotations, proceedings from the ground (or model) system to the photo system of coordinates are ω primary, ϕ secondary, and κ tertiary.

Equation (1) may be written in determinant form as,

$$F_1 = \begin{bmatrix} b_X & b_Y & b_Z \\ X_{1i} & Y_{1i} & Z_{1i} \\ X_{2i} & Y_{2i} & Z_{2i} \end{bmatrix} = 0 \qquad (2)$$

Now, let $K_1 = K_2 = 1$ and $x_c = y_c = 0$. Then, using photo 2 for the dependent method of relative orientation,

$$\omega_1 = \phi_1 = \kappa_1 = b_{y1} = b_{z1} = 0$$

Here

$b_Y = b_{y2} - b_{y1}$ and $b_Z = b_{z2} - b_{z1}$

The vectors $\vec{R}_{1i}$ and $\vec{R}_{2i}$ are then reduced to

$$\vec{R}_{1i} = \begin{bmatrix} X_{1i} \\ Y_{1i} \\ Z_{1i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1i} \\ y_{1i} \\ -f \end{bmatrix} = \begin{bmatrix} x_{1i} \\ y_{1i} \\ -f \end{bmatrix} \quad (3)$$

$$\vec{R}_{2i} = \begin{bmatrix} X_{2i} \\ Y_{2i} \\ Z_{2i} \end{bmatrix} = \begin{bmatrix} x_{2i}\cos\phi\cos\kappa - y_{2i}\cos\phi\sin\kappa - f\cdot\sin\phi \\ x_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) + y_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) + f\cdot\sin\omega\cos\phi \\ x_{2i}(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa) + y_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) - f\cdot\cos\omega\cos\phi \end{bmatrix}$$

(4)

Note here that $\omega, \phi,$ and κ are for camera 2.

Substituting Eqs (3) and (4) in Eq (1) and then expanding and rearranging, the mathematical model, F_1 is given by:

$$\begin{aligned} F_1 = {} & [b_x y_{1i} - b_{y2} x_{1i}][x_{2i}(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa) + y_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) - f\cdot\cos\omega\cos\phi] \\ & + [b_x f + b_{z2} x_{1i}][x_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) + y_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) + f\cdot\sin\omega\cos\phi] \\ & + [b_{y2} f + b_{z2} y_{1i}][y_{2i}\cos\phi\sin\kappa - x_{2i}\cos\phi\cos\kappa + f\cdot\sin\phi] = 0 \text{..................}(5) \end{aligned}$$

LINEARIZATION OF THE COPLANARITY CONDITION EQUATION

The coplanarity condition equation (eq. (5)), is linearized into the general form:

$$[A_i](V_i) + [B_i](\Delta) + (F_{Oi}) = 0 \quad (6)$$

Where

$[A_i] = \partial(F_i)/\partial$ (observed quantities)

$[B_i] = \partial(F_i)/\partial$ (parameters)

$(F_{Oi}) = F_i$ (evaluated with observations and approximate parameters)

$(V_i) =$ (a vector of residuals)

$(\Delta) =$ (a vector of corrections to approximate parameters)

Here:-

$$[A_i] = \begin{bmatrix} \frac{\partial F_i}{\partial x_{1i}} & \frac{\partial F_i}{\partial y_{1i}} & \frac{\partial F_i}{\partial x_{2i}} & \frac{\partial F_i}{\partial y_{2i}} \end{bmatrix} \quad (7)$$

$$\begin{aligned} \frac{\partial F_1}{\partial x_{1i}} &= \begin{bmatrix} b_x & b_y & b_z \\ m_{11}^1 & m_{12}^1 & m_{13}^1 \\ X_{2i} & Y_{2i} & Z_{2i} \end{bmatrix} \\ &= \begin{bmatrix} b_x & b_y & b_z \\ 1 & 0 & 0 \\ X_{2i} & Y_{2i} & Z_{2i} \end{bmatrix} \\ &= (b_z \cdot Y_{2i} - b_y \cdot Z_{2i}) \quad (8) \end{aligned}$$

Substituting from Eq.(4):-

$$\frac{\partial F_i}{\partial x_{1i}} = b_z\left[X_{2i}(\cos\omega\sin\kappa+\sin\omega\sin\phi\cos\kappa)+y_{2i}(\cos\omega\cos\kappa-\sin\omega\sin\phi\sin\kappa)+f\sin\omega\cos\phi\right]-$$
$$b_y\left[X_{2i}(\sin\omega\sin\kappa-\cos\omega\sin\phi\cos\kappa)+y_{2i}(\sin\omega\cos\kappa+\cos\omega\sin\phi\sin\kappa)-f\cos\omega\cos\phi\right]\text{...............(9)}$$

Similarly,

$$\frac{\partial F_i}{\partial y_{1i}} = \begin{bmatrix} b_x & b_y & b_z \\ m_{21}^1 & m_{22}^1 & m_{23}^1 \\ X_{2i} & Y_{2i} & Z_{2i} \end{bmatrix}$$
$$= \begin{bmatrix} b_x & b_y & b_z \\ 0 & 1 & 0 \\ X_{2i} & Y_{2i} & Z_{2i} \end{bmatrix}$$

$$= (b_x \cdot Z_{2i} - b_z \cdot X_{2i}) \qquad (10)$$
$$= b_x\left[x_{2i}(\sin\omega\sin\kappa-\cos\omega\sin\phi\cos\kappa)+y_{2i}(\sin\omega\cos\kappa+\cos\omega\sin\phi\sin\kappa)-f\cos\omega\cos\phi\right]$$
$$-b_z\left[x_{2i}\cos\phi\cos\kappa-y_{2i}\cos\phi\sin\kappa-f\sin\phi\right]\text{....................(11)}$$

Furthermore,

$$\frac{\partial F_i}{\partial x_{2i}} = \begin{bmatrix} b_x & b_y & b_z \\ X_{1i} & Y_{1i} & Z_{1i} \\ m_{11}^1 & m_{12}^1 & m_{13}^1 \end{bmatrix}$$
$$= \begin{bmatrix} b_x & b_y & b_z \\ x_{1i} & y_{1i} & -f \\ \cos\phi\cos\kappa & \cos\omega\sin\kappa+\sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa-\cos\omega\sin\phi\cos\kappa \end{bmatrix}\text{...............(12)}$$

$$= (b_x\cdot y_{1i}-b_y\cdot x_{1i})(\sin\omega\sin\kappa-\cos\omega\sin\phi\cos\kappa)+(b_x\cdot f+b_y\cdot x_{1i})(\cos\omega\sin\kappa+\sin\omega\sin\phi\cos\kappa)$$
$$-(b_y\cdot f+b_z\cdot y_{1i})\cos\phi\cos\kappa\text{..................(13)}$$

$$\frac{\partial F_i}{\partial y_{2i}} = \begin{bmatrix} b_x & b_y & b_z \\ X_{1i} & Y_{1i} & Z_{1i} \\ m_{21}^2 & m_{22}^2 & m_{23}^2 \end{bmatrix}$$
$$= \begin{bmatrix} b_x & b_y & b_z \\ x_{1i} & y_{1i} & -f \\ -\cos\phi\cos\kappa & \cos\omega\sin\kappa-\sin\omega\sin\phi\sin\kappa & \sin\omega\sin\kappa+\cos\omega\sin\phi\sin\kappa \end{bmatrix}\text{...............(14)}$$

$$= (b_x\cdot y_{1i}-b_y\cdot x_{1i})(\sin\omega\cos\kappa+\cos\omega\sin\phi\sin\kappa)+(b_x\cdot f+b_z\cdot x_{1i})(\cos\omega\cos\kappa-\sin\omega\sin\phi\sin\kappa)$$
$$+(b_y\cdot f+b_z\cdot y_{1i})\cos\phi\sin\kappa\text{......................(15)}$$

And:-

$$[B_i] = \left[\frac{\partial F_i}{\partial b_{y2}} \quad \frac{\partial F_i}{\partial b_{z2}} \quad \frac{\partial F_i}{\partial \omega_2} \quad \frac{\partial F_i}{\partial \phi_2} \quad \frac{\partial F_i}{\partial \kappa_2} \right] \tag{16}$$

$$\frac{\partial F_i}{\partial b_{y2}} = -\begin{bmatrix} X_{1i} & Z_{1i} \\ X_{2i} & Z_{2i} \end{bmatrix} \quad .(17)$$

From equation (4):-

$$\frac{\partial F_i}{\partial b_{y2}} = x_{1i}\left[x_{2i}(\cos\omega\sin\phi\cos\kappa - \sin\omega\sin\kappa) - y_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) + f\cdot\cos\omega\cos\phi\right]$$
$$- f\cdot\left[x_{2i}\cos\phi\cos\kappa - y_{2i}\cos\phi\sin\kappa - f\cdot\sin\phi\right]........................(18)$$

Similarly,

$$\frac{\partial F_i}{\partial b_{z2}} = \begin{bmatrix} X_{1i} & Y_{1i} \\ X_{2i} & Y_{2i} \end{bmatrix}$$
$$= x_{1i}\left[x_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) + y_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) + f\cdot\sin\omega\cos\phi\right]$$
$$- y_{1i}\left[x_{2i}\cos\phi\cos\kappa - y_{2i}\cos\phi\sin\kappa - f\cdot\sin\phi\right].......................(19)$$

$$\frac{\partial F_i}{\partial \omega_2} = \begin{bmatrix} b_x & b_y & b_z \\ X_{1i} & Y_{1i} & Z_{1i} \\ \frac{\partial X_{2i}}{\partial \omega_2} & \frac{\partial Y_{2i}}{\partial \omega_2} & \frac{\partial Z_{2i}}{\partial \omega_2} \end{bmatrix}$$
$$= \begin{bmatrix} b_x & b_y & b_z \\ x_{1i} & y_{1i} & -f \\ \frac{\partial X_{2i}}{\partial \omega_2} & \frac{\partial Y_{2i}}{\partial \omega_2} & \frac{\partial Z_{2i}}{\partial \omega_2} \end{bmatrix} \quad .(20)$$

From equation (4):

$$\frac{\partial X_{2i}}{\partial \omega_2} = 0$$
$$\frac{\partial Y_{2i}}{\partial \omega_2} = x_{2i}(\cos\omega\sin\phi\cos\kappa - \sin\omega\sin\kappa) - y_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) + f\cdot\cos\omega\cos\phi$$

And

$$\frac{\partial Z_{2i}}{\partial \omega_2} = x_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa) + y_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) + f\cdot\sin\omega\cos\phi$$

Now, substituting values in eq. (20):

$$\frac{\partial F_i}{\partial \omega_2} = \left(b_x \cdot y_{1i} - b_y \cdot x_{2i}\right)\left(x_{2i}\left(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa\right) + y_{2i}\left(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa\right) + f \cdot \sin\omega\cos\phi\right)$$
$$+\left(b_x \cdot f + b_z \cdot x_{1i}\right)\left(f \cdot \cos\omega\cos\phi + x_{2i}\left(\cos\omega\sin\phi\cos\kappa - \sin\omega\sin\kappa\right) - y_{2i}\left(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa\right)\right)$$
.......................(21)

$$\frac{\partial F_i}{\partial \phi_2} = \begin{bmatrix} b_x & b_y & b_z \\ X_{1i} & Y_{1i} & Z_{1i} \\ \frac{\partial X_{2i}}{\partial \phi_2} & \frac{\partial Y_{2i}}{\partial \phi_2} & \frac{\partial Z_{2i}}{\partial \phi_2} \end{bmatrix} = \begin{bmatrix} b_x & b_y & b_z \\ x_{1i} & y_{1i} & -f \\ \frac{\partial X_{2i}}{\partial \phi_2} & \frac{\partial Y_{2i}}{\partial \phi_2} & \frac{\partial Z_{2i}}{\partial \phi_2} \end{bmatrix} \quad (22)$$

From eq. (4)

$$\frac{\partial X_{2i}}{\partial \phi_2} = -x_{2i}\sin\phi\cos\kappa + y_{2i}\sin\phi\sin\kappa - f \cdot \cos\phi$$
$$\frac{\partial Y_{2i}}{\partial \phi_2} = x_{2i}\sin\omega\cos\phi\cos\kappa - y_{2i}\sin\omega\cos\phi\sin\kappa - f \cdot \sin\omega\sin\phi$$
$$\frac{\partial Z_{2i}}{\partial \phi_2} = -x_{2i}\cos\omega\cos\phi\cos\kappa + y_{2i}\cos\omega\cos\phi\sin\kappa + f \cdot \cos\omega\sin\phi$$

Substituting, values into eq. (22):

$$\frac{\partial F_i}{\partial \phi_2} = \left(b_x \cdot y_{1i} - b_y \cdot x_{1i}\right)\left(f \cdot \cos\omega\sin\phi + y_{2i}\cos\omega\cos\phi\sin\kappa - x_{2i}\cos\omega\cos\phi\cos\kappa\right) + \left(b_x \cdot f + b_z \cdot x_{1i}\right)\cdot$$
$$\left(x_{2i}\sin\omega\cos\phi\cos\kappa - y_{2i}\sin\omega\cos\phi\sin\kappa - f \cdot \sin\omega\sin\phi\right) + \left(b_y \cdot f + b_z \cdot y_{1i}\right)\cdot$$
$$\left(x_{2i}\sin\phi\cos\kappa - y_{2i}\sin\phi\sin\kappa + f \cdot \cos\phi\right)$$
......................(23)

$$\frac{\partial F_i}{\partial \kappa_2} = \begin{bmatrix} b_x & b_y & b_z \\ X_{1i} & Y_{1i} & Z_{1i} \\ \frac{\partial X_{2i}}{\partial \kappa_2} & \frac{\partial Y_{2i}}{\partial \kappa_2} & \frac{\partial Z_{2i}}{\partial \kappa_2} \end{bmatrix}$$

$$= \begin{bmatrix} b_x & b_y & b_z \\ x_{1i} & y_{1i} & -f \\ \dfrac{\partial X_{2i}}{\partial \kappa_2} & \dfrac{\partial Y_{2i}}{\partial \kappa_2} & \dfrac{\partial Z_{2i}}{\partial \kappa_2} \end{bmatrix} \quad .(24)$$

From eq. (4):

$$\frac{\partial X_{2i}}{\partial \kappa_2} = -x_{2i}\cos\phi\sin\kappa - y_{2i}\cos\phi\cos\kappa$$

$$\frac{\partial Y_{2i}}{\partial \kappa_2} = x_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) - y_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa)$$

$$\frac{\partial Z_{2i}}{\partial \kappa_2} = x_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) - y_{2i}(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa)$$

Now, substituting the values into eq. (24):

$$\frac{\partial F_i}{\partial \kappa_2} = (b_x \cdot y_{1i} - b_y \cdot x_{1i})(x_{2i}(\sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa) - y_{2i}(\sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa)) +$$
$$(b_x \cdot f + b_z \cdot x_{1i}) \cdot (x_{2i}(\cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa) - y_{2i}(\cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa)) +$$
$$(b_y \cdot f + b_z \cdot y_{1i})(x_{2i}\cos\phi\sin\kappa + y_{2i}\cos\phi\cos\kappa) \text{.........................}(25)$$

Here in our case, we use two overlapped photos with six points appear on each one, so the dimensions of the matrices will be:

$$A_{(6\times24)} = \begin{bmatrix} [A_1]_{(1\times4)} & 0 & 0 & 0 & 0 & 0 \\ 0 & [A_2]_{(1\times4)} & 0 & 0 & 0 & 0 \\ 0 & 0 & [A_3]_{(1\times4)} & 0 & 0 & 0 \\ 0 & 0 & 0 & [A_4]_{(1\times4)} & 0 & 0 \\ 0 & 0 & 0 & 0 & [A_5]_{(1\times4)} & 0 \\ 0 & 0 & 0 & 0 & 0 & [A_6]_{(1\times4)} \end{bmatrix}$$

$$B_{(6\times5)} = \begin{bmatrix} [B_1]_{(1\times5)} \\ [B_2]_{(1\times5)} \\ [B_3]_{(1\times5)} \\ [B_4]_{(1\times5)} \\ [B_5]_{(1\times5)} \\ [B_6]_{(1\times5)} \end{bmatrix}$$

$$F_{O(6\times1)} = \begin{bmatrix} F_{O1} \\ F_{O2} \\ F_{O3} \\ F_{O4} \\ F_{O5} \\ F_{O6} \end{bmatrix}$$

$$\Delta_{(5\times1)} = \begin{bmatrix} \Delta b_y \\ \Delta b_z \\ \Delta\omega \\ \Delta\phi \\ \Delta\kappa \end{bmatrix}$$

$$V_{(24\times1)} = \begin{bmatrix} [V_1]_{(4\times1)} \\ [V_2]_{(4\times1)} \\ [V_3]_{(4\times1)} \\ [V_4]_{(4\times1)} \\ [V_5]_{(4\times1)} \\ [V_6]_{(4\times1)} \end{bmatrix}$$

Here

The numbers, (24) refers to the number of observed quantities, (6) refers to the number of condition equations where observed quantities and unknown quantities are present, and (5) refers to the number of unknown quantities.

LEAST SQUARE SOLUTION OF THE COPLANARITY CONDITION EQUATION

Coordinate observations at five selected points give a unique solution of the parameters ($b_{y2}, b_{z2}, \omega_2, \phi_2$ and κ_2). However, when redundant observations are made, an adjustment situation arises, and the principles of least squares is applied to minimize the sum of the squares of the residuals.

The solution vector (Δ) is given by

$$\Delta = -\left(B^T M^{-1} B\right)^{-1} B^T M^{-1} F_O \qquad (26)$$

Where

$M = AW^{-1}A^T$

$W =$ weight matrix associated with the observations

$$V^T WV = -KL^T F_O \qquad .(27)$$

Where

$KL = -M^{-1}\left(B\Delta + F_O\right)$

The unit variance m_o^2 is given by

$$m_o^2 = \frac{V^T WV}{r-u} \qquad .(28)$$

Where

$r =$ number of condition equations
$u =$ number of unknown quantities
$(r - u) =$ the degree of freedom
The weight coefficient matrix of Δ can be written as

$$Q_\Delta = \left(B^T M^{-1} B\right)^{-1} \quad (29)$$

The variance-covariance matrix of unknown parameters is

$$\sum \Delta = m_o^2 Q_\Delta \quad .(30)$$

The corrections Δ are added to the approximate values of the parameters which were used in computing the coefficients of $F_i, [A_i]$, and $[B_i]$. It is sometimes necessary to iterate the solution until the corrections are negligible. Quantities related to both the parameters and the observations, should be updated for each iteration. The number of required iterations depends on the initial approximations, the total number of parameters, and the geometric strength of the model. Here, we used 6 iterations depending on the conditions above. There are several criteria, one of which may be used to terminate the iteration in a particular case, e.g., minimum variances of 0.00001rad for the angular orientation elements (as used by NOAA).

INTERSECTION BY VECTORS

The intersection of five pairs of rays $\vec{R}_1$ and $\vec{R}_2$ is the condition for relative orientation. After the relative orientation one may find, however, that the rays fail to intersect, i.e., there may be residual parallaxes. Therefore, one must define a point which will represent the location of intersection (acceptable for model point coordinates). A suitable point is one mid way along the vector $\vec{D}$ between vectors $\vec{R}_1$ and $\vec{R}_2$ (see **Fig. (2)**) in the region where the rays come closest together.

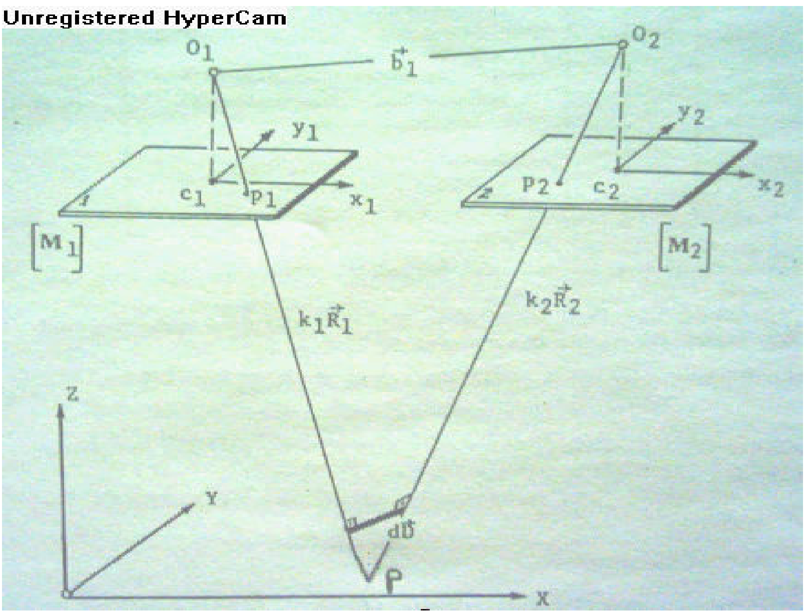

Fig. (2) The intersection concept in coplanarity condition

The direction (but not the length) of vector $\vec{D}$ which is perpendicular to both $\vec{R}_1$ and $\vec{R}_2$ is given by:

$$\vec{D} = \vec{R}_1 \times \vec{R}_2 \quad \ldots\ldots\ldots\ldots\ldots(31)$$

From this it is apparent that

$$K_1 \cdot \vec{R}_1 + d \cdot \vec{D} + K_2 \cdot \vec{R}_2 = \vec{b} \qquad (32)$$

Where K_1, K_2 and d are three unknown scalar multipliers (scale factors). Equation (32) has three components and, therefore, it may be solved for the three scalars.

The vectors $\vec{R}_1$ and $\vec{R}_2$ are determined using Eqs. (3) & (4) after evaluating the final matrices $[M_1]$ and $[M_2]$. The base components (b_x, b_y, b_z) are also determined after the relative orientation procedure.

Equation (31) can be written in the form:

$$\vec{D} = \begin{bmatrix} D_X \\ D_Y \\ D_Z \end{bmatrix} = \begin{bmatrix} Y_1 Z_2 - Z_1 Y_2 \\ Z_1 X_2 - X_1 Z_2 \\ X_1 Y_2 - Y_1 X_2 \end{bmatrix} \qquad .(33)$$

Since the triangulation is performed in the X direction (of strip), it is possible, as a harmless approximation of this condition, to choose for $\vec{D}$ the unit vector along the Y direction (i.e.,Y Parallax in the model space). In this case the scalars K_1 and K_2 are given by:

$$K_1 = \frac{R_{2Z} \cdot b_x - R_{2X} \cdot b_z}{R_{2Z} \cdot R_{1X} - R_{2X} \cdot R_{1Z}} \qquad (34)$$

And

$$K_2 = \frac{R_{1X} \cdot b_z - R_{1Z} \cdot b_x}{R_{2Z} \cdot R_{1X} - R_{2X} \cdot R_{1Z}} \qquad .(35)$$

Here, the coordinates of the required point P are:

$$\left.\begin{aligned} X_P &= X_{O1} + K_1 \cdot R_{1X} \\ Y_P &= 1/2\left[\left(Y_{O2} + K_2 \cdot R_{2Y}\right) + \left(Y_{O1} + K_1 \cdot R_{1Y}\right)\right] \\ Z_P &= Z_{O1} + K_1 \cdot R_{1Z} \end{aligned}\right\} \qquad (36)$$

The residual Y parallax which is the scalar d_1 is given by:

$$d_1 = \left(Y_{O2} + K_2 \cdot R_{2Y}\right) - \left(Y_{O1} + K_1 \cdot R_{1Y}\right) \qquad (37)$$

RESULTS

The results that have been gained in this research are:-

1- Relative Orientation:-

	Y_{L2} **(m.)**	Z_{L2} **(m.)**	ω **(rad.)**	ϕ **(rad.)**	κ **(rad.)**
value	2360.129	3699.116	-0.0093	-0.0099	2.1259
σ^2	1.8415	1.7578	5.143*10^-8	1.259*10^-7	1.625*10^-8

2- Space Intersection:-

point	X (m.)	Y (m.)	Z (m.)	Parallax (mm.)
1	5985.340	153.897	1079.300	0.532
2	7824.516	709.731	1089.001	0.400
3	8500.109	1568.394	1087.120	0.009
4	7911.999	2839.722	1104.580	0.987
5	7054.110	2061.984	1088.110	0.031
6	5610.441	930.651	1081.125	1.631

CONCLUSION

Methods for recovering the relative orientation of two cameras with respect to each other are of importance in aerotriangulation problem. An usual iterative method for finding the relative orientation parameters then easily computed the ground coordinate points that appear in the coplane between two pairs of stereo photographs, has been described here. This method does not use the traditional colliniarity condition equations, even in intersection problem; but it uses the coplanarity condition equation, which is rather hard equation (after linearization) as compared with the colliniarity. The results that have been gained were so good, and encourage to apply on more than two photos.

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