OPTIMIZATION OF THE OPERATION OF A COMPLEX WATER RESOURCES SYSTEM

PART -I : ANALYSIS OF THE CONVERGENCE CRITERION IN A SOLUTION BY THE DISCRETE DIFFERENTIAL DYNAMIC PROGRAMMING

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ABSTRUCT

An iterative – solution procedure necessarily involves pre – specified convergence criteria to stop iteration. The Discrete Differential Dynamic Programming procedure to solve optimization problems formulated by the Dynamic Programming is an iterative – solution procedure which, in its traditional form, involves two convergence criteria, namely, (α) and (β).

The research used the optimum operation of an existing complex water – resources system as a case study. The objective function was formulated as the maximum real monetary return. The formulated optimization model was run for a total of (194) different operation cases. Beside the traditional (α) and (β), seven new styles for a unique convergence criterion were examined in the solution.

Considering the monetary return and the number of performed iterations as the bases of comparison, the research showed that the new (γ) convergence criterion was the favorite among the tested convergence criteria.

الخلاصة

إن طريقة الحل التكراري تتضمن بالضرورة معايير تقارب محددة مسبقاً لإيقاف التكرار. إن طريقة البرمجة الدينامية التفاضلية المتقطعة هي طريقة حل تكراري، وهي بصيغتها التقليدية تتضمن معيارين للتقارب هما (α) و(β).

استخدم البحث التشغيل الأمثل لنظام موارد مائية مركب حقيقي كحالة تطبيقية، وصيغت دالة الهدف كأعلى عائد مالي حقيقي. تم تشغيل نموذج الأمثلية لما مجموعه (194) حالة تشغيلية مختلفة. وبالإضافة الى (α) و(β) التقليديتين، فقد اختبرت في الحل سبعة أشكال جديدة من معيار وحيد للتقارب.

وباعتماد مقدار العائد المالي وعدد دورات الحل التكراري كأساس للمقارنة، فقد بيّن البحث بأن معيار التقارب الجديد (γ) كان الأفضل بين معايير التقارب المختبرة.

KEY WORDS
Reservoir Operation; Dynamic Programming; Discrete Differential Dynamic Programming; The Convergence Criterion.

INTRODUCTION

The mathematical model which simulates the optimal operation of a single reservoir, (j = 1), or a set of jointly - operated reservoirs, (j = 1,2,..., J), is called a reservoir - operation problem, ROP. The solution of such a problem implies establishing the optimal inflow - storage - outflow relationship, i.e., finding the appropriate operation policy and, consequently, the respective trajectory, that yields the optimal outcome. The operation policy, $\{\hat{R}(j, t)\}$, is the set of feasible reservoirs releases, R(j , t), during the consecutive time stages, (t = 1,2, ... , T). The trajectory, $\{\hat{S}(j, t)\}$, is the set of feasible reservoirs storages, S(j , t), at the beginning of the considered stages. The obtained solution corresponds solely to the specified inputs and constraints.
A complex water - resources system denotes a multi - objective, multi - reservoir system. The complexity of the system comes from the complexity of the solution of its ROP.
In [AL-DELEWY: 1995], the discrete deterministic model has been identified as the appropriate practical general formulation for ROPs of the same style as the one under consideration in this research. Moreover, after reviewing several currently - used methodologies for formulating such ROP, particularly the Linear Programming [DANTZIG: 1963], the Linear Decision Rule [REVELLE et al.: 1969], and the Dynamic Programming [BELLMAN: 1957], the forward algorithm of a Dynamic - Programming formulation was identified as the most appropriate one. Furthermore, after reviewing several currently - applied procedures for solving the aforementioned formulation, particularly the Discrete Dynamic Programming [BELLMAN and DREYFUS: 1962], Bellman's successive approximations [BELLMAN: 1961], the Incremental Dynamic Programming [BERNHOLTZ and GRAHAM: 1960], (quoted in TURGEON: 1982]), the Discrete Differential Dynamic Programming, DDDP, [HEIDARI et al.: 1971], and the Multi - Level Incremental Dynamic Programming [NOPMONGCOL and ASKEW: 1976], the DDDP was identified as the most appropriate procedure of solution.

THE CONVERGENCE CRITERIA :

The DDDP is an iterative computation procedure. The measure of success of an iterative solution is how quick it converges to a certain target. In DDDP, the target is the global maximum return. Iteration is terminated when a certain pre-specified measure (criterion) of the aimed success is reached, or when a pre-specified total number of interations (K) has been performed.
For a total objective function, F[.], expressed in monetary terms (discounted return), then, {F[k] - F[k-1]} shows the improvement in the return of iteration step (k-1) due to iteration step (k). Similar to other iterative procedures, the DDDP involves a certain convergence criterion, CC , for the aforesaid improvement. The one commonly used is the duple CC of [CHOW and RIVERA: 1974], namely:

For computation cycles other than the last one, i. e., Cy < CY.

$$\frac{F[k] - F[k-1]}{F[1] - F[0]} \leq \alpha \qquad [1]$$

For the last computation cycle, i. e., Cy = CY:

$$\frac{F[k] - F[k-1]}{F[k-1]} \leq \beta \qquad [2]$$

The values used by [CHOW and RIVERA: 1974] are: $\alpha = 0.1$; $\beta = 0.001$. This CC has been used by [ALI: 1978], and others.

The objective of this research is to analyze the CC which is used in the DDDP solution of the formulated ROP, aiming at obtaining an improved CC as compared to the one traditionally used. The analysis will be based on the results of a case study that reflects a real operation process.

THE CASE STUDY

The case study is shown schematically in **Fig.(1).** It involves two, serially - connected reservoirs, denoted as RS1 and RS2. The system serves two distinct areas, denoted as MA and LA.

According to the considerations set forth in [AL-DELEWY: 1995], the following has been adopted in the research:

1- Stages, (t), are calendar months ; the operation horizon is one year, i.e., (t = 1, 2 , … , 12).

2- The objectives considered in the objective function, the constraints imposed on the system, and the values for inflow, demands, losses, and schedule of prices, are as given in [AL-DELEWY: 1995].

3- The dynamics of the system are as follows:

For RS1: $S(1,t+1) = S(1,t) + I(1,t) - R(1,t) - Ev(1,t)$ [3]

For RS2: $S(2,t+1) = S(2,t) + U(2,t) - R(2,t) - Ev(2,t)$ [4]

where: $U(2,t) = R(1,t) + IC(1,t) - OC(1,t)$ [5]

4- The overall single - stage objective function, representing the net return of the system, is given by [AL-DELEWY: 1995] as :

$$Z[t] = \sum_{j-1}^{2} [ZI(j,t) + ZP(j,t)] \qquad [6]$$

where: $ZI(j,t) = ZIM(j,t) - LSpI(j,t) - LEvI(j,t)$ [7]

and $ZP(j,t) = ZPM(j,t) - LSpP(j,t) - LEvP(j,t)$ [8]

RUNNING THE MODEL

The general designation for the operation scenarios is "X a b c d e f g"; (X) denotes the basic group,where (X=A) denotes operating RS1 assuming non-existence of RS2; (X=B) denotes the reverse of (X=A); (X=C) denotes the operating of the complex system, i.e., operating RS1 and RS2 conjointly. The parameters (a) through (f) stand for the following: (a) denotes specified sub-group of (X=C), (b) denotes flood-control criteria; (c) for inflow data; (d) for the initial trial trajectory; (e) for the state of the ends of the trajectory; and (f) for the selected values of (K), (CY), and (Cw(Cy)). The parameter (g) denotes the applied convergence criteria. Eight styles of convergence criteria were used, namely:

g = 1 ⇒ CC involves both (α) and (β) [as given in Eqs. (1) and (2), respectively];

g = 2 ⇒ CC involves (β) only;

g = 3 ⇒ CC is the new (γ) criterion (proposed in the research), where:

$F^*[k] - F^*[k-1] \leq \gamma$ [9]

$\gamma = 0.00001 \times FO \times CY^2 / Cy^2$ [10]

g = 4 (g=1) but (α) is replaced by ($\alpha1 = 2\alpha$);
g = 5 (g=1) but (α) is replaced by ($\alpha2 = 0.5\alpha$);
g = 6 (g=3) but (γ) is replaced by ($\gamma1 = 2\gamma$);
g = 7 (g=3) but (γ) is replaced by ($\gamma2 = 0.5\gamma$);
g = 8 (g=3) but (γ) is replaced by ($\gamma3$), where:

$$\gamma3 = 0.00002 \times FO \times CY / Cy \qquad [11]$$

A total of (194) operation scenarios were run, covering a variety of operation cases. In respect to the convergence criterion, the numbers of the run scenarios are as given in **Table (1)**.

RESULTS AND ANALYSES :

Results of running the aforementioned operating scenarios showed the following:

The (α) and (β) Criteria:

1- With all scenarios involving the (α) and (β) criteria together, i.e., scenarios (g=1) , (g=4) and (g=5), the (β) criterion is satisfied in one iteration step only.
2- When (α) criterion is used alone, i.e., without (β) criterion for the last computation cycle, the results were mostly the same as if (β) criterion was in use.

The Comparative Analyses

Comparative analyses based on ranking have been performed. The ranking was based mainly on the magnitude of the global optimal return (F*), with a consideration to the total number of executed iteration steps (TK). The results are summarized in **Table (2)**

Moreover:

1- Analysis (1) indicates that the results of (g=2) are the poorest as compared to those of (g=1) and (g=3). Sample comparison of the results is illustrated in **Fig. (2)**.
2- In Analysis (2), the improvement in the results by (g=4) as compared to those of (g=2) is almost insignificant.
3- In Analysis (3), the improvement over (g=3) by both (g=6) and (g=7) was mainly a slight reduction in (TK).
4- In Analysis (4), the results of seven scenarios from (g=3) and (g=8) were identical.
5- In Analysis (5), the differences between the results of the respective scenarios of sub-groups (g=1) and (g=3) of group (X=C) ranged from [equal (F*) and a difference of one iteration in (TK)] to [(F*=3,264,205 th. ID; TK=64) and (F*=3,267,254 th. ID; TK=58)], respectively, which is in favour of (g=3).
6- The global optimal solution for the basic C - scenarios is F**=3,240,714 th. ID; TK=30), given by a scenario from the sub-group (g=3).

CONCLUSION

Analysis of the results, as given hereinbefore , indicates that the (γ) criterion is the favorite among the applied convergence criteria, including that of [CHOW and RIVERA: 1974]. Consequently, rather than the duple convergence criterion, (α) and (β), the established single criterion (γ) is recommended.

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NOMENELATURE :

Cy : counter for performed computation cycles; its limiting value is CY.
Ev(j,t) : evaporation loss from the j-th reservoir during stage (t).
F[k] : total discounted return (thousand Iraqi Dinars) from the system as a result of performing the k-th iteration step; however, $F_0 = F[0] = 0$.
I(j,t) : total inflow to the j-th reservoir during (t).
IC(j,t) : natural inflow to the j-th channel during (t).
LEvI(j,t) : loss (th. ID) due to evaporation, evaluated as if it is (SpI).
LEvP(j,t) : ditto, evaluated as if it is (SpP).
LSpI(j,t) : loss (th. ID) due to spilling beyond irrigation demand.
LSpP(j,t) : ditto, beyond hydropower generation.
OC(j,t) : water consumption taken directly from the channel.
U(j,t) : additional inflow to a reservoir from an artificial source.
Z[t] : single - stage objective function.
ZI(j,t) : single - stage discounted return from irrigation.
ZIM(j,t) : ditto, of a sub - system.
ZP(j,t) : single - stage discounted return from hydropower generation.
ZPM(j,t) : ditto, of a sub - system.

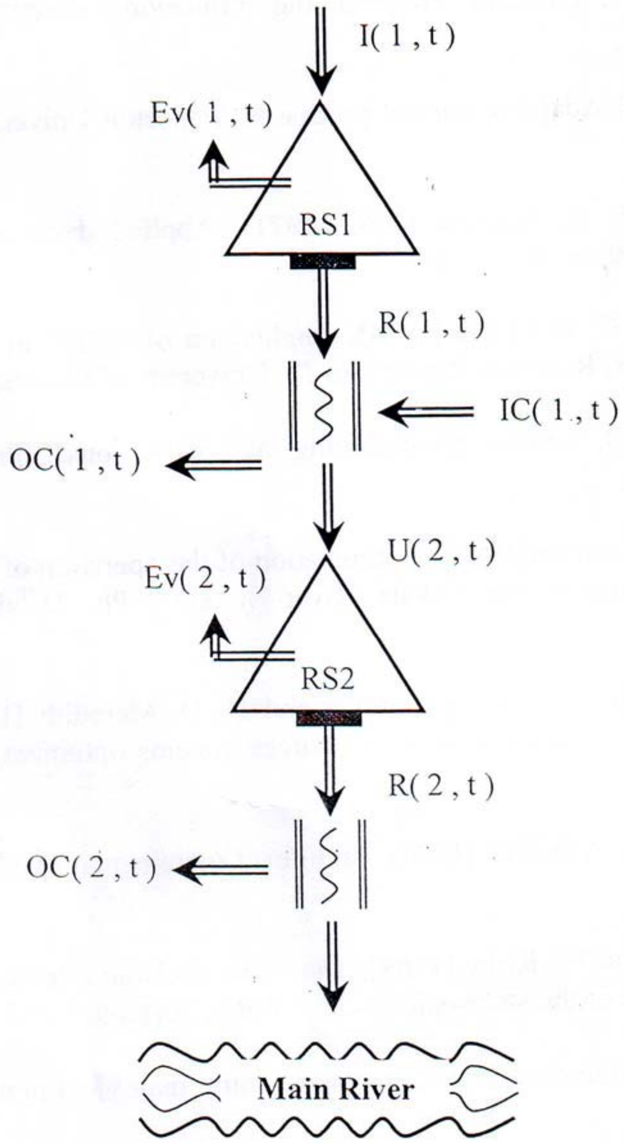

Fig. (1) : Schematic representation of the case study.

Table (1) Categories of the applied operation scenarios .

Parametric Category	Sub - group	No. in the basic group			Total No.
		A	B	C	
α + β	g = 1	14	14	36	64
β	g = 2	8	8	4	20
γ	g = 3	14	14	38	66
α1 + β	g = 4	2	2	2	6
α2 + β	g = 5	2	2	2	6
γ1	g = 6	2	2	2	6
γ2	g = 7	2	2	2	6
γ3	g = 8	8	8	4	20

Table(2) Summary of the comparative analyses .

ANALYSIS No.		1			2			3				4		5	
SUB-GROUP		g1	g2	g3	g1	g4	g5	g3	g6	g7	g8	g3	g8	g1	g3
NUMBER OF SCENARIOS	NUMBER USABLE IN THE ANALYSIS	20	20	20	6	6	6	6	6	6	6	20	20	64	64
	FIRST RANK	9	---	11	2	4	1	1	3	3	1	9	4	29	35
	SECOND RANK	11	---	9											
	THIRD RANK	---	20	---											

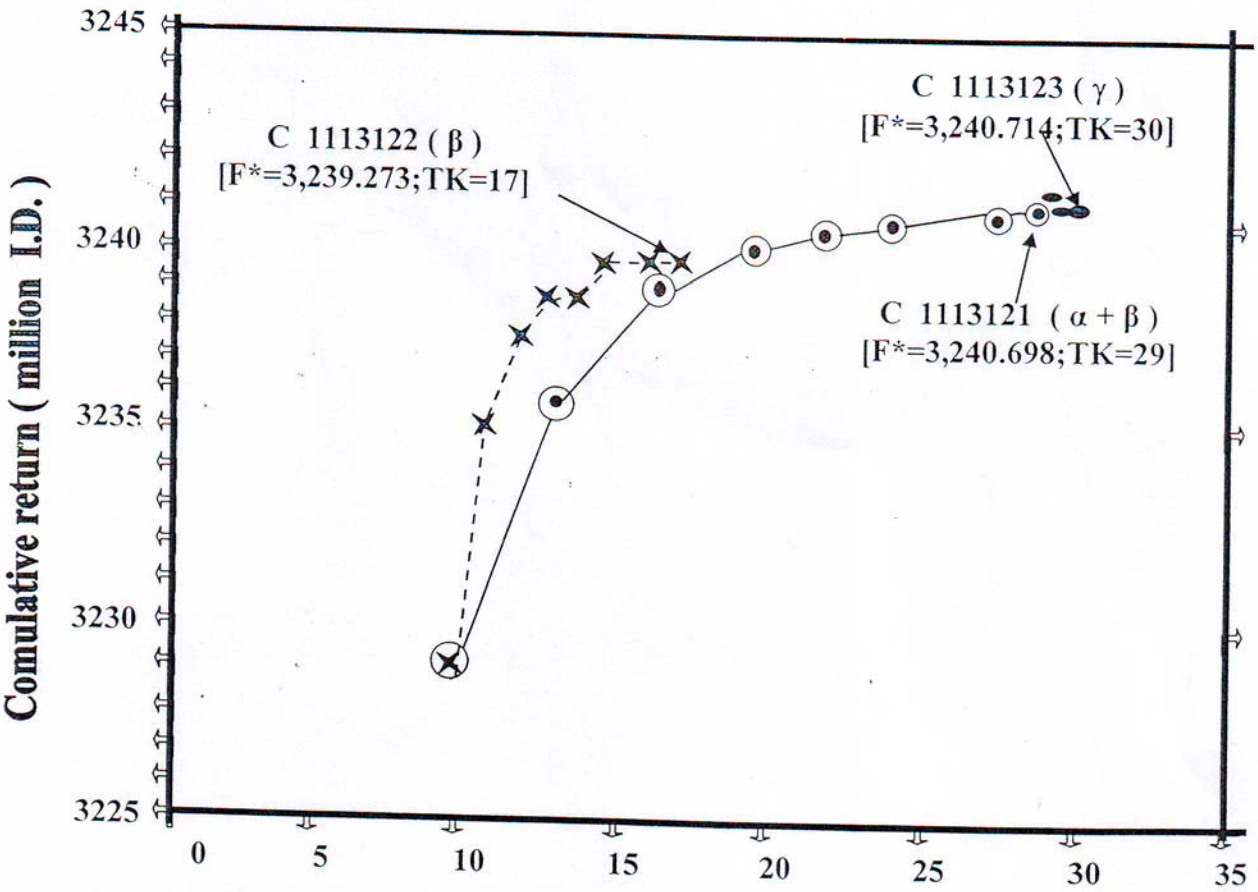

Fig. (2) Comparative results of applying the convergence criteria (α + β) , (β), and (γ).